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Relativistic Stellar Models with Quadratic Equation of State

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Abstract. In this paper, we have obtained and presented new relativistic stellar configurations considering an anisotropic fluid distribution with a charge distribution and a gravitational potential $Z(x)$ that depends on an adjustable parameter. The quadratic equation of state based on Feroze and Siddiqui viewpoint is used for the matter distribution. The new solutions can be written in terms of elementary and polynomial functions. We have investigated that the radial pressure, metric coefficients, energy density, anisotropy factor, charge density, mass function have been well defined and are regular in the interior of these new models, which satisfy all physical properties in a realistic star.

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1. Introduction

The description of the structure of relativistic stars and the research on gravitational collapse is one of the most fundamental important problems in astrophysics according to the formulation of general theory of relativity [1, 2]. One of the fundamental problems in theoretical physics is finding exact solutions of the Einstein field equations [3]. The exact solutions as physical model of compact stars was studied for Delgaty and Lake [4] who have constructed several analytical solutions that describe static spherically symmetric perfect fluid that satisfy all the necessary conditions to be physically interesting.

In the development of models of compact stars that are important subjects to study in Astrophysics, we can refer to the pioneering works of Schwarzschild [5], Tolman [6], Oppenheimer and Volkoff [7]. Schwarzschild [5] found analytical solutions that have allowed us to describe a star with uniform density, Tolman [6] developed a method to find solutions of static spheres of fluid and Oppenheimer and Volkoff [7] used Tolman's solutions to study the gravitational balance of neutron stars. It is important to mention that Chandrasekhar's contributions [8] has been studied in the model production of white dwarfs

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in the presence of relativistic effects and the researches of Baade and Zwicky [9] who propose the concept of neutron stars and identify an astronomic dense object known as supernovas.

The description of the gravitational collapse and evolution of the compact objects has been an important topic in general relativity. Recent experimental results in binary pulsars suggest that some compact objects can be quark stars [10]. The existence of quark stars in hydrostatic equilibrium was first suggested by Itoh [11] in a seminal treatment. The study of strange stars with quark matter has been an interesting topic in the last decades so that this can represent the most energetically favorable state of baryon matter [12].

Stellar models consisting of spherically symmetric distribution of matter with presence of anisotropy in the pressure have been widely considered in the frame of general relativity [13-25]. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [26] or another physical phenomenon by the presence of an electrical field [27]. Many researchers have used a vast variety of mathematical techniques to try in order to obtain solutions of the Einstein-Maxwell field equations for anisotropic relativistic stars since it has been demonstrated by many researchers as Komathiraj and Maharaj [28], Thirukkanesh and Maharaj [29], Maharaj et al. [30], Thirukkanesh and Ragel [31,32], Feroze and Siddiqui [33,34], Sunzu et al.[35], Pant et al. [36] and Malaver [37-40]. These analyses indicate that the system of Einstein-Maxwell equations are very important in the description of ultra-compacts objects.

In order to integrate field equations analytically, the choice of the appropriate equation of state allows obtained models of compact stars physically acceptable [41]. Komathiraj and Maharaj [12], Malaver [42], Bombaci [43], Thirukkanesh and Maharaj [29], Dey et al [44] and Usov [27] assume linear equation of state for quark stars. Feroze and Siddiqui [33] consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj [45] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel [10] have obtained particular models of anisotropic fluids with polytropic equation of state, which are consistent with the reported experimental observations. Malaver [46] generated new exact solutions to the Einstein-Maxwell system by considering Van der Waals modified equation of state with polytropic exponent. Rocha et al. [41] presented a new model with anisotropic pressure and an equation of state that describes the internal structure of a compact star made of strange matter in the color flavor locked (CFL) phase.

In this research paper, we generate a new class of anisotropic matter with quadratic equation of state proposed for Feroze and Siddiqui [33] in a static spherically symmetric space-time using a gravitational potential $Z(x)$ which depends on an adjustable parameter η. We obtain some new class of static spherically symmetrical models for a charged anisotropic matter distribution where the variation of the parameter modifies the radial pressure, energy density, stellar radius and the mass of the compact objects. This article is organized as follows: In Section 2, we present Einstein´s field equations. In Section 3, we make a particular choice of gravitational potential $Z(x)$ that allows solving the field equations and we have obtained new models for charged anisotropic matter. In Section 4, physical acceptability conditions are discussed. In section 5, a physical analysis of the new solutions is performed. Finally, in Section 6, we make a conclusion about obtained and discussed results.

