Journal of
Linear and Topological Algebra
Vol. 09*, No.* 03*,* 2020*,* 213- 220

S-metric and fixed point theorem

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Received 18 May 2020; Revised 25 June 2020; Accepted 15 July 2020.
Communicated by Ghasem Soleimani Rad

Abstract. In this paper, we prove a general fixed point theorem in S-metric spaces for maps satisfying an implicit relation on complete metric spaces. As applications, we get many analogues of fixed point theorems in metric spaces for S-metric spaces.

Keywords: Fixed point, implicit relation.

2010 AMS Subject Classification: 54H25; Secondary 47H10.

1. Introduction

This article presents some new fixed point results for mappings. In section 2, we present an implicit relation and some examples for this relation. In section 3, some fixed point theorem for maps are proved using the implicit relation. In section 4, we give an application of the integral equation result.

In [12], Sedghi et al. have introduced the notion of an S-metric space as follows.

Definition 1.1 [12, Definition 2.1] Let X be a nonempty set. An *S-metric* on X is a function $S : X^3 \longrightarrow [0, \infty)$ that satisfies the following conditions for all $x, y, z, a \in X$.

(1) $S(x, y, z) = 0$ if and only if $x = y = z$,

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(2) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$.

The pair (X, S) is called an *S-metric space.*

Immediate examples of such a $S - metric$ space are:

(a) If $X = \mathbb{R}^n$, then we define $S(x, y, z) = ||y + z - 2x|| + ||y - z||$.
(b) If $X = \mathbb{R}^n$, then we define $S(x, y, z) = ||x - z|| + ||y - z||$.
(c) If d is the ordinary metric on X, then we define $S(x, y, z) = d(x, z) + d(y, z)$.

This notion is a generalization of a G-metric space [8] and a D^*-metric space [13]. For the fixed point problem in generalized metric spaces, many results have been proved, see [1, 5–7] for example. In [12], the authors have proved some properties of S-metric spaces. Also, they have been proved some fixed point theorems for a self-map on an S-metric space.

In this paper, we prove a general fixed point theorem in S-metric spaces which is a generalization of [12, Theorem 3.1]. As applications, we get many analogues of fixed point theorems in metric spaces for S-metric spaces.

Now we recall some notions and lemmas which will be useful later.

Definition 1.2 [12] Let (X, S) be an S-metric space. For $r > 0$ and $x \in X$, we define the *open ball* $B_S(x, r)$ and the *closed ball* $B_S[x, r]$ with center x and radius r as follows.

$$B_S(x, r) = \{y \in X : S(y, y, x) < r\} \text{ and } B_S[x, r] = \{y \in X : S(y, y, x) \leq r\}.$$

The *topology induced by the S-metric* is the topology generated by the base of all open balls in X.

Definition 1.3 [12] Let (X, S) be an S-metric space.

(1) A sequence $\{x_n\} \subset X$ *converges* to $x \in X$ if $S(x_n, x_n, x) \to 0$ as $n \to \infty$. That is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ we have $S(x_n, x_n, x) < \varepsilon$. We write $x_n \to x$ for brevity.
(2) A sequence $\{x_n\} \subset X$ is a *Cauchy sequence* if $S(x_n, x_n, x_m) \to 0$ as $n, m \to \infty$. That is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n, m \geq n_0$ we have $S(x_n, x_n, x_m) < \varepsilon$.
(3) The S-metric space (X, S) is *complete* if every Cauchy sequence is a convergent sequence.

Lemma 1.4 [12, Lemma 2.5] In an S-metric space, we have

(1) $S(x, x, y) = S(y, y, x)$ for all $x, y \in X$,
(2) $S(x, y, y) \leqslant 2S(y, y, x)$ for all $x, y \in X$.

Lemma 1.5 [12, Lemma 2.12] Let (X, S) be an S-metric space. If $x_n \to x$ and $y_n \to y$ then $S(x_n, x_n, y_n) \to S(x, x, y)$.

As a special case of [12, Examples in page 260] we have the following.

