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A High Order Sliding Mode Controller Design for the Quadruple Tank Process with a Time Delay Compensator in The Presence of Uncertainties

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Abstract

In this article for controlling the height of the liquids in bottom tanks of the quadruple tank process system which is a prototype and an appropriate model of industrial processes, a second-order linear sliding mode controller is designed. The design of the linear second-order sliding mode controller which is stable is done to utilize the benefits of the sliding mode controllers and also reducing the high benefit and chattering in the control signal. To design an applicable and reliable controller, the real present criteria in the industry such as the presence of delay in performance of the pumps and liquid movement in pipes are also considered. To this end, the compensator of the time delay, based on the first order Pade estimation is designed for the model of “quadruple tank system with delay”. Using the controllers of the sliding model with the compensator of the time delay provides a control system that is not only persistent against the external perturbation and uncertainties, but also can provide an appropriate performance by reducing high benefits and chattering in the presence of current delays in the system.

Keywords: Quadruple tank process system, advanced process control, second-order sliding model control, non-linear control, Pade estimation and time delay compensator, persistent control.
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1. Introduction

With the advance of technology and creation of industrial computers, human has gained the ability to control most systems. But even with these considerable advancements, the control of systems that have intrinsically non-linear behavior or the systems that are influenced by the internal and external variables directly and indirectly, and have multiple inputs and outputs and the intrinsically slow systems, is still an interesting subject for researchers. Another challenge for controlling the mentioned systems is the practical experiment and investigation of developed control algorithms on the real system. If this investigation is done on the real system, it might create height expenditure in terms of time and finances. Therefore, the investigation of the laboratory system will provide considerable help in the development of the control algorithm. The quadruple tank is an intrinsically nonlinear laboratory system, multivariable, and has high interference between its modes, and due to the movement of liquid inside of it, it is intrinsically with delay. This system has many features of the real system in different industries. The design and creation of this laboratory system are for two purposes of trading and researching for the development of the control algorithms with the least possible expenditure. The process of quadruple tank system was designed and built-in 1996 for the first time in the Lund Institute of Technology in Stockholm of Sweden, to show the importance of the placement of multivariable zeros in the control system [1,2,3,4]. The proposal and creation of this design were done in the doctoral thesis of Henrik Johnson. After the creation of this system, due to its easy application, it was widely used in the control courses of Lund Institute. [5] has provided the application of second-order sliding mode for non-linear systems of MIMO and had developed the proposed model on the quadruple tank system as a porotype of the industrial systems. [6] has also benefitted from the high order sliding mode controller for solving the problem of chattering in the classic sliding mode controller. It is proved that the proposed model will remove the chattering and stabilizes the closed circle system in the Lyapunov function. In [7], the regular quadruple tank process system is controlled by using the classic sliding mode control. In this reference for removing the chattering process, instead of the inconsistent section in the sliding mode controller, a proportional controller-comparative derivative is used. [8] investigated the development of an estimation technique for the process and compensation of its effect and designing a persistent strategy with high-speed convergence, without chattering and without increasing the control signal domain in the presence of uncertainties in the system for the quadruple tank system. In this research, the effect of delays in liquid movement in pipes and pumps is modeled. In [9], also, for controlling the liquid level in two bottom tanks in a quadruple tank system, a sliding mode controller is used in a way that parameters of these controllers and application of them is done with the intelligent phase approach.