2. Einstein-Maxwell field equations

We consider a spherically symmetric, static and homogeneous space-time. In Schwarzschild coordinates, the metric is given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where $\nu(r)$ and $\lambda(r)$ are two arbitrary functions.

The Einstein field equations for the charged anisotropic matter are given by

$$\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\lambda'}{r}e^{-2\lambda} = \rho+\frac{1}{2}E^2 \tag{2}$$

$$-\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\nu'}{r}e^{-2\lambda} = p_r-\frac{1}{2}E^2 \tag{3}$$

$$e^{-2\lambda}\left(\nu''+\nu'^2+\frac{\nu'}{r}-\nu'\lambda'-\frac{\lambda'}{r}\right) = p_t+\frac{1}{2}E^2 \tag{4}$$

$$\sigma = \frac{1}{r^2}e^{-\lambda}(r^2E)' \tag{5}$$

where ρ is the energy density, p_r is the radial pressure, E is electric field intensity, p_t is the tangential pressure and primes denote differentiations with respect to r. Using the transformations, $x = cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and $c > 0$, suggested by Durgapal and Bannerji [47], the Einstein field equations can be written as:

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c}+\frac{E^2}{2c} \tag{6}$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c}-\frac{E^2}{2c} \tag{7}$$

$$4xZ\frac{\ddot{y}}{y} + (4Z+2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c}+\frac{E^2}{2c} \tag{8}$$

$$p_t = p_r + \Delta \tag{9}$$

$$\frac{\Delta}{c} = 4xZ\frac{\ddot{y}}{y} + \dot{Z}\left(1+2x\frac{\dot{y}}{y}\right) + \frac{1-Z}{x} - \frac{E^2}{c} \tag{10}$$

$$\sigma^2 = \frac{4cZ}{x}\left(x\dot{E}+E\right)^2 \tag{11}$$

σ is the charge density, $\Delta = p_t - p_r$ is the anisotropy factor and dots denote differentiation with respect to x. With the transformations of [47], the mass within a radius r of the sphere takes the form

$$M(x) = \frac{1}{4c^{3/2}}\int_0^x \sqrt{x}\left(\rho^* + E^2\right)dx \tag{12}$$

where

$$\rho^* = \left(\frac{1-Z}{x} - 2\dot{Z}\right)c$$

In this paper, we assume the following quadratic equation of state:

$$p_r = \alpha\rho^2 + \beta\rho - \gamma \tag{13}$$

proposed for Feroze and Siddiqui [33]. In eq. (13), α, β and γ are arbitrary constants, ρ is the energy density.

3. Solutions of the Einstein-Maxwell field equations

In this research, we have chosen the gravitational potential as $Z(x) = (1-\eta a x)^2$ where is a real constant and η is an adjustable parameter. This potential is well behaved and regular at the origin in the interior of the sphere. Following Ngubelanga et al. [48] for the electric field intensity, we take the form:

$$\frac{E^2}{2c} = x\left(a + bx\right) \tag{14}$$

This electric field is finite at the center of the star and remains continuous in the interior. We have considered the cases for $\eta = 1, 2$.

For the case $\eta = 1$, using $Z(x)$ and Eq. (14) in Eq. (6), we obtain:

$$\rho = c\left[6a - \left(5a^2 + a\right)x - bx^2\right] \tag{15}$$

Substituting eq. (15) in eq. (13), the radial pressure can be written in the form:

$$p_r = \alpha c^2\left[6a - \left(5a^2 + a\right)x - bx^2\right]^2 + \beta c\left[6a - \left(5a^2 + a\right)x - bx^2\right] - \gamma \tag{16}$$

Using eq. (15) in eq. (12), the expression of the mass function is:

$$M(x) = \frac{\left[70a - \left(35a^2 + 7a\right)x - 5bx^2\right]x^{3/2}}{70\sqrt{c}} \tag{17}$$

With eq. (14) and substituting $Z(x)$ in eq. (11), the charge density is:

$$\sigma^2 = \frac{2c^2\left(1 - ax\right)^2\left(3a + 4bx\right)^2}{\left(a + bx\right)} \tag{18}$$

Replacing (14), (16) and $Z(x)$ in eq. (7), we have:

$$\frac{\dot{y}}{y} - \frac{(2a - a^2 x)}{4\left(1 - ax\right)^2} = \frac{\alpha c\left[6a - \left(5a^2 + a\right)x - bx^2\right]^2}{4\left(1 - ax\right)^2} + \frac{\beta\left[6a - \left(5a^2 + a\right)x - bx^2\right]}{4\left(1 - ax\right)^2} - \frac{\gamma}{4c\left(1 - ax\right)^2} - \frac{x\left(a + bx\right)}{4\left(1 - ax\right)^2} \tag{19}$$

