

# **Robust Model for Networked Control System with Packet** Loss

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#### **Abstract**

The Networked Control System in modern control widely uses to decrease the implementation cost and increasing the performance. NCS in addition to its advantages is inevitable. Nevertheless they suffer of some limitations and deficiencies. Packet loss is one of the main limitations which affect the control system in different conditions and finally may lead to system instability. For this reason, it is important to model the system properly. In this paper, a new model has been proposed that is very simple and independent from the main system. This model based on Robust Theory structure. Robust theory is a branch of control theory that explicitly deals with uncertainty in its approach to controller design. Robust control methods are designed to function properly so long as uncertain parameters or disturbances are within some (typically compact) set.

Keywords: Network Control System (NCS), Packet Loss, Robust Theory.

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# 1. Introduction

Control systems based on control networks are widely applied in the modern industrial control in attention to the extension of the control system scale and fast development of network technologies, which have advantages of less wires, high reliability, good flexibility and sharing of information resources, etc. However, this system has some phenomenon like time delay, multi-packet transmission and data dropouts. The inherent characteristics of network such as limited bandwidth, carrying capacity and service ability make them [1].

The wireless communications technological advances and the fabrication of inexpensive embedded electronic devices, are creating a new paradigm where a large number of systems are interconnected, thus providing an unprecedented opportunity for totally new distributed control

applications, commonly referred as networked control system [2].

The time delay and data dropouts are the main phenomenon and often exist at the same time in the NCS. However, a study considering the two problems at the same time is scarce. Compared with conventional point-to-point control systems NCSs have the advantages of low installation cost, reduced wiring, easy maintenance and diagnosis, and so on. Examples of NCSs are available in manufacturing systems, intelligent vehicle highway systems, teleoperation of robots, aircraft systems, etc.[2,3]

## 2. Network System

A Networked Control System (NCS) is a control system wherein the control loops are closed through a real-time network. The defining feature of an NCS is that control and feedback signals are exchanged among the system's components in the form of information packages through a network. A network system can divide into layers. Easy to manage the system and divide tasks to special layers are the benefits of system layering. So packet loss related to special layers and attention to this layer can lead to propose an acceptable model for packet loss. Network system contains sensor, actuator, controller and network link [3].

All sensors and actuators can behave in two states. Clock driven and event driven are two main states of elements behavior. Time driven elements are working with system sample time. But Event driven element wait for signal to act. At most NCS, sensors and actuators act clock driven because packet loss in these systems can cause a big fault for event driven elements [4, 10].

# 3. NCS phenomenon

The Network Control System has some phenomenon like timeout, packet loss and packet disordering. One of the main phenomena is packet loss because of losing data that can have important information.

#### 3.1. Packet Loss

One of the most common problems in networked control systems, especially in wireless sensor networks, is packet loss, i.e. If the controller is placed in a remote location and is not co-located with the sensors and the actuators, then both sensor measurement packets and control packets can be lost due to communication noise, interference, or congestion [1].

In the past decade, much attention has been paid to the impact of the delayed data packets of NCSs.

In attention to the critical real-time requirement in control systems, the transmission packet dropouts are the potential source of instability and poor performance in NCSs. Therefore the impact of packet dropouts is an important aspect in the analysis and synthesis of NCSs and this issue has received wide attention recently [3].

# 3.2. Packet loss models

In general, in most of the literature two different strategies are considered for dealing with packet drops. In the first one, which we refer as zero-input, when the control packet from the controller to the actuator is lost, the actuator input to the plant is set to zero, while in the second, which we refer as hold-input, when a packet is lost, the latest control input stored in the actuator buffer is used. These are not the only strategies that can be adopted. In fact, if smart actuators are available, i.e. if actuators are

provided with computational resources, then the whole controller or a compensation filter can be placed on the actuator [1]. The model predictive controller is another strategy. It sends not only the current input but also a finite window of future control inputs into a single packet so that if a packet is lost the actuator can pop up from its buffer the corresponding predicted input from the latest received packet [4].

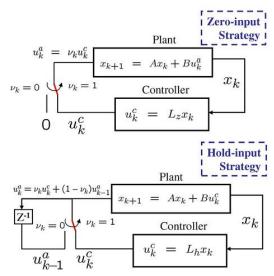


Fig.2. Compensation approaches for actuators with no computational resources when a control packet is lost: zero-input approach (top) and hold-input approach (bottom) [1].

