



# Probabilistic GENCOs Bidding Strategy in Restructured Two-Side Auction Power Markets

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## Abstract

As a matter of course, power market uncertainties escalation is by product of power industry restructure on one hand and the unrivalled penetration of renewable energies on the other. Generally, the decision making process in such an uncertain environment faces with different risks. In addition, the performance of real power markets is very close to oligopoly markets, in which, some market players exercise market power to influence the power market and this matter brings some risks to other players. Hence, each market player must consider these market features to choose his best decision. So, in case of such an uncertain environment, GENCO's bidding strategy would be a complicated and error-prone process. This paper aims to ease this issue suggesting the use of probabilistic bidding strategy of generation units by the unscented transformation (UT) method. The proposed method can consider the correlation between variables, where it models the coalition between market participants. Using the proposed methodology, a market participant can choose a desired range of profit; then, set his decision to manage his profit by reducing his risks. Finally, the proposed methodology is examined through some case studies done in a standard test system. Simulation results show that executing market power by some market players disturbs the competition in the market.

Keywords: GENCOs market power, optimal bidding strategy, uncertainty modeling, solar generation, wind generation.

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## Nomenclature

The notation used throughout the text is defined as below; others will be explained as required in the manuscript.

$\alpha(i, t)$	No load cost coefficient of $i$ th thermal unit at time $t$ [\$/h]
$b(i, t)$	Linear cost coefficient of $i$ th thermal unit at time $t$ [\$/MWh]
$c(i, t)$	Quadratic cost coefficient of $i$ th thermal unit at time $t$ [\$/((MW) <sup>2</sup> h)]
$C_W$	Weibull scale parameter [m/sec].
$k$	Index of samples, running from 1 to $n$
$K_W$	Weibull shape parameter.
$n$	The number of uncertain variables
$\mathbf{P}_{ZZ}$	Covariance matrix of the input variables
vector	
$\mathbf{P}_{YY}$	Covariance matrix of output variables
vector	

$P^{\min}(i)$	Minimum generation of $i$ th unit [MW]
$P^{\max}(i)$	Maximum generation of $i$ th unit [MW]
$P_{rs}$	SCG's rated power [MW]
$P_{rw}$	WTG's rated power [MW]
$P_{SCG}$	SCG output power [MW]
$P_{WTG}$	WTG output power [MW]
$R$	Solar radiation [ $W/m^2$ ]
$R_C$	A certain radiation point, usually 150 $W/m^2$
$R_{STD}$	Solar radiation in the standard radiation, usually 1000 $W/m^2$

$v$	Wind speed [m/sec]
$V_i$	Wind turbine cut- in speed [m/sec].
$V_o$	Wind turbine cut- out speed [m/sec].
$V_r$	Wind turbine rated speed [m/sec].
$W^k$	Weight associated with the $k$ th sample point.
$\mathbf{Z}$	Vector of uncertain input variables.
$\bar{\mathbf{Z}}$	Vector of mean value for input variables
$\mathbf{Z}^k$	The $k$ th sample point
$\mathbf{Y}$	Vector of uncertain output variables
$\bar{\mathbf{Y}}$	Vector of mean value for output variables
$\mathbf{Y}^k$	Vector of transformed sample point of $\mathbf{Z}^k$
$\rho_{x,y}$	Correlation coefficient between variables $x$ and $y$
$\alpha_\beta$	Beta shape parameter [kW/m <sup>2</sup> ]
$\beta_\beta$	Beta shape parameter [kW/m <sup>2</sup> ]
$\pi$	Market energy price [\$/MWh]
$\Omega_i$	The profit of GENCO $i$ [\\$]

## 1. Introduction

Restructuring process of power markets in recent years caused the bulk power system operation tend to change from a traditional and vertically integrated system to a competitive one and consequently, highlighting the power market concepts more and more. As a matter of fact, power systems and consequently the power markets are always faced with a variety of uncertainties. The system security, reliability and market efficiency may be influenced by these uncertainties which can deteriorate the system performance. In recent years, as a direct result of environmental issues and especially after the oil shock at 1973 which caused the energy prices to increase dramatically, the attention is more concentrated on renewable energies (REs) exploitation. These energies have an instable behavior and are not expected to have a certain trend in a given time; therefore, have their own uncertainties added to other uncertain system parameters. So, scurrying the power industry reregulation juxtaposed with the unrivalled utilization of uncertain REs faces the power system operation with severe uncertainties [1]. As a result, assessment of system and market behaviors considering these uncertainties is of significant importance. The problem of GENCO's optimal bidding strategy is one of restructured power market problems faced with generation companies. In this problem, GENCO's

must provide their selling bids for the future hours with the knowledge of market behavior, opponents decision, the model of demand, market mechanism, and power system operational conditions. These types of problems are included in the category of game theory problems [2]. Nash equilibrium is the most commonly accepted solution concept in game theory [3]. In order to have a safe profit margin, rational GENCOs bid at Nash equilibrium point. As the profit of these market participants relates to the energy sold and the price of their bids, finding their optimal bidding strategy considering market uncertainties is of significant interest.

