New Applications on Linguistic Mathematical Structures and Stability Analysis of Linguistic Fuzzy Models

Jafar Tavoosi, Sajjad Rahmati

Electrical Engineering Department, Islamic Azad University, Ilam Branch, Ilam, Iran, jtavoosi@aut.ac.ir

Abstract

In this paper some algebraic structures for linguistic fuzzy models are defined for the first time. By definition linguistic fuzzy norm, stability of these systems can be considered. Two methods (normed-based & graphical-based) for stability analysis of linguist fuzzy systems will be presented. At the follow a new simple method for linguistic fuzzy numbers calculations is defined. At the end two simple (stable and unstable) systems are modeled by linguistic fuzzy logic then stability of them by both methods are checked.

1. Introduction

In mathematics, and more specifically in abstract algebra, the term algebraic structure generally refers to an arbitrary set with one or more operations defined on it. Common examples of algebraic structures include groups, rings, modules and lattices. There are a lot of literatures about fuzzy vector spaces form 1970s and we briefly consider some of them.

In 1977, Katsaras and Liu [1] formulated and studied the concept of a fuzzy subspace of a vector space. Since then, a host of mathematicians are involved in extending the basic concepts and results from the theory of crisp vector spaces to the broader framework of the fuzzy setting. However, not all the results can be fuzzified. This reference introduced the notion of a fuzzy subspace of a vector space and obtained some fundamental results pertaining to this notion. In [2], among other concepts and results, the fuzzy cosset of a fuzzy subspace is defined and the algebraic nature of fuzzy subspaces under homomorphism is studied. In [3, 4] are shown that such space is a generalization of the usual topological vector space and fuzzy topological vector space. In [5] many definitions such as linguistic fuzzy matrix and vector, fuzzy linguistic space and etc. are presented.

For finding a practical way studies have been done to analyze fuzzy control systems [15]-[16]. Tanaka and et al [17] proposed a method for stability analysis of TSK model by finding a common Lyapunov function, Percup and et al [19] used the center of manifold theory for fuzzy system stability analysis. The authors of this paper proposed a sufficient condition for stability of TSK fuzzy model [18], Farinawata [19] and Linder [20] separately worked on robust stability controller design. Many other efforts, which have been done in the TSK stability analysis, are found in [21]. They use classical approaches for stability analysis of fuzzy systems. Unfortunately these approaches conflict with the simplicity idea, which was the main aim of Zadeh when presenting his fuzzy system approach. For simplicity in system analysis and humanity interface, a linguistic model is offered [22]. Some authors have done some incomplete researches on this area [23]-[24]. Recently Margaliot and Langholz [25]-[26] have proposed some approaches for linguistic nonlinear systems. The basic idea is using crisp equation of Lyapunov for the stability analysis in their approaches. Furuhashi et. al [23]-[24] have proposed a definition for equilibrium in fuzzy

linguistic model. In their definition some things seems not to be quite right. Until now the stability analysis of linguistic systems is an open problem [16]. Here we will extend a stability analysis method [27] to fuzzy linguistic systems associated with a class of applied nonlinear systems.

Unfortunately almost of the above literature have pure mathematical view of the fuzzy logic [13, 14]. So in this paper we have applied view to linguistic fuzzy systems and define some properties of norm (pseudo norm) vector spaces then we use them to stability analysis of linguistic fuzzy systems. First we redefine some algebraic structures for linguistic fuzzy logic. In the following some algebraic structures are introduced for the first time in this paper and then we develop them to linguistic fuzzy logic and stability of linguistic fuzzy systems.

2. Linguistic Mathematical Structures

In this section some basic mathematical structures for linguistic fuzzy logic are investigated.

A) Simple structures(no binary operation)

– *Fuzzy Linguistic Set: a collection of distinct objects*

A set is introduced in linguistic fuzzy logic as like as classical logic. For example the below collection is a set: $A = \{large, small, medium\}$

B) Group-like structures(One binary operation)

A group is a set of elements together with an operation that combines any two of its elements to form a third element also in the set while satisfying four conditions called the group axioms, namely closure, associatively, identity and inevitability:

– *Closure*

For all a, b in G, the result of the operation, a • b, is also in G.

