ISSN: 2251-9246
EISSN: 2345-6221

pp. 149:158

Investigating and Improving the Uncertainty of Control Systems Using Fuzzy Differential Equations

Fateme Arab

Department of Electrical Engineering, Central Tehran Branch, Islamic Azad University, Tehran, Iran, Arabfatemeh723@gmail.com

Abstract

Fuzzy differential equations have been widely used in recent years to model the uncertainty of mathematical models, and modeling has always been one of the practical ways to better understand and avoid wasting time in biological experiments. In this paper, in addition to HIV biology, the necessity of using fuzzy systems in modeling has been investigated, the main methods in solving fuzzy differential equations according to the available sources have been introduced, and the advantages and disadvantages of each method have been explained. The second part of the article examines the results of modeling HIV-infected particles in an inaccurate environment using fuzzy logic in both analytical and numerical methods and simulates the model.

Keywords: Fuzzy differential equations, fuzzy control systems, fuzzy logic
Article history: Received 10-Jan-2020; Revised 21-Feb-2021; Accepted 03-Mar-2021.

1. Introduction

Differential equations have always played an important role in various sciences. In general, these equations are divided into definite and indefinite parts. The uncertain part is divided into two categories of random and ambiguous (fuzzy) problems according to the nature of the given problems and models. When we want to study fuzzy differential equations, we also encounter the limitations and at the same time the specific variations of fuzzy sets such as type of metric, type of derivative, types of acceptable answers to the problems posed. So far, methods and ideas Various solutions have been proposed to solve these problems numerically. In this dissertation, it is intended to discuss the uncertainty of control systems using fuzzy differential equations. The reason for using fuzzy systems Decisions based on qualitative values as well as qualitative inferences from quantitative values are a reasonable reason to use fuzzy systems. To introduce the reasons for choosing fuzzy systems, we refer to the sources of uncertainty production:

A) Fuzzy measurement

Any measurement of physical or biological processes has a natural uncertainty. This means that there is no perfect accuracy in the measurements. Any numbers that are extracted from the measurement are firstly influenced by the expressions of the measuring instruments because the measurement process is from the interaction of the process and the measuring instruments, and secondly, the values used by them are evaluated under the influence of the observer. Which is more due to [1].

B) Quality of models

In all modeling, the fundamental point of modeling is that the symbols and relations of the mathematical model are not elements that flow within the physical phenomenon, but an analogy between the physical phenomenon and the mathematically defined relations expresses the basis of modeling. The mathematical variables of the model are each an indication (and not itself) of an independent and desired quantity, and for this reason simulation is the axis of modeling, which in turn is a qualitative matter [1].

2. Method for solving fuzzy differential equations in systems

A) Derivative of Hakuhara

One of the first suggestions for defining the concept of derivability for fuzzy functions was to study fuzzy differential equations. The Hakuhara derivative becomes more obscure over time. Recently, there have been two extensions of the concept of Hakohara derivative with different differences. Performs a different definition of the Hakohara derivative to the surface cutting system.

In this method (Hakohara derivative) the parameters are fuzzyly placed in the problem. That is, the fuzzy parameters are placed according to the membership functions in the equation and the equation becomes two ordinary differential equations according to the alpha-cut variables and the time variable (an equation for boundaries When is the bottom and an equation for the upper edges)[2].

B) Fuzzy differential inclusion

The fuzzy differential Schumol method is a novel innovation in the field of fuzzy differential equations. In fuzzy differential chamomile, the study is written in another form in which the equal is placed by the inclusion[2].

C) Solve second order fuzzy differential equations by fuzzy Laplace transform method

We develop and use this method to solve second-order fuzzy linear differential equations under Hakohara generalized derivatives [2].

D) Differential equations of FDEs

A suitable set for mathematical modeling of real world problems. There are several ways to study FDE. The first and most popular method is to use the Hakohara derivative [3].One drawback of this method, however, is that the solution becomes more obscure over time (more fuzzy). Hence, the fuzzy solution behaves quite differently from the definitive solution. The use of various models such as mathematical models and computer simulations in all systems, including computer systems, can greatly reduce the time and cost of these experiments and research by predicting some problems and targeting them, and in many Avoid aimless and costly experiments and researches, or even offer new ways for medical researchers in some cases.

According to the mentioned scientific and practical necessities, in general, the purpose of this research is to obtain mathematical models to show the behavior of control systems based on fuzzy model so that we can interpret the interactions of the system and how they affect Let's examine each other.

In this article, the biology of HIV's defense mechanism and the stages of HIV infection and diagnostic tests for HIV infection are examined.

3. Biology of HIV and it's mathematical models

HIV stands for (Human Immunodeficiency Virus), meaning human immunodeficiency virus. The virus, considered a group of microbes, is a very small living particle that can multiply and spread, but needs another living thing to live. When the virus prepares a cell, it begins to multiply inside the cell, eventually damaging the cell. HIV is a group of retroviruses and a subfamily of lentiviruses that attack the cells of the immune system.