In the performance of the industries, the inconsistency of the model with the factory is an important issue. The quadruple tank performance (QTP) which is a multi-input and multi-output system in both the minimum phase level and non-minimum phase, is configurable. Even so, in NMP, the control of QTP is a challenge which in this article solving this challenge by a robust nonlinear model predictive control with a new control (NMPC) is investigated, and then by the genetic algorithm of NMPC variable, they have been improved. Due to the intrinsic ability of the wavelet neural control in the placement of synchronized signal with two modes of time and frequency, QTP models are used based on these networks [10]. In article [11] a control technique is provided which is based on an artificial network and it is used for controlling the level of the tank and its performance is compared to PID classic controllers. PID controller based on the Ziegler–Nichols approach was assessed correctly and the neural network was trained according to holding the dynamic system in an open circle. In [12] for controlling the inconsistent control systems of two tanks, the combination of phase inconsistent controller and proportionally of PI integral is used which is the combination of feedback of a suitable state and an integral action about the tracking error and all the gains of the control are acquired using the convergence improving approach in linear matrix inconsistencies (LMI). In [13] a stable controller against error for controlling the level of the individual tank or multi-sided against error is designed using artificial intelligence techniques such as phase logic and neural networks which have a better and more precise performance compared to classic controllers. In reference [14], between H-infinity and control of phase logic type 2, there is a lot of sameness and they even complement each other, and a new hybrid controller with the combination of their capacities is provided. In [15] this controller is implemented on a linear structure of the quadruple system. Finally, by comparing these algorithms together they concluded that super-innovative algorithms have better performance compared to IMC. In [16] the weights of the former MPC controllers and phase control are set using the PSO algorithm automatically to control the level of the quadruple tank. [17] by considering the matter of reducing the performance of the linear controllers in facing the systems whose parameters change in time, the comparative approach is used for improving the performance of this controller. [18] provided the inconsistent sliding mode controller for controlling the challenging system of the quadruple tank process. In the proposed control approach of this reference all four cases of the system (liquid height in all quadruple tank systems), are estimated using the stable filter H_∞, and it is designed based on the estimated values of the inconsistent sliding mode controller. [19] has practically investigated the types of controllers on quadruple tank process systems and these controllers are assessed from different perspectives. In [20] for quadruple tank process system, the proportional-integral linear controller and linear order 2 regulator controller with the ratio-integral controller are investigated both unfocused for each circle and focused for multi-input-multi-output.

2. System Modelling

The diagram view of this process is presented in figure 1, this process aims to control the level in two bottom tank using two electrical pumps. Inputs of this process are v_1 and v_2 which are the voltages for these two pumps and outputs are y_1 and y_2 which are the outputs of the level measurement sensors. According to Bernoulli's equation and mass balance, the equations of the quadruple tank are as follow:

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2) k_2}{A_3} v_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1) k_1}{A_4} v_1
\end{aligned} \tag{1}$$

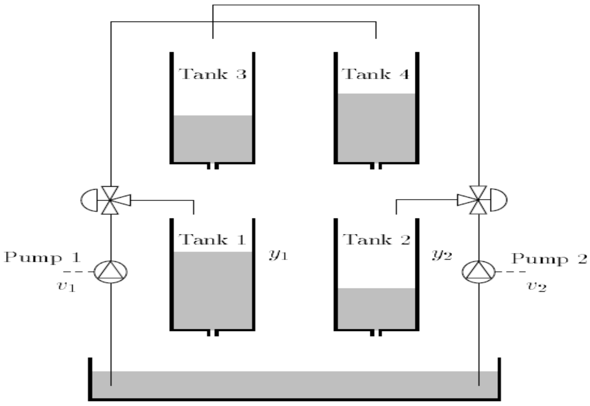

Fig. 1. The diagram of the quadruple tank process [3].

In which A_i is the bottom surface of the tank, i, a_i are the surface of the output hole and h_i is the water level of tank i. The voltage that is given to the ith pump, is shown with v_i and its respective flow is shown with $k_i v_i$. γ_1 and γ_2 parameters which have a number between 0 and 1, show which one is more important in transferring the rate. The input flow to the first tank is $\gamma_1 k_1 v_1$, and the input flow to the fourth tank $(1-\gamma_1)k_1 v_1$ is similar to tanks 2 and 3. Another existing parameter in modeling is the gravitational acceleration which is shown with g. Measurement of the tank surface is done using sensors and their output is measured using signal $y_1 = k_c h_1$ and $y_2 = k_c h_2$. The function matrix of the linear system is as follow:

$$G(s) = \begin{bmatrix} \frac{\gamma_1 c_1}{1+sT_1} & \frac{(1-\gamma_2)c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} \quad (2)$$

In which the time constants are as follow:

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}} \quad i = 1, \ldots\ldots, 4. \qquad c_1 = \frac{T_1 k_1 k_c}{A_1}, \quad c_2 = \frac{T_2 k_2 k_c}{A_2} \quad (3)$$

It should be noted that γ_1 and γ_2 parameters are entering the system in the function matrix of inversion. If we give these two parameters a value of one, the triangulated inverted function matrix will not have a limited zero and this means that input flow to the upper tank equals the input flow to the bottom tank [2, 3, and 4].

As it was shown, in a quadruple tank system using the related values we can change the system zeros and transfer them from right to left and vice versa. Replacement of these zeros is done using γ_1 and γ_2 values. In the following, we will investigate the physical changes of zeros in the quadruple tank system.

3. Design of level controller and compensation of delay in the level

This paper aims to design a level controller to keep the level of the two bottom tanks in the quadruple tank process system, in the desired value in a way that the proposed controller can practically implement and place in the real environment situation.