– *Associatively*

For all a, b and c in G, (a • b) • c = a • (b • c).

– *Identity element*

There exists an element e in G, such that for every element a in G, the equation e • a = a • e = a holds. Such an element is unique and thus one speaks of the identity element.

– *Inverse element*

For each a in G, there exists an element b in G such that a • b = b • a = e.

– *Fuzzy Linguistic Magma:*

For all a, b in A, the result of the operation $a.b$ is also in A. For example for linguistic fuzzy logic :

$A = \{large, small, medium\}$

if '".""min $\rightarrow$ *'large' . 'small' =*

– *Fuzzy Linguistic Semigroup:*

an algebraic structure consisting of a set together with an associative binary operation. For all a, b, c in A, the equation $(a.b).c = a.(b.c)$ holds. for linguistic fuzzy logic :

if '".""min $\rightarrow$ *('medium' . 'large 'medium' . ('large*

– *Fuzzy Linguistic Monoid :*

A monoid is a set, S, together with a binary operation "•" (pronounced "dot" or "times") that satisfies closure, associatively and identity:

*Closure: $\forall a, b \in A : a.b \in A$

*Associatively: $\forall a, b, c \in A : (a.b).c = a.(b.c)$

*Identity element: $\exists e \in A : \forall a \in A : e.a = a.e = a$

In linguistic fuzzy logic Closure and Associatively are proved in Magma and Semigroup respectively. For Identity element we can choose $e = \max(A)$
$\min(e, a) = \min(a, e) = \min(\max(A), a) = a$

C) 2.3. Ring-like structures

Two binary operations, often called addition and multiplication, with multiplication distributing over addition.

– *Fuzzy Linguistic Semi-ring:*

A semi-ring is a set R equipped with two binary operations + and ·, called addition and multiplication, such that:
1. $(R, +)$ is a commutative monoid with identity element 0:
    1. $(a + b) + c = a + (b + c)$
    2. $0 + a = a + 0 = a$
    3. $a + b = b + a$
2. $(R, \cdot)$ is a monoid with identity element 1:
    1. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
    2. $1 \cdot a = a \cdot 1 = a$
3. Multiplication left and right distributes over addition:
    1. $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$
    2. $(a + b) \cdot c = (a \cdot c) + (b \cdot c)$
4. Multiplication by 0 annihilates R:
    1. $0 \cdot a = a \cdot 0 = 0$

Linguistic fuzzy sets are semiring if we choose $'+' = max$ and $'.' = min$ and two replaces:

$'1' = \max(A)$ in $1. a = a. 1 = a$
$'0' = \min(A)$ in $0. a = a. 0 = 0$

– *Fuzzy Linguistic Quasigroup :*

A quasigroup (A, *) is a set A with a binary operation * (that is, a magma), such that for each a and b in A, there exist unique elements x and y in A such that:
$a * x = b$ and $y * a = b$.
If $\min(a,b) = b$, and $A = \{large, small, medium\}$ and we choose $x = y = b$ then A is a quasigroup.

– *Fuzzy Linguistic Comparability:*

Any two elements *a* and *b* of a set *A* that is partially ordered by a binary relation ≤ are comparable when either $x \le y$ or $y \le x$.
We can define linguistic operations of them, "better" of two terms or "maximum" of two terms likewise "minimum" or 'lesser' of the two terms.

$\min(large, small) = small$
$\max(large, small) = large$
$\min(large, medium) = medium$
$\max(large, medium) = large$
$\min(small, medium) = small$
$\max(small, medium) = medium$
We have a chain or total order and even we can call this {A, max, min} as a fuzzy linguistic chain lattice; $large \ge medium \ge small$.

– *THEOREM [1]:*

Let A be the fuzzy linguistic set. If every pair of elements is comparable, then A is a fuzzy linguistic chain lattice.
Suppose A be a fuzzy linguistic set, we can define metric on A as follows which will be known as "fuzzy linguistic metric" or "fuzzy distance" and denoted by d_F.
If we choose M = {0, near, very near, overlap, more over lap, partly near, little far, just near, very far, far, partly far, not comparable} in which the fuzzy linguistic terms are the ones which shows comparison between them; We have
$d_F(large, small) = far$ or $(veryfar, ...)$
$d_F(medium, small) = near$ or $(partlynear, ...)$
$d_F(small, small) = overlap$
M can also have any other equivalent linguistic terms which compares the 'fuzzy distance' that is the shade of difference between two terms.