***Example* 1.6** Let $C[a, b] = \{f | f : [a, b] \longrightarrow \mathbb{R} \text{ is a continuous function}\}$. If set $||f||_\infty = \sup_{x \in [a,b]} \{|f(x)|\}$. Then $S(f, g, h) = ||f - h||_\infty + ||g - h||_\infty$ for all $f, g, h \in C[a, b]$ is an S-metric on $C[a, b]$ and $(C[a, b], S)$ is an complete metric space.

Lemma 1.7 Let (X, S) be a S- metric space. If there exist sequences $\{x_n\}$ and $\{y_n\}$

such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then

$$\limsup_{n\longrightarrow\infty} S(a, x_n, y_n) \leqslant S(a, a, x) + S(x, x, y)$$

for every $a \in X$. In particular, if $\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = x$, then

$$\limsup_{n\longrightarrow\infty} S(a, x_n, y_n) \leqslant S(a, a, x).$$

Proof. Since $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then for each $\varepsilon > 0$ there exist $n_1, n_2 \in \mathbb{N}$ such that

$$\forall\ n \geqslant n_1 \Rightarrow S(x_n, x_n, x) < \frac{\varepsilon}{2} \ \text{ and } \ \forall\ n \geqslant n_2 \Rightarrow S(y_n, y_n, y) < \frac{\varepsilon}{4}.$$

If set $n_0 = \max\{n_1, n_2\}$, then for every $n \geqslant n_0$ by second condition S-metric we have:

$$\begin{aligned} S(a, x_n, y_n) &\leqslant S(a, a, x) + S(x_n, x_n, x) + S(y_n, y_n, x) \\ &\leqslant S(a, a, x) + S(x_n, x_n, x) + 2S(y_n, y_n, y) + S(x, x, y). \end{aligned}$$

Taking the upper limit as $n \to \infty$, we obtain

$$\begin{aligned} \limsup_{n\longrightarrow\infty} S(a, x_n, y_n) &\leqslant S(a, a, x) + \frac{\varepsilon}{2} + \frac{\varepsilon}{2} + S(x, x, y) \\ &= S(a, a, x) + S(x, x, y) + \varepsilon. \end{aligned}$$

Taking $\varepsilon \longrightarrow 0$, we have

$$\limsup_{n\longrightarrow\infty} S(a, x_n, y_n) \leqslant S(a, a, x) + S(x, x, y).$$

■

2. Implicit relations

Implicit relations on metric spaces have been used in many articles. For examples, see [2–4, 9–11]. Let $\mathbb{R}_+$ be the set of nonnegative real numbers and let $\mathcal{F}$ be the set of all functions $F : \mathbb{R}_+^7 \to \mathbb{R}$ satisfying the following conditions:

F_0 $F(\lim_{n\to\infty} p_n) = \lim_{n\to\infty} F(p_n)$ for any $p_n \in \mathbb{R}_+^7$, where $\lim_{n\to\infty} p_n$ means component-wise lim.
F_1 $F(t_1, ..., t_7)$ is nonincreasing in $t_2, ..., t_7$.
F_2 there exists a h with $0 < h < 1$ such that the inequality $F(u, v, v, v, v, 2u, 2u) \leqslant 0$ implies $u \leqslant hv$.

***Example* 2.1** $F(t_1, ..., t_7) = t_1 - h\max\{t_2, t_3, t_4, t_5, \frac{1}{2}t_6, \frac{1}{2}t_7\}$, where $0 < h < 1$.
F_0 and F_1 : Obviously.
F_2 : Let $u, v > 0$ and $F(u, v, v, v, v, 2u, 2u) = u - h\max\{u, v\} \leqslant 0$. If $u \geqslant v$, then $u \leqslant hu$, a contradiction. Thus $u < v$ and $u \leqslant hv$. If $u = 0$, then $u \leqslant hv$. Thus F_2 is satisfied.

***Example* 2.2** $F(t_1, ..., t_7) = t_1 - a\max\{t_2, t_3, t_4, t_5\} - b(t_6 + t_7)$, where $a, b > 0$ and $0 < \frac{a}{1-4b} < 1$.