The most important and first working requirement in the real environment, for the quadruple tank process system, is the existing delay in the system. This delay comes from the system and the operators and its high value. By changing the diameters of the pipes and the power of the pumps, the amount of delay is changeable. Therefore investigation of the effect of delay and external perturbation is one of the main focuses in studying the design of the controller and assessing the operation metric.In this study, the high order sliding mode controller is used for controlling the level in the quadruple tank system in a way that the errors in the classic sliding mode controller solve. Also, there are few references, which assessed the effect of delay in the quadruple tank process system. Compensation of delay effect using approaches such as its approximation is done using Taylor series [8] and/ or using Smith Predictor [21]. In this study, also, for compensating the considered delay effect in the system, the first order Pade approximation is using and also the stable performance of the system is provided using the sliding mode controller.

A) Investigation of delay effect on system modeling

The non-linear model of the quadruple tank process system regardless of the intrinsic delay of the operators and modes is shown in (1-1). For considering the external perturbation and effect of system delays, figure 2 is presented.

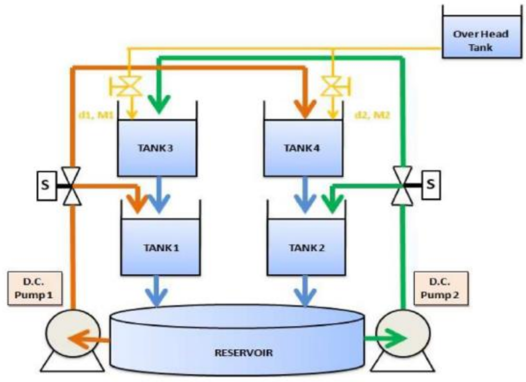

Fig. 2. The diagram of the quadruple tank process system considering the external perturbation [8]

In this model, the effect of external perturbation by M1 and M2 which is connected to the perturbation effect of d_1 and d_2 in the upper tank is given to the system.

In case of considering the delay of operators and also the effect of liquid movement delay from tanks to each other and the effect of external perturbation, the quadruple tank system model with delay and perturbation is achieved as follow:

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3(t-t_1)} + \frac{\gamma_1 k_1}{A_1}v_1(t-t_3) \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4(t-t_2)} + \frac{\gamma_2 k_2}{A_2}v_2(t-t_4) \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2(t-t_5) + d_1 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1(t-t_6) + d_2
\end{aligned} \tag{4}$$

In which A_i is the surface of the ith tank, is a_i ، the bottom surface of the output connector pipes of ith tank. t_3 and t_6 are the required time for transferring the liquid from the first pump to the first and fourth tanks and also t_4 and t_5 are the required time for transferring the liquid from the second pump to the second and third tanks, t_1 is the delay in transferring liquid from the third tank to the first tank and also t_2 is the delay in transferring liquid from the fourth tank to the second tank. d_1 and d_2 are also the perturbation effect of the input flow to tank 3 and 4 by the upper tank. Considering the equality of diameter and length of the pipes of the tank connectors in the quadruple tank system, without losing the general subject we can consider the system delays as follow:

$$\begin{aligned}
t_1 &= t_2 = \tau_1 \\
t_3 &= t_4 = \tau_2 \\
t_5 &= t_6 = \tau_3
\end{aligned} \tag{5}$$

Doing this, the required times for transferring water from upper tanks to bottom tanks, and also the time of arriving liquid from each pump to connected tanks will be equal, and by replacing (5) in (4) the model of the system is attained with mentioned delays which can be written as mode space. Describing the new system in the mode space is as follow [8]:

$$\dot{h}(t) = A_0 h(t) + A_1 h(t-\tau_1) + B_0 u(t-\tau_2) + B_1 u(t-\tau_3) + Dd(t) \tag{6}$$

In which $\mathbf{h} \in R^n$ is the vector of system modes, $\mathbf{u} \in R^m$ is the vector of control input which is made of voltage. A_0 and D, are the mode B_1 ، B_0 ، A_1 ، matrixes, input, and perturbation of the system which are defined as follow. In which $T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}$ belongs to i = 1,2,3,4 and $h_i^0 = \frac{h_{\max}}{2}$.

$$A_0 = \begin{bmatrix} -\frac{1}{T1} & 0 & 0 & 0 \\ 0 & -\frac{1}{T2} & 0 & 0 \\ 0 & 0 & -\frac{1}{T3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T4} \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 0 & \frac{a_3}{A_1 T_3} & 0 \\ 0 & 0 & 0 & \frac{a_4}{A_2 T_4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$B_0 = \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} \tag{7}$$

Matrix D is also the Matrix of the input perturbation which shows the coefficient of the perturbation in modes 3 and 4.

