



# Calculating the Eigenvalues of the Quantum Dot Operator Method and Exact Diagonalization Method

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## Abstract

In this paper, first, we introduced a quantum system and how to create a quantum dot-pair and then with regard to the calculation of a quantum system uses quantum dots define a Hamiltonian for a Coulomb potential is caused by an electric dipole special functions particularly well as it could have obtained the relevant amounts. Finally, using a numerical method to compare the two discussed.

Keywords: Quantum, Hamiltonian, dipole

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## 1. Introduction

Quantum dot consist of two metal pages that one of them made of GaAs (gallium arsenide) and another page of ALGaAs (aluminum gallium arsenide). There is a space between the two plates, which is covered by a two-dimensional electron gas. The initial and lower screen that is made of GaAs should be full of free electrons. If a high potential difference is applied to both ends of these pages, electrons moved to other parts of the page and only a few electrons remain in the space. The reason for the closure is that if we consider the force of attraction and repulsion caused by the coulomb electron is surrounded on all sides to be applied and thus electrons at one point encased in the form of a dot-one's will [1].

If the number of electrons in the area is surrounded by 2 will have dual dot in dot-double in some cases have data mobility. Now, according to preliminary explanations must be said that the Schrödinger equation for the desired dot-wrote and consider the Hamiltonian system [2].

It should be noted that the equation could not be solved by analytical and numerical methods to solve equations. To do this we can use computer programs and programming and advanced

applications. To do this, two methods of exact diagonalization method, Finite elements we use [1].

Quantum dots or nanocrystals placed in a class of semiconductors. Today, the new semiconductor electronics industry and in devices such as light emitting diodes and home computers has been used. The importance of semiconductors in the electrically conductive material can be changed by external stimuli such as electric field or light up when the insulator to a conductor become and act as the key. The conductor on one of the critical components of electrical circuits and optical devices as well. Because of the small size of quantum dots, a unique set of Semiconductor and the width is between 2 and 10 nanometres are equivalent to get together 10 to 50 atoms. At these tiny dimensions, materials behave differently and behave differently these unprecedented capabilities in scientific and technical applications of quantum dots give. Performance wavelength of the quantum dots is adjustable for maximum light intensity radiates [3]. When the quantum dots with ultraviolet light stimulus to induce the emission wavelength of the quantum dots determines the color. The wavelength of the quantum dots is very sensitive to gender and

size and new techniques in nanotechnology to many manufacturers the ability to control the wavelength is given. Electrons in a semiconductor material in much larger of 10 nm periods of their energy. When a different energy electron energy levels of electrons in one another is said to be different. An intrinsic property of the electron that makes it more than two electrons can be in an energy balance. In a large mass of energy levels very close semiconductor material that are so close to being described as a continuous range of the energy difference between adjacent balance is zero [2].

Other properties of semiconductor materials are that the chat or there is a continuous range of its energy electrons are allowed to hold power in this chat. the electrons occupy the lower levels "valence band valence electrons in" conduction electrons in the conduction band electrons high levels of chat are said. In semiconductor materials in bulk in a small percentage of electrons in the conduction band and are much more electron in the valence band are so that they are almost full. The same phenomenon that makes semiconductor materials insulator in electric current are normal. If you want more electrons in the conduction band are have enough energy to get up the energy gap. Light electric field or heat can stimulate electrons from the valence band to the conduction band to send a number of

in this case is the level of capacity that hole is empty because during this Event a temporary hole in the valence band is formed [4,5]. Excitable electrons from the valence band to the conduction band, causing mutations and pitting over a wide energy gap should be and the width of the energy gap in semiconductors mass is constant that depends only on the composition of the material and electrons are excited into the conduction band valence band returned after a while again.

Most mutations occur in the return of the electrons through thermal vibration energy is transmitted to the rest mass of material drawn as a result energy is at its lowest level in the conduction band and valence band are transferred to the radiation energy as light. Such as lead sulphide, zinc sulphide, indium phosphate, etc. depending on the size of the wavelength or colour of light after excitation of electrons using an external source to emit [8].

To stretch and relax the skin of a drum could affect its sound. This feature is available in graphene the electrical properties of graphene is changed so as to drag it. If graphene is subject to strain, such as the magnetic field is to produce quantum dots. Semiconductor material that is used extensively in electrical equipment. Graphene, a single layer of carbon material that has high electrical conductivity of the material can be used in the production of high-

speed transistors. Graphene has very low electrical resistance; this material also has no bandgap, band gap and lack of space in which there is no electronic bandgap of the material would not be used for building computers.