The investigation of power system uncertainties was proposed in the early seventies [4], [5]. As yet, many probabilistic methods have been proposed to study the uncertainty in power system problems such as Cumulant method [6]- [8], point estimation method (PEM) [9]-[12], and Unscented Transformation (UT) method [1], [13]-[15], to name a few. In [16], the bidding strategy of GENCOs under the uncertainty of opponents' information is studied. In [17], the fuzzy modeling is used to model the uncertainty of input variables in the bidding strategy. In [18]-[19], the problem of bidding strategy under the uncertainty using probability theory is investigated. However, there is not a comprehensive work on the problem of GENCOs bidding strategy considering different kind of uncertainties and this motivates the researchers in this field.

The main contribution of this work is to propose a framework for probabilistic bidding strategy in deregulated two-side auction markets using the Unscented Transformation (UT) method considering different kind of uncertainties. As the coalition between market players is probable in power markets; the coalition modeling and assessing its effects on the market behavior is of significant importance. As another contribution, it is proposed to model the coalition between market participants using the concept of correlation between uncertain decision variables of the market players. Using the proposed method of this work, each market player i.e. GENCOs or system loads, can define their profit margins and the method determines their decision variable margins to ensure the expected profit margin.

The rest of the paper is organized as follows. Section II formulates the GENCOs bidding strategy problem and discusses about the solution of this problem. The game theory and finding Nash equilibrium point through particle swarm optimization (PSO) method is discussed. In Section III, the procedure of uncertainty modeling in the bidding strategy is given. Section IV introduces the UT method and its formulation. In Section V, the probabilistic framework for GENCOs optimal bidding strategy and its flowchart is proposed. Section VI

describes the case studies, thereafter; the obtained results are presented for each case study.

## 2. GENCOs Bidding Strategy

### A) Bidding Strategy

In deregulated two side auction power markets, the generation units and loads participate in the market to sell/buy energy and other market commodities. In such a system, the generation units prepare their offers in an increasing order of price and the loads prepare their bids in a decreasing order to the independent system operator (ISO). Then, the market clearing price (MCP) is obtained by sorting and the intersection of the aggregated supply and demand curves. Once the energy market is cleared, each GENCO will be paid according to pricing mechanism of the market. Generally, there are two pricing mechanisms: uniform and pay-as-bid [20]. In a power market, GENCOs may prepare their strategic bids according to the four known models in imperfect competition included are Cournot, Bertrand, Stackelberg, and Supply Function Equilibrium (SFE) [3]. In the SFE model, GENCOs compete through the simultaneous choice of supply functions. In SFE model, GENCOs are able to link their bidding price with the bidding quantity of their product. This model is close to the actual behavior of players in the actual power market [2].

The cost function of a generation unit can be modeled as a quadratic function relating the operation cost to the amount that is generated.

$$C_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad (1)$$

In a perfect competitive energy market, with many number of generation units, GENCOs try to bid their generations according to their marginal cost as:

$$\rho_i = MC_i = 2a_i P_i + b_i \quad (2)$$

In the real energy market with limited number of players, oligopoly is the base of energy market performance. In such a market, some GENCOs act as price makers and the others as price takers. A player will behave as a price taker in the market if it bid at its marginal cost. If all of GENCOs bid according to their marginal cost, the market power will not produce. So each price maker player tries to increase its profit using its market power opportunity. In such a market, GENCOs use their mark-up marginal cost to construct their bids as (3):

$$\rho_i = k_i MC_i = k_i (2a_i P_i + b_i) \quad (3)$$

$$k_i \geq 1$$

Where,  $k_i$  is the bidding strategy of  $i$ th GENCO. In such an oligopoly market, GENCOs try to maximize their profit by choosing their best bidding strategy.

$$\Omega_i = R_i - C_i(P_i) = \pi P_i - C_i(P_i) \quad (4)$$

$$\begin{aligned} \text{Max } \Omega_i &= \text{Max } [R_i - C_i(P_i)] = \\ \text{Max } [\pi P_i - C_i(P_i)] \end{aligned} \quad (5)$$

In a pool-co system, the minimum and maximum limits of generation for all units must be considered.