B) *Compensation of time delay effect using Pade approximation:*

The Laplace transform term in the domain of time causes exponential multiplication in the frequency domain:

$$L\{h(t-\tau_1)\} = e^{-\tau_1 s}H(s) = \frac{e^{\frac{-\tau_1 s}{2}}}{e^{\frac{\tau_1 s}{2}}}H(s) \tag{8}$$

By using the first-order Pade approximation, the exponential term will be removed and turns to a fraction. By defining new variable $h'(t)$, the following will be achieved:

$$H'(s) - H(s) = \frac{1-\frac{\tau_1 s}{2}}{1+\frac{\tau_1 s}{2}}H(s) \tag{9}$$

By multiplying and removing denominators together, we will have the following term:

$$H(s) = H'(s) + \frac{\tau_1 s}{2}H'(s) - H(s) \tag{10}$$

By calculating the mirror Laplace, we will have:

$$\dot{h}'(t) = -\frac{2}{\tau_1}h'(t) + 2\frac{2}{\tau_1}h(t) \tag{11}$$

In other words, for compensating the delay effect in system modes, an auxiliary variable $h'(t)$ is defined. It should be mentioned that this variable is not a new case for the system and it does not mean a real state and it is only an auxiliary variable and it is without a unit that can be used instead of $h(t-\tau)$ and as $h'(t) - h(t)$.

When $\dot{h}'(t)$ reaches its balance mode and its changes become zero, $h'(t) = 2h(t)$ and the difference of $h'(t) - h(t)$ will be the value of $h(t)$. Time for $h'(t)$ to reach its final value will be achieved by differential equation (11). Therefore we have the following:

$$h(t-\tau) = h'(t) - h(t) = h(t)(2e^{-\frac{\tau_1}{2}t} - 1) \tag{12}$$

From this equation in zero time, $h(t-\tau)|_{t=0} = h(t)$ is achieved and this shows the effectiveness of the delay compensator. For input variable, $u(t-\tau_2)$ and $u(t-\tau_3)$, again we have:

$$u(t-\tau_2) = u_1'(t) - u(t)$$
$$\dot{u}_1'(t) = -\frac{2}{\tau_2}u_1'(t) + 2\frac{2}{\tau_2}u(t) \tag{13}$$
$$u(t-\tau_3) = u_2'(t) - u(t)$$
$$\dot{u}_2'(t) = -\frac{2}{\tau_3}u_2'(t) + \frac{4}{\tau_3}u(t) \tag{14}$$

By replacing (12), (13), and (14) in (6), equation of system mode with delay compensator and considering the delay, will be achieved as follow:

$$\dot{h}(t) = (A_0 - A_1)h(t) + A_1 h'(t) + B_0 u_1'(t) + (B_0 - B_1)u(t) + B_1 u_2'(t) + B_1 D d(t) \tag{15}$$

For controlling introduced system in (3-17) in a way that perturbation effect and uncertainties in the system are overcome, the classic sliding mode controller and high order controller are used and in the following sections, the effectiveness of these controllers are compared and contrasted against each other.

C) *Design of controller of tank height using the second-order sliding mode approach*

In first-order sliding mode controllers, the controller aims to achieve the sliding variable S to zero and in these controllers, the derivative behavior of the sliding surface is not considered. Therefore, even little changes in the sliding surface of the variable lead to the creation of chattering and high utilization in the control variable which in turn will to high amortization of the operator. The basic aim of the second-order sliding mode controllers is to achieve the sliding variable S and its derivative $\dot{S}$ to zero.

One of the most common approaches in second-order sliding mode is the torque sliding mode. The torque controller is designed as the first second-order sliding mode controller. The characteristic of this controller is torsion around the source point of phase level $S\emptyset\dot{S}$ (figure 3). This means that the movement path of the sliding surface is as a torsion around the source point and convergence toward it.