– *Linguistic Fuzzy Pseudo Vector*

A set "*A* " is a Linguistic Fuzzy Pseudo Vector if all array elements of A be Linguistic Fuzzy number. $A = \{large, small, medium\}$
As like as in mathematics, vector addition is the operation of adding two vectors by adding the corresponding entries together, In Linguistic Fuzzy Pseudo Vector we can add two pseudo vector.

$A = \{large, small, medium\}$
$B = \{very\ large, small, very\ small\} =$

$A + B = \{max(lar$
$max(small, small$

Also we can define pseudo vector product (cross product and dot product):

$$AB^T = [large, small, medium]\begin{bmatrix} very\ large \\ small \\ very\ small \end{bmatrix}$$
$= \min(large, very\ lare) + \min(small, small) + \min(medium, very\ small)$
$= \{large, small, very\ small\}$

3. Linguistic Fuzzy Norm

In linear algebra, functional analysis and related areas of mathematics, a norm is a function that assigns a strictly positive length or size to each vector in a vector space. We get extend this concept to linguistic fuzzy vector. "*A*" and "*B*" are two linguistic fuzzy sets as follows:
$A = \{medium, small, large\}$
$B = \{small, large, very\ large\}$
A norm is a way of measuring the length of a vector. Let A_F be a vector space. A *norm* on A_F is a function $\|\cdot\|: A_F \to [0, \infty)$ satisfying
(i) $\|A\| = 0$ *iff* $A = 0$
(ii) $\|\alpha A\| = |\alpha|\|A\|$ for $\alpha \in F$, $A \in A_F$
(iii) $\|A + B\| \le \|A\| + \|B\|$ for $A, B \in A_F$

A) A New Method for Linguistic Fuzzy Numbers Calculations

There are a lot of literatures about calculations with fuzzy numbers. Some of them are about comparability [6,7,8], Addition [9,10], Multiplication [11,12] and etc. In [6], a centroid-based distance method was suggested for ranking fuzzy numbers. The method utilizes the Euclidean distances from the origin to the centroid point of each fuzzy number to compare and rank the fuzzy numbers. In [7] found that the distance method could not rank fuzzy numbers correctly if they are negative and therefore suggested using the area between the centroid point and the origin to rank fuzzy numbers. In this paper we define linguistic fuzzy numbers in very simple method for a class of applications on fuzzy logic. Mathematical operations on this linguistic fuzzy numbers are easier than the methods which are presented in the mentioned references.

B) A New Definition of Fuzzy Numbers

In this section we introduce a new formula of fuzzy numbers based on centroid point. The width of a fuzzy number is $\pm\%20$ of its center. For example : $\tilde{5} * \widetilde{10} = \widetilde{50}$

Where the center and width of $\widetilde{50}$ are 50 and [40,60]. Consider the membership degrees in 40 and 60 are 0.1.

C) Fuzzy Linguistic Pseudo Field

In algebraic structure, a field is a set F that is a commutative group with respect to two compatible operations, addition and multiplication.

The most common way to formalize this is by defining a field as a set together with two operations, usually called addition and multiplication, and denoted by $+$ and $\cdot$, respectively, such that the following axioms hold; subtraction and division are defined implicitly in terms of the inverse operations of addition and multiplication. We define F is a Fuzzy linguistic real numbers. For example: $F = \{\tilde{1}, \widetilde{-1}, \frac{\tilde{1}}{\tilde{7}}, \widetilde{0.25}, \dots\}$ Where the symbol "~" denote fuzzy numbers.

D) Closure of F under addition and multiplication

In crisp numbers world, for all a, b in F, both $a + b$ and $a \cdot b$ are in F (or more formally, $+$ and $\cdot$ are binary operations on F). So, here in fuzzy world this section is established. For example $\widetilde{-1} + \widetilde{0.25} = \widetilde{-0.75} \in F$ and $\widetilde{-1} * \widetilde{0.25} = \widetilde{-0.25} \in F$.