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad (6)$$

In an imperfect power market, the MCP is influenced by the market players' decisions. So, the profit of each GENCO depends on his decision  $k_i$  and decision of his opponents  $k_{-i}$ . This matter can be stated as:

$$\Omega_i = \Omega_i(X_i, X_{-i}) \quad (7)$$

$$\Omega_i = \Omega_i(k_i, k_{-i}) \quad (8)$$

Note that as the profit of each GENCO depends on its production and the market price, the maximum profit can be obtained when

$$\frac{d\Omega_i}{dk_i} = 0 \quad (9)$$

So, finding the solution of this problem would be a multi-player optimization problem. Game theory is one of the approaches that may be used for this purpose.

### B) Game Theory and Nash Equilibrium

Generally, the multi-player decision making problem is studied by game theory [2]. Usually, three main mathematical forms are used in the study of games included are the strategic form, the extensive form, and the coalitional form [3]. They are different in the amount of detail on the play of the game. The behavior of electricity markets is near to strategic form [3]. Nash equilibrium is the most commonly accepted problem solution of the game theory. A strategy has the characteristics of Nash equilibrium for a player if that player cannot increase its own payoff by choosing any strategy other than its equilibrium strategy, given the strategy choice of its rivals. In this case, each player will decrease its payoff if it deviates from its Nash equilibrium strategy, assuming all other players continue to play their existing strategies. As a result, a Nash equilibrium point is the "best response", which means that no player has an incentive to change its strategy choice, given all other player's strategy choices. As mentioned, the problem of finding a Nash equilibrium point can be formulated as a problem of finding the global optimum of a real valued function. In this work, the particle swarm optimization method is used to find this point.

### C) Particle Swarm Optimization (PSO) Method for Optimal Bidding

Particle swarm optimization (PSO) is a swarm based evolutionary algorithm [21]. It can reliably and accurately solve very complex constrained optimization problems [21]-[24]. In this method, each particle which is a potential solution moves in multi-dimensional problem space with a given velocity. Each particle updates its velocity according to its flying experiences and the others. The  $i$ th particle in swarm at iteration  $k$  has a position represented by a  $d$ -dimensional vector such as (10). Its velocity is calculated from (12); where  $V_d(i, k)$  is the velocity of particle  $i$  in the  $d$ th dimension. The best position of particle  $i$  obtained until iteration  $k$  is named as particle best (PB) that represented by  $PB(i, j, t, k-1)$ . The best previous position among all the particles in iteration  $k$  is recorded and called global best (GB) that represented by  $GB(j, t, k-1)$ . Particles' position is updated by (13).

$$X(i, k) = [x_1(i, k), x_2(i, k), \dots, x_d(i, k)] \quad (10)$$

$$V(i, k) = [v_1(i, k), v_2(i, k), \dots, v_d(i, k)] \quad (11)$$

$$V(i, j, t, k) = w \times v(i, j, t, k-1) + c_1 \times rand_1 \times [PB(i, j, t, k-1) - x(i, j, t, k-1)] + c_2 \times rand_2 \times [GB(j, t, k-1) - x(i, j, t, k-1)] \quad (12)$$

$$x(i, j, t, k) = x(i, j, t, k-1) + v(i, j, t, k) \quad (13)$$

It must be noted that in (12),  $v(i, j, t, k-1)$  is the particle's current velocity and the second term indicates the cognitive part of PSO in which the given particle updates its velocity based on its own experiences. The social part of PSO is given by the third part in which the particle uses the experiences of other particles to update its velocity. Specific weights are devoted to each term. Note that  $i$  indicates particle number,  $j$  represents generating unit/vehicle,  $t$  is the time and  $k$  shows the iteration number.