D) *Even second level sliding mode controller*

Even second level sliding mode controller is a modified algorithm from torque second-order sliding mode. The only difference is that in this algorithm, there is a more even control signal compared to torque second-order sliding mode and on the other hand the stable limited time related to that regardless of the uncertainties in the system is proved using consistent systems theory. Control rule in this approach is provided as follow:

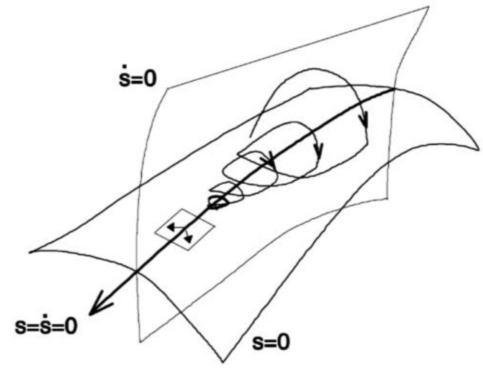

Fig. 3. Movement path of the sliding surface in the torque sliding mode controller [5].

$$u = u_{eq} - \alpha|S|^{\frac{1}{2}}Sgn(S) - \beta\int|S|^{\frac{1}{3}}Sgn(S)\,dt \tag{16}$$

In which α, β are the gain of the sliding mode and they are set in a way that stabilization of system is assured. For proving this approach see reference [22]. But the important point in practice is that parameters are not achieved based on equation 16. But it does not mean that term 16 is not true, it means that in practice there is no exact knowledge of that. Therefore the most practical way for achieving parameters for the controller is to use computer simulation for finding the best answer (this solution is used in all sliding mode approaches). According to provided explanations and finding the second-order derivative from the switching surface ($s = L_1 e + L_2\int_0^t e(\tau)d\tau$), the designed controller is achieved by giving the zero value to sliding surface derivative and finally $\dot{u}$ is integrated and will be as follow:

$$u_{eq}(t) = -(B^T B)^{-1}B^T\int\left[A\dot{h}(t) + A_1\dot{h}'(t) + B_0\,\dot{u}'_1(t) + B_1\dot{u}'_2(t) + {L_1}^{-1}L_2\dot{e}\right] \tag{17}$$

In which we have $A = A_0 - A_1$ and $B = B_0 - B_1$. The control rule in the even second-order sliding mode controller is calculated according to equations 16 and 17.

E) *Insuring the stabilization of closed circle system*

For ensuring the stabilization of the closed circle system, the candidate function of Lyapunov is considered as follow:

$$u_{eq}(t) = -(B^T B)^{-1}B^T\int\left[A\dot{h}(t) + A_1\dot{h}'(t) + B_0\,\dot{u}'_1(t) + B_1\dot{u}'_2(t) + {L_1}^{-1}L_2\dot{e}\right] \tag{18}$$

In which we have $\delta = -\beta\int|S|^{\frac{1}{2}}Sgn(S)\,dt$ and $\xi = S$. Derivation of the above Lyapunov function is as follow:

$$\dot{V} = \beta|\xi|^{\frac{1}{2}}Sgn(\xi)\dot{\xi} + \delta\dot{\delta} = -\alpha\beta|\xi|^{\frac{5}{6}} - \beta Sgn(\xi)\delta - \beta Sgn(\xi)\delta = -\alpha\beta|\xi|^{\frac{5}{6}} \quad (19)$$

As it can be observed from the above equation, the derivative of the Lyapunov function for $\alpha\beta > 0$ is a certain negative half, and the system is stabilized. Considering that it a certain negative half, it is required to investigate the Lassalle .theorem Therefore, this system will be stable and convergence of $\delta = -\beta\int|S|^{\frac{1}{2}}Sgn(S)\,dt$ and $\xi = S$ will be guaranteed.

Therefore for stabilization of system, it required that α, β are obtained in a range in which the $\alpha\beta > 0$ criteria is met. In this range for designing the sliding mode controller, the range of inserted perturbation should be clear, simulation results for the designed controller are shown in the next chapter, and for different working circumstances, they are compared to other controllers.

4. Simulation

In this section, the design will be evaluated. For getting the simulation close to reality, designed controllers based on the compensation model are applied to the real model of the system with its delays. Also, circumstances of the real environment such as the existence of perturbation, the existence of different desired values, and changing the system parameters are tested. For the final results of the test to be fair, the tests are compared to the proportional-integral controller which is widely used. Since the proportional-integral controller is industrial and its easy application, is one of the most applied controllers that due to its structure, is stable against perturbation and uncertainty. Therefore a comparison of final results with the proportional-integral controller can show the effectiveness of designed controllers in the industry.

Parameters of the applied system in the simulation are presented in table 1. Existing controllers are designed based on these parameters.

