A Bi-level Formulation for Centralized Resource Allocation DEA Models

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Abstract. In this paper, the common centralized DEA models are extended to the bi-level centralized resource allocation (CRA) models based on revenue efficiency. Based on the Karush–Kuhn–Tucker (KKT) conditions, the bi-level CRA model is reduced to a one-level mathematical program subject to complementarity constraints (MPCC). A recurrent neural network is developed for solving this one-level mathematical programming problem. Under a proper assumption and utilizing a suitable Lyapunov function, it is shown that the proposed neural network is Lyapunov stable and convergent to an exact optimal solution of the original problem. Finally, an illustrative example is elaborated to substantiate the applicability and effectiveness of the proposed approach.

1. Introduction

Data envelopment analysis (DEA), originally developed by Charnes et al. [5], is a linear programming methodology for assessing relative efficiency and productivity of multiple inputs and outputs decision making units (DMUs). In its recent developments, DEA has had more of a planning orientation for resource allocation problems. The use of DEA provides an alternative to the resource allocation problem because it is possible in this method to consider feasible production plans and trade-offs between the inputs/outputs based on the empirical characterization of a production possibility set [13].

The main limitation of a traditional DEA model in resource allocation is that it analyzes one unit at a time independently. Many authors have developed a number of centralized DEA-based models from different perspectives [1,2,8,10-12,15]. However, most papers in the literature have proposed centralized resource allocation models from the technical efficiency perspective.

Since the computing time needed to solve a DEA problem greatly depends on its dimension and structure, traditional algorithms cannot evaluate the efficiency of large

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scale data sets. In general, the large-scale DEA problems with negative data are challenging problems. Unlike traditional algorithms, artificial neural networks have massively paralleled distributed computation, fast convergence and robust solution. Therefore, artificial neural networks can be considered as a promising approach to solve the large-scale DEA problem in real time.

Neural networks for solving mathematical programming problems were first introduced in the 1980s by Hopfield and Tank [16]. The main feature of these neural networks is that its equilibrium point coincides with the solution of the underlying optimization problem.

Motivated by the above discussions, in this paper, we extend the common centralized DEA models to the bi-level CRA models based on revenue efficiency and a cost analysis across a set of DMUs under a centralized decision-making environment. Also, we present a recurrent neural network for solving the proposed bi-level CRA model. This neural network is proved to be globally stable by constructing a suitable Lyapunov function and the solution trajectory can converge to an optimal solution of the original optimization problem.

The rest of the paper is organized as follows. In Section 2, we introduce the traditional revenue efficiency DEA formulations. In Section 3, we develop a bi-level centralized DEA model based on revenue efficiency for resource reallocation also present selected extensions to the model. In Section 4, a neural network model is designed for solving the proposed bi-level CRA model and it's stability is analyzed. An empirical example is given in Section 5. Finally, some conclusions are drawn in Section 6.

2. Preliminaries

Before formulating the models, the required notation needs to be introduced. Let

n	Number of observed units
x_j	Input vector of DMU_j
x_{ij}	Value of input i of unit j
y_j	Output vector of DMU_j
y_{rj}	Value of output r of unit j
p	Price vector of outputs
p_{rj}	Price of output r of unit j
λ_j	Intensity variable for DMU_j

2.1. *Traditional revenue efficiency model*

We assume that there are n DMUs and that each DMU uses m inputs to produce s outputs. For each DMU_j ($j = 1,\ldots,n$), we denote the input and the output vectors as (x_j, y_j), where $x_j = (x_{1j},\ldots,x_{mj})$ and $y_j = (y_{1j},\ldots,y_{sj})$. Arranging the data set in matrices $X = (x_j)$ and $Y = (y_j)$, and assuming that $X > 0$ and $Y > 0$, the production possibility set under a Variable Returns to Scale (VRS) technology is generally denoted as follows:

$$T = \left\{ (x,y) \mid x \geq \sum_{j=1}^{n} x_j \lambda_j, \ y \leq \sum_{j=1}^{n} y_j \lambda_j, \ \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0, \ j = 1,\ldots,n \right\}.$$