### 3. Uncertain Parameters and Uncertainty Modeling

As formerly mentioned, optimal bidding strategy is an optimization problem with some constraints. In an optimization problem, the lagrangian multipliers stand for lack of resources; so, they change as the right hand side of constraints vary. MCP in the power market is the incremental operation cost of the system due to increase of the demand by 1MWh. The locational marginal prices (LMPs) of power systems with competitive market form comprises from different components each standing for marginal energy price, marginal loss price, and marginal congestion price [25]. In case of restructured

oligopoly market, there would be an extra term affecting the MCP, named as the bidding strategy of market participants. So, with the variation of lagrangian multipliers associated with these parts, the MCP would be variable. In fact, the existence of some uncertainties such as the estimated load, generation, system element conditions, and the bidding strategy of market participants, the right hand sides of the problem constraints vary and consequently, the value of MCP changes. In the following, the considered uncertainties in this work and their modeling are presented.

### D) Uncertain Parameters

With the introduction of power markets' restructuring concepts, the process of energy generation has experienced huge revolutions from centralized governmental ownership to distributed private form on one hand and from traditional thermal units to modern renewable based ones on the other hand. In such markets, market behavior factors are highlighted more and more. Therefore, the momentousness of optimal bidding strategy for power generation companies has been glanced considerably. Generally, optimal bidding strategy is known as a deterministic market problem with fixed model parameters and input variables. However, many uncertain factors may exist within such uncertain power market operation due to system parameter forecasting errors or system elements outage as well as price spikes. These uncertainties introduce errors in the optimal decisions when deterministic data are used; therefore, probabilistic decision making must be considered. In fact, power markets are faced with variety of uncertainties but we mostly focus on the load, wind, and solar power generation uncertainties as system uncertainties and the bidding strategy of energy sellers/buyers as market participants' uncertainties.

### E) Uncertainty Modeling

Certainly, the load as the most challenging uncertain variable fluctuates as a function of time, weather conditions, and electricity price among the rest. In the literature review, it is commonly used to model the load uncertainty through the normal probability density function (PDF) with specific mean and STD values obtained from historical data [25], [26]. In this study, the load is divided to price sensitive (interruptible) and insensitive (fixed) load. The fixed load is modeled through a normal distribution function with  $\mu$ , the mean value equal to the base load, and  $\sigma$ , the STD equal to  $\pm 5\%$  of its mean. For the price sensitive loads, the price bids are modeled through normal distribution.

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \quad (14)$$

In our study, some buses are assumed to have integrated wind farms which have uncertain output powers. Wind speed varies both in time and location and its PDF is modeled by weibull PDF in the respected literature [27], [28]. So, the wind speed is modeled with the weibull distribution function. The modeling of uncertainty for wind power generation in our problem is summarized as follows:

**Step1:** The wind speed is modeled by an appropriate PDF such as weibull. The weibull distribution function is represented by (15).

$$f(v) = \frac{K_w}{C_w} \left(\frac{v}{C_w}\right)^{K_w-1} \exp\left[-\left(\frac{v}{C_w}\right)^{K_w}\right] \quad (15)$$

**Step2:** In order to model the uncertainties, the problem is evaluated several times to cover at least the most important or probable conditions [15]. In order to model the WTG's output power uncertainty, the wind speed samples are generated in each evaluation by an appropriate manner.

**Step3:** The wind speed-power curve is used to convert wind speed samples to wind turbine output power.

$$P_{WTG} = \begin{cases} 0 & \text{if } v \leq V_i \text{ or } v \geq V_o \\ P_{rw} \frac{v - V_i}{V_r - V_i} & \text{if } V_i < v < V_r \\ P_{rw} & \text{if } V_r \leq v < V_o \end{cases} \quad (16)$$

**Step4:** When the wind power generation is calculated for each iteration, the next step would be the uncertainty modeling of offered price for this power that is done using normal distribution.

The solar power generation is the other uncertain variable. It has a high degree of uncertainty. It varies as a function of several factors such as environmental conditions, time of day, month, season, and orientation of the solar cell generator (SCG) to the sun radiation and so on. The solar radiation PDF is modeled by beta distribution function [28], as the following:

$$f(R; \alpha_\beta, \beta_\beta) = \frac{\Gamma(\alpha_\beta + \beta_\beta)}{\Gamma(\alpha_\beta)\Gamma(\beta_\beta)} R^{\alpha_\beta-1} (1-R)^{\beta_\beta} \quad (17)$$

SCG's output power is related to the solar radiation; therefore, its output power modeling requires the solar radiation modeling. The SCG's output power as a function of radiation is stated as radiation-power curve [28]:

$$P_{SCG} = \begin{cases} P_{rs} \left(\frac{R^2}{R_{STD} R_C}\right) & \text{if } 0 \leq R < R_C \\ P_{rs} \frac{R}{R_{STD}} & \text{if } R_C \leq R < R_{STD} \\ P_{rs} & \text{if } R_{STD} \leq R \end{cases} \quad (18)$$

The procedure of uncertainty modeling for solar generation would be the same as that of the wind generation. Stress again that the uncertainty of bidding strategy for market participants (offer and bid prices) can be modeled using the normal distribution

function. So, the price of offers for thermal, wind, and solar units as well as the price of bids for interruptible loads can be modeled using normal PDF.

