



Randomness, Coherence and Noise Robustness in Compressive Sensing

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Abstract

The theory of compressive sensing (CS) in contrast with well-known Nyquist sampling theorem was proposed. Sensing matrix and sparse matrix have key roles in perfect signal reconstruction by using either greedy algorithms like orthogonal matching pursuit (OMP) or ℓ_1 -norm based methods. In this paper, different pairs as sensing and sparse matrices are evaluated in terms of randomness and coherence. Noiselet as a complex measurement matrix has low coherence with Haar wavelet, and so the recovered images by OMP in comparison with other measurement-sparse matrices are appropriate. But, because of complexity, it cannot be used for big size images. However, the pair structured random sensing matrix with values 0, 1 and Fourier sparse matrix which got the second rank in terms of coherence, approved to be a noise robust pair and showed a great potential to be used in CS.

Keywords: Coherence, Compressive Sensing (CS), Noise Robust, Noiselets, Randomness.

1. INTRODUCTION

In 1949, Shannon [1] presented that any band-limited time-varying signal with ‘ f ’ Hertz highest frequency component can be reconstructed perfectly by sampling the signal at regular intervals of at least $1/2f$ seconds. Accordingly, some applications, such as synthetic aperture radar (SAR) results in great number of samples. So, for big data,

compressing before storage or transmission is obligation. Compressive Sensing (CS) [2], is an alternative to Shannon/Nyquist sampling theorem for the acquisition of sparse or compressible signals. In fact, instead of using a periodic impulse for sampling, CS uses random matrices for sensing such that sampling and compressing are performed simultaneously. Although CS may disregard the Nyquist rate, it was proved

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that by meeting some circumstances, the signal could be perfectly recovered. In essence, CS combines the sampling and compression into one step by measuring minimum samples that contain maximum information about a signal [1]: this eliminates the need to acquire and store large number of samples. CS signal recovery is usually based on ℓ_1 -norm [3], or greedy algorithms such as Matching Pursuit (MP) [4], Orthogonal Matching Pursuit (OMP) [5] and Compressive Sampling Matching Pursuit (CoSaMP) [6]. In CS framework, measurement and basis matrices play a key role in perfect signal recovery, so they should be designed carefully in order to satisfy some specific requirements. For perfect reconstruction in CS framework, two important factors should be satisfied; 1) sparsity of signal which is usually explored using some sparsity basis like Fourier transform [7], discrete Cosine transform [8], and wavelet transform [9], 2) coherence between the measurement and the sparse matrices. Coherence, which measures the largest correlation between vectors of measurement and sparsifying matrices [10] is an important factor in CS applications. Precisely, the less coherence between measurement and sparsifying matrices, the less measured samples are needed to recover the signal perfectly. Well known measurement matrices such as Random Gaussian and Bernoulli matrices [2], have been largely used in CS framework whereas Noiselets [11], [12] are not well known yet. On the other hand, sensing matrices usually require huge memory for storage and high computational cost for signal reconstruction. Many structured sensing

matrices have been proposed recently to simplify the sensing scheme and the hardware implementation in practice [13] such as the one used in [14], where the structured 0 and 1 sensing matrix is used in CS framework to simultaneously recover and denoise real SAR data.

In signal and image processing applications, those methods that are noise robust and also inherently remove noise are of interest by researchers. In this paper, we demonstrate under choosing appropriate measurement and sparse matrices, by considering coherence and randomness, the CS recovery algorithms do the two tasks simultaneously. That means, CS would not only be noise robust but also do noise reduction without any extra cost.

In this paper, we are going to evaluate the CS precisely in terms of coherence between the measurement and sparse matrices, randomness of the sensing matrix, different recovery algorithms, and the size of input signal. In addition, we recommend a measurement and sparse matrix pair in order to show the CS noise robustness as well as inherently noise removing.

The paper is organized as follows. In Section 2, CS theory, OMP Recovery algorithm, Randomness and Coherence are explained. Then in Section 3, Noiselets as a new sampling matrix which has a good incoherence with Haar wavelet basis, are presented. In Section 4, different scenarios have been explored to properly compare the Noiselets with other measurement matrices especially with structured 0 and 1 random matrix and finally we have conclusion in Section 5.