Since the substrate reduces the speed of the electrons, "Nikolai Klimov" holes in the graphene on silicon dioxide is drawn that this has created a structure like drums. To measure the properties of graphene, the research team used a scanning microscope.

The measured results showed that when the tip moves over the surface of the graphene because graphene Van der Waals attraction towards the tip, which tends to weaken the electric power it [11]. Researcher's simulations show that graphene tip upward due to the attraction that looks like a circus tent. The researchers found that by using the specific layers of the graphene sheet can be pushed to the following. The results showed that in this case does not change the electrical properties of graphene. The team found that when graphene comes in the form of tents. The point is that the head acts as a quantum dot. You can create graphene structures such as quantum dots containing fast-moving electrons and band gap is important that both of them are important to the field of computing. In normal mode to create a small part of Graphene quantum dots should be cut off, quantum dots are nanoscale crystals that emit light. The wavelength of light emitted from them depends on the size of the crystal. Because the crystal structure of quantum dots called them to do the same [9].

Quantum confinement effects in three dimensions, optical and electronic properties of semiconductor quantum points (as particles) to create relatively to a different bulk material. Quantum dot encapsulated in the regime of strong electron-hole pairs is a species of atomic levels.

Due to the special properties of these nanostructures range of applications in the fields of information technology, cavity quantum electrodynamics, quantum optics, neoclassical light sources, nonlinear optics and lasers are multi-colored. In many cases, the phenomena of scattering, absorption and emission, and the result of the interaction between the electromagnetic field and quantum point is important. Useful and common approach to the study of the interaction of light matter interaction is purely quantum reviews Hamiltonian system is divided into three parts. Possession of released free Hamiltonian and Hamilton are two such it constitutes the third such interactions and observable variables used in dynamic testing. as a macroscopic approach in the study of the interaction of light - matter that it is quantized electromagnetic field in the presence of

matter and as a result the boundaries between the material and the pair below is taken into account [2].

## 2. Vector magnetic dipole moment and coulomb potential

The magnetic dipole moment of the dipole moment.

$$\vec{L} = \sum_i m_i \vec{r}_i \times \vec{v}_i \quad (1)$$

$$d\vec{\mu} = \int dq \vec{r} \times \vec{v} \quad (2)$$

$$\text{if } \vec{\mu} = 1/2 \sum_i q_i \vec{r}_i \times \vec{v}_i \quad (3)$$

$$\mu = 1/2 \sum_i q_i / m_i \vec{L} \quad (4)$$

If we assume that one's a dot inside a potential well placed and the coulomb interaction force to enter:

$$V(x_1, x_2) = V e^2 / 2a e^{-(x_1 - x_2)/a} \quad (5)$$

As for a particle or a quantum system using a finite potential well Schrödinger equation could particularly relevantly value and the Hamiltonian system using special functions for an electron confined in a box we have. Now we can say that if potential defined in the Hamiltonian consider the coulomb potential for exponential function is defined and increased. Of course, the coulomb potential is very similar to the finite potential well in the coulomb potential of the electric dipole is a pair of quantum dots is captured inside a chat. It should be noted that along with the potential components of the magnetic field and spin angular momentum is defined in this potential the impact of the Hamiltonian on special functions, in particular the resulting values very similar to the eigenvalues is finite potential well [15].

In fact, we can say that this component of the coulomb interaction force is in place.

$$1/4\pi\epsilon * \left( \frac{e^2}{x_1 - x_2} \right) \quad (6)$$

If we consider that [15]:

$$H = H_1 + H_2 + \vartheta_{(x_1, x_2)} \quad (7)$$

$$\begin{aligned} H = & -\hbar^2/2m_1 \partial^2/\partial x_1^2 - \hbar^2/2m_2 \partial^2/\partial x_2^2 + u(x_1) + \\ & u(x_2) + e^g/2mc \vec{B} * \vec{S}_1 + e^g/2mc \vec{B} * \vec{S}_2 + \\ & 2e^2 \alpha / \hbar^2 P_{x_1} S_{z_1} + 2e^2 \alpha / \hbar^2 P_{x_2} S_{z_2} + \vartheta_{(x_1, x_2)} \end{aligned} \quad (8)$$

$$\begin{aligned} H_1 = & -\hbar^2/2m \partial^2/\partial x^2 + u(x) + \\ & 2e^2 \alpha / \hbar^2 P_x S_z \end{aligned} \quad (9)$$