The standard DEA model based on this assumption is called the BCC model [3]. Given the common unit price vector $p = (p_1,\ldots,p_n)$ for the output y, the maximum revenue

for DMU_o is obtained as follows [12,13]:

$$\max \; p^T y + \varepsilon.\mathbf{1}^T s^-$$
$$\text{s.t.} \quad X\lambda + s^- = x_o,$$
$$Y\lambda = y, \tag{1}$$
$$\mathbf{1}^T \lambda = 1,$$
$$s^- \geq 0, \; \lambda \geq 0,$$

where $\mathbf{1} = [1,\ldots,1]^T \in \mathbb{R}^n$ and ε represents a non-Archimedean infinitesimal. The slack vector $s^- = (s_1^-, s_2^-, \ldots, s_m^-) \in \mathbb{R}^m$ represents the actual inputs that can be reduced. $\lambda = (\lambda_1, \ldots, \lambda_n)^T$ is a non-negative vector in $\mathbb{R}^n$, and y is the revenue maximizing level produced by DMU_o given the output price p and the input level x_o. Let the optimal solution of (1) be (y^*, λ^*). Then, the revenue efficiency of DMU_o is defined as the ratio of maximum revenue to observed revenue via the following formula:

$$RE_o \text{ (Revenue Efficiency)} = \frac{p^T y^*}{p^T y_o}.$$

2.2 Resource reallocation based on revenue efficiency

We assume that all of the units operate under a central unit with the control of some decision parameters, such as resources of those units. To allocate the input resources to a set of existing units so that the total output revenue will be maximized, Fang and Li [8] proposed the revenue allocation model based on revenue efficiency within the original production possibility set as follows:

$$\max \varphi = \sum_{k=1}^{n} \sum_{r=1}^{s} p_{rk} \bar{y}_{rk} + \varepsilon \sum_{i \notin U} \sum_{k=1}^{n} s_{ik}^-$$
$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_{jk} x_{ij} \leq x_{ik}, \; k=1,\ldots,n, \; i \in U,$$
$$\sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^- = \bar{x}_{ik}, \; k=1,\ldots,n, \; i \notin U,$$
$$\sum_{j=1}^{n} \lambda_{jk} y_{rj} = \bar{y}_{rk}, \; k=1,\ldots,n, \; r=1,\ldots,s, \tag{2}$$
$$\sum_{k=1}^{n} \bar{x}_{ik} = \sum_{r=1}^{n} x_{ik}, \; i=1,\ldots,m, \; i \notin U,$$
$$\sum_{j=1}^{n} \lambda_{jk} = 1, \; k=1,\ldots,n,$$
$$\lambda_{jk} \geq 0, \; k=1,\ldots,n, \; j=1,\ldots,n,$$

where $j,k = 1,\ldots,n$ are the indexes for the DMUs, and U represents the set of un-reallocatable variables. $\begin{pmatrix} \bar{x}_{ik} \\ \bar{y}_{rk} \end{pmatrix}$ describes the input-output target after the reallocation $(\lambda_{1k}, \lambda_{2k}, \ldots, \lambda_{nk})$ represents the vector for projecting the DMU_k. s_{ik}^- indicates the slack of input i for DMU_k. Also, ε is a non-Archimedean element smaller than any positive real number.