Integrating eq. (19), we have:

$$y(x) = c_1\left(ax - 1\right)^{A*} e^{\frac{Bx^4 + Cx^3 + Dx^2 + Ex + F}{12a^5(ax - 1)}} \tag{20}$$

Where for convenience, we have let:

$$A* = -\frac{\left(10a^6 - 8a^5 - 2a^4 - 6a^3 b - 6a^2 b - 4b^2\right)\alpha c + \left(5a^5 + a^4 + 2a^2 b\right)\beta + a^5 + a^4 + 2a^2 b}{4a^5} \tag{21}$$

$$B = a^4 b^2 \alpha c \tag{22}$$

$$C=\left(15a^6b+3a^5b+2a^3b^2\right)c\alpha \tag{23}$$

$$D=\left(75a^8+30a^7+3a^6+9a^5b+9a^4b+6a^2b^2\right)\alpha c-3a^4b\beta-3a^4b \tag{24}$$

$$E=\left(-75a^7-30a^6-3a^5-24a^4b-12a^3b-9ab^2\right)\alpha c+3a^3b\beta+3a^3b \tag{25}$$

$$F=\left(-3a^6+6a^5-3a^4+6a^3b-6a^2b-3b^2\right)\alpha c \\ +\left(3a^4-3a^5+3a^2b\right)\beta-3a^5+3a^4+3a^2b \tag{26}$$

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as:

$$e^{2\lambda}=\frac{1}{(1-ax)^2} \tag{27}$$

$$e^{2\nu}=A^2c_1^2(ax-1)^{2A*}e^{\frac{Bx^4+Cx^3+Dx^2+Ex+F}{6a^5(ax-1)}} \tag{28}$$

and for the anisotropy factor Δ, we have

$$\Delta=4xc(1-ax)^2\left[\begin{array}{l}\frac{\left(A*^2-A*\right)a^2}{(ax-1)^2}+\frac{2aA}{(ax-1)}\left(\frac{4Bx^3+3Cx^2+2Dx+E}{12a^5(ax-1)}-\frac{Bx^4+Cx^3+Dx^2+Ex+F}{12a^4(ax-1)^2}\right)\\ +\frac{12Bx^2+6Cx+2D}{12a^5(ax-1)}-\frac{4Bx^3+3Cx^2+2Dx+E}{6a^4(ax-1)^2}+\frac{Bx^4+Cx^3+Dx^2+Ex+F}{6a^3(ax-1)^3}\\ +\left(\frac{3aBx^4+2(aC-2B)x^3+(aD-3C)x^2-2Dx-(aF+E)}{12a^5(ax-1)^2}\right)^2\end{array}\right] \\ -2ac(1-ax)\left[1+2x\left(\frac{aA*}{ax-1}+\frac{4Bx^3+3Cx^2+2Dx+E}{12a^5(ax-1)}-\frac{Bx^4+Cx^3+Dx^2+Ex+F}{12a^4(ax-1)^2}\right)\right] \\ 2ac-a^2cx-2xc(a+bx) \tag{29}$$

The metric for this model is

$$ds^2=-A^2c_1^2\left(acr^2-1\right)^{2A*}e^{\frac{Bx^4+Cx^3+Dx^2+Ex+F}{6a^5(ax-1)}}dt^2+\frac{dr^2}{\left(1-acr^2\right)^2}+r^2(d\theta^2+\sin^2\theta d\varphi^2) \tag{30}$$

With η=2, the expression for the energy density is

$$\rho=c\left[12a-\left(20a^2+a\right)x-bx^2\right] \tag{31}$$

Replacing eq.(31) in eq. (13), we have the radial pressure as follows:

$$p_r=\alpha c^2\left[12a-\left(20a^2+a\right)x-bx^2\right]^2+\beta c\left[12a-\left(20a^2+a\right)x-bx^2\right]-\gamma \tag{32}$$

and the mass function is:

$$M(x)=\frac{\left[140a-\left(140a^2+7a\right)x-5bx^2\right]x^{3/2}}{70\sqrt{c}} \tag{33}$$

Substituting eq. (14) and $Z(x)$ in eq. (11), we obtain the charge density as follows:

$$\sigma^2 = \frac{2c^2\left(1-2ax\right)^2\left(3a+4bx\right)^2}{\left(a+bx\right)} \tag{34}$$