2. BACKGROUND

In this Section, a brief review of CS theory and its recovery algorithms are presented.

2.1. CS Theory

Suppose $x \in \mathbf{R}^{N \times 1}$ is a signal to be sensed (sampled). It can be expressed based on a linear non-adaptive measurement as:

$$y = \varphi x \quad (1)$$

where x denotes the signal or data of interest with finite dimension N , φ is the sampling or measurement matrix with size $M \times N$ and y with size $M \times 1$ is the observed data. As said before, in CS theory, there are two fundamental requirements that need to be fulfilled, the ‘sparsity’ and the ‘coherence’. A signal x is k -sparse if it has k nonzero or big elements. As most of the signals are not sparse inherently so the sparsity is explored generally in ψ domain. In other words, the signal x is k -sparse if it could be expressed as the combination of just k columns of ψ matrix and it would be compressible if $k \ll N$. For example, a Dirac or a Spike is sparse in time domain, a sinusoid is sparse in frequency domain and a chirp is sparse in fractional Fourier transform domain. The k -sparse signal x based on basis matrix, ψ , is expressed as:

$$x = \psi s \quad (2)$$

where $s \in \mathbf{R}^{N \times 1}$ with k nonzero elements denotes the sparse representation of signal x and ψ with size $N \times N$ is the sparse basis matrix. The second CS requirement called ‘coherence’ means creating φ and ψ matrices such that they are maximally

incoherent to each other. Satisfying the coherence guaranties the signal perfect recovery with few measurement samples. Combining Eq. (1) and Eq. (2), the observed signal is:

$$y = A s \quad (3)$$

where $A = \varphi \psi$ with size $M \times N$ is called the dictionary. The main challenges for CS applications [15] are 1) using a stable measurement matrix φ such that the salient information in any k -sparse or compressible signal is not damaged by the dimension reduction from $x \in \mathbf{R}^{N \times 1}$ to $y \in \mathbf{R}^{M \times 1}$ 2) finding a sparse domain which enables the high incoherence between measurement and representation matrices and 3) using an efficient algorithm which is able to recover x from only M observed data y for $M < N$.

2.2. CS Recovery Algorithms

In general, the CS recovery algorithms try to find a unique solution for underdetermined or ill-posed problem in Eq. (3). However, according to CS theory, the original signal can be exactly recovered by solving the linear programming problem as long as x is sparse in some domain. So far, different recovery algorithms have been developed for solving the underdetermined or sparse approximation problems which lie in two categories: the first group are norm-based algorithms [3]. Between the three norms used so far for CS recovery i.e. ℓ_0 , ℓ_1 and ℓ_2 norms; ℓ_1 -norm showed to be powerful and reliable tool to find the sparse solution which is expressed as:

$$\hat{s} = \mathbf{arg\,min}_s \|\hat{s}\|_{\ell_1} \quad \text{subject to} \quad y = A \hat{s} \quad (4)$$

where ℓ_1 norm, defined as $\|s\|_1 = \sum_n |s[n]|$. Often the observed vector includes additive noise, so it is modeled as:

$$y = As + e \quad (5)$$

where e is stochastic or deterministic error with bounded energy $\|e\|_2 < \varepsilon$ and ℓ_2 -norm defined as $\|s\|_2 = \sqrt{\sum_n |s[n]|^2}$. Then, the sparsest s is obtained by solving the following optimization problem:

$$\hat{s} = \arg \min_s \|s\|_{\ell_1} \quad \text{subject to} \quad \|y - A\hat{s}\|_{\ell_2} < \varepsilon \quad (6)$$

The second group includes the greedy algorithms [4-6] which are well-known due to low complexity framework and fast run time. The core idea of greedy algorithms is finding the highest correlation between the residual of signal and the dictionary columns. In this paper, as we use the orthogonal matching pursuit (OMP) [5] recovery algorithm in our simulations, so it is briefly explained in following.