F_0 and F_1 : Obviously.
F_2 : Let $u, v > 0$ and $F(u, v, v, v, v, 2u, 2u) = u - av - 4bu \leqslant 0$. Then $u \leqslant \frac{a}{1-4b} v = hv$. Thus F_2 is satisfied.

***Example* 2.3** $F(t_1, ..., t_7) = t_1^2 - h \max\{t_1 t_2, t_2 t_3, t_3 t_4, t_4 t_5\}$, where $0 < h < 1$.
F_0 and F_1 : Obviously.
F_2 : Let $u, v > 0$ and $F(u, v, v, v, v, 2u, 2u) = u^2 - h \max\{uv, v^2\} \leqslant 0$. If $u \geqslant v$, then $u^2 \leqslant huv$, a contradiction. Thus $u < v$ and $u \leqslant \sqrt{h} v$. If $u = 0$, then $u \leqslant \sqrt{h} v$. Thus F_2 is satisfied.

***Example* 2.4** $F(t_1, ..., t_7) = t_1 - h t_2$, where $0 < h < 1$.
F_0 and F_1 : Obviously.
F_2 : Let $u, v > 0$ and $F(u, v, v, v, v, 2u, 2u) = u - hv \leqslant 0$. Thus F_2 is satisfied.

3. Fixed point theory

Theorem 3.1 Let (X, S) be a complete S-metric space and $T_n : X \longrightarrow X$ be a self map, for every $n \in \mathbb{N}$. Suppose, for all $x, y, z \in X$

$$F\begin{pmatrix} S(T_i x, T_j y, T_k z), S(x, y, z), S(T_i x, T_j y, x), S(T_i x, T_j y, y), \\ S(T_i x, T_j y, z), S(T_i x, z, T_k z), S(T_j y, z, T_k z) \end{pmatrix} \leqslant 0$$

for every $i, j, k \in \mathbb{N}$ and $F \in \mathbb{F}$. Then there exists unique $x \in X$ with $T_n x = x$, for every $n \in \mathbb{N}$.

Proof. Let $x_0 \in X$, then we can choose $x_n \in X$ with $T_{n+1} x_n = x_{n+1}$.

$$F\begin{pmatrix} S(T_n x_{n-1}, T_n x_{n-1}, T_{n+1} x_n), S(x_{n-1}, x_{n-1}, x_n), S(T_n x_{n-1}, T_n x_{n-1}, x_{n-1}), \\ S(T_n x_{n-1}, T_n x_{n-1}, x_{n-1}), S(T_n x_{n-1}, T_n x_{n-1}, x_n), S(T_n x_{n-1}, x_n, T_{n+1} x_n), \\ S(T_n x_{n-1}, x_n, T_{n+1} x_n) \end{pmatrix} \leqslant 0.$$

Hence

$$\begin{aligned} &F\begin{pmatrix} S(x_n, x_n, x_{n+1}), S(x_{n-1}, x_{n-1}, x_n), S(x_n, x_n, x_{n-1}), S(x_n, x_n, x_{n-1}), \\ S(x_n, x_n, x_n), S(x_n, x_n, x_{n+1}), S(x_n, x_n, x_{n+1}) \end{pmatrix} \\ &= F\begin{pmatrix} S(x_n, x_n, x_{n+1}), S(x_{n-1}, x_{n-1}, x_n), S(x_{n-1}, x_{n-1}, x_n), S(x_{n-1}, x_{n-1}, x_n), \\ S(x_n, x_n, x_n), S(x_n, x_n, x_{n+1}), S(x_n, x_n, x_{n+1}) \end{pmatrix} \leqslant 0. \end{aligned}$$

