

Solving a Class of Partial Differential Equations by Differential Transforms Method

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Abstract. In this work, we find the differential transforms of the functions \tan and \sec , and then we applied this transform on a class of partial differential equations involving \tan and \sec .

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Index to information contained in this paper

- 1 Introduction
- 2 Differential Transform
- 3 Differential Transform for a Special Class of Functions
- 4 Numerical Examples
- 5 Conclusions

1. Introduction

The differential transform (one-dimensional) method first introduced by Zhou [10] has been used for obtaining a series solutions to a wide class of linear and nonlinear ordinary differential equations. Based on the same methodology, Chen and Ho [4] recently developed the two-dimensional differential transform method for solving linear and nonlinear partial differential equations (PDEs). Recently, a new algorithm has been established for calculating the one-dimensional differential transform of nonlinear functions [3]. The developed technique depends only on the fundamental operation properties of differential transform and calculus. Based on the same concept, a simple and reliable algorithm used to calculate the two-dimensional differential transform of nonlinear functions in an easy way is introduced in this paper.

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2. Differential Transform

The basic definitions and fundamental Theorem of two-dimensional differential transform are defined and proved in [4] and will be stated in brief in this section.

Two-dimensional differential transform of a function $u(x, y)$ is defined as follows:

$$U(k, h) = \frac{1}{k!h!} \left[\frac{\partial^{k+h} u(x, y)}{\partial x^k \partial y^h} \right]_{\substack{x=0 \\ y=0}} \quad (1)$$

Where $u(x, y)$ is the original function and $U(k, h)$ is the transformed function, which is also called T-function. In this paper, the lowercase and uppercase letters represent the original and transformed functions, respectively. The inverse differential transforms of $U(k, h)$ is defined as:

$$u(x, y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} U(k, h) x^k y^h \quad (2)$$

By substituting (2) into (1), we have

$$u(x, y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \left[\frac{\partial^{k+h} u(x, y)}{\partial x^k \partial y^h} \right]_{\substack{x=0 \\ y=0}} \frac{x^k y^h}{k!h!} \quad (3)$$

The fundamental theorem of two-dimensional differential transform is

THEOREM 2.1 *Two-dimensional differential transform have the following properties:*

- If $w(x, y) = u(x, y) \pm v(x, y)$, then $W(k, h) = U(k, h) \pm V(k, h)$.
- If $w(x, y) = cu(x, y)$, then $W(k, h) = cU(k, h)$.
- If $w(x, y) = \frac{\partial^{m+n} u(x, y)}{\partial x^m \partial y^n}$, then

$$W(k, h) = \frac{(k+m)!(h+n)!}{k!h!} U(k+m, h+n).$$

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- If $w(x, y) = u(x, y)v(x, y)$, then

$$W(k, h) = \sum_{m=0}^k \sum_{n=0}^h U(m, h-n) V(k-m, n).$$

- If $w(x, y) = x^m y^n$, then

$$W(k, h) = \delta(k-m)\delta(h-n) = \begin{cases} 1, & k=m \text{ and } h=n \\ 0, & \text{otherwise} \end{cases}$$

in which m and n are nonnegative integers and c is a constant.

3. Differential Transform for a Special Class of Functions

In [2], $\sin(au)$, $\cos(au)$, e^{au} , $\ln(au)$ and hyperbolic functions have been investigated.

In this work we present the differential transform of the functions $\sec(au)$ and $\tan(au)$.

THEOREM 3.1 *The transformation function of $l(u) = \sec(au)$ is as follows:*

$$\left\{ \begin{array}{l} L(0, 0) = \sec(aU(0, 0)) \\ \sum_{r=0}^k \sum_{s=0}^h G(r, h-s)L(k-r+1, s) \\ -a \sum_{r=0}^k \sum_{t=0}^{k-r} \sum_{p=0}^h \sum_{s=0}^{h-p} (k-r-t+1)F(r, h-s-p)L(t, s)U(k-r-t+1, p) = 0, \quad k \geq 1 \\ \sum_{r=0}^k \sum_{s=0}^h L(r, h-s+1)G(k-r, s) \\ -a \sum_{r=0}^k \sum_{t=0}^{k-r} \sum_{s=0}^h \sum_{p=0}^{h-s} (h-s-p+1)U(r, h-s-p+1)L(t, s)F(k-r-t, p) = 0, \quad h \geq 1 \end{array} \right.$$

THEOREM 3.2 *The transformation function of $l(u) = \tan(au)$ is as follows:*

$$\left\{ \begin{array}{l} H(0, 0) = \tan(aU(0, 0)) \\ (k+1)H(k+1, h) \\ = a \sum_{r=0}^k \sum_{t=0}^{k-r} \sum_{s=0}^h \sum_{p=0}^{h-s} (k-r-t+1)L(r, h-s-p)L(t, s)U(k-r-t+1, p), \quad k \geq 1 \\ (h+1)H(k, h+1) \\ = a \sum_{r=0}^k \sum_{t=0}^{k-r} \sum_{s=0}^h \sum_{p=0}^{h-s} (h-s-p+1)U(r, h-s-p+1)L(t, s)L(k-r-t, p), \quad h \geq 1 \end{array} \right.$$

4. Numerical Examples

Consider the problem

$$4u_x + u_{tx} = 0 \tag{4}$$

Subject to the initial condition

$$u(x, 0) = \tan(2x). \tag{5}$$

The transformed version of the Equation (4) is:

$$\sum_{r=0}^k G(r, 0)L(k-r+1, 0) - 2 \sum_{r=0}^k \sum_{t=0}^{k-r} (k-r-t+1)F(r, 0)L(t, 0)U(k-r-t+1) = 0 \tag{6}$$

Applying the differential transform to this PDE and using the initial condition, we obtain

$$\begin{aligned} u(x, t) &= \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} x^k t^h = [1 - 4t + (4t)^2/2 - \dots][2x + (2x)^3/3 + (2x)^5/5 + \dots] \\ &= e^{-4t} \tan(2x) \end{aligned}$$

5. Conclusions

In this work, differential transform method (DTM) is applied to the functions sec and tan. The present study has confirmed that the differential transformation method offers significant advantages in terms of its straightforward applicability, its computational effectiveness and accuracy.

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