OMP Recovery Algorithm

- Compute the inner product $\mathbf{g}_j = \mathbf{A}^T \mathbf{r}_{j-1} \in \mathbf{R}^N$.
- Find the index k where $k = \arg \max_{i=1, \dots, N} |\mathbf{g}_j[i]|$.
- Update the index set and matrix of chosen atoms $\Lambda_j = \Lambda_{j-1} \cup \{k\}$,
 $\mathbf{A}_{\Lambda_j} = \mathbf{A}_{\Lambda_{j-1}} \cup \mathbf{A}_k$.
- Obtain the new estimate $\tilde{\mathbf{x}} = (\mathbf{A}_{\Lambda_j}^T \mathbf{A}_{\Lambda_j})^{-1} \mathbf{A}_{\Lambda_j}^T \mathbf{y}$. Note that the size of $\tilde{\mathbf{x}}$ is growing while the number of iterations is increasing. Compute the coefficient vector $\mathbf{x}_j[\Lambda_j] = \tilde{\mathbf{x}}$.
- Update the residual $\mathbf{r}_j = \mathbf{y} - \mathbf{A}\mathbf{x}_j$.

Consider the iteration number, j , or obtain the residual norm value, $\|\mathbf{r}_j\|_{\ell_2}$. Stop the algorithm if j is greater or $\|\mathbf{r}_j\|_{\ell_2}$ is less than the pre-defined value. Otherwise go through the first step.

It should be noted that the main issues regarding recovery algorithms are the sparsity of the solution and the consumed time.

2.3. CS Randomness

Randomness of a measurement matrix is important for CS implementation. Although both random Gaussian and Bernoulli matrices were recommended [16], they have limited use in practice due to the fact that the structure is imposed on the measurement matrix by many measurement technologies [17], [18]. In this Section, the matrix randomness is evaluated in terms of entropy. In general, the entropy of matrix p with N elements is [19]:

$$h(p) = -\sum_{i=1}^N p_i \ln p_i \quad (7)$$

It is notified that the entropy value is always positive [19] with range of $[0, \ln N]$. The minimum value, $h(p) = 0$, is achieved when only one p_i equals 1 and others equal 0 whereas the maximum value is achieved when all p_i 's are equal $1/N$. Generally, it is concluded [19] that the entropy could be used as a measure of matrix randomness. In this way, the low entropy is equivalent to high randomness. Before using the entropy as a measure of randomness, elements of measurement matrix should be normalized. As an example, if

$\varphi = [\varphi_{11}, \dots, \varphi_{1N}; \dots; \varphi_{N1}, \dots, \varphi_{NN}]$ with size $N \times N$ is an arbitrary matrix, the corresponding normalized matrix is $\varphi_{norm} = [\varphi'_{11}, \dots, \varphi'_{1N}; \dots; \varphi'_{N1}, \dots, \varphi'_{NN}]$ where

$$\varphi'_{ij} = \varphi_{ij} / \sqrt{\sum_{i=1}^N \sum_{j=1}^N \varphi_{ij}^2}.$$

Obviously, the randomness interval value of any arbitrary matrix depends on the two parameters, i.e. elements probability and the matrix size. So, randomness of different matrices is incomparable. For example, zero is expected for the randomness of matrix with repetitive elements, but the calculated randomness by means of entropy doesn't support this idea. So, in order to have an ideal number for comparison of randomness between different matrices, we have proposed a method which is explained as follows.

At first, the input matrix φ' is normalized by the mean value; i.e. $\varphi'' = \varphi' / G$ where G is the average value of the matrix. Then the entropy of new matrix is obtained and considered as the measure of randomness:

$$\begin{aligned} \text{Randomness} &= -\sum_{i=1}^N \sum_{j=1}^N \varphi''_{ij} \ln \varphi''_{ij} \\ &= -\sum_{i=1}^N \sum_{j=1}^N \frac{\varphi'_{ij}}{G} \ln \frac{\varphi'_{ij}}{G} \\ &= -\sum_{i=1}^N \sum_{j=1}^N \frac{\varphi'_{ij}}{G} (\ln \varphi'_{ij} - \ln G) \\ &= -\sum_{i=1}^N \sum_{j=1}^N \frac{\varphi'_{ij}}{G} \ln \varphi'_{ij} + \sum_{i=1}^N \sum_{j=1}^N \frac{\varphi'_{ij}}{G} \ln G \end{aligned} \quad (8)$$