Since F is nonincreasing in $t_2, ..., t_7$, therefore

$$F(u, v, v, v, v, 2u, 2u) \leqslant F(u, v, v, v, 0, u, u) \leqslant 0,$$

and by property F_2, exist $0 < h < 1$ so that

$$u = S(x_n, x_n, x_{n+1}) < hv = hS(x_{n-1}, x_{n-1}, x_n).$$

In the other hand, for $n \in \mathbb{N}$, we have

$$S(x_n, x_n, x_{n+1}) \leqslant h^n S(x_0, x_0, x_1).$$

Thus we have

$$\begin{aligned} S(x_n, x_n, x_m) &\leqslant 2\sum_{i=n}^{m} S(x_i, x_i, x_{i+1}) \\ &\leqslant 2\sum_{i=n}^{m} h^i S(x_0, x_0, x_1) \\ &= 2\frac{(h^n - h^m)}{1-h} S(x_0, x_0, x_1) \leqslant 2\frac{h^n}{1-h} S(x_0, x_0, x_1) \longrightarrow 0. \end{aligned}$$

Therefore $\{x_n\}$ is a Cauchy sequence. Thus there exists $u \in X$ with $x_n \to u$. It remains to show $T_n u = u$. For every $n \in \mathbb{N}$, we have

$$\begin{aligned} & F\begin{pmatrix} S(T_n u, T_n u, T_m x_{m-1}), S(u, u, x_{m-1}), S(T_n u, T_n u, u), S(T_n u, T_n u, u), \\ S(T_n u, T_n u, x_{m-1}), S(T_n u, x_{m-1}, T_m x_{m-1}), S(T_n u, x_{m-1}, T_m x_{m-1}) \end{pmatrix} \\ = & F\begin{pmatrix} S(T_n u, T_n u, x_m), S(u, u, x_{m-1}), S(T_n u, T_n u, u), S(T_n u, T_n u, u), \\ S(T_n u, T_n u, x_{m-1}), S(T_n u, x_{m-1}, x_m), S(T_n u, x_{m-1}, x_m) \end{pmatrix} \leqslant 0. \end{aligned}$$

Taking the upper limit as $m \to \infty$ we obtain

$$\begin{aligned} & \limsup_{m \longrightarrow \infty} F\begin{pmatrix} S(T_n u, T_n u, x_m), S(u, u, x_{m-1}), S(T_n u, T_n u, u), S(T_n u, T_n u, u), \\ S(T_n u, T_n u, x_{m-1}), S(T_n u, x_{m-1}, x_m), S(T_n u, T_n u, x_m) \end{pmatrix} \\ = & F\begin{pmatrix} S(T_n u, T_n u, u), S(u, u, u), S(T_n u, T_n u, u), S(T_n u, T_n u, u), \\ S(T_n u, T_n u, u), \ \limsup\limits_{m \longrightarrow \infty} S(T_n u, x_{m-1}, x_m), \ \limsup\limits_{m \longrightarrow \infty} S(T_n u, x_{m-1}, x_m) \end{pmatrix} \leqslant 0. \end{aligned}$$

By Lemma 1.7, we have:

$$\begin{aligned} & F\begin{pmatrix} S(T_n u, T_n u, u), S(u, u, u), S(T_n u, T_n u, u), S(T_n u, T_n u, u), \\ S(T_n u, T_n u, u), S(T_n u, T_n u, u), S(T_n u, T_n u, u) \end{pmatrix} \\ \leqslant & F\begin{pmatrix} S(T_n u, T_n u, u), S(u, u, u), S(T_n u, T_n u, u), S(T_n u, T_n u, u), \\ S(T_n u, T_n u, u), \ \limsup\limits_{m \longrightarrow \infty} S(T_n u, x_{m-1}, x_m), \ \limsup\limits_{m \longrightarrow \infty} S(T_n u, x_{m-1}, x_m) \end{pmatrix} \leqslant 0. \end{aligned}$$

Since F is nonincreasing in $t_2, ..., t_7$, therefore

$$F(t, t, t, t, t, 2t, 2t) \leqslant F(t, 0, t, t, t, t, t) \leqslant 0.$$

So, from F_2, we have

$$t = S(T_n u, T_n u, u) \leqslant hS(T_n u, T_n u, u) = ht,$$

therefore, $T_n u = u$.