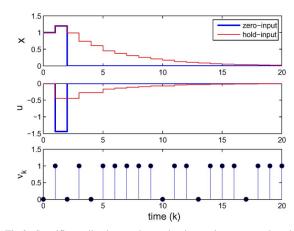


Fig.3. Specific realization under optimal zero-input control and optimal hold-input control [1].

NCS with a short time delay can be modeled as a linear switching model with certain switching rules and the periodic dynamic output feedback controller [2]. The packet transmission process is stochastic and satisfies the Markov characteristic because the packet transmission sequence of sensor nodes is non-deterministic. Fig.1 present two type of system that in Fig.2 their responsibility are compared to random lossy input.

In attention to packet loss can exist in two ways, sensor to controller and controller to the actuators, the best model is a system that model loss in two way independently.

## 3-3 Mathematic lost models

General type of NCS models assumes that the packet dropouts follow certain probability distributions with packet dropouts via stochastic models, such as Markov chains or binary switching sequences. The Bernoulli binary distributed white sequence takes the values of 0 and 1 with certain probability. Recently, there have been some control methods presented on such a model [4]. Another stochastic model approach is to view the packet dropouts as the Markov chains to represent random packet dropout models for the NCSs, a few results have been developed recently [5-7].

"A Markov chain (discrete-time Markov chain or DTMC) named after Andrei Markov, is a mathematical system that undergoes transitions from one state to another, between a finite or countable number of possible states. It is a random process usually characterized as memoryless the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of "memorylessness" is called the Markov property. Markov chains have many applications as statistical models of real-world processes [8-10]. "In probability and statistics, a Bernoulli process is a finite or infinite sequence of binary random variables, so it is a discrete-time stochastic process that takes only two values canonically 0.

timite or infinite sequence of binary random variables, so it is a discrete-time stochastic process that takes only two values, canonically 0 and 1. The component Bernoulli variables  $X_i$  are identical and independent. Prosaically, a Bernoulli process is a repeated coin flipping, possibly with an unfair coin (but with consistent unfairness). Every variable  $X_i$  in the sequence is associated with a Bernoulli trial or experiment. They all have the same Bernoulli distribution. Much of what can be said about the Bernoulli process can also be generalized to more than two outcomes (such as the process for a six-sided die); this generalization is known as the Bernoulli scheme."

Bernoulli binary is a good method for loss modeling because the loss occurs according to bandwidth limitation or small packet size.

### 4. Packet Loss Models

There are various models to describe packet loss. Some models have been proposed for their simplicity. These models describe the packet loss in different methods and the best method will be selected for its simplicity, independence and structure.

#### *4.1 Additive uncertainty*

In attention to the packet loss effect on system input, additive uncertainty is a good model for this phenomenon affect. This model of uncertainty is a usual model of unstructured uncertainty family. The real parametric uncertainty is utilized if the structure of the system is known but its actual physical parameters are not. On the contrary, unstructured uncertainty does not require even knowledge of the structure (order) of the model. Parametric uncertainty is defined through intervals which the imprecisely known parameters lie within. The unstructured uncertainty description is based on the restriction of the area of the possible appearance of frequency characteristics [11]. In the framework of unstructured uncertainties, we only know that  $\Delta$  belongs to set

$$\sum_{\Delta} = \left\{ \Delta | \bar{\sigma} \big( \Delta(j\omega) \big) < 1 \forall \omega \right\} \tag{1}$$

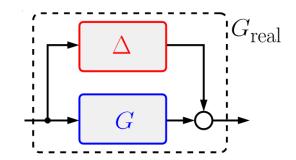


Fig.4. Additive Uncertainty Block Diagram

The System is described exactly by  $G_{real}$ , and G is nominal model. So System model can described by (2).

$$G_{real} = G + \Delta \tag{2}$$

Infinity norm needed for using this method to model packet loss. (3) is used for calculating error.

$$Y - \Delta = error \Rightarrow \|\Delta\|_{\infty} = \|Y - error\|_{\infty}$$
 (3)

# 4-2. Uncertainty in state space

System uncertainty exists in system input and output or exists in system dynamics. In attention to the packet loss effect in Input and output, dynamic uncertainty can neglect.[12] So system state space equation equal (4).