#### 4. Unscented Transformation (UT) Method

The UT is a probabilistic analysis method that has been developed to remove the drawbacks associated with linearization-based probabilistic methods. It is very simple and easy to code. Till now, several extensions of the method have been implemented to different uncertain problems and have shown exceptional performances [29]. The UT is a reliable method to calculate the statistics of output random variables undergoing a set of nonlinear transformations [29]. It is based on the fact that it is easier to approximate a probability distribution than an arbitrary nonlinear transformation function [30]. It easily produces appropriate samples of input variables to maintain sufficient information about the input variable's PDF. Assume that  $\mathbf{Z}$  is a vector of  $n$ -dimensional input random variables with mean  $\bar{\mathbf{Z}} = \mathbf{m}$ , and covariance matrix  $\mathbf{P}_{ZZ}$ . Assume the other random variable  $\mathbf{Y}$  relates to the input variable through a nonlinear function as (19).

$$\mathbf{Y} = f(\mathbf{Z}) \quad (19)$$

Where, both  $\mathbf{Z}$  and  $\mathbf{Y}$  are random variables and  $f$  can be a set of nonlinear transformation functions. With the UT method, the mean and covariance of the output variable  $\bar{\mathbf{Y}}$  and  $\mathbf{P}_{YY}$  can be obtained through the following steps [29].

**Step 1:** Obtain  $2n+1$  sample points using (20) to (22).

$$\mathbf{Z}^0 = \mathbf{m} \quad (20)$$

$$\mathbf{Z}^k = \mathbf{m} + \left(\sqrt{\frac{n}{1-W^0} \mathbf{P}_{ZZ}}\right)_k, \quad k = 1, 2, \dots, n \quad (21)$$

$$\mathbf{Z}^{k+n} = \mathbf{m} - \left(\sqrt{\frac{n}{1-W^0} \mathbf{P}_{ZZ}}\right)_k, \quad k = 1, 2, \dots, n \quad (22)$$

**Step 2:** The associated weight of each  $\mathbf{Z}$  is calculated using (23) to (25) .

$$W^0 = W^0 \quad (23)$$

$$W^k = \frac{1-W^0}{2n}, \quad k = 1, 2, \dots, n \quad (24)$$

$$W^{k+n} = \frac{1-W^0}{2n}, \quad k+n = n+1, \dots, 2n \quad (25)$$

It must be noted that the sum of associated weights must be equal to 1.

$$\sum_{k=0}^{2n} W^k = 1 \quad (26)$$

Note that  $(\sqrt{\frac{n}{1-W^0}} \mathbf{P}_{ZZ})_k$  is the  $k^{\text{th}}$  row or column of matrix square root  $(\frac{n}{1-W^0}) \mathbf{P}_{ZZ}$ . The matrix square root of positive definite matrix  $\mathbf{P}$  means that there is a matrix  $\mathbf{A} = \sqrt{\mathbf{P}}$  such that  $\mathbf{P} = \mathbf{A}\mathbf{A}^T$ . Numerically efficient and stable methods such as the Cholesky decomposition must be used to calculate the matrix square root [29]. Here,  $w^0$  is the weight associated with the point  $\bar{\mathbf{Z}} = \mathbf{m}$ , named zeroth point. It controls the location of other points around the mean value of  $\mathbf{Z}$  [29].

**Step 3:** Each sample point is fed to the nonlinear function to obtain a set of transformed sample points as:

$$\mathbf{y}^k = f(\mathbf{Z}^k) \quad (27)$$

In this method, the nonlinear transformation function is considered as a black box; therefore, no simplification or linearization is required. This issue is shown in Fig. 1.