With the eq. (14), eq. (32) and Z(x), the eq. (7) becomes:

$$\frac{\dot{y}}{y} - \frac{(4a-4a^2x)}{4\left(1-2ax\right)^2} = \frac{\alpha c\left[12a-\left(20a^2+a\right)x-bx^2\right]^2}{4\left(1-2ax\right)^2} + \frac{\beta\left[12a-\left(20a^2+a\right)x-bx^2\right]}{4\left(1-ax\right)^2} - \frac{\gamma}{4c\left(1-2ax\right)^2} - \frac{x\left(a+bx\right)}{4\left(1-ax\right)^2} \tag{35}$$

Integrating eq. (35), we obtain:

$$y(x) = c_2\left(2ax-1\right)^G e^{\frac{Hx^4+Ix^3+Jx^2+Kx+L}{384a^5(2ax-1)}} \tag{36}$$

Again, for convenience we have let:

$$G = -\frac{\left(160a^6-32a^5-2a^4-12a^3b-3a^2b-b^2\right)\alpha c+\left(40a^5+2a^4+2a^2b\right)\beta+8a^5+2a^4+2a^2b}{32a^5} \tag{37}$$

$$H = 16a^4b^2\alpha c \tag{38}$$

$$I = \left(960a^6b+48a^5b+16a^3b^2\right)c\alpha \tag{39}$$

$$J = \left(19200a^8+1920a^7+48a^6+288a^5b+72a^4b+24a^2b^2\right)\alpha c - 48a^4b\beta - 48a^4b \tag{40}$$

(41)

$$K = \left(-9600a^7-960a^6-24a^5-384a^4b-48a^3b-18ab^2\right)\alpha c+24a^3b\beta+24a^3b \tag{42}$$

$$L = \left(-192a^6+96a^5-12a^4+48a^3b-12a^2b-3b^2\right)\alpha c+\left(24a^4-96a^5+12a^2b\right)\beta - 96a^5+24a^4+12a^2b+\frac{48a^4}{c}\gamma \tag{43}$$

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as

$$e^{2\lambda} = \frac{1}{\left(1-2ax\right)^2} \tag{43}$$

$$e^{2\nu} = A^2c_2^2\left(2ax-1\right)^{2G} e^{\frac{Hx^4+Ix^3+Jx^2+Kx+L}{192a^5(2ax-1)}} \tag{44}$$

and for the anisotropy factor Δ, we have:

$$\Delta = 4xc\left(1-2ax\right)^2\left[\begin{array}{l}\frac{\left(G^2-G\right)a^2}{2\left(ax-1\right)^2}+\frac{4aG}{(2ax-1)}\left(\frac{4Hx^3+3Ix^2+2Jx+K}{384a^5\left(2ax-1\right)}-\frac{Hx^4+Ix^3+Jx^2+Kx+L}{192a^4\left(2ax-1\right)^2}\right)\\ +\frac{12Hx^2+6Ix+2J}{384a^5\left(2ax-1\right)}-\frac{4Hx^3+3Ix^2+2Jx+K}{96a^4\left(2ax-1\right)^2}+\frac{Hx^4+Ix^3+Jx^2+Kx+L}{48a^3\left(2ax-1\right)^3}\\ +\left(\frac{4Hx^3+3Ix^2+2Jx+K}{384a^5\left(2ax-1\right)}-\frac{Hx^4+Ix^3+Jx^2+Kx+L}{192a^4\left(2ax-1\right)^2}\right)^2\end{array}\right] \tag{45}$$

$$\frac{-4ac\left(1-2ax\right)\left[1+2x\left(\frac{2aG}{2ax-1}+\frac{4Hx^3+3Ix^2+2Jx+K}{384a^5\left(2ax-1\right)}-\frac{Hx^4+Ix^3+Jx^2+Kx+L}{192a^4\left(2ax-1\right)^2}\right)\right]}{4ac-4a^2cx-2xc\left(a+bx\right)}$$

The metric for this model is:

$$ds^2=-A^2c_2^2\left(2acr^2-1\right)^{2G}e^{\frac{Hx^4+Ix^3+Jx^2+Kx+L}{192a^5(ax-1)}}dt^2+\frac{dr^2}{\left(1-2acr^2\right)^2}+r^2(d\theta^2+\sin^2\theta d\varphi^2) \tag{46}$$

4. Elementary criteria for physical acceptability

A physically acceptable interior solution of the field equations must comply with the certain physical conditions [10,49,50]:

(i) Regularity of the gravitational potentials is in the stellar interior and at the origin.
(ii) The radial pressure should be positive and a decreasing function of radial coordinate.
(iii) The energy density should be well defined, positive and a decreasing function of the radial parameter.
(iv) $p_r > 0$ and $\rho > 0$ in the origin.
(v) Any physically acceptable solution must satisfy the causality condition where the radial speed of sound should be less than speed of light throughout the model, i.e. $0 \le dp_r/d\rho \le 1$
(vi) For the anisotropic case, the radial and the tangential pressure are equal to zero at the centre $r=0$, i.e. $\Delta(r=0)=0$.
(vii) In the surface of the sphere, it should be matched with the Schwarzschild exterior solution, for which the metric is given by

$$ds^2=-\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2\left(d\theta^2+\sin^2 d\varphi^2\right) \tag{47}$$

through the boundary *r=R* where *M* is the total mass of the star.

The conditions (ii), (iii) and (iv) mean that radial pressure and energy density should must reach a maximum at the centre and decreasing towards the surface of the sphere.

5. Physical features of the models

For the case $\eta=1$, we have $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=A^2c_1^2\left(-1\right)^{2A*}e^{-F/6a^5}$ in the origin $r=0$ and $\left(e^{2\lambda(r)}\right)'_{r=0}=\left(e^{2\nu(r)}\right)'_{r=0}=0$. This shows that the potential gravitational factor is regular in the origin. In the centre $r=0$, $\rho(0)=6ac$ and $p_r(0)=36\alpha a^2c^2+6a\beta c-\gamma$, both are positive if $a,\alpha,\beta>0$. In the surface of the star $r=R$, we have $p_r(r=R)=0$ and

$$R=\frac{\sqrt{2\alpha b\left(-5a^2\alpha c-a\alpha c+\sqrt{25a^4\alpha^2c^2+10a^3\alpha^2c^2+a^2\alpha^2c^2+24a\alpha^2bc^2+2\alpha b\beta c+2\alpha bc\sqrt{4\alpha\gamma+\beta^2}}\right)}}{2\alpha bc}$$

For the pressure and density gradients for all $0 < r < R$, we can obtain respectively

$$\frac{d\rho}{dr}=-2c^2\left(5a^2+a\right)r-4bc^3r^3<0 \tag{48}$$

$$\begin{aligned}\frac{dp_r}{dr}=&-2\alpha c^2\left[2c\left(5a^2+a\right)r+4bc^2r^3\right]\left[6a-\left(5a+a\right)cr^2-bc^2r^4\right]\\&-\beta c\left[2c\left(5a^2+a\right)r+4bc^2r^3\right]<0\end{aligned} \tag{49}$$

and according to the equations eq. (48) and eq. (49), the energy density and radial pressure decrease from the centre to the surface of the star.

From eq. (17), we have:

$$M(r)=\frac{\left[70a-\left(35a^2+7a\right)cr^2-5bc^2r^4\right]cr^3}{70\sqrt{c}} \tag{50}$$

and the total mass of the star is:

$$M(r=R)=\frac{\left[\begin{aligned}&70a-\left(35a^2+7a\right)c\left(\frac{2\alpha b\left(-5a^2c-a\alpha c+\sqrt{25a^4\alpha^2c^2+10a^3\alpha^2c^2+a^2\alpha^2c^2+24a\alpha^2bc^2+2\alpha bc\sqrt{4\alpha\gamma+\beta^2}+2\alpha b\beta c}\right)}{4\alpha^2b^2c^2}\right)\\&-5bc^2\left(\frac{4\alpha^2b^2\left(-5a^2c-a\alpha c+\sqrt{25a^4\alpha^2c^2+10a^3\alpha^2c^2+a^2\alpha^2c^2+24a\alpha^2bc^2+2\alpha bc\sqrt{4\alpha\gamma+\beta^2}+2\alpha b\beta c}\right)^2}{16\alpha^4b^4c^4}\right)\end{aligned}\right]}{70\sqrt{c}}\left[\frac{\left(2\alpha b\left(-5a^2\alpha c-a\alpha c+\sqrt{25a^4\alpha^2c^2+10a^3\alpha^2c^2+a^2\alpha^2c^2+24a\alpha^2bc^2+2\alpha bc\sqrt{4\alpha\gamma+\beta^2}+2\alpha b\beta c}\right)\right)^{3/2}}{8\alpha^3b^3c^3}\right]c \tag{51}$$