To prove the uniqueness, let $v \in X$ with $v \neq u$ such that $v = T_n v$. Then

$$F\begin{pmatrix} S(T_i u, T_j u, T_k v), S(u, u, v), S(T_i u, T_j u, u), S(T_i u, T_j u, u), \\ S(T_i u, T_j u, v), S(T_i u, v, T_k v), S(T_i u, v, T_k v) \end{pmatrix} \leqslant 0$$

Hence

$$F\begin{pmatrix} S(u,u,v), S(u,u,v), S(u,u,u), S(u,u,u), \\ S(u,u,v), S(u,v,v), S(u,v,v) \end{pmatrix} \leqslant 0.$$

Since F is nonincreasing in $t_2, ..., t_7$, and by Lemma 1.4, we have

$$\begin{aligned} &F\begin{pmatrix} S(u,u,v), S(u,u,v), S(u,u,v), S(u,u,v), \\ S(u,u,v), 2S(u,u,v), 2S(u,u,v) \end{pmatrix} \\ \leqslant\ &F\begin{pmatrix} S(u,u,v), S(u,u,v), 0, 0, \\ S(u,u,v), S(u,v,v), S(u,v,v) \end{pmatrix} \leqslant 0. \end{aligned}$$

So, from F_2, we have $t = S(u,u,v) < hv = hS(u,u,v)$, that is $u = v$. ■

Corollary 3.2 Let T be a self-map on a complete S-metric space (X, S) and

$$F\begin{pmatrix} S(Tx,Ty,Tz), S(x,y,z), S(Tx,Ty,x), S(Tx,Ty,y), \\ S(Tx,Ty,z), S(Tx,z,Tz), S(Ty,z,Tz) \end{pmatrix} \leqslant 0$$

for all $x, y, z \in X$ and $F \in \mathbb{F}$. Then there exists unique $x \in X$ with $Tx = x$.

Proof. The assertion follows from using Theorem 3.1 with $T_n = T$ for some $n \in \mathbb{N}$. ■

Corollary 3.3 Let (X, S) be a complete S-metric space and $T_n : X \longrightarrow X$ be a self map, for every $n \in \mathbb{N}$. Suppose, for all $x, y, z \in X$

$$S(T_i x, T_j y, T_k z) \leqslant h \begin{Bmatrix} \max\{S(x,y,z), S(T_i x, T_j y, x), S(T_i x, T_j y, y), \\ S(T_i x, T_j y, z), \frac{1}{2}S(T_i x, z, T_k z), \frac{1}{2}S(T_j y, z, T_k z) \end{Bmatrix}$$

for every $i, j, k \in \mathbb{N}$. Then there exists unique $x \in X$ with $T_n x = x$, for every $n \in \mathbb{N}$.

Proof. The assertion follows from using Theorem 3.1 with

$$F(t_1, \cdots, t_7) = t_1 - h\max\{t_2, \cdots, t_5, \frac{1}{2}t_6, \frac{1}{2}t_7\}.$$

■

Corollary 3.4 Let (X, S) be a complete S-metric space and $T : X \longrightarrow X$ be a self map. Suppose, for all $x, y, z \in X$, we have

$$S(Tx,Ty,Tz) \leqslant h \begin{Bmatrix} \max\{S(x,y,z), S(Tx,Ty,x), S(Tx,Ty,y), \\ S(Tx,Ty,z), \frac{1}{2}S(Tx,z,Tz), \frac{1}{2}S(Ty,z,Tz) \end{Bmatrix}.$$

Then there exists unique $x \in X$ with $Tx = x$.