2.4. Coherence

The coherence property is defined between the measurement or sensing matrix, Φ , with size $M \times N$, and the basis or sparse basis matrix, Ψ with size $N \times N$, as:

$$\mu(\Phi, \Psi) = \sqrt{N} \max_{1 < z, j < N} |\langle \Phi_z, \Psi_j \rangle| \quad (9)$$

where Φ_z is the z -th row of Φ and Ψ_j refers to the j -th column of Ψ and $\mu(\varphi, \psi) = 1$ means maximum incoherence.

so when matrix φ' has repetitive elements, the average G is equal to the value of every φ' elements, then according to the definition, Eq. (8), the randomness of matrix φ' is zero. Satisfaction of restricted isometry property (RIP) [15] guaranties the maximum incoherence between the sampling matrix, φ , and the sparse basis matrix ψ . According to CS theory, the matrices Φ and Ψ should have maximum incoherence. It was shown that random matrices are largely incoherent with any fixed basis [15].

3. NOISELETS

The Noiselet basis, originally presented in [11], has received interest recently due to the following facts: 1) being maximally incoherent to the Haar basis, 2) having a fast implementation algorithm. Thus, they have been employed in CS to sample signals that are sparse in wavelet domain where Haar is the sparse matrix [20]. Noiselets family on the interval $[0,1)$ are constructed as follows:

$$\begin{aligned} f_1(z) &= \chi_{[0,1)}(z), \\ f_{2n}(z) &= (1-i)f_n(2z) + (1+i)f_n(2z-1), \\ f_{2n+1}(z) &= (1+i)f_n(2z) + (1-i)f_n(2z-1) \end{aligned} \quad (10)$$

where $\{f_j\}$ is a basis, and $\chi_{[0,1)}(z) = 1$ on the definition interval $[0,1)$ and 0 otherwise. In order to generate the Noiselets matrix, Noiselets functions should be extended and discretized [21].

It is started with a 1×1 matrix $N_1 = [1]$, then a sequence of Noiselet matrices N_2, N_4, \dots, N_{2^m} with sizes $2 \times 2, 4 \times 4, \dots, 2^m \times 2^m$, are generated. So, the Noiselet matrix with size $n \times n$ is built up recursively according to:

$$N_n(k, *) = \frac{1}{2}(1-i \ 1+i) \otimes N_{n/2}\left(\frac{k}{2}, *\right) \quad (11)$$

$, k = 0, 2, 4, \dots, n-2.$

$$N_n(k, *) = \frac{1}{2}(1+i \ 1-i) \otimes N_{n/2}\left(\frac{k-1}{2}, *\right) \quad (12)$$

$, k = 1, 3, 5, \dots, n-1.$

where \otimes denotes the Kronecker product [21] and $N_n(k, *)$ denotes the row vector of N_n .

It should be noted that the Noiselet matrices are not real valued. As an example, N_2 and N_4 by using Eqs. (11)-(12) are:

$$N_2 = \frac{1}{2} \begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix}, \quad (13)$$

$$N_4 = \frac{1}{2} \begin{bmatrix} -i & 1 & 1 & i \\ 1 & i & -i & 1 \\ 1 & -i & i & 1 \\ i & 1 & 1 & -i \end{bmatrix}$$

In addition, the imaginary part element summation in every Noiselet matrix is always equal to zero, or in other words, the average value of imaginary part of Noiselet matrix, irrespective of its size, is zero. As mentioned in [2], well known random matrices like Gaussian, which are mostly used in CS, are largely incoherence with any basis ψ with size $N \times N$ and the coherence is about $\sqrt{2 \log N}$. Noiselets which are not as popular as random sampling matrices, also have good incoherence with fixed basis ψ matrices with size $N \times N$ like Fourier and wavelets [2].