Retroviruses are viruses whose genetic material or genome is made up of RNA. Therefore, in order to replicate, they need an enzyme called the reverse transcriptor, which copies their RNA genome into DNA so that it can enter the host cell genome with the help of the enzyme integrase, thus allowing the virus to replicate. HIV has different genes that encode its structural proteins. The general genes for retroviruses are gag, pol, env, and the specific genes tat and rev.

HIV causes AIDS by infecting a large group of cells in the immune system called CD4 + T lymphocytes. These cells are a subset of white blood cells that naturally regulate the immune response to infection. HIV uses T cells to multiply and spread throughout the body, reducing the number of these cells. The body needs these cells to defend itself. When the CD + 4T cell count drops to a certain level, an HIV-infected person becomes susceptible to a range of diseases that the body is usually unable to control. These opportunistic infections cause the death of the patient.

Fighting HIV is difficult for a number of reasons. Among other things, HIV is an RNA virus that uses the reverse fraternal enzyme to convert its own RNA to DNA. This process makes them more likely to mutate in HIV than viruses with DNA. Therefore, it is possible to develop rapid resistance to treatment.The human body eliminates pathogens and foreign agents in two ways, non-specific defense and specific defense, and prevents the occurrence of diseases[4]. As the immune system weakens, over time, the body prepares for infections and cancers that are not normally seen in ordinary people. The symptoms of HIV infection are very complex. There are generally two main hypotheses for treating HIV infection. One is the complete removal of the virus from the body by stimulating the virus genome, making the wrong protein, and injecting non-toxic prodrugs that become toxic in the infected cell. Another hypothesis is to prevent the virus from replicating or slowing down.

After the HIV virus became known and the dangers that AIDS posed to communities, many efforts were made to treat and control the disease. In this regard, many mathematical models for HIV infection have been proposed and in recent years, many efforts have been made to develop these models. These models are expressed to express different modes of this infection. The goal of the AIDS virus is to kill CD_4^+lymphocytes in the blood, and because these cells are the target of the HIV virus, a decrease in the density of CD_4^+ blood is an important sign in detecting the presence of the virus.

In 1989, Perlson [5] developed a simple model for the interaction between the immune system and the HIV virus. In 1996, Haraba and Dolza [6] proved that the loss ofCD_4^+ lymphocytes is the most important harm to the immune system in HIV infection, and based on this, they proposed models of differential equations that describe the stages of lymphocyte depletion. They were. Researchers later combined time latency with these biological models. In general, delayed differential equations express complexities more than ordinary differential equations and are more practical. The following is an example of these models.

A) The first example of the HIV model

The following model of HIV infection is presented in [7].

$$\begin{cases} \dot{T} = s - dT - \beta TV \\ \dot{T}^* = \beta TV - \mu_2 T \\ \dot{V} = T^* - \mu_1 V \end{cases} \quad (1)$$

B) The second example of the HIV model

The model of HIV infection presented in this section is presented in [8]. This nonlinear model has typical fifth-order differential equations that focus on the cytotoxic lymphocyte (CTL) response to HIV infection as a mediator. This model is a reduced-order model of a developed model that considers the reaction between different types, including: antigen-presenting cells and free viruses, and free virus activation pathways. Under certain conditions the model described in [9] is reduced to the following model.

$$\begin{cases} \dot{X} = \lambda - dx - \beta(1-u)xy \\ \dot{Y} = \beta(1-u)xy - ay - p_1 z_1 y - p_2 z_2 y \\ \dot{Z}_1 = p_1 z_1 y - b_1 z_1 \\ \dot{W} = c_2 xyw - c_2 qyw - b_2 w \\ \dot{Z}_2 = c_2 qyw - hz_2 \end{cases} \quad (2)$$

C) The third example of the HIV model

Another type of model used to describe HIV infection is the model presented in [10], which includes three state variables as follows:

$$\begin{aligned} \dot{X}_1 &= S - dx - (1-u_1)\beta x_1 x_3 \\ \dot{X}_2 &= (1-u_1)\beta x_1 x_3 - \mu x_2 \\ \dot{X}_3 &= (1-u_2)kx_2 - cx_3 \end{aligned} \quad (3)$$

D) The fourth example of the HIV model

This model discusses how chaotic dynamics can develop in a recently proposed model of the interaction between the human immunodeficiency virus HIV and the human immune system.

4. Fuzzy systems

Fuzzy systems have long been a common practice in most fields of engineering and science. This method is very compatible with the method of human thinking due to its inclusion of inaccurate and qualitative methods. Major fuzzy systems include socio-economic systems, biological systems and information technology systems [10]. The logic of fuzzy systems, in parallel with their birth and development, requires another branch of knowledge for the analysis of fuzzy systems, which is called algebra and analysis of fuzzy systems.

The subject of algebra and analysis of fuzzy systems is an issue that has received more attention in recent years. Meanwhile, a number of these experts have also addressed the problem of fuzzy differential equations [10]. The methods used to solve the problems have obvious strengths and weaknesses that are not hidden from their view and are approved by experts. The principles of Ka and the basis of all their computational methods are based on the leveling of fuzzy numbers with membership. Figure 1 shows two fuzzy numbers. Leveling them based on membership is the first step in performing the four basic functions of algebra.