E) Associativity of addition and multiplication

In crisp numbers world, for all a, b, and c in F, the following equalities hold: $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. So, here in fuzzy world this section is established. For example $\widetilde{-1} + (\widetilde{0.25} + \tilde{1}) = \widetilde{0.25} = (\widetilde{-1} + \widetilde{0.25}) + \tilde{1}$ and $\widetilde{-1} * (\widetilde{0.25} * \tilde{1}) = \widetilde{-0.25} = (\widetilde{-1} * \widetilde{0.25}) * \tilde{1}$

F) Commutatively of addition and multiplication

In crisp numbers world, for all a and b in F, the following equalities hold: $a + b = b + a$ and $a \cdot b = b \cdot a$. So, here in fuzzy world this section is established.
$(\widetilde{0.25} + \tilde{1}) = (\tilde{1} + \widetilde{0.25}) = \widetilde{1.25}$
$(\widetilde{0.25} * \tilde{1}) = (\tilde{1} * \widetilde{0.25}) = \widetilde{0.25}$

G) Existence of additive and multiplicative identity elements

There exists an element of F, called the additive identity element and denoted by 0, such that for all a in F, $a + 0 = a$. Likewise, there is an element, called the multiplicative identity element and denoted by 1, such that for all a in F, $a \cdot 1 = a$. To exclude the trivial ring, the additive identity and the multiplicative identity are required to be distinct. So, here in fuzzy world this section is established.
$(\widetilde{0.25} + \tilde{0}) = \widetilde{0.25}$
$(\widetilde{0.25} * \tilde{1}) = \widetilde{0.25}$

H) Existence of additive inverses and multiplicative inverses

For every a in F, there exists an element $-a$ in F, such that $a + (-a) = 0$. Similarly, for any a in F other than 0, there exists an element a^{-1} in F, such that $a \cdot a^{-1} = 1$. In other words, subtraction and division operations exist. So, here in fuzzy world this section is established.
$\widetilde{0.25} + (\widetilde{-0.25}) = \tilde{0}$
$(\widetilde{0.25} * \tilde{4}) = \tilde{1}$

I) Distributivity of multiplication over addition

In crisp numbers world, for all a, b and c in F, the following equality holds: $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$. So, here in fuzzy world this section is established.
$\widetilde{-1} * (\widetilde{0.25} + \tilde{1}) = \widetilde{-1.25}$
$\qquad = (\widetilde{-1} * \widetilde{0.25}) + (\widetilde{-1} * \tilde{1})$

Using above properties we can conclude that F is a linguistic fuzzy field.

In (ii),(properties of norm), F is linguistic field that it have some property of field and all element (such as α) are linguistic fuzzy (mentioned previous).

The pair $(A_F, \|\cdot\|)$ is called a pseudo linguistic Fuzzy *normed* linear space (or pseudo linguistic Fuzzy normed vector space).

A class of vector norms, called a 1-norm and denoted $\|\cdot\|_1$, is defined as:
$\|A\|_1 = |A_1| + |A_2| + \cdots + |A_n|$

We can define 1-norm (or ∞-norm) linguistic fuzzy. For example:
$A = \{medium(\tilde{2}), small(\tilde{1}), large(\tilde{3})\}$
$B = \{small(\tilde{1}), large(\tilde{3}), very\ large(\tilde{4})\}$
$\|A\|_1 = \tilde{2} + \tilde{1} + \tilde{3} = \tilde{6}$
$\|B\|_1 = \tilde{1} + \tilde{3} + \tilde{4} = \tilde{8}$
(i) is satisfied because $\|A\|_1 = |A_1| + |A_2| + \cdots + |A_n| = 0$ *if* $|A_1| = |A_2| = \cdots = |A_n| = 0$ *then* $\|A\|_1 = 0$.
$(ii)\|\alpha A\| = |\alpha|\|A\|$ *for* $\alpha \in F$, $A \in A_F$
Using distributive of multiplication over addition this property is satisfied. For example
$\|A\|_1 = \tilde{2} + \tilde{1} + \tilde{3} = \tilde{6}$
$\alpha = \widetilde{-1}$
$\|\alpha A\| = \|\widetilde{-1}(\tilde{2}, \tilde{1}, \tilde{3})\| = \|\widetilde{-2}, \widetilde{-1}, \widetilde{-3}\|$
$\qquad = |\widetilde{-2}| + |\widetilde{-1}| + |\widetilde{-3}| = \tilde{2} + \tilde{1} + \tilde{3}$
$\qquad = |\alpha|\|A\|$