The coherence between Noiselets and Haar wavelets is equal $\sqrt{2}$ and that between

Noiselets and Daubechies-4 (D4) and D8 wavelets is, respectively, about 2.2 and 2.9. So, this gave us motivation to analyze the performance of different CS recovery algorithms when Noiselet is used as the measuring matrix.

4. EXPERIMENTAL RESULT

In this Section, at first, we are going to calculate the randomness of three measurement matrices, including: Gaussian, Bernoulli and Noiselet. Despite the real part of Noiselet matrix that contain positive elements, imaginary part has both negative and positive elements that are same valued and equal in numbers.

This feature of imaginary part makes the matrix average value be 0; hence, in our proposed method for calculating the randomness, the division of elements to the average number would be infinity, so it is not reported in Table 1. Comparing the square sized matrices, the randomness of Gaussian matrix irrespective of size is always greater than both Bernoulli and Noiselet. Fig. 1 shows the three mentioned matrices with sizes 128×128 and 512×512 for structure comparison. As it is seen, Gaussian has fully random shape whereas Noiselet has repetitive pattern. Whereas it seems that Noiselet cannot be used in CS framework according to the randomness value (Table. 1) and repetitive pattern shape (Fig. 1), one should bear in mind that measurement matrix in CS framework is not square but with size of $M \times N$ where $M \ll N$.

So, in second scenario, we are going to compare the abovementioned matrices when they are in size $M \times N$. As the Noiselet is a

Table 1. Comparing the randomness of Gaussian, Bernoulli, and Noiselet with different matrix sizes.

Matrix size	Randomness		
	Gaussian	Bernoulli	Noiselet (Real part)
64×64	4.7621×10^5	2.8491×10^3	2.1092×10^4
128×128	2.1445×10^6	1.1385×10^4	1.8334×10^5
256×256	4.1836×10^7	4.5354×10^4	7.4316×10^5
512×512	1.8408×10^8	1.8142×10^5	6.2192×10^6

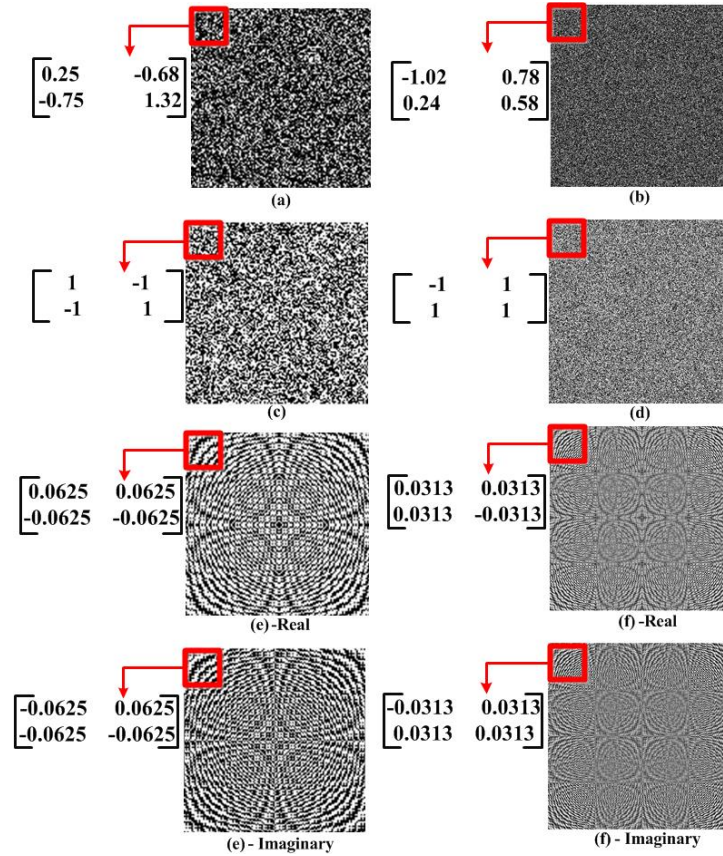


Fig 1. The pattern of measurement matrices: Gaussian (a)-(b), Bernoulli (c)-(d), and Noiselet (e)-(f). First Column matrices are with size 128×128 and second column with 512×512.