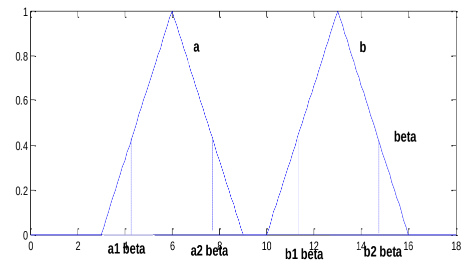

Fig. 1. Display of two fuzzy numbers

Almost all measurements of physical or biological processes have natural uncertainties. This means that there is no perfect accuracy in the measurements. Any kind of numbers that are extracted from measurements are firstly affected by the measuring instruments because the measurement process is due to the interaction of the intended process and the measuring instruments. Second, the values used are evaluated under the influence of the observer's understanding, which is further due [11], [12] and [13].

5. Quality of models and calculations

In all modeling, the fundamental point of modeling is that the symbols and relations of the mathematical model are not elements that flow within the physical phenomenon, but an analogy between the physical phenomenon and the mathematically defined relations underlies the modeling. The mathematical variables of the model are each an indication (and not itself) of an independent and desired quantity, and for this reason simulation is the axis of modeling, which in turn is qualitative. The signs and relationships used in the usual analysis of phenomena (provided they are acknowledged) all generate answers that are a subset of acceptable answers and are therefore accepted in everyday applications [11] and [12].

In addition to the quality of measurements and modeling, there is another fundamental issue called the quality of calculations. Most analysis problems are solved by the assumption that in the phenomena in question, in addition to the continuity of events, each component in question of the system can be defined to infinitesimally small dimensions.

In this article, the outlines of cases that cause problems in the inconsistency of the results of calculations of real numbers, fuzzy numbers and natural and logical expectations of phenomena will be addressed. In this field, the main source of error generation is the current algebra of fuzzy numbers, which also causes unavoidable errors in fuzzy number analysis. This chapter is divided into two separate sections: 1- Analysis of fuzzy problems and 2- Dissemination of uncertainties and their effect on fuzzy number analysis. There are some Methods for solving fuzzy differential equations.

A) Derivative of Hakuhara

The first method is the Hakohara derivative method. In this method, the parameters are fuzzy in the problem, ie the fuzzy parameters are placed in terms of membership functions in the equation and the equation becomes two ordinary differential equations in terms of alpha-cut variables and time variables (one equation for lower boundaries and one equation for upper boundaries) [14].

B) Fuzzy differential inclusion

Then the fuzzy differential inclusion method was proposed. The differential inclusion method is a novel innovation in the field of fuzzy differential equations.

A number of papers have presented the method of distance differential equations [15] and [16]. In this method, the region of uncertainty depends on the number of fuzzy parameters. The region of uncertainty due to the two fuzzy parameters is rectangular or rectangular and the region of uncertainty due to the three parameters is in the shape of a cube.

In this method, for each sample time, a series of calculations are required and for each sample time, the upper and lower bands are determined by the maximum and minimum. At moment t, the region with uncertainty is determined, and then at moment t + dt, it is examined at what interval each of these uncertainties resembles. In this method, there is no problem of divergence, but the volume of calculations is high [15].

In the first two methods, parameters or initial conditions that have uncertainty need a membership function for modeling, but this method does not have the problem of determining the membership function, but due to the dependence of the uncertainty area on the number of fuzzy parameters has its own problems.

Considering the advantages and disadvantages of the solution methods and considering the advantages of fuzzy differential inclusion method in repeating brittle and non-fuzzy results and the capability of this method in producing and disseminating uncertainties, fuzzy differential inclusion method is used in modeling in this project. It does not have the problem of divergence of the first method and can be used to determine the uncertainty region of all equations with fuzzy parameters, and is not limited to a specific category of equations.

6. Modeling of HIV-infected particles in inaccurate environment using fuzzy logic

The presence of inaccurate parameters and variables in scientific applications in the field of bio-mathematical modeling is the subject of new research in uncertainty modeling. Accordingly, the differential equations describing a system can also be expressed accurately or inaccurately. The concept of inaccuracy in general can include uncertainty in parameters or polynomial coefficients, and it can also include uncertainty in initial conditions. In other words, research related to solving inaccurate differential equations includes three basic categories:

- Solve differential equations with inaccurate parameters
- Solve differential equations with inaccurate initial conditions
- Solve differential equations with inaccurate parameters and initial conditions

On the other hand, there are two general methods for solving inaccurate differential equations, which are:

- Analytical methods
- Nnumerical methods

In this paper, modeling of HIV defective population using fuzzy logic is formulated and simulations are presented along with the analysis of the results. To this end, the classical model of a defective HIV population is first introduced. Then, its modeling is presented in two different environments, which are:

- Modeling of defective HIV population in fuzzy environment
- Modeling of defective HIV population in intermittent environment

The model for AIDS is derived from the classic Anderson model, which is, in fact, a macroscopic model of AIDS. This model can be explained as follows [2]:

$$\frac{dx(t)}{dt} = -\lambda(t)x(t)$$
$$\frac{dy(t)}{dt} = \lambda(t)x(t) = \lambda(t)\left(1-y(t)\right) \quad (4)$$

In the AIDS descriptive model, the parameters and initial fuzzy conditions are:

- Transfer rate between infected particles and infected particles that have promoted AIDS as a fuzzy parameter
- Population of defective cells as initial fuzzy conditions

In the following, modeling of HIV virus infected particles in fuzzy environment is formulated for three different cases and its simulations are presented.