And,

$$\hbar_{x,s}(x) = \sqrt{\frac{2}{L}} \exp\left(\frac{-iasx}{a_B}\right) \sin(n\pi x/L) \chi_s \quad (10)$$

$$H\Psi = E\Psi \quad (11)$$

If we consider that:

$$\begin{aligned} \Rightarrow & \sqrt{\frac{2}{L}} e^{-(iasx/a_B)} \cos(n\pi x/L) \left[ -\hbar^2/m S/a_B + \right. \\ & \left. e^2 \right] n\pi\alpha/iL \\ E_n = & \frac{n^2 \pi^2 \hbar^2}{2mL^2} - \frac{\alpha^2 m e^4}{2\hbar^2} \end{aligned} \quad (12)$$

$E_n$  : Special value is infinite potential well.

$$\begin{aligned} \text{if } s = -1 \Rightarrow & \alpha^2 e^2/2 + E_n \quad \alpha^2 e^2/2 + E_n + \\ & \alpha^2 e^2/a_B \Rightarrow \alpha^2 e^2(1/2 + 1/a_B) + E_n \\ \text{if } s = +1 \Rightarrow & \alpha^2 s^2 e^2/2 + E_n \Rightarrow \alpha^2 e^2/2 + E_n \Rightarrow \\ & \alpha^2 e^2/2 + E_n + s\alpha^2 e^2/a_B \\ \Rightarrow & \alpha^2 e^2/2 + E_n - \alpha^2 e^2/a_B \Rightarrow |(1/2 - 1/a_B)\alpha^2 e^2 + \\ & E_n| \end{aligned} \quad (13)$$

## 3. Exact diagonalization method

In this way, if we consider the double dot it can be said that the total Hamiltonian system is composed of two parts, the first and second dot particles are divided. Here it must be said that the total Hamiltonian system with kinetic energy and potential can be said that in this case we assume that the potential of being applied in dot These include: the potential spin-circuit uniform magnetic field and magnetic potential which in the end fully described relations and calculations [10]. In this way, if we assume that the Hamiltonian matrix to consider what we can say the  $n \times n$  matrix and consider the larger number of rows and columns, and the matrix considered the accuracy of calculation is more accurate to say Hamiltonian systems and appropriate method for calculating the components of the Hamiltonian system. If we take into equation(11) we can say:

$$\begin{aligned} \text{if } H_{\alpha\beta} = & \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ -1 & 0 & 1 \end{bmatrix} \quad u_{\alpha\beta} \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \\ \begin{cases} H_{11}\Psi_1 - Eu_{11}\Psi_1 + H_{12}\Psi_2 - Eu_{12}\Psi_2 + H_{13}\Psi_3 - Eu_{13}\Psi_3 = 0 \\ H_{21}\Psi_1 - Eu_{21}\Psi_1 + H_{22}\Psi_2 - Eu_{22}\Psi_2 + H_{23}\Psi_3 - Eu_{23}\Psi_3 = 0 \\ H_{31}\Psi_1 - Eu_{31}\Psi_1 + H_{32}\Psi_2 - Eu_{32}\Psi_2 + H_{33}\Psi_3 - Eu_{33}\Psi_3 = 0 \end{cases} \end{aligned} \quad (14)$$

**4. Method operator**

In this way by defining and fits with the quantum values  $|n, s_n, \ell, s_\ell\rangle$  and  $|m, s_m, k, s_k\rangle$  special functions related to a quantum system, the operator method. In this way, according to the Hamiltonian using the method of multiplying operator, special values corresponding to each section of the General Hamiltonian [1].

$$\text{if } e^{g/2mc} \vec{B} * \vec{S} \quad \text{if } \vec{B} = B\vec{i} \quad \vec{B} * \vec{S} = B s_x \quad (15)$$

At first obtains eigenvalues of the influence of the magnetic field [1,15].