The reallocation of input resources among the DMUs may incur costs, which includes the transportation costs, miscellaneous labor costs, etc. Therefore, Fang and Li [8] developed the following linear programming problem in which the reallocation amounts are taken to their minimum cost:

$$\min \xi = \sum_{i \notin U} \sum_{k=1}^{n} c_{ik}^{-} \gamma_{ik}^{-} + \sum_{i \notin U} \sum_{k=1}^{n} c_{ik}^{+} \gamma_{ik}^{+}$$

$$\text{s.t.} \ \sum_{j=1}^{n} \lambda_{jk} x_{ij} \le x_{ik}, \ k = 1,...,n, \ i \in U,$$

$$\sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = \overline{x}_{ik}, \ k = 1,...,n, \ i \notin U,$$

$$\sum_{j=1}^{n} \lambda_{jk} y_{rj} = \overline{y}_{rk}, \ k = 1,...,n, \ r = 1,...,s,$$

$$\sum_{k=1}^{n} \sum_{r=1}^{s} p_{rk} \overline{y}_{rk} + \varepsilon \sum_{i \notin U} \sum_{k=1}^{n} s_{ik}^{-} = \varphi^{*} \tag{3}$$

$$\sum_{k=1}^{n} \overline{x}_{ik} = \sum_{r=1}^{n} x_{ik}, \ i = 1,...,m, \ i \notin U,$$

$$\overline{x}_{ik} + \gamma_{ik}^{-} - \gamma_{ik}^{+} = x_{ik}, \ k = 1,...,n, \ i \notin U,$$

$$\sum_{j=1}^{n} \lambda_{jk} = 1, \ k = 1,...,n,$$

$$\lambda_{jk} \ge 0, \ \gamma_{ik}^{-} \ge 0, \ \gamma_{ik}^{+} \ge 0, \ k = 1,...,n, \ j = 1,...,n, \ i \notin U,$$

where φ^{*} is the optimal value for the model (2). γ_{ik}^{-} and γ_{ik}^{+} denote the negative deviation and the positive deviation from the current level of input i for DMU_{k}, respectively, and c_{ik}^{-}, c_{ik}^{+} denote the cost of moving one unit of input i from DMU_{k} and into DMU_{k}, respectively.

3. A bi-level formulation for CRA based on the cost-revenue analysis

In this section, we present a bi-level DEA-based model for centralized resource allocation based on revenue efficiency and a cost analysis across a set of decision making units (DMUs) under a centralized decision-making environment. The upper-level model is concerned with determining the minimum reallocation cost while input resources and output targets are evaluated in the lower-level model.

The bi-level programming problem (BPP) corresponding to the linear program (2) and (3) can be described as follows:

$$(UP) \min \ \overline{\xi} = \sum_{i \notin U} \sum_{k=1}^{n} c_{ik}^{-} \gamma_{ik}^{-} + \sum_{i \notin U} \sum_{k=1}^{n} c_{ik}^{+} \gamma_{ik}^{+}$$

$$\text{s.t.} \ \ \overline{x}_{ik} + \gamma_{ik}^{-} - \gamma_{ik}^{+} = x_{ik}, \ k = 1,...,n, \ i \notin U,$$

$$\gamma_{ik}^{-} \ge 0, \ \gamma_{ik}^{+} \ge 0, \ k = 1,...,n, \ i \notin U,$$

$$(LP) \max \ \overline{\varphi} = \sum_{k=1}^{n} \sum_{r=1}^{s} p_{rk} \overline{y}_{rk} + \varepsilon \sum_{i \notin U} \sum_{k=1}^{n} s_{ik}^{-}$$

$$\text{s.t.} \ \sum_{j=1}^{n} \lambda_{jk} x_{ij} \le x_{ik}, \ k = 1,...,n, \ i \in U,$$

$$\sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = \overline{x}_{ik}, \ k = 1,...,n, \ i \notin U, \tag{4}$$

$$\sum_{j=1}^{n} \lambda_{jk} y_{rj} = \overline{y}_{rk}, \ k = 1,...,n, \ r = 1,...,s,$$

$$\sum_{k=1}^{n} \overline{x}_{ik} = \sum_{r=1}^{n} x_{ik}, \ i = 1,...,m, \ i \notin U,$$

$$\sum_{j=1}^{n} \lambda_{jk} = 1, \ k = 1,...,n,$$

$$\lambda_{jk} \ge 0, \ \ k = 1,...,n, \ j = 1,...,n.$$

The term (UP) is called the upper level problem and (LP) is called the lower level problem.