A) Modeling of HIV virus infected particles in fuzzy environment

Modeling in this section is based on three different modes, which are:

- Infectious particles mean that the initial condition is a fuzzy number.
- The transfer rate means that the coefficients are a fuzzy number.

Infected particles and transfer rates mean that both the initial conditions and the coefficients are fuzzy numbers.

B) First mode:

Under these conditions, the dynamics of the HIV particle model will be as follows:

$$\frac{dx_1(t,\alpha)}{dt} = -atx_2(t,\alpha)$$
$$\frac{dx_2(t,\alpha)}{dt} = -atx_1(t,\alpha)$$
$$\frac{dy_1(t,\alpha)}{dt} = at\left(1-y_2(t,\alpha)\right)$$
$$\frac{dy_2(t,\alpha)}{dt} = at\left(1-y_1(t,\alpha)\right) \quad (5)$$

Where the initial conditions will be equal to:

$$\left[x_1(t_0,\alpha),x_2(t_0,\alpha)\right] = \left[x_{01}(\alpha),x_{20}(\alpha)\right] \quad (6)$$

$$\left[y_1(t_0,\alpha),y_2(t_0,\alpha)\right] = \left[0,0\right] \quad (7)$$

The equations presented in 5 can be rewritten in the following matrix form:

$$\begin{bmatrix} \frac{dx_1(t,\alpha)}{dt} \\ \frac{dx_2(t,\alpha)}{dt} \\ \frac{dy_1(t,\alpha)}{dt} \\ \frac{dy_2(t,\alpha)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -at & 0 & 0 \\ -at & 0 & 0 & 0 \\ 0 & 0 & 0 & -at \\ 0 & 0 & -at & 0 \end{bmatrix} \begin{bmatrix} x_1(t,\alpha) \\ x_2(t,\alpha) \\ y_1(t,\alpha) \\ y_2(t,\alpha) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ at \\ at \end{bmatrix} \quad (8)$$

Figure 2 shows a view of the above fuzzy membership function per and two examples of fuzzy cutting. To calculate the initial fuzzy condition, the following equation based on fuzzy section is used:

$$A_\alpha = \left[a+\alpha(b-a), c-\alpha(c-b)\right], \ \forall\alpha\in[0,1] \quad (9)$$

Given the basic conditions $x(0) = (0.7,1,1.4)$ and $y(0) = 0$ and $a = 0.237$ Numerical values of system state variables for different values of fuzzy cut α and t=2 Table 1 shows:

$$x_{10}(\alpha) = a+\alpha(b-a), \ x_{20}(\alpha) = c-\alpha(c-b), \ \forall\alpha\in[0,1] \quad (10)$$

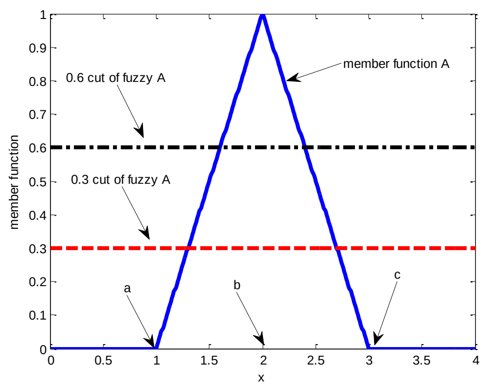

Fig. 2. View of the fuzzy membership function with fuzzy cut

Table.1.
Numerical values of system states for different values of fuzzy section

$y_2(t,\alpha)$	$y_1(t,\alpha)$	$x_2(t,\alpha)$	$x_1(t,\alpha)$	α
0.3775	0.3775	1.2159	0.0914	0
0.3775	0.3775	1.1516	0.1445	0.1
0.3775	0.3775	1.0972	0.1445	0.2
0.3775	0.3775	1.0379	0.1976	0.3
0.3775	0.3775	0.9875	0.2507	0.4
0.3775	0.3775	0.9192	0.3038	0.5
0.3775	0.3775	0.8599	0.3569	0.6
0.3775	0.3775	0.8005	0.4101	0.7
0.3775	0.3775	0.7412	0.4632	0.8
0.3775	0.3775	0.6818	0.5694	0.9
0.3775	0.3775	0.6225	0.6225	1

Also, the behavior of model mode variables over time $t \in [0,2]$ It is presented in Figures 3 to 8.