The designed controllers are designed for the minimum state of the system phase $1 < \gamma_1 + \gamma_2 < 2$, and by changing the controller's coefficient and resetting them, it is possible to use them in the non-minimum system phase $0 < \gamma_1 + \gamma_2 < 1$. Parameters of the model of the system with delay compensator (15) according to table 1 are obtained as follows.

$$A_0 = \begin{bmatrix} -0.1553 & 0 & 0 & 0 \\ 0 & -0.0882 & 0 & 0 \\ 0 & 0 & -0.0897 & 0 \\ 0 & 0 & 0 & -0.0623 \end{bmatrix} \quad B_0 = \begin{bmatrix} 1.0938 & 0 \\ 0 & 1.0938 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (20)$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0.0199 & 0 \\ 0 & 0 & 0 & 0.0111 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0.4688 \\ 0.4688 & 0 \end{bmatrix}$$

Table.1.
Parameters of quadruple tank system

Unit	Value	Sign	Parameter name
cm^2	0.32	$A_{1,..,4}$	The bottom surface of first to fourth tanks
cm^2	0.071	$a_{1,3}$	The bottom surface of output pipes for tank 1 and 3
cm^2	0.057	$a_{2,4}$	The bottom surface of output pipes for tank 2 and 3
second	10	t_1	Delay in liquid transference from tank 3 to 1
second	10	t_2	Delay in liquid transference from tank 4 to 2
second	4	t_3	Delay in liquid transference from pump 1 to tank 1
second	4	t_4	Delay in liquid transference from pump 2 to tank 2
second	8	t_5	Delay in liquid transference from pump 1 to tank 4
second	8	t_6	Delay in liquid transference from pump 2 to tank 3

The designed PI controller in figure 4, is set for regulatory issue for controlling the level in 10 cm and its gains are presented in table 2. It is clear that due to existing delay in the system, increasing the gain is not possible, because it will lead to oscillation behavior in the system and consequently it will lead to instability in it.

Table.2.
Parameters of PI controllers

PI₁	*P*	*0.05*
	I	*0.01*
PI₂	*P*	*0.05*
	I	*0.01*

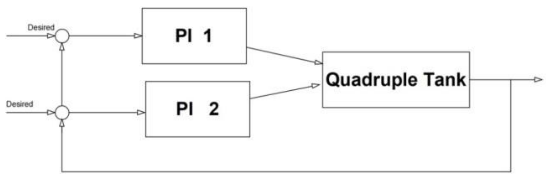

Fig. 4. Structure of PI Controller

The second-order sliding mode controller (SOSMC) is implanted based on (16) and for the regulatory issue, it is set like other controllers. The obtained parameters for this controller are as follow:

$$L_1 = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 50 \end{bmatrix} \quad L_2 = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \quad (21)$$

A) Simulation and analyzing the results

This Step response without the presence of perturbation. In this experiment, the controllers are asked to move the system states from primary values [10 20 30 40] to desired values [10 10 0 0]. The controller's effectiveness metrics are time response and a control signal which are investigated in the following. Figures 5 and 6 show the liquid heights in the first and second tanks.

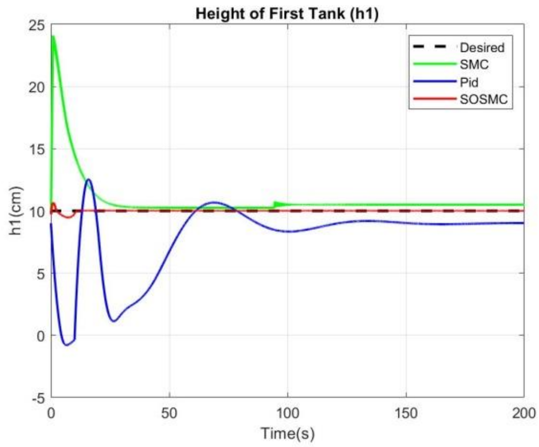

Fig. 5. Liquid height in the first tank for following the step response

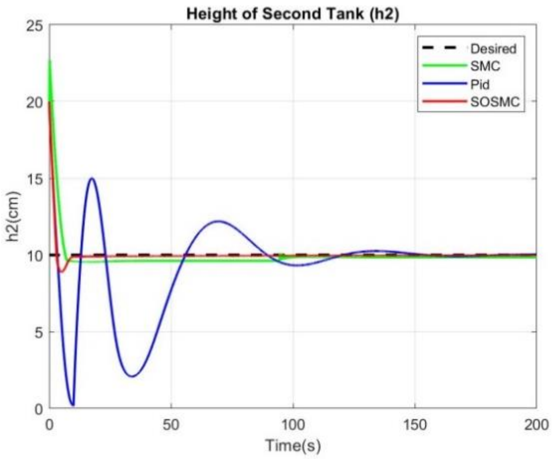

Fig. 6. Liquid height in the second tank in the test of the step response

As it can be observed from the figures, the PI controller reaches its final value in a long time, and in this time, also, it behaved like a mortal system and close to oscillation. This behavior and the big falling time comes from existing delays in the system that avoids increasing the system gain. But sliding mode controller's response due to designing based on the model with delay compensator has higher gain and it reaches its final value in a shorter time. Figure 7 shows the produced control signal by controllers.