At the UP, the decision maker has to choose first a vector $(\gamma_{ik}^-, \gamma_{ik}^+)^T$, $k = 1, ..., n$, $i \in U$, to minimize his objective function $\bar{\xi}$; then under this decision the LP decision maker has to select the decision vector $(\bar{x}_{rk}, \bar{y}_{rk}, \lambda_{jk}, s_{ik}^-)^T$, $k = 1, ..., n$, $j = 1, ..., n$, $i \notin U$, that minimizes his own objective $\bar{\varphi}$.

We can reduce the BPP (4) to the one-level programming problem by replacing the lower-level problem with its KKT optimality condition.

4. Neural network model

In this section, we introduce a recurrent neural network for solving (4). Let $w \in W \subset \mathbb{R}^t$, $z \in Z \subset \mathbb{R}^h$, $F : W \times Z \to \mathbb{R}$, $f : W \times Z \to \mathbb{R}$. Without loss of generality, we consider the following bi-level programming problem:

$$\begin{aligned} \text{(UP)} \quad &\min_{w \in W} \quad F(w, z) = c_1^T w + d_1^T z, \\ &\text{s.t.} \quad A_1 w + B_1 z \le b_1, \\ \text{(LP)} \quad &\min_{z \in Z} \quad f(w, z) = c_2^T w + d_2^T z, \\ &\text{s.t.} \quad A_2 w + B_2 z \le b_2, \\ &\quad c_1, c_2 \in \mathbb{R}^t, d_1, d_2 \in \mathbb{R}^h, b_1 \in \mathbb{R}^p, b_2 \in \mathbb{R}^q, \\ &\quad A_1 \in \mathbb{R}^{p \times t}, B_1 \in \mathbb{R}^{p \times h}, A_2 \in \mathbb{R}^{q \times t}, B_2 \in \mathbb{R}^{q \times h}. \end{aligned} \tag{5}$$

Assumption 1. The constraint region of the above BPP
$$S = \{(w, z) : w \ge 0, \ z \ge 0, \ A_1 w + B_1 z \le b_1, \ A_2 w + B_2 z \le b_2\},$$
is nonempty and compact.

Following the above assumption, we can reduce the BPP (5) to the one-level programming problem:

$$\begin{aligned} \min \quad &F(w, z) \\ \text{s.t.} \quad &A_1 w + B_1 z \le b_1, \\ &A_2 w + B_2 z \le b_2, \\ &d_2 + B_2^T u - v = 0, \\ &u^T (b_2 - A_2 w - B_2 z) = 0, \\ &w \ge 0, \ z \ge 0, \ u \ge 0, \ v \ge 0, \end{aligned} \tag{6}$$

where $u \in \mathbb{R}^q$ and $v \in \mathbb{R}^h$. Problem (6) is non-convex and non-smooth problem, and it is not good for using the neural network approach to solve problem (6). But fortunately, smoothing method is also presented in [6,9] for MPCC (6). Following this smoothing method, we can propose a neural network approach for problem (5).

Definition 4.1. A function $\phi : \mathbb{R}^2 \to \mathbb{R}$ is called an NCP function if it satisfies
$$\phi(a, b) = 0 \Leftrightarrow a \ge 0, b \ge 0, ab = 0.$$

A popular NCP-function is the perturbed Fischer-Burmeister (FB) function, which is defined as
$$\phi_{FB}^\varepsilon(a, b) = \sqrt{a^2 + b^2 + \varepsilon} - a - b, \quad \varepsilon \to 0^+.$$

The important property of ϕ_{FB}^ε can be stated in the following result.