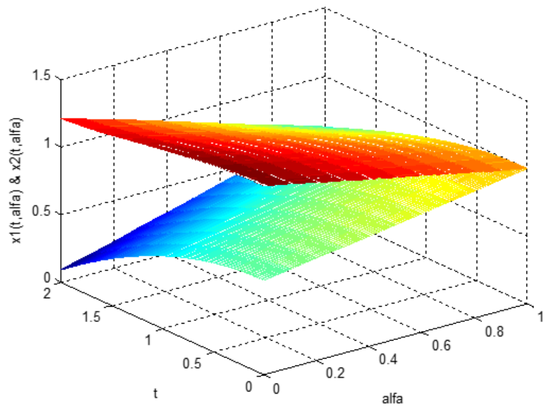

Fig. 3. Behavior of mode variables $x_1(t,\alpha)$ and $x_2(t,\alpha)$ In the interval $t \in [0,2]$

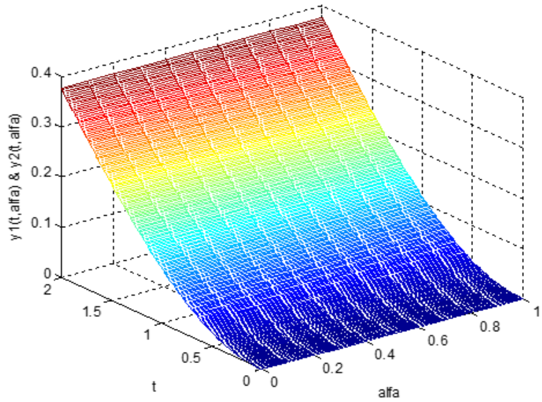

Fig. 4. Behavior of state variables $y_1(t,\alpha)$ and $y_2(t,\alpha)$ In the interval $t \in [0,2]$

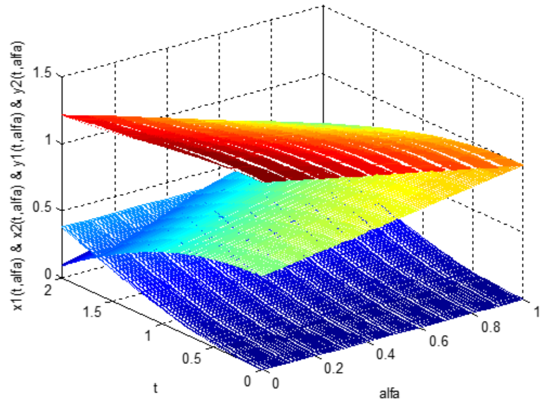

Fig. 5. Behavior of state variables $x_1(t,\alpha)$ and $x_2(t,\alpha)$ and $y_1(t,\alpha)$ and $y_2(t,\alpha)$ In the interval $t \in [0,2]$

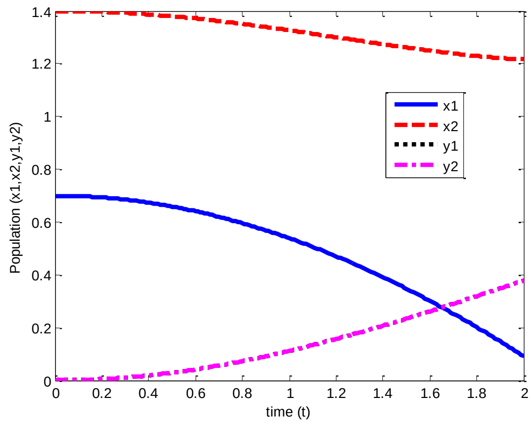

Fig. 6. Fuzzy answer of model mode variables per $\alpha = 0$

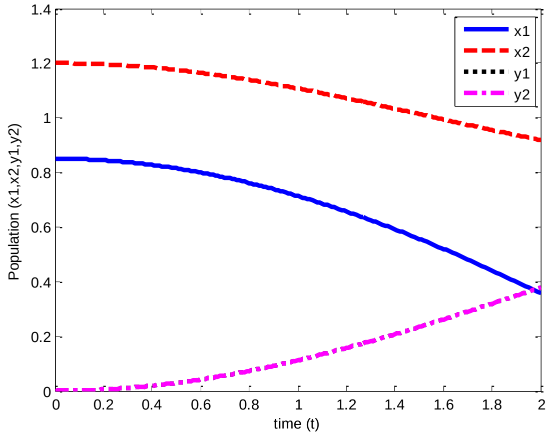

Fig. 7. Fuzzy answer of model mode variables per $\alpha = 0.5$

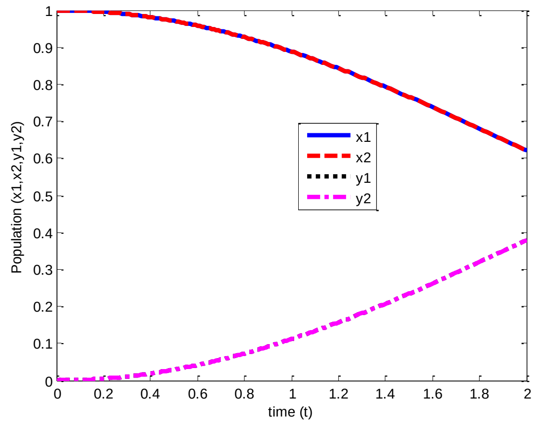

Fig. 8. Fuzzy answers of model mode variables per $\alpha = 1$

According to the results presented on the page, it can be seen that $x_1(t,\alpha)$ and $x_2(t,\alpha)$ They are decreasing and increasing functions, respectively. while $y_1(t,\alpha)$ and $y_2(t,\alpha)$ are explicit functions in t=2. Based on the results presented in three dimensions, it can be seen that the function $x_1(t,\alpha)$ actually describes the fuzzy answer of the triangular shape.