Matching conditions for $r=R$ can be written as

$$\left(1-\frac{2M}{R}\right)=A^2y^2\left(cr^2\right) \tag{52}$$

$$\left(1-\frac{2M}{R}\right)^{-1}=\frac{1}{\left(1-acR^2\right)^2} \tag{53}$$

In order to maintain of causality, the radial sound speed defined as $v_{sr}^2=dp_r/d\rho$ should be within the limit $0\le v_{sr}^2\le 1$ in the interior of the star [4]. In this model, we have:

$$v_{sr}^2=\frac{dp_r}{d\rho}=2\alpha c\left[6a-\left(5a^2+a\right)x-bx^2\right]+\beta \tag{54}$$

and for the eq. (34), we can impose the condition:

$$0 \leq 12a\alpha c + \beta - 2a\alpha\left(1+5a\right)c^2r^2 - 2b\alpha c^3r^4 \leq 1 \tag{55}$$

With $\eta = 2$, we have $e^{2\lambda(0)} = 1$, $e^{2\nu(0)} = A^2c_2^2\left(-1\right)^{2G}e^{-L/192a^5}$ in the origin and $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0.$ Again, the gravitational potential factor is regular in $r = 0.$ In the centre $\rho(0) = 12ac$ and $p_r(0) = 144\alpha a^2c^2 + 12a\beta c - \gamma,$ both are positive if $a, \alpha, \beta > 0.$ In the boundary of the star $r = R$, we have $p_r\left(r = R\right) = 0$ and

$$R = \frac{\sqrt{2\alpha b\left(-20a^2\alpha c - a\alpha c + \sqrt{400a^4\alpha^2c^2 + 40a^3\alpha^2c^2 + a^2\alpha^2c^2 + 48a\alpha^2bc^2 + 2\alpha b\beta c + 2\alpha bc\sqrt{4\alpha\gamma + \beta^2}}\right)}}{2\alpha bc}$$

If α=1/10, β=1/5 and γ=0, then we obtain for the stellar radius

$$R = \frac{1}{2}\sqrt{\frac{2bc\left(-20a^2 - a + \sqrt{400a^4 + 40a^3 + a^2 + 48ab}\right)}{bc}}.$$

This is a new value found for the radius of the star. As the radial pressure and the energy density decrease from the centre to the surface of the star, we have that for all $0 < r < R$.

$$\frac{d\rho}{dr} = -2c^2\left(20a^2 + a\right)r - 4bc^3r^3 < 0 \tag{56}$$

$$\begin{aligned}\frac{dp_r}{dr} = &-2\alpha c^2\left[2c\left(20a^2 + a\right)r + 4bc^2r^3\right]\left[12a - \left(20a + a\right)cr^2 - bc^2r^4\right]\\ &-\beta c\left[2c\left(20a^2 + a\right)r + 4bc^2r^3\right] < 0\end{aligned} \tag{57}$$

From (33), we get

$$M(r) = \frac{\left[140a - \left(140a^2 + 7a\right)cr^2 - 5bc^2r^4\right]cr^3}{70\sqrt{c}} \tag{58}$$

For α=1/10, β=1/5 and γ=0, the total mass of the star is

$$M(r = R) = \frac{\left[\begin{array}{l}140a - \left(140a^2 + 7a\right)c\left(\dfrac{2bc\left(-20a^2 - a + \sqrt{400a^4 + 40a^3 + a^2 + 48ab}\right)}{4\alpha^2b^2c^2}\right)\\ -5bc^2\left(\dfrac{2bc\left(-20a^2 - a + \sqrt{400a^4 + 40a^3 + a^2 + 48ab}\right)^2}{16b^4c^4}\right)\end{array}\right]}{70\sqrt{c}}\left[\frac{\left(2bc\left(-20a^2 - a + \sqrt{400a^{42} + 40a^3 + a^2 + 48ab}\right)\right)^{3/2}}{8\alpha^3b^3c^3}\right]c \tag{59}$$

Matching conditions for $r = R$ can be written as:

$$\left(1-\frac{2M}{R}\right)=A^2y^2\left(cr^2\right) \quad \text{and} \quad \left(1-\frac{2M}{R}\right)^{-1}=\frac{1}{\left(1-2acR^2\right)^2}.$$

For this case, the condition $0 \le v_{sr}^2 \le 1$, also implies that:

$$0 \le 24a\alpha c+\beta-2a\alpha\left(1+20a\right)c^2r^2-2b\alpha c^3r^4 \le 1 \tag{60}$$

The figures 1,2,3,4,5 and 6 represent the graphs of p_r, ρ, $M(x)$, σ^2, Δ and v^2_{sr}, respectively with η=2, α=1/10, β=1/5, γ=0 , a=0.2, b=0.005, c=1 and a stellar radius of r=1.5 km.