The third property of norm must be satisfied, so:
(iii) $\quad \|A + B\| \le \|A\| + \|B\|$ *for* $A, B \in A_F$ is satisfied because:
$\|A + B\| = (\tilde{2} + \tilde{1}) + (\tilde{1} + \tilde{3}) + (\tilde{3} + \tilde{4}) = \widetilde{14}$
$\|A\| + \|B\| = (\tilde{2} + \tilde{1} + \tilde{3}) + (\tilde{1} + \tilde{3} + \tilde{4}) = \widetilde{14}$

4. Graphical Based Stability Analysis of First Order Linguistic Fuzzy Systems

In this section a new method for stability analysis of first order linguistic fuzzy systems is presented. To explain this method, assume that a linguistic fuzzy model is as follows,

$R1$: if $x(k)$ $L1$ then $x(k+1)$ is $L2$
$R2$: if $x(k)$ $L2$ then $x(k+1)$ is $L3$
⋮

Then the $[x(k), x(k+1)]$ plain is drawn. The next step is locating the centres of all region in $[x(k), x(k+1)]$ plain. The final step is using least squares method to calculation of the straight line (Fig.1). If the ramp of obtained line is larger than 1 then system is unstable because the state is strongly increasing else system is stable.
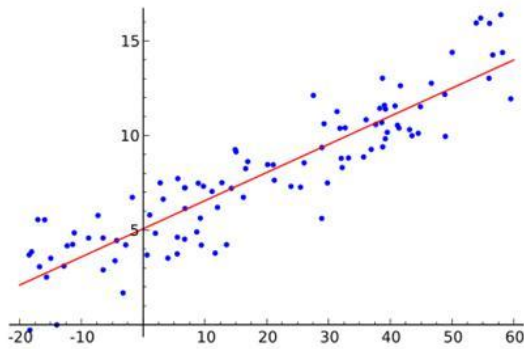

Fig. 1.   Least squares method

So if $R > 1$ then system is unstable and if $R \leq 1$ then system is stable. For example assume that a linguistic fuzzy system with three rules is as follows,

$R1$: if $x(k)$ $L1$ then $x(k+1)$ is $L1$
$R2$: if $x(k)$ $L2$ then $x(k+1)$ is $L2$
$R3$: if $x(k)$ $L3$ then $x(k+1)$ is $L3$

The $[x(k), x(k+1)]$ plain of mentioned system is shown in Fig. 2.

As it can be seen from Fig. 2, Least squares method uses the center of the boxs to find a straight line. The next step is to find the slope of the line. Slope of the line indicates stability or instability of the system as mentioned in section 4.

5. Simulation

In this section two systems (stable and unstable) with linguistic fuzzy rules are modeled them stability of them are checked. At the follow performance of graphical − based method for stability analysis of first order linguistic system is evaluated. Fig.3 and Fig.4 show an unstable and a stable system respectively.

Unstable system is: $x_1(k+1) = 1.05 * x_1(k)$
Stable system is : $x_1(k+1) = 0.95 * x_1(k)$

Linguistic fuzzy membership functions for unstable and stable systems are shown in Fig.5. Where the linguistic variables are as follows:

$L1 = \tilde{0}$, $L2 = \widetilde{0.1}$, $L3 = \widetilde{0.3}$, $L4 = \widetilde{0.7}$

$L5 = \widetilde{1.5}, L6 = \tilde{3}, L7 = \tilde{5}, L8 = \tilde{8}$

$L9 = \widetilde{12}, L10 = \widetilde{18}, L11 = \widetilde{25}, L12 = \widetilde{40}$
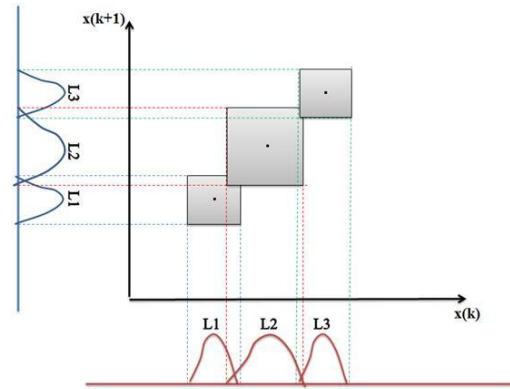