7. Solve the fuzzy differential equation of the HIV model using a numerical method

Numerical methods are another category of methods for solving fuzzy differential equations, in which they determine the approximate answer for a differential equation of any order without the need for analytical solution of differential equations and only by using equations directly. The most important numerical methods can be mentioned as follows [3]:

– The Adam-Bashforth two-step method
– The Adam-Bashforth three-step method
– The two-step Edam-Multon method
– The Adam-Multon three-step method
– Predictive-corrective three-step method

In this article, we discuss one of the above methods. Numerical values of system state variables for different values of fuzzy cut α and t=2 In the three-step Adam-Bashworth method, it is presented in Table 2.

Table.2.
Numerical values of system states for different values of fuzzy cut

$y_2(t,\alpha)$	$y_1(t,\alpha)$	$x_2(t,\alpha)$	$x_1(t,\alpha)$	α
0.3775	0.3775	1.5965	0.0993	0
0.3775	0.3775	1.5174	0.1701	0.1
0.3775	0.3775	1.4385	0.2409	0.2
0.3775	0.3775	1.3596	0.3117	0.3
0.3775	0.3775	1.2808	0.3826	0.4
0.3775	0.3775	1.2019	0.4534	0.5
0.3775	0.3775	1.1230	0.5242	0.6
0.3775	0.3775	1.0441	0.5950	0.7
0.3775	0.3775	0.9653	0.6659	0.8
0.3775	0.3775	0.8864	0.7367	0.9
0.3775	0.3775	0.8075	0.8075	1

Also, the behavior of the model state variables in the time interval $t \in [0,2]$ is presented in Figures 9 to 14.

8. Conclusion

In this paper, the aim is to model HIV-infected particles in an inaccurate environment using fuzzy differential equations. AIDS is an immune system disease caused by the human immunodeficiency virus, HIV. The virus attacks the cells of the immune system. HIV infection is released by the acute spread of the virus in the blood and the virus initially multiplies in large quantities. After this stage, the number of viruses is greatly reduced and a period called clinical latent begins.

The HIV virus infects a large number of immune cells, leading to the development of AIDS. These cells, which play an important role in providing the body with immunity, are gradually reduced and the body becomes susceptible to many diseases that it can no longer fight. As a result, a person with AIDS is more likely to die.

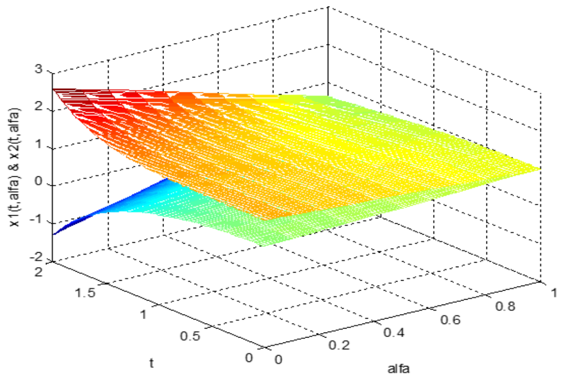

Fig. 9. Behavior of state variables $x_1(t,\alpha)$ and $x_2(t,\alpha)$

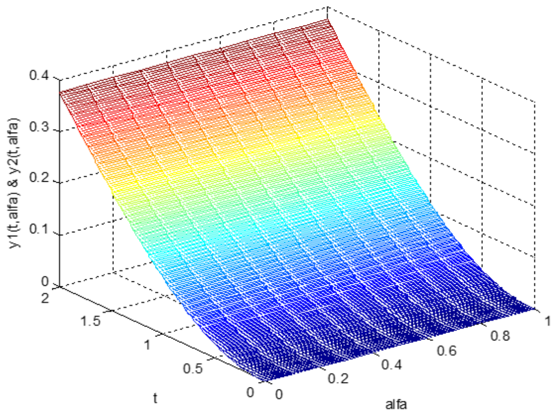

Fig. 10. Behavior of state variables $y_1(t,\alpha)$ and $y_2(t,\alpha)$

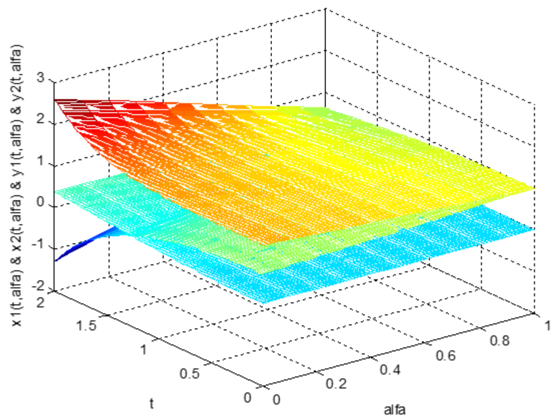

Fig. 11. Behavior of state variables $x_1(t,\alpha)$, $x_2(t,\alpha)$, $y_1(t,\alpha)$ and $y_2(t,\alpha)$

One way to fight AIDS is to identify the correct dynamic model of HIV behavior in the body. The differential equations describing HIV in this study include three basic variables which are:

– The rate of transmission between infected particles and infected particles that have promoted AIDS.
– The proportion of particles susceptible to infection that do not yet have symptoms of AIDS.