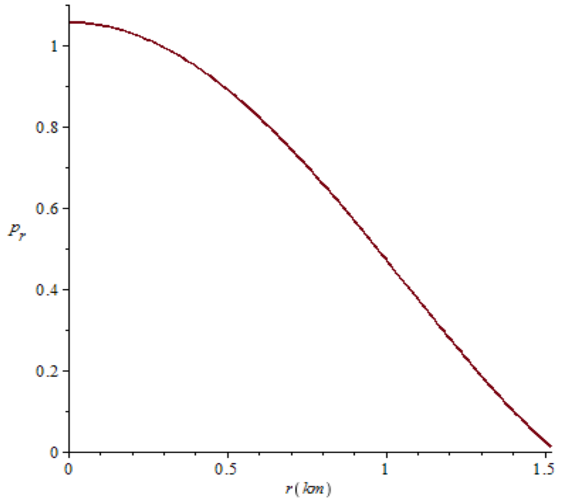

Figure 1. Radial pressure vs radial coordinate for η=2, α=1/10, β=1/5, γ=0 where a=0.2, b=0.005 *and* c=1.

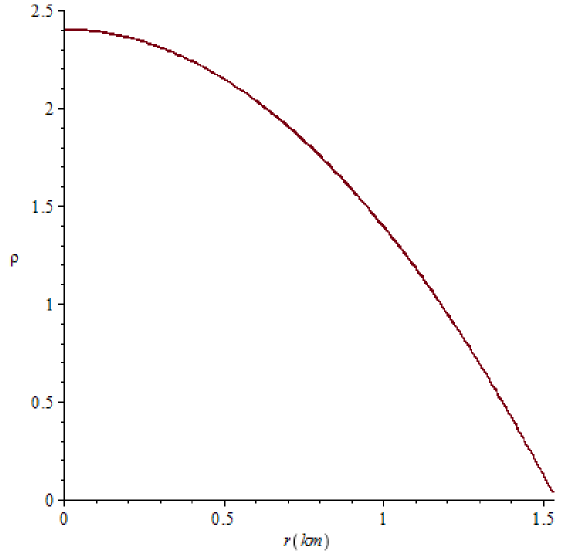

Figure 2. Energy density vs radial coordinate for η=2, α=1/10, β=1/5, γ=0 where a=0.2,

b=0.005 *and* c=1.

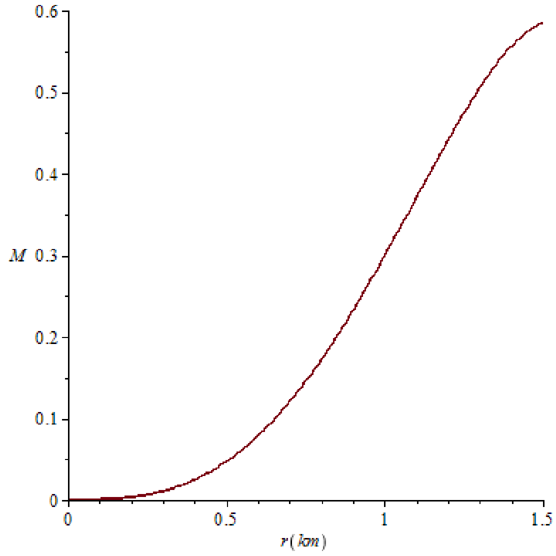

Figure 3. Mass function vs radial coordinate for η=2, α=1/10, β=1/5, γ=0 where a=0.2, b=0.005 *and* c=1.

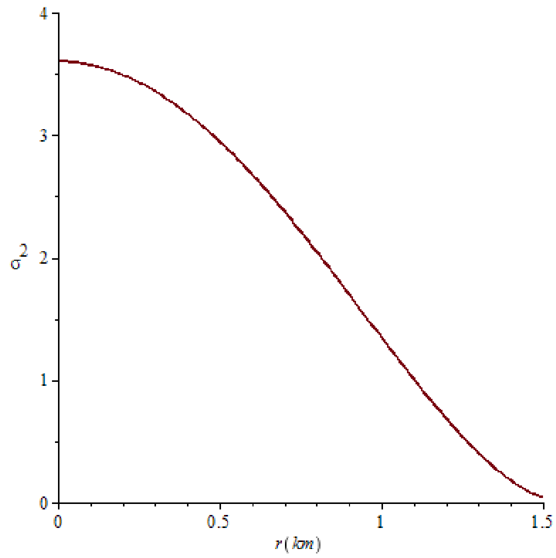

Figure 4. Charge density vs radial coordinate for η=2, α=1/10, β=1/5, γ=0 where a=0.2, b=0.005 and c=1.