$$\langle m, s_m, k, s_k | e^{g/2mc} \vec{B} * \vec{S} | n, s_n, \ell, s_\ell \rangle$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\hbar}{L} e^{-i s_n \alpha x_1 / a_B} \sin\left(\frac{n \pi x_1}{L}\right) e^{-i s_\ell \alpha x_2 / a_B} \\ \sin\left(\frac{n \pi x_2}{L}\right) \chi_{-s_n}^{(1)} \chi_{s_n}^{(2)} \chi_{-s_\ell}^{(1)} \chi_{s_\ell}^{(2)} - \frac{\hbar}{L} e^{-i s_n \alpha x_2 / a_B} \\ \sin\left(\frac{n \pi x_2}{L}\right) e^{-i s_\ell \alpha x_1 / a_B} \sin\left(\frac{n \pi x_1}{L}\right) \chi_{-s_n}^{(1)} \chi_{s_n}^{(2)} \chi_{-s_\ell}^{(1)} \chi_{s_\ell}^{(2)} \end{bmatrix} \quad (16)$$

If you want the ultimate amount related  $\langle m, s_m, k, s_k | e^{g/2mc} \vec{B} * \vec{S} | n, s_n, \ell, s_\ell \rangle$  to the special impact of the following form will include:

$$\langle m, s_m, k, s_k | e^{g/2mc} \vec{B} * \vec{S} | n, s_n, \ell, s_\ell \rangle$$

The other component of the overall Hamiltonian considers and then the result of the Coulomb interaction force on our special review  $|n, s_n, \ell, s_\ell\rangle$  and  $|m, s_m, k, s_k\rangle$  the special functions and values we calculate it [8,9].

$$\text{if } \vartheta_{(x_1, x_2)} = -\vartheta e^2 / 2a e^{-(x_1 - x_2) / a}$$

$$\langle m, s_m, k, s_k | V_{(x_1, x_2)} | n, s_n, \ell, s_\ell \rangle$$

$$\Rightarrow \langle m, s_m, k, s_k | \vartheta e^2 / 2a e^{-(x_1 - x_2) / a} | n, s_n, \ell, s_\ell \rangle \quad (17)$$

$$\Rightarrow -V e^2 / 2a \langle m, s_m, k, s_k | e^{-(x_1 - x_2) / a} | n, s_n, \ell, s_\ell \rangle$$

The final answer to the eigenvalues of the Coulomb potential impact are follows:

Part one:

$$\Rightarrow V e^2 / 2a \frac{1}{\sqrt{2}} \frac{4}{L^2} \begin{bmatrix} \int_0^L \int_0^L \sin\left(\frac{n \pi x_1}{L}\right) \sin\left(\frac{m \pi x_1}{L}\right) \\ \sin\left(\frac{\ell \pi x_2}{L}\right) \sin\left(\frac{k \pi x_2}{L}\right) \\ e^{-(x_1 - x_2) / a} dx_1 dx_2 \end{bmatrix} \delta_{s_n, s_m} \delta_{s_\ell, s_k} \quad (18)$$

Part two:

$$\Rightarrow V e^2 / 2a \frac{1}{\sqrt{2}} \frac{4}{L^2} \begin{bmatrix} \int_0^L \int_0^L \sin\left(\frac{k \pi x_1}{L}\right) \sin\left(\frac{n \pi x_1}{L}\right) \\ \sin\left(\frac{m \pi x_2}{L}\right) \sin\left(\frac{\ell \pi x_2}{L}\right) \\ e^{-(x_1 - x_2) / a} dx_1 dx_2 \end{bmatrix} \delta_{s_m, s_\ell} \delta_{s_n, s_k} \quad (19)$$

Part three:

$$\Rightarrow V e^2 / 2a \frac{1}{\sqrt{2}} \frac{4}{L^2} \begin{bmatrix} \int_0^L \int_0^L \sin\left(\frac{m \pi x_1}{L}\right) \sin\left(\frac{\ell \pi x_1}{L}\right) \\ \sin\left(\frac{k \pi x_2}{L}\right) \sin\left(\frac{n \pi x_2}{L}\right) \\ e^{-(x_1 - x_2) / a} dx_1 dx_2 \end{bmatrix} \delta_{s_n, s_k} \delta_{s_m, s_\ell} \quad (20)$$

Part four:

$$\Rightarrow V e^2 / 2a \frac{1}{\sqrt{2}} \frac{4}{L^2} \begin{bmatrix} \int_0^L \int_0^L \sin\left(\frac{k \pi x_1}{L}\right) \sin\left(\frac{\ell \pi x_1}{L}\right) \\ \sin\left(\frac{m \pi x_2}{L}\right) \sin\left(\frac{n \pi x_2}{L}\right) \\ e^{-(x_1 - x_2) / a} dx_1 dx_2 \end{bmatrix} \delta_{s_n, s_m} \delta_{s_\ell, s_k} \quad (21)$$