$$x(k+1) = Ax(k) + Bu(k)$$
  
 $x(k+1) = Ax(k) + Bu(k-1)$  (4)

So by defining a variable in format of vector in the system, Packet loss effect will be defined. The equations will be:

$$X(k+1) = AX + BU$$

$$U = \alpha U + \beta U(k-1)$$

$$\propto = \bar{\beta}$$

$$\begin{bmatrix} X(k+1) \\ U(k) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} X(k+1) \\ U(k-1) \end{bmatrix} + \begin{bmatrix} B \\ \alpha \end{bmatrix} U(k)$$
(5)

# 4.3 Multiplicative Input Uncertainty

Another form of unstructured uncertainty is multiplicative uncertainty. In this method  $\Delta$  is parallel with input line and manipulate that. Fig.5 represents the block diagram of multiplicative input uncertainty [12].

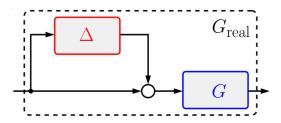


Fig.5. Multiplicative Input Uncertainty Block Diagram

So system model can describe by (6)
$$G_{real} = G(I + \Delta)$$
 (6)

#### 5. The Best Candidate Model for Packet Loss

In attention to various models of uncertainty, it seems that the best candidate to model Packet loss is multiplicative input uncertainty. With a simple binary vector manipulate input, packet loss can be modelled. To investigate this idea, a system has been proposed that based on many assumptions. These assumptions are memorial system with clock driven sensors and actuators and the Controller continuously processes with proper step time. The Block diagram of networked control system has been represented in Fig.5.

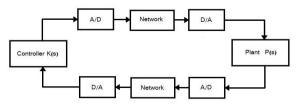


Fig.6. NCS Block Diagram

Analog to digital converter process can model with zero order hold conversion. ZOH transfer function represented in (7)

$$H_n(s) = \frac{1 - e^{-s(nT_0)}}{s(nT_0)} \tag{7}$$

 $T_0$  is system sample time and n is an uncertain number between 0 and  $N_{max}$ . Packet loss occurs in random time so  $t=n(t)T_0$ . Indeed if  $h_{min}< h(t)< h_{max}$  the system will be stable.

At last Packet Loss transfer function obtain independent from plant and with multiplicative input uncertainty unstructured. When there is no packet loss G=1. Indeed G is a family of models.

$$G(s) = 1 + W_m(s)\Delta(s)$$
(8)

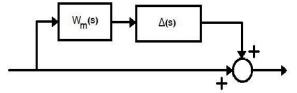


Fig.7. Packet Loss Model

 $W_{\rm m}$  is a weight function that results to  $\Delta$  infinity norm less than one. (9) describe appropriate family for  $H_{\rm n}$ .

$$1 + W_{m}(s)\Delta(s) = H_{n}(s) = \frac{1 - e^{-s(nT_{0})}}{s(nT_{0})}$$

$$\Rightarrow \Delta(s) = \frac{1}{W_{m}(s)} \times \left[ \frac{1 - e^{-s(nT_{0})}}{s(nT_{0})} - 1 \right]$$

$$\left| \Delta(s) \right| < 1$$

$$\left| \frac{1 - e^{-s(nT_{0})} - s(nT_{0})}{s(nT_{0})} \right| < \left| W_{m}(s) \right|$$
(9)

Thus,  $W_{\rm m}$  must calculate that the size function is an upper bound of all functions. So to find  $W_{\rm m}$  function, all bode size functions per every n must draw and calculate such that its size diagram become the upper bound of all bode size diagrams.

$$M_{n}(s) = \frac{1 - e^{-s(nT_{0})} - s(nT_{0})}{s(nT_{0})}$$

$$\Rightarrow M_{n}(j\omega) = \frac{1 - e^{-j\omega(nT_{0})} - j\omega(nT_{0})}{j\omega(nT_{0})} = \frac{1 - j\omega(nT_{0}) - \left[\cos(\omega nT_{0}) - j\sin(\omega nT_{0})\right]}{j\omega nT_{0}}$$

$$= \frac{1 - \cos(\omega nT_{0}) - j\omega nT_{0} + j\sin(\omega nT_{0})}{j\omega nT_{0}} = \frac{\left[\sin(\omega nT_{0}) - 1\right] + j\left[\cos(\omega nT_{0}) - 1\right]}{\omega nT_{0}}$$
(10)