complex measurement matrix, the corresponding randomness is calculated for both the real and imaginary parts. Here it

should be noted that as Noiselets are generated in square sizes, so, to generate the matrix in equivalent sampling size, we have

generated the nearest square Noiselet matrix which is bigger than M at first, then it is cut into 30, 40, 60% of its rows size to have a $M \times N$ measurement matrix. For example, for an original matrix with size of 128×128 , when $\frac{M}{N} = 40\%$, the measurement matrix size is 6553×16384 . The computed randomness is written in Table 2 where Noiselet is the best. However, as explained in Section 2, randomness and coherence are two important properties for signal perfect reconstruction in CS. The ideal coherence between the measurement and sparse matrix is equal one. Here, for coherence comparison, three sensing matrices called Gaussian (G),

Bernoulli (B), Noiselet (N) and three sparse matrices called Fourier (F), DCT (D) and Haar wavelet (H) are selected. In addition, the coherence between the random sensing matrix with values 0, 1 with Fourier (F) sparse matrix is also computed and the results are written in Table. 3. Coherence for every sensing and sparse pair matrices are obtained under following set up:

- Because of computer memory limitation, the coherence is computed for images with size 64×64 pixels.
- All values are computed using 20 iterations, that means the different rows of sensing matrices are chosen randomly 20 times.

Table 2. Comparing the randomness of Gaussian, Bernoulli, and Noiselet in different sizes with various sampling rates.

Sampling Rate (%)	Size	Randomness			
		Gaussian	Bernoulli	Noiselet (Real part)	Noiselet (Imaginary part)
30	32×32	1.3208×10^9	2.1877×10^5	7.7868×10^6	1.4107×10^{25}
	64×64	3.3669×10^{10}	3.4857×10^6	2.4602×10^8	9.0397×10^{26}
40	32×32	4.7362×10^9	2.9056×10^5	9.9987×10^6	2.1773×10^{25}
	64×64	1.2652×10^{11}	4.6511×10^6	3.2789×10^8	1.2095×10^{26}
60	32×32	4.9995×10^{10}	4.3730×10^5	4.4306×10^5	4.2001×10^{24}
	64×64	7.9506×10^{11}	6.9804×10^6	4.9207×10^8	1.2626×10^{26}

Table3. Comparison of different measurement and sparsifying basis matrices.

Sampling Rate	Coherence									
	G-D	G-F	G-H	B-H	B-F	B-D	N-H	N-F	N-D	coded 0 & 1 with Fourier
0.1	5.04	3.62	4.84	4.12	3.71	5.28	1	28.12	26.76	1.41
0.5	5.13	4.37	5.16	5.25	3.94	5.41	1	30.87	28.79	2.23
0.9	5.62	3.85	5.15	4.59	3.89	5.64	1	30.87	28.79	2.44