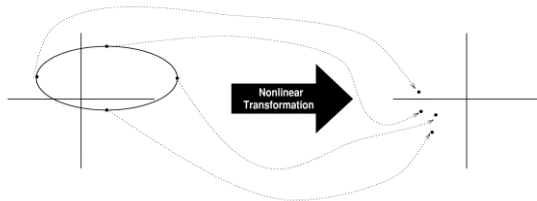


Fig. 1. The principle of the UT method [29]

**Step 4:** The mean and covariance of output variable  $\mathbf{Y}$  is calculated using (28) and (29), respectively.

$$\bar{\mathbf{Y}} = \sum_{k=0}^{2n} \mathbf{w}^k \mathbf{Y}^k \quad (28)$$

$$\mathbf{P}_{YY} = \sum_{k=0}^{2n} \mathbf{w}^k (\mathbf{Y}^k - \bar{\mathbf{Y}})(\mathbf{Y}^k - \bar{\mathbf{Y}})^T \quad (29)$$

The UT method has two special properties that make it easy and operative. Firstly, the sample points are not selected randomly; they are so chosen to have a predefined mean and covariance. Secondly, the associated weights on the selected points do not have to be in the range [0, 1]; they may have positive or negative values but they must meet the situation of (26) [29]. For more details and extensions of the UT method, interested reader is referred to [29].

## 5. Proposed Probabilistic Bidding strategy Method

In order to execute the probabilistic bidding strategy of generation units in a restructured two sided auction market, we propose the following steps:

- Define the market players and list the uncertain parameters that can be modeled using probability

theory. In this study, we consider the following players:

- Thermal units: their offering price is unknown and is modeled by normal distribution
- Wind and solar units: their generation and offering prices are unknown. Wind and solar power generation uncertainties are modeled by weibull and beta distributions, respectively. Their offering prices are modeled by normal distribution.
- Fixed (price insensitive) loads: the demand of these variables is unknown and their uncertainty can be modeled by normal distribution.
- Interruptible (price sensitive) loads: their bid prices are unknown and are modeled by normal distribution.
- Define the probabilistic method and the optimization method to find the Nash equilibrium point and use the simplified flowchart depicted in Fig. 2 Note that the UT method is used in this study as the probabilistic method.
- Select the appropriate sampling of uncertain variables by the probabilistic method using equations (20) - (22).
- For each selected sample of the method, find the Nash equilibrium point and save the results. Note that finding the Nash equilibrium point needs an optimization process which has some inner loops and is not depicted in this figure in order to avoid the crowded figure. The PSO method is used as the optimization technique to find the Nash equilibrium point of each sample point of the UT method.
- Obtain the statistical data of decision variables by probabilistic method using equations (28), (29).

## 6. Case Studies and Discussion

In this section, the proposed methodology is examined in the IEEE 9-Bus test system. This system has three generation units in buses 1, 2, and 3 with the maximum generation capacity of 250, 80, and 80 MW, respectively [31]. There are 3 loads in buses 5, 7, and 9 of the system. The loads in buses 5, 7 participate in the market by submitting their bids to the ISO and the load in bus 9 is not price sensitive and assumed to be an aggregated unit of household consumers. It is also assumed that a wind farm with an installed capacity of 15 MW and wind speed weibull scale and shape parameters equal to 8 [m/sec.] and 3 is located at bus 7. Let assume that a solar farm with an installed capacity of 10 MW and solar radiation beta shape parameters equal to 0.5 [kw/m<sup>2</sup>] and 0.3 [kw/m<sup>2</sup>] is located at bus 8. Here, unit 1 is selected to be probabilistically studied on the

market and that chooses its optimal bidding strategy associated with uncertainties listed in the following:

- Generation unit in bus 2: its offering price modeled by normal PDF
- Generation unit in bus 3: its offering price modeled by normal PDF
- Wind unit in bus 7: its generated power modeled by weibul PDF of wind speed
- Wind unit in bus 7: its offering price modeled by normal PDF
- Solar unit in bus 8: its generated power- modeled by beta PDF of solar radiation
- Loads in buses 5, 7: their bidding prices- modeled by normal PDF
- Load in bus 9: its power consumption- modeled by normal PDF

**Base Case**

Assume that GENCO 1 estimates the generator offers and price sensitive load bids mean values according to Table 1.

Note that STDs of all normally distributed variables are considered to be %5 of their mean values. The market clearing mechanism is based on pay as bid and the bidding strategy of market players and their obtained profit are considered as output variables. Performing the probabilistic market study by unit 1 using the proposed methodology, gives the results outlined in Table 3.