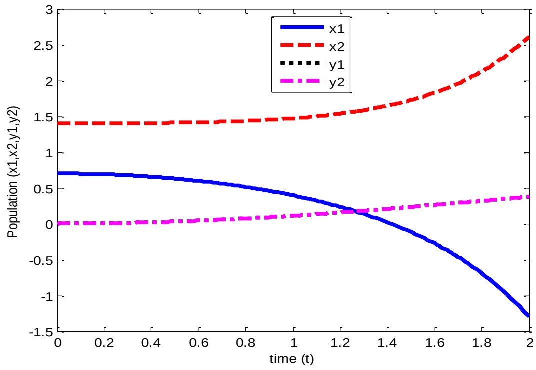

Fig. 12. Fuzzy answers of model mode variables per $\alpha = 0$

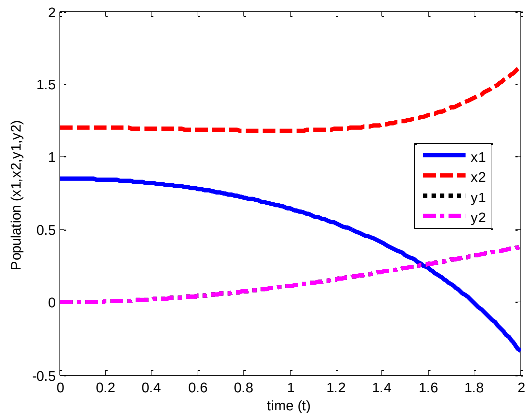

Fig. 13. Fuzzy answer of model mode variables per $\alpha = 0.5$

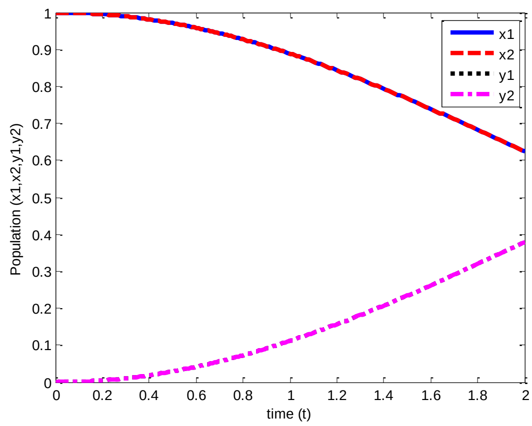

Fig. 14. Fuzzy answer of model mode variables per $\alpha = 1$

– The proportion of particles that have developed the symptoms of AIDS.

In the dynamic model used, two factors are fuzzy in nature and can not be considered definitively in the general case. These two factors are:

– Transfer rate between infected particles and infected particles that have promoted AIDS as a fuzzy parameter

- Population of defective cells as initial fuzzy conditions

Given the initial fuzzy parameters or conditions, the dynamics describing the HIV virus model can be explained as fuzzy differential equations. Accordingly, in this research, two analytical and numerical approaches have been used to solve fuzzy differential equations.

Accordingly, first in this article, the importance of fuzzy systems and the concept of qualitative models and fuzzy measurement were mentioned. Then, there was a reference to the quality of calculations based on fuzzy logic. At the end of this article, a reference was made to the classical algebra of fuzzy logic and the problems related to this field. In the analytical phase of solving the fuzzy differential equations of the HIV virus model, the HIV virus infectious particles were modeled in two fuzzy and interval environments. In each of these two environments, the following conditions were considered:

- Infectious particles means that the initial condition is a number of intervals.
- Transfer rate means that the coefficients are an interval number.
- Infectious particles and transfer rates mean that both the initial condition and the coefficients are interval numbers

To solve the fuzzy differential equations analytically, concepts such as fuzzy set, α of fuzzy set, triangular fuzzy number, Hakohara number derivative and Hakohara function derivative were used. In the numerical phase of solving the fuzzy differential equations of the HIV virus model, the following three three-step numerical methods were used:

- Predictor-corrector method
- The Adam-Bashforth method
- The Adam-Multon method

All three methods are able to find the answer to the dynamic problem recursively without the need for analytical solution of fuzzy differential equations. Of course, it should be noted that the analytical method can be used for both initial conditions and fuzzy coefficients. While numerical methods can still be developed for fuzzy initial conditions. Based on the results and analyzes, the following general conclusions can be expressed:

- The analytical methods used in this research are able to solve the fuzzy differential equation of the HIV virus particle model in both fuzzy and interval environments.
- Analytical methods are able to find the exact answer of fuzzy differential equations both in spite of the initial fuzzy conditions, fuzzy coefficient and their combination.
- The larger the cut-off value of the fuzzy set, the closer the fuzzy solutions are to the explicit solutions of the classical equations.
- Solving fuzzy equations in this research was based on the relationship between cutting a fuzzy set and a triangular fuzzy number.
- Fuzzy derivative approximation is done with the help of Hakohara numerical derivative.
- In the interval environment, the fuzzy differential equation is described by the coefficients of the interval values and the interval value function.
- The solutions of the fuzzy differential equations of HIV-infected particles are one with decreasing behavior and the other with increasing behavior.
- As the amount of fuzzy cut increases, the difference between the two solutions of the fuzzy differential equations decreases.
- For cutting a fuzzy set equal to 1, the fuzzy answer equals the explicit answer.
- If the level of the system increases, it will be very difficult to find analytical answers.
- If the system is highly nonlinear in nature, it seems very difficult to solve the fuzzy differential equation analytically.
- Generally, analytical methods have been developed for first-order differential equations.
- Numerical methods determine approximate answers independent of the order and dynamic nature of the system.
- Numerical methods solve the problem by having fuzzy initial conditions and recursive method.
- Numerical methods have been developed for fuzzy initial conditions only.
- Numerical methods do not require a closed-ended answer
- In low phase $\alpha = 0, 0.1$, the Adam-Bashforth method has the highest adaptation to the analytical method and the worst adaptation is related to the Adam-Molton method.
- In fuzzy sections 0.2 to 0.9, the Adam-Molton method has the highest degree of compliance with the numerical method.
- The values calculated by the predictive-correction methods are very close to the values calculated by the Adam-Molton method.
- The Adam-Bashforth method has accurate answers only in very small sections.

References

[1] Y. Chalco-Cano, W. A., Lodwick, and B. Bede, "Fuzzy differential equations and Zadeh's extension principle," In Fuzzy Information Processing Society (NAFIPS), 2011 Annual Meeting of the North American (pp. 1-5). IEEE, 2011.

[2] T., Allahviranloo, and L. Hooshangian, "Fuzzy generalized H-differential and applications to fuzzy differential equations of second-order," Journal of Intelligent & Fuzzy Systems, 26(4), 1951-1967, 2014.

[3] B.Bede,I.J.Rudas and A.L.Bencsik, First order linear fuzzy differential equations under generalized differentiability, Inf.Sci.177 (2007) 1648–1662.

[4] P. Darabi, "Generalized H-differentiability for solving second order linear fuzzy differential equations," International Journal of Industrial Mathematics, 8(3), 293-301, 2016.

[5] L.A.Zadeh and Toward a generalized theory of uncertainty (GTU) – an outline, Information Sciences 172 (2005) 1–40.

[6] L. Stefanini, A generalization of Hukuhara difference for interval and fuzzy arithmetic, in: D. Dubois, M.A. Lubiano, H. Prade, M. A. Gil, P. Grzegorzewski, O. Hryniewicz (Eds.), Soft Methods for Handling Variability and Imprecision, in: Series on Advances in Soft Computing, 48 (2008).

[7] L. Stefanini, B. Bede, Generalized Hukuhara differentiability of interval-valued functions and interval differential equations, Nonlinear Analysis, 71 (2009) 1311-1328.54

[8] L.C. Barros , R.C. Bassanezi and P.A. Tonelli , Fuzzy modelling in population dynamics, Ecological Modelling 128 (2000) 27–33.

[9] M.L. Puri and D.A. Ralescu, Differentials of fuzzy functions, Journal of Mathematical Analysis and Application 91 (1983) 552–558.

[10] M.Chen,C.Wu,X.Xue and G.Liu, On fuzzy boundary value problems, Inf.Sci. 178 (2008) 1877–1892.

[11] Omer Akın and Omer Oruc, A Prey predator model with fuzzy initial values, Hacettepe Journal of Mathematics and Statistics, Volume 41 (3) (2012), 387 – 395.

[12] Normah Maan, Khabat Barzinji, and Nor'aini Aris, Fuzzy Delay Differential Equation in Predator-Prey Interaction: Analysis on Stability of Steady State, Proceedings of the World Congress on Engineering 2013 Vol I,WCE 2013, July 3 - 5, 2013, London, U.K.

[13] M. Najariyan, M. H. Farahi and M. Alavian, Optimal Control of HIV Infection by using Fuzzy Dynamical Systems, The Journal of Mathematics and Computer Science Vol .2 No.4 (2011) 639-649

[14] L. Stefanini, B. Bede, Generalized Hukuhara differentiability of interval-valued functions and interval differential equations, Nonlinear Analysis, 71 (2009) 1311-1328.54 .

[15] O. Kaleva, Fuzzy differential equations, Fuzzy Sets and Systems 24 (1987) 301–317.

[16] P. Diamond and P. Kloeden, Metric Spaces of Fuzzy Sets, World Scientific, Singapore, 1994.

[17] Purnima Pandit and Payal Singh, Prey Predator Model with Fuzzy Initial Conditions, International Journal of Engineering and Innovative Technology (IJEIT) Volume 3, Issue 12, June 2014.