And finally impact the total Hamiltonian defined in the special functions is computed. If the Hamiltonian of the first quantum dot system with the wave function defined to consider, It could be said that [6]:

$$\text{For the first particle } \Rightarrow \langle m, s_m, k, s_k | H_{01} | n, s_n, \ell, s_\ell \rangle$$

$$\left( \frac{-\alpha^2 m e^4}{\hbar^2} + (n^2 + \ell^2) \left( \frac{\pi^2 \hbar^2}{2m\ell^2} \right) [\delta_{s_m, s_n} \delta_{s_\ell, s_k} \delta_{mn} \delta_{\ell k} - \delta_{s_m, s_\ell} \delta_{s_n, s_k} \delta_{kn} \delta_{m\ell}] \right) \quad (22)$$

Since the wave function  $(|n, s_n, \ell, s_\ell\rangle)$  to operator mode  $H_{01}$  is particularly the result of the special operation mode  $H_{01}$  is a special mode has special value.

$$\varepsilon_n = n^2 \pi^2 \hbar^2 / 2mL^2 - \alpha^2 m e^4 / 2\hbar^2$$

$$\varepsilon_\ell = \ell^2 \pi^2 \hbar^2 / 2mL^2 - \alpha^2 m e^4 / 2\hbar^2$$

$$\Rightarrow \varepsilon_n + \varepsilon_\ell = n^2 \pi^2 \hbar^2 / 2mL^2 - \alpha^2 m e^4 / 2\hbar^2 + \ell^2 \pi^2 \hbar^2 / 2mL^2 - \alpha^2 m e^4 / 2\hbar^2 \quad (23)$$

Finally:

$$\Rightarrow -2 \alpha^2 m e^4 / 2\hbar^2 + (n^2 \pi^2 \hbar^2 / 2mL^2 + \ell^2 \pi^2 \hbar^2 / 2mL^2) \Rightarrow -\alpha^2 m e^4 / 2\hbar^2 + (n^2 + \ell^2) \left( \frac{\pi^2 \hbar^2}{2m\ell^2} \right) \quad (24)$$

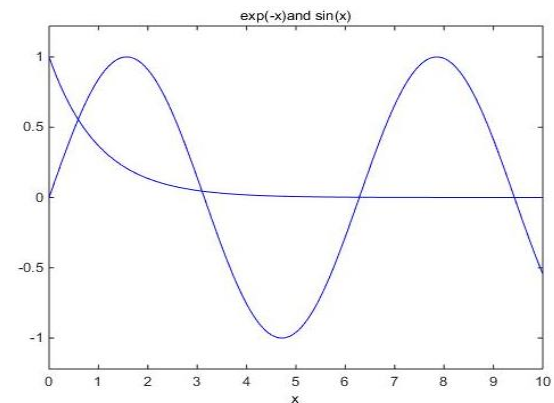


Fig. 1. Diagram of the diagonalization method is described using diagonal matrix

We can say to explain this using diagonalization method and under consideration, by foreseeing able to Hamiltonian system related to a particular quantum system and define their functions and components of a matrix of rows and columns for matrix numerically different values defined in each sector and special functions have changed as a

result, the Hamiltonian has changed Special functions and finally to the dimensions of rows and columns for final values that we find in the matrix to a matrix-hand column [12].

In other words, it can be said that the plot for a periodic variation or change the sine curve according to the column matrix eigenvalues of matrix column to a constant value we've found.

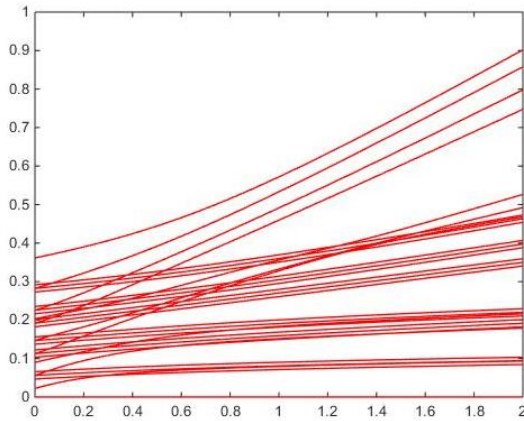


Fig. 2. Diagram of the special values described the result of the impact of the total Hamiltonian using integral equations