Proof. The assertion follows from using Corrollary 3.4 with $T_n = T$ for some $n \in \mathbb{N}$. ■

***Example* 3.5** Let $X = [0, \frac{\pi}{2}]$ and

$$S(x,y,z) = \begin{cases} 0 & \text{if } x = y = z \\ \max\{x,y,z\} & \text{otherwise .} \end{cases}$$

Then, it is easy to see that (X, S) is a complete S-metric space. Let $T_n x = \sin(h^n x)$, for $x \in X$ and $0 < h < 1$. For all $x, y, z \in [0, \frac{\pi}{2}]$ we have

$$\begin{aligned} S(T_i x, T_j y, T_k z) &= \max\{\sin(h^i x), \sin(h^j y), \sin(h^k z)\} \\ &\leqslant \max\{h^i x, h^j y, h^k z\} \\ &\leqslant h \max\{x, y, z\} \\ &= hS(x, y, z) \\ &\leqslant h \left\{ \begin{array}{l} \max\{S(x, y, z), S(T_i x, T_j y, x), S(T_i x, T_j y, y), \\ S(T_i x, T_j y, z), \frac{1}{2}S(T_i x, z, T_k z), \frac{1}{2}S(T_j y, z, T_k z) \end{array} \right\}. \end{aligned}$$

Thus, by Corrollary 3.3, it is clear that $x = 0$ is the unique fixed point of T_n for every $n \in \mathbb{N}$.

4. An application to the integral equation

In this section, we give an application of the integral equation. Let

$$C[a, b] = \{f | f : [a, b] \longrightarrow \mathbb{R} \;\; is \;\; a \;\; continuous \;\; function\}.$$

If set

$$f(x) = g(x) + \int_0^1 k(x, t) f(t) dt,$$

where $f, g \in C([0, 1])$ and $k(x, t)$ which is continuous on the squared region $[0, 1] \times [0, 1]$ with $|k(x, t)| < h(h < 1)$. Then there exists a unique $f_0 \in C([0, 1])$ such that

$$f_0(x) - g(x) = \int_0^1 k(x, t) f_0(t) dt.$$

Since, for every $f \in C([0, 1])$ if we define $T : C([0, 1]) \longrightarrow C([0, 1])$ by $T(f) = T_f$ such that for every $x \in [0, 1]$, we have

$$T_f(x) = g(x) + \int_0^1 k(x, t) f(t) dt.$$

As in Example 1.6, let

$$S(f, g, h) = ||f - g||_\infty + ||g - h||_\infty$$

for every $f, g, h \in C([0, 1])$. Similarly, now we define the function $S : C[0, 1] \times C[0, 1] \times C[0, 1] \longrightarrow [0, \infty)$ by

$$S(T_f, T_g, T_h) = \sup_{x \in [0,1]} |T_f(x) - T_h(x)| + \sup_{x \in [0,1]} |T_g(x) - T_h(x)|$$

for all $f, g, h \in C[0,1]$. Then,

$$\begin{aligned} S(T_f, T_g, T_h) &= \sup_{x\in[0,1]} |T_f(x) - T_h(x)| + \sup_{x\in[0,1]} |T_g(x) - T_h(x)| \\ &\leqslant \sup_{x\in[0,1]} \int_0^1 |k(x,t)|(|f(t) - h(t)|)dt + \sup_{x\in[0,1]} \int_0^1 |k(x,t)|(|g(t) - h(t)|)dt \\ &\leqslant h \int_0^1 |f(t) - h(t)|dt + h \int_0^1 |g(t) - h(t)|dt \\ &\leqslant h \sup_{x\in[0,1]} |f(x) - h(x)| \int_0^1 dt + h \sup_{x\in[0,1]} |g(x) - h(x)| \int_0^1 dt \\ &\leqslant h||f - h||_\infty + h||g - h||_\infty \\ &= hS(f, g, h) \\ &\leqslant h \left\{ \begin{array}{l} S(f,g,h), S(T_f, T_g, f), S(T_f, T_g, g), \\ S(T_f, T_g, h), \frac{1}{2}S(T_f, h, T_h), \frac{1}{2}S(T_g, h, T_h) \end{array} \right\}. \end{aligned}$$

Hence, the assertion follows from using Corollary 3.4, there exists a unique $f_0 \in C([0,1])$ such that $T(f_0) = T_{f_0} = f_0$; that is

$$f_0(x) - g(x) = \int_0^1 k(x,t) f_0(t)dt$$

for every $x \in [0,1]$.

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