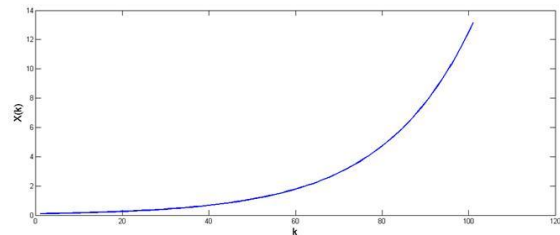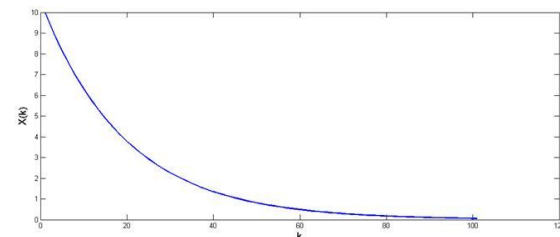Fig. 2.   [x(k), x(k + 1)] plain

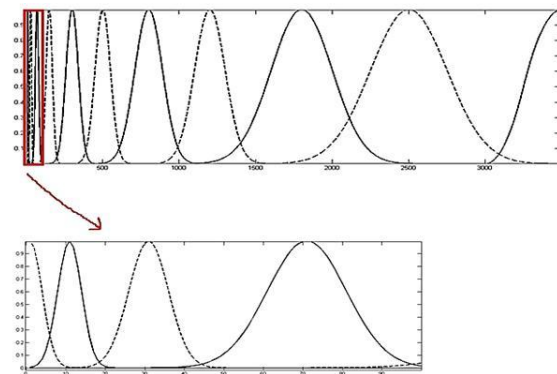Fig. 3.   Unstable system

Fig. 4.   Stable system

Fig. 5. Linguistic fuzzy membership functions for unstable and stable systems

Linguistic fuzzy rules for unstable and stable systems are shown in Table 1.

Table.1.
Linguistic fuzzy rules for unstable and stable systems

| unstable | | stable | |
$x(k)$	$x(k+1)$	$x(k)$	$x(k+1)$
L1	L1	L9	L7
L2	L3	L7	L6
L3	L4	L6	L5
L4	L5	L5	L4
L5	L5	L4	L3
L6	L7	L3	L2
L7	L7	L2	L2
L8	L8	L1	L1
L9	L9	-	-
L10	L11	-	-
L11	L12	-	-
L12	L12	-	-

We calculate 1-norm of both systems. So for unstable system the 1- norm is

$$\|x(k)\|_1 = \widetilde{48}$$

$$\|x(k+1)\|_1 = \widetilde{78}$$

So the norm has increased then we can say this system is unstable. For stable system the 1-norm is

$$\|x(k)\|_1 = \widetilde{22.6}$$

$$\|x(k+1)\|_1 = \widetilde{10.7}$$

It can be seen that this norm has decreased so system is stable. Two example of the second order system (stable & unstable) are considered as follow:

−   *Stable system(shows in Fig.6):*

$$x_1(k+1) = 0.4 * x_1(k) + 0.5 * x_2(k)$$
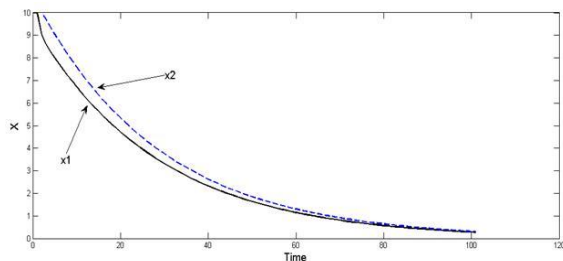$$x_2(k+1) = 0.3 * x_1(k) + 0.7 * x_2(k)$$