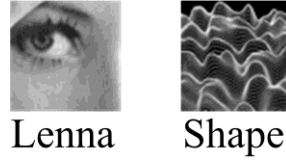
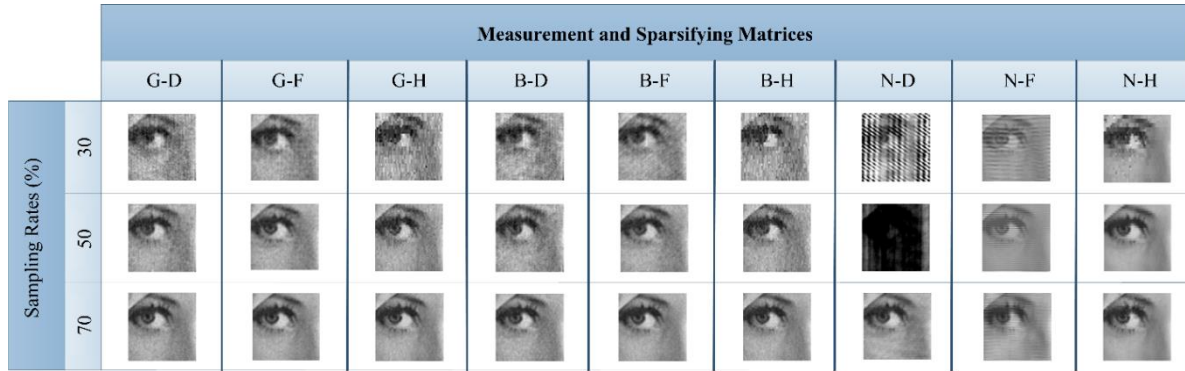
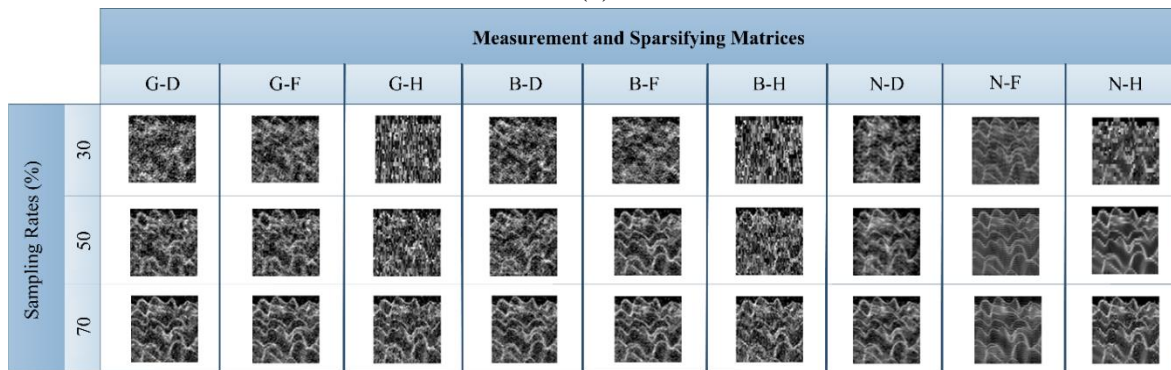


Fig. 2. Original test images.



(a)



(b)

Fig. 3. Recovered images (a) Lenna, (b) Shape by using OMP with different sensing, sparse matrices and sampling rates.

As a result, the random sensing matrix with values 0, 1 with Fourier (F) and pair N-H got the least coherence, so they are recommended for CS implementation.

Now, for visual evaluation, two test images shown in Fig. 2 with sizes 64×64 pixels are chosen and recovered by using OMP algorithm in CS framework with different sensing and sparse matrices and also different sampling rates. The recovered images are shown in Fig. 3. Also, in order to

support the visual conclusion, two image assessments, peak signal to noise ratio (PSNR) [22] and structural similarity index (SSIM) [23] are obtained for the recovered images. The achieved PSNR and SSIM values are written in Table 4 for OMP recovery algorithm.

According to visual results shown in Fig. 3, it can be concluded that the visual quality of recovered images improves by increasing the measurement rates.

Table 4. PSNR and SSIM values of recovered Lenna and Shape images by using OMP algorithm.

Lenna		G-D	G-F	G-H	B-D	B-F	B-H	N-D	N-F	N-H
0.3	PSNR	29.40	30.65	28.99	29.51	30.41	29.13	29.49	29.14	34.08
	SSIM	0.40	0.57	0.35	0.39	0.57	0.33	0.15	0.55	0.73
0.5	PSNR	31.33	33.62	30.87	30.49	32.74	30.58	27.19	29.00	37.66
	SSIM	0.63	0.75	0.59	0.55	0.72	0.56	0.22	0.55	0.95
0.7	PSNR	33.54	36.82	34.22	33.26	36.25	34.02	34.89	31.74	36.59
	SSIM	0.77	0.87	0.80	0.75	0.86	0.78	0.85	0.74	0.92