Proposition 4.1. ([9]) For every $\varepsilon \in \mathbb{R}$, we have

$$\phi_{FB}^{\varepsilon}(a,b) = 0 \Leftrightarrow a > 0,\ b > 0,\ ab = \frac{\varepsilon}{2}.$$

By the Proposition 4.1, MPCC (6) can be rewritten as follows:

min $\quad F(w,z)$

s.t. $\quad A_1 w + B_1 z \leq b_1,$

$\qquad d_2 + B_2^T u - v = 0,$

$\qquad \sqrt{u_i^2 + (b_2 - A_2 w - B_2 z)_i^2 + \varepsilon} - u_i - (b_2 - A_2 w - B_2 z)_i = 0,\ i = 1,\ldots,q,$ $\quad$ (7)

$\qquad \sqrt{v_j^2 + z_j^2 + \varepsilon} - v_j - z_j,\ j = 1,\ldots,h,$

$\qquad w \geq 0.$

For notational convenience, introducing

$$\tilde{w} = (w, z, u, v),$$

$$G(w,z,u,v) = \begin{pmatrix} A_1 w + B_1 z - b_1 \\ -x \end{pmatrix},$$

$$H(w,z,u,v) = \begin{pmatrix} d_2 + B_2^T u - v \\ \phi_{FB}^{\varepsilon}(u_i, (b_2 - A_2 w - B_2 z)_i),\ i = 1,\ldots,q \\ \phi_{FB}^{\varepsilon}(v_j, z_j),\ j = 1,\ldots,h \end{pmatrix},$$

(7) can be rewritten as the following equivalent problem:

min $\quad F(\tilde{w}) = c_1^T w + d_1^T z$

s.t. $\quad G_l(\tilde{w}) \leq 0,\ l = 1,\ldots,t + p,$ $\qquad\qquad$ (8)

$\qquad H_o(\tilde{w}) = 0,\ o = 1,\ldots,2h + q,$

Definition 4.2. Let $\tilde{w}$ be a feasible point of problem (8) and $L = \{l : G_l(\tilde{w}) = 0,\ l = 1,\ldots t + p\}$. We say that $\tilde{w}$ is a regular point of problem (8) if the gradients $\nabla H_o(\tilde{w}),\ o = 1,\ldots,2h + q,$ and $\nabla G_l(\tilde{w}),\ l \in L$ are linearly independent.

Theorem 4.1. Let $\{\tilde{w}_\varepsilon^*\}$ be a sequence of solutions of (8). Suppose that the sequence $\{\tilde{w}_\varepsilon^*\}$ converges to some $\bar{w}$ for $\varepsilon \to 0^+$. If $\bar{w}$ is a regular point, then $\bar{w}$ solves the problem (5).

Now, consider the Lagrangian function associated with (8) as follows:

$$L(\tilde{w}, \mu, \alpha, \beta) = F(\tilde{w}) + \sum_{o=1}^{2h+q} \alpha_o H_o(\tilde{w}) + \sum_{l=1}^{t+q} \beta_l [G_l(\tilde{w}) + \mu_l^2],$$

where the term μ is slack variable, and the terms α, β are referred as Lagrange multipliers.

The aim is to design a continuous-time dynamical system for solving the problem (5). The proposed neural network model is described as follows:

$$\begin{cases} \dfrac{d\tilde{w}}{dt} = -\nabla_{\tilde{w}} L(\tilde{w}, \mu, \alpha, \beta), \\[2mm] \dfrac{d\mu}{dt} = -\nabla_{\mu} L(\tilde{w}, \mu, \alpha, \beta), \\[2mm] \dfrac{d\alpha}{dt} = \nabla_{\alpha} L(\tilde{w}, \mu, \alpha, \beta), \\[2mm] \dfrac{d\beta}{dt} = \nabla_{\beta} L(\tilde{w}, \mu, \alpha, \beta). \end{cases} \qquad (9)$$

Similarly, a recurrent neural network can be defined for solving the problem (4). Now, we study stability properties of the neural network whose dynamics is described by the nonlinear differential equations (9).

Theorem 4.2. Let $(\tilde{w}^*, \mu^*, \alpha^*, \beta^*)$ be the equilibrium of the neural network (9), and assume that $\tilde{w}^*$ is a regular point of problem (8). Then the equilibrium of the neural network solves problem (8).