Fig. 6.   Stable system

−   *Unstable system(shows in Fig.7):*

$$x_1(k+1) = 0.6 * x_1(k) + 0.5 * x_2(k)$$
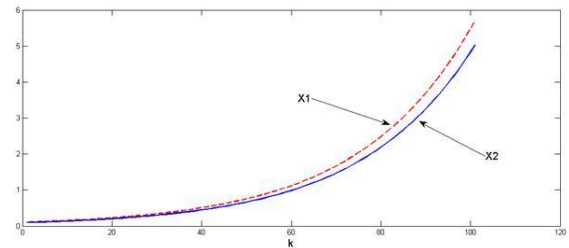
$$x_2(k+1) = 0.3 * x_1(k) + 0.7 * x_2(k)$$

Fig. 7.   Unstable system

Linguistic fuzzy rules for stable system are shown in Table 2.

Table.2.
Linguistic fuzzy rules for stable system

| stable | | | |
$x_1(k)$	$x_1(k+1)$	$x_2(k)$	$x_2(k+1)$
L1	L1	L2	L1
L2	L2	L3	L2
L3	L3	L4	L4
L4	L4	L5	L4
L5	L5	L5	L5
L5	L5	L6	L6
L6	L6	L6	L6
L6	L6	L7	L6
L7	L7	L7	L7

Where the linguistic variables are as follows

$$L1 = \tilde{1} \quad , \quad L2 = \tilde{2} \quad , \quad L3 = \tilde{3} \quad ,$$

$$L4 = \tilde{4} \quad , \quad L5 = \tilde{5} \quad , \quad L6 = \tilde{6} \quad , \quad L7 = \tilde{7}$$

$$\frac{\|x_1(k)\|_1 + \|x_2(k)\|_1}{2} = \frac{\widetilde{43} + \widetilde{47}}{2} = \widetilde{45}$$

$$\frac{\|x_1(k+1)\|_1 + \|x_2(k+1)\|_1}{2} = \frac{\widetilde{43} + \widetilde{43}}{2} = \widetilde{43}$$

As you can see the norm has reduced. Linguistic fuzzy rules for unstable system are shown in Table 3.

Table.3.
Linguistic fuzzy rules for unstable system

| stable | | | |
$x_1(k)$	$x_1(k+1)$	$x_2(k)$	$x_2(k+1)$
L1	L1	L1	L1
L2	L2	L1	L1
L2	L3	L2	L2
L3	L3	L2	L3
L3	L4	L3	L3
L4	L5	L3	L4
L5	L5	L4	L4
L5	L6	L5	L5
L6	L6	L5	L5

The linguistic variables are same as stable system. As you can see the norm has increased:

$$\frac{\|x_1(k)\|_1 + \|x_2(k)\|_1}{2} = \frac{\widetilde{31} + \widetilde{26}}{2} = \widetilde{28.5}$$

$$\frac{\|x_1(k+1)\|_1 + \|x_2(k+1)\|_1}{2} = \frac{\widetilde{35} + \widetilde{28}}{2} = \widetilde{31.5}$$

6. Conclusion

Stability analysis is clearly important. However, the lack of satisfactory formal techniques for studying the stability of fuzzy control systems especially linguistic fuzzy systems has been considered a major drawback of fuzzy application. This paper presented a new stability analysis method for linear systems modelled by linguistic (mamdani) fuzzy systems. This method is deduced from stability study of fuzzy systems based on vector norms. We used norm 1 to stability analysis of linguistic fuzzy systems. Using norm to stability analysis is simple way to understanding and implementation. Another method for stability analysis of linguistic fuzzy systems was graphical-based method that it was presented.

References

[1] A. K. Katsaras, D. B. Liu,"Fuzzy vector spaces and fuzzy topological spaces", J. Math. Anal. Appl, vol.58 ,1977.

[2] R. Kumar,"Fuzzy vector spaces and fuzzy cosets", Fuzzy Sets Syst, vol. 45, 1992.

[3] A.K. Katsaras, "Fuzzy Topological Vector Spaces I", Fuzzy Sets and Systems,vol. 6 ,1981.

[4] A.K. Katsaras, "Fuzzy Topological Vector Spaces II", Fuzzy Sets and Systems,vol. 12 ,1984.