In figure 1, it is observed that the radial pressure is finite and decreasing from the center to the surface of the star. In figure 2, the energy density is continuous, also is finite and monotonically decreasing function. In figure 3, the mass function is strictly increasing, continuous and finite. In figure 4, the charge density is non-singular at the origin, non-negative and decreases. In figure 5, the measure of anisotropy is continuous in the stellar interior and Δ vanishes at the center and this means that the radial and tangential pressures should be equal in $r = 0$. The figure 6 shows that the condition $0 \leq v_{sr}^2 \leq 1$ is maintained throughout the interior of the star and satisfy the causality, which is a physical requirement for the construction of a realistic star [4].

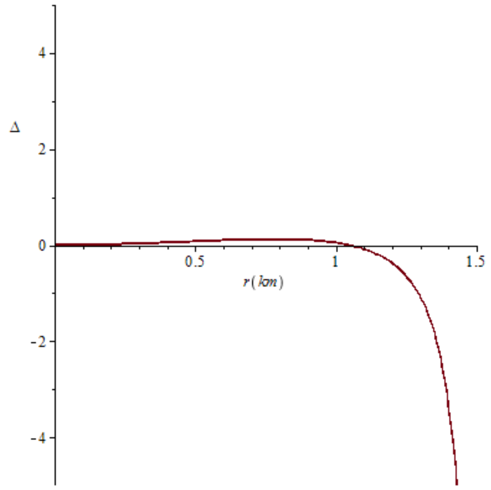

Figure 5. Anisotropy vs radial coordinate for η=2, α=1/10, β=1/5, γ=0 where a=0.2, b=0.005 *and* c=1.

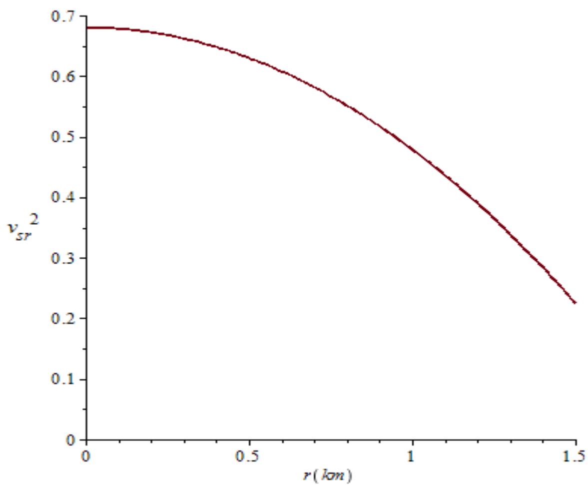

Figure 6. Radial speed sound vs radial coordinate for η=2, α=1/10, β=1/5, γ=0 where a=0.2, b=0.005 and c=1.

In addition, it needs to be considered that Einstein Field Equations lie in the category of Systems of Differential equations and many new analytical and approximate methods can be suggested to solve these types of equations [51-55].

6. Conclusion

Considering a particular form of gravitational potential factor depending on an adjustable parameter and an electric field intensity, we have generated a physically valid category of exact solutions to the Einstein-Maxwell system of equations with a quadratic equation of state that correspond to anisotropic compact sphere. The radial pressure, energy density, anisotropy, mass function, charge density and all the metric coefficients behave well inside the stellar interior and are free of singularities. All the obtained models are physically reasonable and satisfy the physical characteristics of a realistic star as are the regularity of the gravitational potentials at the origin, cancellation of anisotropy in $r=0$, radial pressure finite and decreasing of the energy density and the radial pressure from the center to the surface of the star. These solutions match with the Schwarzschild exterior metric at the boundary for each value of adjustable parameter.

The values calculated for mass and stellar radius could correspond to compact objects with real existence. With $\eta = 2$, it may be possible to explain the stability of compact stars with masses ~ $0.6M_{sol,}$ as some kinds of white dwarfs with $r = 1.5$ km.

The constants α, β and γ have been chosen in order to maintain the causality condition and the regularity of metric potentials inside the radius of the star. The MIT bag model can be recovered as a particular case of this work by taking $\beta = 0$, $\gamma \neq 0$ in eq. (13) and generates families of exact solutions for the Einstein-Maxwell field equations for modeling relativistic compact objects, strange stars and configurations with anisotropic matter distribution.

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