Theorem 4.3. Let $(\tilde{w}^*, \mu^*, \alpha^*, \beta^*)$ be the equilibrium of the neural network (9). If $\tilde{w}^*$ is the regular point of problem (8), then $(\tilde{w}^*, \mu^*, \alpha^*, \beta^*)$ is an asymptotically stable point of the neural network.

5. Illustrative example

In order to demonstrate the effectiveness and efficiency of the proposed approach, in this section, we analyze an empirical data set that is extracted from real application.

The data set is extracted from a real life case, 20 fast-food restaurants located in the city of Hefei, AnHui Province, China, presented by Du et al. [7]. These fast-food restaurants belong to the same chain, which has a central unit with the authority to supervise the operations of all branches and allocate resources among them. Table 1 shows the input and output dataset for 20 fast-food restaurants.

Table 1: Inputs and outputs for 20 fast-food restaurants.

	Inputs				outputs		
DMU	Man-hour $10^3 h$	Shop Size $10^2 m^2$	Meat dish 10^3 servings	Vegetable dish 10^3 servings	Soup 10^3 servings	Noodle 10^3 servings	Beverage 10^3 servings
1	3.20	2.00	2.24	2.46	1.22	3.12	0.96
2	3.40	2.10	2.12	2.52	1.34	3.08	0.88
3	3.10	1.80	2.08	2.25	1.05	2.85	0.74
4	3.80	2.20	2.45	2.10	1.30	2.96	0.79
5	4.20	2.60	2.80	2.78	1.42	3.48	1.05
6	4.10	2.50	2.65	2.95	1.38	3.25	0.98
7	3.80	2.30	2.60	2.24	1.15	3.18	0.95
8	3.80	2.20	2.50	2.15	1.10	3.20	0.82
9	2.90	1.60	2.10	2.04	0.98	2.88	0.72
10	4.20	2.80	2.90	2.85	1.52	3.36	1.12
11	3.40	2.10	2.60	2.45	1.36	3.32	0.82
12	4.00	2.40	2.78	2.66	1.18	3.15	0.98
13	3.80	2.60	2.84	2.38	1.25	3.29	0.85
14	3.40	1.90	2.33	2.20	1.06	2.99	0.82
15	2.80	1.60	2.00	2.18	1.96	2.84	0.71
16	3.50	2.20	2.40	2.25	1.26	2.93	0.74
17	4.20	2.50	2.68	2.50	1.46	3.22	0.92
18	3.30	1.80	2.05	2.20	1.12	3.02	0.78
19	3.60	1.90	2.00	2.16	1.02	2.89	0.74
20	3.10	1.70	2.05	2.12	0.94	2.90	0.68
Total	71.6	42.8	48.17	47.44	25.07	61.91	17.05

Five output variables and two input variables are considered. The term 'man-hours' refers to the labor force used within a certain period, and the shop size is the total rental floor space of the restaurant used for serving customers. The outputs are the sales of meat dishes, vegetables dishes, soup, noodles and beverages. The output price vector is $p = (32, 15, 10, 6, 3)^T$. The current total revenue is 2926.35.

Because these fast food restaurants belong to the same organization, a central decision-maker can simultaneously control all of the DMUs. Note that the input shop size is a non-reallocatable variable. We assume that the central decision maker can reallocate the man-hour resources among these 20 fast-food restaurants to maximize the total revenue. The results from model (2) and (3) are given in Table 2, where reallocation of current man-hour resources was allowed. The total revenue is 3105.36.

The results of bi-level model (4) and proposed neural network (9) are shown in Table 3 below in an example that allowed for current resources man-hour reallocation. The total revenue is 3109.12 and increases by 3109.12 / 2926.35-1=6.24% compared with the original total revenue.

Table 2: Results of resources reallocation by Fang and Li [8].