For the sake of simplicity and without loss of generality, it is assumed that wind and solar units construct their offers as %40, %30, and %30 of their power generation with mean offer prices of Table 2. The mean value of load at bus 9 is assumed to be 125 MW. High mean and low STD value of the bidding strategy show that the benefit of unit 1 from market power is very large. Note that due to scarce of generation in the system, two other units may benefit from such condition tend to increase their bids. In order to confirm this reality, one may study the market under a more competitive condition by increasing the generation capacity of units 2 and 3.

**Case 1: More Competitive condition**

Assume that the market in the base case has been more competitive by doubling the generation capacity of units 2 and 3. Note that the generation offers for units 2 and 3 in Table 1 has been doubled with the same prices, where the results of probabilistic market studies are outlined in Table 4.

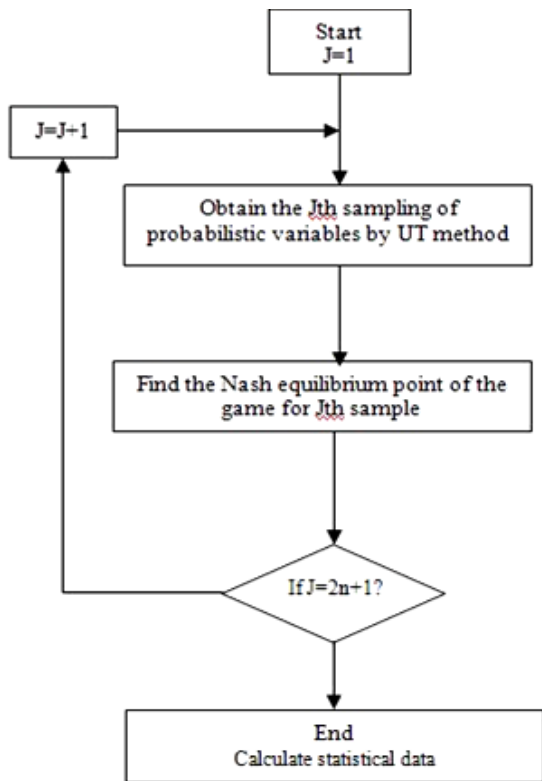


Fig. 2. The principle of the UT method [29]

Table.1. Generator Offers and Load Bids

Generator/ load	Block1	Block 2	Block3
	MW@\$/MWh	MW@\$/MWh	MW@\$/MWh
Unit@1	50@13	80@15.5	120@19.5
Unit@2	10@16	30@17	40 @18
Unit@3	10 @16	30 @18	40 @19
Load@5	30@80	30@60	30@40
Load@7	10@100	5@80	15@60

Table.2. Wind and Solar Generator Offers According to Their Generation

Generator	Block1	Block 2	Block3
	% of Gen. @ \$/MWh	% of Gen. @ \$/MWh	% of Gen. @ \$/MWh
Wind Unit	%40@12	%30@13	%30@15
Solar Unit	%40@12	%30@14	%30@16

Table.3. Generation Units' Statistical Data- Base Case

Unit	Bidding Strategy (Ki)		Profit [1e3\$]	
	Mean	STD	Mean	STD
Unit@1	4.7649	0.0645	2.304	0.0705
Unit@2	3.9038	3.4875	3.7353	2.1954
Unit@3	3.6299	2.966	4.3128	4.2666
Unit@7	4.661	3.7939	0.2506	0.2285
Unit@8	4.3813	3.5667	0.3556	0.3187

Table.4. Generation Units' Statistical Data- Case 1

Unit	Bidding Strategy		Profit [1e3\$]	
	Mean	STD	Mean	STD
Unit@1	1.673	0.1624	0.4122	0.1055
Unit@2	1.2783	0.4689	1.0756	0.6698
Unit@3	1.3011	0.3820	0.9892	0.8369
Wind@7	1.4695	0.6927	0.0673	0.0442
Solar@8	1.3857	0.6419	0.0992	0.0674

Comparing the results of Tables 3 and 4 reveals that the market has been more competitive as the mean values of bidding strategy for all GENCOs are close to each other and the STD for GENCO 1 is increased while for the others are decreased. It means that the decision risk for GENCO 1 is higher than the base case, where it may lose its market share under improper decision. As another observation, the profit of GENCO 1 has been decreased drastically.

In Fig. 3, the inverse cumulative distribution function (CDF) of profit and bidding strategy for GENCO 1 that helps to have the same horizontal axis is shown. Having Fig. 3, GENCO 1 can consider a desired range of profit and optimally choose the best range of decision variable according to the range of desired profit. The probability of achieving the desired profit can be obtained by subtracting the probability of lower bound profit from the upper one. For an instance, if GENCO 1 targeting a profit between 0.2 and 0.5 [1e3\$], his bidding strategy i.e. K1, is between 1.346 and 1.81, where the probability of profit would be 0.7795.