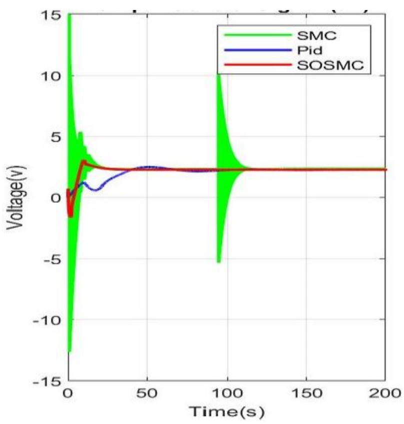

Fig. 7. The produced control signal in regulatory issue

The second chattering wave for the SMC controller was created due to the existence of a delay in the system. As it can be observed in the above image, the produced control signal by the SMC controller has chattering and the high gain in the first seconds is limited by the restrictor of pump voltages. This is while two controllers SOSMC and PI had control signals with limited domains and under 5 V.

4-1-2 Step response with the presence of perturbation and uncertainty of the model

In this experiment values related to the bottom surface of the tanks are doubled and they have increased from 0.32 cm^2 to 0.64 cm^2. For controlling this new system some of the previously designed controllers based on the previous system model and provided gains are used (1-4, 2-4, 3-4). The test scenario for this experiment is applying the desired signal 10 to bottom tanks and applying 10 cm perturbation to the system in the 200th second of the simulation.

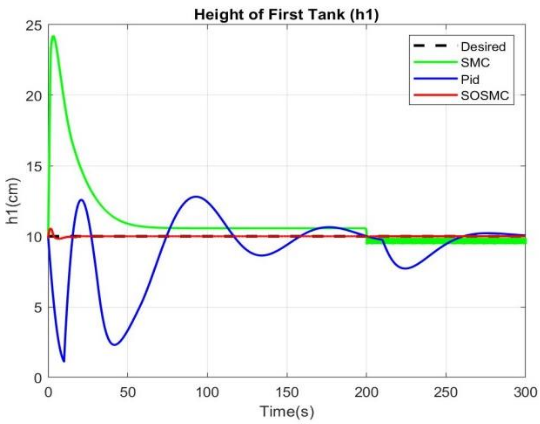

Fig. 8. Liquid height in the first tank in the presence of uncertainty and perturbation of the model

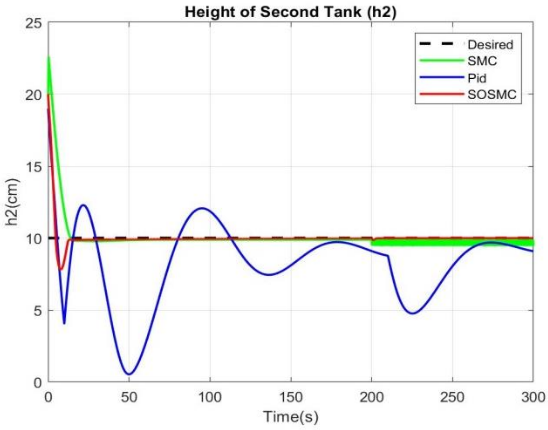

Fig. 9. Liquid height in the second tank in the presence of uncertainty and perturbation of the model

Figures 8 and 9 show liquid height in the first and second tanks. As it can be observed from the figures, the PI controller in the presence of parameters of the model loses its proper performance and its behavior becomes more oscillation-like which consequently makes the falling time longer. This is while model uncertainty does not have an effect on SOSMC controller performance and this controller is still persistent against facing perturbation. Meanwhile, the SMC controller also has a few changes as increasing the falling time which is considerable. After applying perturbation in step order to the system, the controllers will hold the values of upper tanks on zero so that bottom tanks stay in their desired value. Figure 10 shows the produced control signal in this experiment.