DMU	Inputs	outputs				
	Man-hour $10^3 h$	Meat dish 10^3 servings	Vegetable dish 10^3 servings	Soup 10^3 servings	Noodle 10^3 servings	Beverage 10^3 servings
1	3.28	2.48	2.396	1.48	3.224	0.798
2	3.40	2.60	2.45	1.36	3.32	0.82
3	3.08	2.24	2.288	1.72	3.032	0.754
4	3.80	2.64	2.516	1.372	3.352	0.866
5	4.20	2.80	2.78	1.42	3.48	1.05
6	4.10	2.76	2.714	1.408	3.448	1.004
7	3.80	2.68	2.582	1.384	3.384	0.912
8	3.80	2.64	2.516	1.372	3.352	0.866
9	2.90	2.00	2.18	1.96	2.84	0.71
10	4.20	2.90	2.85	1.52	3.36	1.12
11	3.40	2.60	2.45	1.36	3.32	0.82
12	3.88	2.72	2.648	1.396	3.416	0.958
13	4.20	2.80	2.78	1.42	3.48	1.05
14	3.16	2.36	2.342	1.60	3.128	0.776
15	2.80	2.00	2.18	1.96	2.84	0.71
16	3.56	2.64	2.516	1.372	2.352	0.866
17	4.04	2.76	2.714	1.408	3.448	1.004
18	3.30	2.24	2.288	1.72	3.032	0.754
19	3.60	2.36	2.342	1.60	3.128	0.776
20	3.10	2.12	2.234	1.84	2.936	0.732
Total	71.6	50.34	49.766	30.672	64.772	17.346

Table 3: Results of resources reallocation for 20 fast-food restaurants using proposed model 4 and neural network (9).

| DMU | Inputs | outputs | | | | |
	Man-hour $10^3 h$	Meat dish 10^3 servings	Vegetable dish 10^3 servings	Soup 10^3 servings	Noodle 10^3 servings	Beverage 10^3 servings
1	3.36	2.32	2.42	1.744	3.096	0.846
2	3.50	2.40	2.48	1.690	3.160	0.880
3	3.10	2.16	2.30	1.852	2.968	0.778
4	2.48	2.64	2.54	1.636	3.224	0.914
5	4.20	2.80	2.78	1.420	3.480	1.050
6	4.10	2.72	2.72	1.747	3.416	1.016
7	3.80	2.56	2.60	1.582	3.288	0.948
8	3.80	2.48	2.54	1.636	3.224	0.914
9	2.90	2.00	2.18	1.960	2.840	0.710
10	4.20	2.90	2.85	1.520	3.360	1.120
11	3.40	2.60	2.45	1.360	3.320	0.820
12	4.00	2.72	2.648	1.396	3.416	0.958
13	4.20	2.80	2.78	1.420	3.480	1.050
14	3.40	2.36	2.342	1.600	3.128	0.776
15	2.80	2.00	2.18	1.960	2.840	0.710
16	3.56	2.64	2.516	1.372	2.352	0.866
17	4.20	2.809	2.772	1.458	3.311	1.073
18	3.20	2.24	2.288	1.720	3.032	0.754
19	3.16	2.36	2.342	1.600	3.128	0.776
20	3.92	2.12	2.234	1.840	2.936	0.732
Total	71.6	49.469	49.962	32.24	63.999	17.691

6. Conclusion

In this paper, we extended the common centralized DEA models to the bi-level CRA models based on revenue efficiency and a cost analysis across a set of DMUs under a centralized decision-making environment. Also, a recurrent neural network has been designed for solving the proposed bi-level CRA model. Based on Lyapunov stability theory, the proposed neural network has been proved to be globally asymptotically stable and capable of generating exact optimal solution of the proposed bi-level CRA model. To further demonstrate the advanced features of our approach and its practical relevance, we analyzed an empirical data set that was extracted from real applications. By making comparisons of the proposed approach with the existing approaches, it has been seen that the proposed approach in this paper can achieve higher total revenue.

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