[5] W. B. Vasantha Kandasamy, Florentin Smarandache and K.Amal "Katsaras, "Fuzzy Linguistic Topological Spaces", Zip publishing, 2012.

[6] Ching-Hsue Cheng "A new approach for ranking fuzzy numbers by distance method" Fuzzy Sets and Systems, vol. 95 ,1998.

[7] TA-CHUNG CHU, CHUNG-TSEN TSAO "Ranking Fuzzy Numbers with an Area between the Centroid Point and Original Point", Computers and Mathematics with Applications, vol. 43 ,2002.

[8] Shan-Huo Chen and Guo-Chin Li, Representation, Ranking, and Distance of Fuzzy Number with Exponential Membership Function Using Graded mean Integration method", Tamsui Oxford Journal of Mathematical Sciences, vol.16 ,2002.

[9] D. Chakraborty & D. Guha "Addition of Two Generalized Fuzzy Numbers", Int. J. Industrial Mathematics, vol. 2 , no.1 ,2010.

[10] Xiu-Gang Shang, Wei-Sun Jiang "Addition of fuzzy numbers based on generalized t-norms", Fuzzy Sets and Systems, vol.98 ,1998.

[11] Shang Gao and Zaiyue Zhang, Cungen Cao "Multiplication Operation on Fuzzy Numbers", Journal of Software, vol. 4, no. 4, 2009.

[12] A.Taleshian, S.Rezvani, "Multiplication Operation on Trapezoidal Fuzzy Numbers", Journal of Physical Sciences, vol. 15, 2011.

[13] J Tavoosi, AA Suratgar, MB Menhaj, Stability analysis of recurrent type-2 TSK fuzzy systems with nonlinear consequent part, Neural Computing and Applications ,2015.

[14] J Tavoosi, AA Suratgar, MB Menhaj, Stability Analysis of a Class of MIMO Recurrent Type-2 Fuzzy Systems, International Journal of Fuzzy Systems ,2016.

[15] M. A. L.Thathachar and P. Viswanath, On stability of fuzzy systems, IEEE Trans. On Fuzzy Systems, vol. 5 , 1997.

[16] E. Kim, A new approach to numerical stability analysis of fuzzy control systems, IEEE Trans. On Syst. Man and Cyber. , Part C, vol. 31 ,2001.

[17] K. Tanaka, T. Ikeda and H. O. Wang, A LMI approach to fuzzy controller designs based on relaxed stability condition, Proc. of the Sixth IEEE International Conf. on Fuzzy Systems., vol. 1 ,1997.

[18] A. A. Suratgar and S. K. Nikravesh, A new sufficient condition for stability of fuzzy systems, ICEE 2002, Tabriz, Iran, 2002.

[19] P. Linder and B. Shafai, Qualitative robust fuzzy control with application to 1992 ACC Benchmark, IEEE Trans. On Fuzzy Systems, vol. 7 ,1999.

[20] S. S. Farinwata, A robust stablizing controller for a class of fuzzy systems, Proc. of IEEE Conf. of Decision and Control, vol. 5,1999.

[21] M. Sugeno, On stability fuzzy systems expressed by fuzzy rules with singleton consequents, IEEE Trans. On Fuzzy Systems, vol. 7 ,1999.

[22] L. A. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, IEEE Transaction on Systems Man and Cybernetics, vol. 3, no. 1 ,1973.

[23] T. Hasegawa, T. Furuhashi and Y. Uchikawa, Stability analysis of fuzzy control systems using petri nets, Proc. of 5-th IEEE Int. Conf. On Fuzzy Systems, 1996.

[24] H. Yamamoto and T. Furuhashi, A new sufficient condition for stable fuzzy control system and its design method, IEEE Trans. On Fuzzy Systems, vol. 9, 2001.

[25] M. Margaliot and G. Langholz, New approaches to fuzzy modelling and control design, World Scientific Press, 2000.

[26] M. Margaliot and G. Langholz, Fuzzy control of a benchmark problem: a computing with words approach, IEEE Conf. ,2001.

[27] A. A. Suratgar and S. K. Nikravesh, Necessary and sufficient conditions for asymptotic stability of a class of applied nonlinear dynamical systems, IEEE Circuit and System Conf., Sharjah, 2003.