#### Case 2: Market Players potential Coalition Modeling

As a well-known fact, the coalition is not an impossible action in modern power markets which usually are oligopolistic. So, the modeling of power markets considering the coalition and its effects on the market behavior is of significant interest. When there is a potential coalition, the decision making in such a power market would be a difficult and error-prone process if the coalition is not modeled. So, the market simulation methods that can model this effect are very appealing. Here, we propose a potential coalition model using the concept of correlation between their decision variables. When the variables are mutually correlated, the variation of one variable affects the others. Generally, this matter is stated through the covariance matrix the so-called correlation coefficient matrix. The correlation coefficients can be obtained through (30).

$$\rho_{x,y} = \frac{\text{cov}(x,y)}{\sigma_x \sigma_y} = \frac{E[(x-\mu_x)(y-\mu_y)]}{\sigma_x \sigma_y} \quad (30)$$

$x, y = 1, 2, \dots, n.$

Assume that GENCO 1 wants to model the effects of coalition between GENCOs 2 and 3 on the market and his profit. Let the correlation coefficient between the offered price of these GENCOs to be +0.6. In this case, the results of probabilistic market studies are listed in Table 5. It can be seen that sum of mean profit for GENCOs 2 and 3 has been increased and their STD for profit has been decreased indicating potential market power. Fig. 4 portrays the inverse CDF of profit and bidding strategy for GENCO 1 in this case.

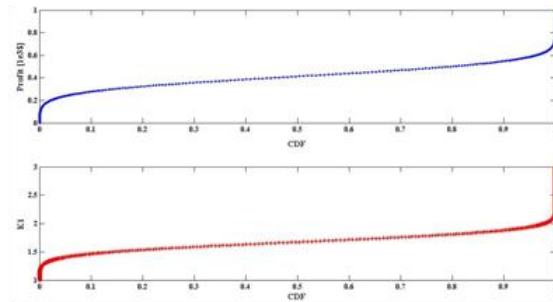


Fig. 3. Inverse CDF of profit and bidding strategy for GENCO 1- Base Case

Table.5.  
Generation Units' Statistical Data- Case 2

Unit	Bidding Strategy		Profit [1e3\$]	
	Mean	STD	Mean	STD
Unit@1	1.6781	0.167	0.4154	0.1084
Unit@2	1.3531	0.4477	1.1849	0.6334
Unit@3	1.3377	0.4561	1.0179	0.7268
Wind@7	1.7293	0.5505	0.094	0.04101
Solar@8	1.6132	0.5118	0.1246	0.04821

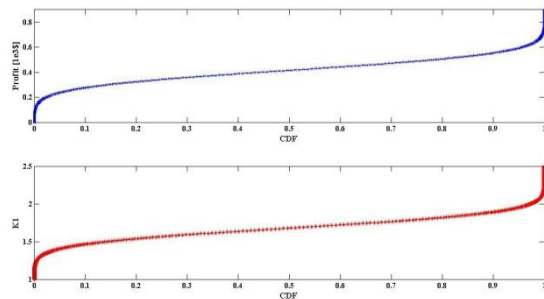


Fig. 4. Inverse CDF of profit and bidding strategy for GENCO 1- Case 2

## 7. Conclusions

Hastening the power industries restructure on one hand and increasing the penetration of renewable energies in the power system on the other hand, intensify the power market uncertainties. Generally, the decision making process in such an uncertain environment faces with different risks. In addition, the performance of real power markets is very close to oligopoly markets, in which, some market players exercise market power to influence the power market and this matter brings some risks to other players. Hence, each market player must consider these market features to choose his best decision. In this paper, we proposed a methodology for probabilistic bidding strategy for GENCOs in two-sided auction power markets. The Unscented Transformation (UT) method is proposed for this purpose. The proposed method can consider the correlation between variables where it models the coalition between market participants. The proposed methodology is examined through case studies done in a 9-Bus test system.



Using the proposed methodology, a market participant can choose a desired range of profit; then, set his decision to manage his profit by reducing his risks. Simulation results show that executing market power by some market players disturbs the competition in the market. The results also indicate that potential coalition between market players may increase their market power and consequently increasing their profit.

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