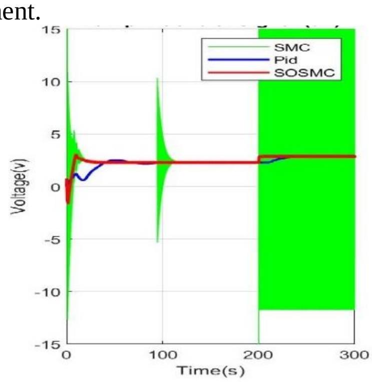

Fig. 10. Control signals in the presence of uncertainty and perturbation of the model

Control signals in this experiment are not that different from step experiments in the presence of perturbation (from existing chattering in SMC control signal to SOSMC control signal behavior). Only the PI controller produced the control signal with more oscillation compared to the previous experiment which was due to changing the model parameters.

5. Conclusion

A quadruple tank is a laboratory system that has features of many multi-input – multi-output industrial systems that are very wide and sensitive. The development of the controller for these industrial systems is very time-consuming, expensive, and it is full of risk. Therefore the quadruple laboratory system allows the development of control algorithms for advanced processes. Delay, external perturbation, and uncertainty in model parameters are the issues that real systems are faced in industrial environments. The controller that can control the system in real circumstances, is desired by industry and it is the research field for most researchers.

Based on obtained results from fourth chapter simulations, it is clear that the PI controller due to existing delay in the system, cannot use a high proportion and if the gain is high, the behavior of the system becomes oscillation-like. Therefore in real systems that are delayed, the PI controller is not the appropriate candidate to use. Also, the low gain of the PI controller leads to reducing the stability of this controller in the time of facing external perturbation and uncertainty in the model. This is while the controllers based on sliding mode, because of being designed using delay compensator, do not have the mentioned disadvantages of PI controller and they have a good gain. Meanwhile, the classic sliding mode controller (SMC) has chattering in control signal which is due to the existence of sign function in control signal and time of implementing that. The existence of chattering in the control signal leads to a lot of waste of energy in the pumps which show themselves as heat, and consequently, it leads to reducing the life of the pumps. After chattering, another disadvantage of the classic sliding mode controller is its high gain in the control system, which was presented for solving the disadvantages of a second-order sliding mode controller with a sliding surface. The control signal of this controller lacked chattering and in the perturbation tests and uncertainties in the model, it performed better than PI and SMC controllers.

Based on the results from the fourth chapter, the SOSMC controller while facing external perturbation has very few falls, while the PI controller had domain fall in many percent. This is while the return time of the signal after applying the perturbation in the PI controller was much longer than SMC and SOSMC. In the issue of uncertainty in the model, also, by changing the surface of the tanks to double of the previous case, we observe the mess in system response with PI and SMC controllers, while system response with SOSMC controller did not have a considerable change.

The summary of results for this study is presented in form of qualitative tables 3 and 4. In this table, three terms of "appropriate", "acceptable", and "inappropriate" are used for describing the performance quality for each of these systems. The evaluation based on the ability of the controller in controlling the level of the tank is presented in table 3.

Table.3.
Qualitative table for describing the controller's performance in controlling the tank level

SOSMC	*SMC*	*PI*	*Controllers criteria*
Appropriate	Appropriate	Acceptable	Stabilization
Appropriate	Appropriate	Acceptable	Step response
Appropriate	Acceptable	Inappropriate	Response to perturbation
Appropriate	Acceptable	Inappropriate	Response to uncertainty

Metrics for this evaluation, time of going up and falling of the output signal, and controller ability in the regulation of output signal, were determined according to reference levels. According to this table, sliding mode controllers due to high gain got the output signal to the source point immediately that meanwhile SOSMC controller compared to SMC has less chattering and higher speed. PI controller, due to its intrinsic delay and the existing delay in the system, has less speed and higher oscillation and compared to the other two controllers, it has a weaker performance.

In table 4, a comparison of controllers is done with the view of the produced control signal. In this comparison produced signals of the PI and SMC controllers have very few oscillations which reduces the waste of power in operators and their high mechanical amortization while controlling SMC due to high-frequency oscillations and high domain in the produced control signal, leads to high amortization in operators and producing high waste of power. At the same time, the high frequency of changes in the control signal, makes the selection of operators face serious limitations. Comparing the regulatory ability of the controller and the quality of produced control signal, the high applicability of SOSMC controllers is determined.

Table.4.
Qualitative table for describing the performance of controllers in the produced control signal

SOSMC	*SMC*	*PI*	*Controllers criteria*
Appropriate	Acceptable	Appropriate	Stabilization
Appropriate	Acceptable	Appropriate	Step response
Appropriate	Inappropriate	Appropriate	Response to perturbation
Appropriate	Inappropriate	Appropriate	Response to uncertainty

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