For example, if  $T_0$ =100msec and packet loss=60% that equal  $N_{max}$ =30 for Bernoulli binary loss model then Wm Obtain as (11) as Upper bound of  $M_n(jw)$ 

$$W_m(s) = \frac{(s+0.01)(s+1.5)(s+140)}{(s^2+2s+1.5)(s+40)}$$
(11)

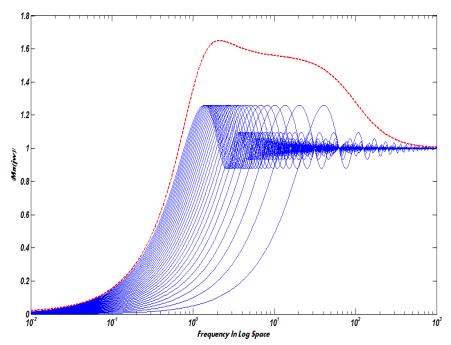


Fig.8. M<sub>n</sub>(jw) size diagrams with their Upper bound

#### 6. Conclusion

Proposing an appropriate model of network control system and analysing their phenomenon in attention to their role in industrial applications is vital. Main phenomenon for these systems is packet loss. In attention to packet loss nature that occur randomly, unstructured uncertainty is a good model for packet loss, because they are simple and independent. In this group of uncertainty and packet mathematical occurring model. multiplicative input model is the best candidate for packet loss modelling. Because in this model uncertainty occur in input way and if system work memorial, the system function becomes zero order hold function. At last with calculating infinity norm of error in special form and draw bode size diagram can lead to a good model that is simple, independent and efficient.

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### References

- Luca Schenato, "To Zero or to Hold Control Inputs With Lossy Links", IEEE Trans. On Automatic Control, Vol.54, No.5, May 2009.
- [2] Huiying Chen, Wanliang Wang, Zuxin Li, "Synthetic Modeling and Control of Networked Control Systems with

- Multi-packet Transmission", International Conference on Advanced Computer Control, Aug. 2012.
- [3] Xiaosheng Fang, Jingcheng Wang, "Stochastic Observerbased Guaranteed Cost Control for Networked Control Systems with Packet Dropouts", Proceedings of the 17th World Congress The International Federation of Automatic Control Seoul, Korea, July 6-11, 2013.
- [4] Ling Q., Lemmon M.D., "Soft real-time scheduling of networked control systems with dropouts governed by a Markov chain", Proc. American Control Conf., pp.4845 – 4850, 2003.
- [5] Seiler P., Sengupta R., "An H approach to networked control", IEEE Trans. Autom. Control, Vol.50, No.3, pp.356 –364, 2005
- [6] Wu J., Chen T., "Design of networked control systems with packet dropouts", IEEE Trans. Autom. Control, Vol.52, No7, pp. 1314–1319, 2007.
- [7] Wang Z., Ho D.W.C., Liu X., "Variance-constrained control for uncertain stochastic systems with missing measurements", IEEE Trans. Syst. Man. And Cybern, Vol.5, No.35, pp.746– 753, 2005.
- [8] Wang Z., Yang F., Ho D.W.C., Liu X., "Robust H1control for networked systems with random packet losses", IEEE Trans. Syst. Man Cybern., Vol.37, No.4, pp.916–924, 2007.
- [9] Yang F., Wang Z.D., Ho W.C., Gani M., "Robust H control with missing measurements and time-delays", IEEE Trans. Autom. Control, Vol.52, No.9, pp.1666–1672, 2007.
- [10] Y. Wang, G. Yang, "H<sub>x</sub> Controller Design for Networked Control Systems via Active-varying Sampling Period Method", Automatica sinica, Jul. 2008.
- [11] Radek Matusu, Roman Prokop, and Libor Pekar, "Parametric and unstructured approach to uncertainty modelling and robust stability analysis", International Journal of Mathematical Models and Methods in Applied Sciences, Feb. 2013.
- [12] S. Lun, S. Wang, "Robust H<sub>∞</sub> cost control for Networked control systems based on T-S model", IEEE Transactions on systems, Apr. 2010.