As seen in the figure occurs in the interval (0,0.1) special values are constant values. In fact, it can be said that if fitted in the equations,  $E_n$  consider the corresponding It must be said that despite the special  $n^2, \ell^2$  parameters of the fitted values define the changes in looked (0,0.1) the values are fixed [1]. but with the increase of the Hamiltonian matrix changes and looked in the previous section we mention and

$(|m, s_m, k, s_k\rangle, |n, s_n, \ell, s_\ell\rangle)$  special functions are defined using the special values we increased and special values for the slope in the form of increased gradually fitted. of course, it must be said that, according to 7-8 equation and the impact of the

$(|m, s_m, k, s_k\rangle, |n, s_n, \ell, s_\ell\rangle)$  special functions in the coulomb potential to looked (0.2, 0.4) changes in integral fitted to create special values lead to increase with the slope of the fitted very fast. In fact, it can be said that Hamilton, who leads the special components result in different values or information which allows data transfer using qubits increases [14].

## 5. Conclusion

As a conclusion, it can be stated that in the brief description of the quantum dots and methods of construction and applications have made it and then with a point or a quantum dot inside a quantum

system examined. In fact stating that a chat or a blank space to create a quantum dot is like creating a suitable environment for the production of graphene.

Because when the holes created, graphene on the surface of silicon dioxide produce and since graphene is made of carbon and therefore its structure can be regarded as a tent. In this case, we can say that the turning point or the culmination of the tent acts as a quantum point. That if we examine the quantum point of fact, using the Hamiltonian of the quantum system that it can be calculated. The Hamiltonian of a different form sentences that include freedom square, free article and interaction Hamiltonian. In fact, the interaction Hamiltonian, coulomb interaction Hamiltonian which can be defined as a particle in the potential well for it.

Then, using diagonalization method, the expression of matrix method where in the definition of the hHamiltonian matrix for special functions in the system, especially the values obtained. Finally using special operators and the definition  $|n, s_n, \ell, s_\ell\rangle$  and  $|m, s_m, k, s_k\rangle$  initial values of the magnetic field  $\langle m, s_m, k, s_k | \frac{eg}{2mc} \vec{B} \cdot \vec{S} | n, s_n, \ell, s_\ell \rangle$

obtains.

Finally, we examined a result of the Coulomb interaction  $\langle m, s_m, k, s_k | \frac{ve^2}{2a} e^{-(x_1-x_2)/a} | n, s_n, \ell, s_\ell \rangle$  force.

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## References

- [1] R. Hanson, L. P. Kouwenhoven, J. R. Petta, S. Tarucha and L. M. K. Vandersypen, Rev. Mod. Phys., Vol. 79, No. 4, October-December 2007
- [2] C.F. Destefani and S.E. Ulloa, Phys. Rev. B 72, 115326 (2005)
- [3] P. Stano and J. Fabian, Phys. Rev. Lett. 96, 186602 (2006)
- [4] P. Stano and J. Fabian, Phys. Rev. B 72, 155410 (2005)
- [5] J. Konemann, R. J. Hung, D. K. Maude, V. I. Fal'ko, and B. L. Altshuler, Phys. Rev. Lett. 94, 226404 (2005)
- [6] G. Burkard and D. Loss, and D. P. Di Vincenzo, Phys. Rev. B 59, 2070 (1999)
- [7] G. Burkard and D. Loss, Phys. Rev. Lett. 88, 047903 (2002)
- [8] D. Stepanenko, N. E. Bonesteel, D. P. Di Vincenzo, G. Burkard, and D. Loss, Phys. Rev. B 68, 115306 (2003)
- [9] M. Florescu and P. Hawrylak, Phys. Rev. B 73, 045304 (2006)
- [10] A. Bagga, P. Pietilainen, and T. Chakraborty, Phys. Rev. B 74, 033313 (2006)
- [11] C.F. Destefani and S.E. Ulloa, and G. E. Marques, Phys. Rev. B 69, 125302 (2004); 70, 205315 (2004)

- [12] Y. A. Bychkov and E. I. Rashba, *J. Phys. C* 17, 6039 (1984)
- [13] K. M. Svore, D. P. Di Vincenzo, and B. M. Terhal, *Quantum Inf.Comput.* 7, 297(2007)
- [14] C.F. Destefani and S.E. Ulloa, *Phys. Rev. B* 71, 161303 (R) (2005)
- [15] M. Elzerman, R. Hanson, L. H. W. Van Beveren, B. Witkamp, L. M. K. Vandersypen, and L. P. Kouwenhoven, *Nature (London)* 403, 431 (2004)