Shape		G-D	G-F	G-H	B-D	B-F	B-H	N-D	N-F	N-H
0.3	PSNR	28.12	28.86	28.22	28.01	28.86	28.34	28.65	29.55	30.28
	SSIM	0.25	0.37	0.16	0.31	0.32	0.19	0.48	0.60	0.47
0.5	PSNR	28.44	29.09	28.21	28.38	29.10	28.24	29.07	29.64	30.60
	SSIM	0.48	0.59	0.31	0.48	0.53	0.34	0.66	0.69	0.78
0.7	PSNR	28.68	29.33	28.72	28.74	29.68	28.51	29.95	29.87	30.79
	SSIM	0.67	0.72	0.57	0.67	0.71	0.54	0.76	0.77	0.78

To validate the power of structured sensing matrices, random sensing matrix with values 0, 1 and Fourier (F) sparse matrix which got the second rank in terms of coherence, is used to recover well-known phantom image with size 256×256 .

The recovered images using three different sampling matrices where ℓ_1 -norm is used as the recovery algorithm are shown in Fig. 4 and the corresponding PSNR and SSIM are written in Table 5.

In addition, the scenario is repeated for phantom image with size 256×256 where the sampling rate is 0.05, and Gaussian and speckle noise with variance 0.1 damage the

image. Using ℓ_1 -norm recovery algorithm, the image is recovered. The achieved results shown in Fig. 5 and written in Table 6 indicate that in spite of existing noise, nearly clean image is recovered.

It should be noted that all of the simulations are done with a 64bit O.S. which has 4GB RAM and core i7 Intel CPU.

As far as Noiselets are complex valued, the recovered images using this measurement will also be complex valued; hence, we have used their absolute value to represent images. As it was expected, N-H overcomes all pairs in terms of subjective and objective criteria.

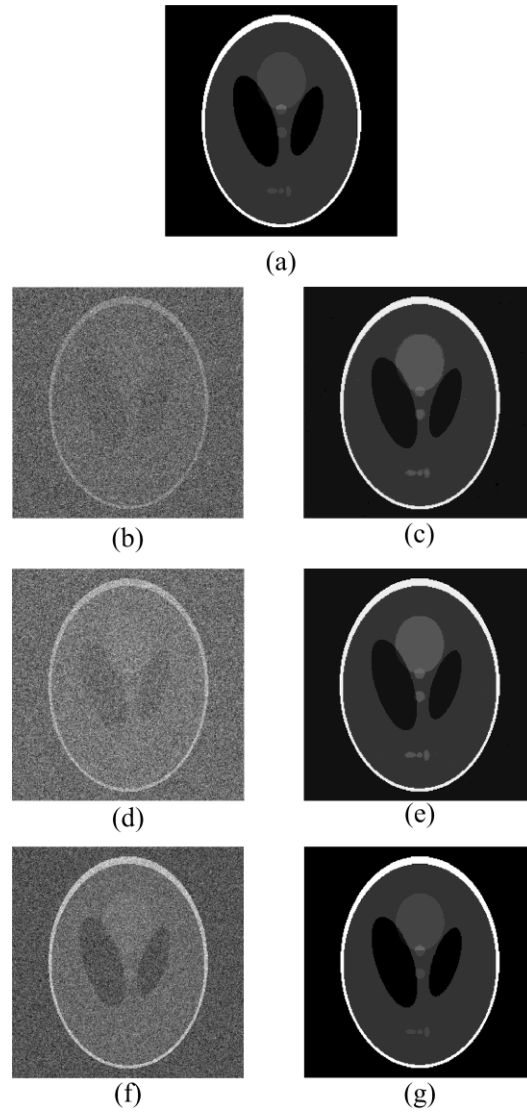


Fig. 4. (a) Original Phantom image with size 256×256 . (b), (d), (f) are undersampled Phantom image with $S.R.=5, 10, 30\%$ in order. (c), (e), (g) are the recovered Phantom image by using ℓ_1 -norm.

Table 5. PSNR, SSIM and cost time of recovered Phantom images.

S.R.	SSIM	PSNR (dB)	Time (s)
5%	0.49	37.69	196
10%	1	inf	41
30%	1	inf	37

Although the size of test images was 64×64 , in real applications where images with bigger sizes are used, using Noiselets because of

being complex valued, enforces implementation problems.

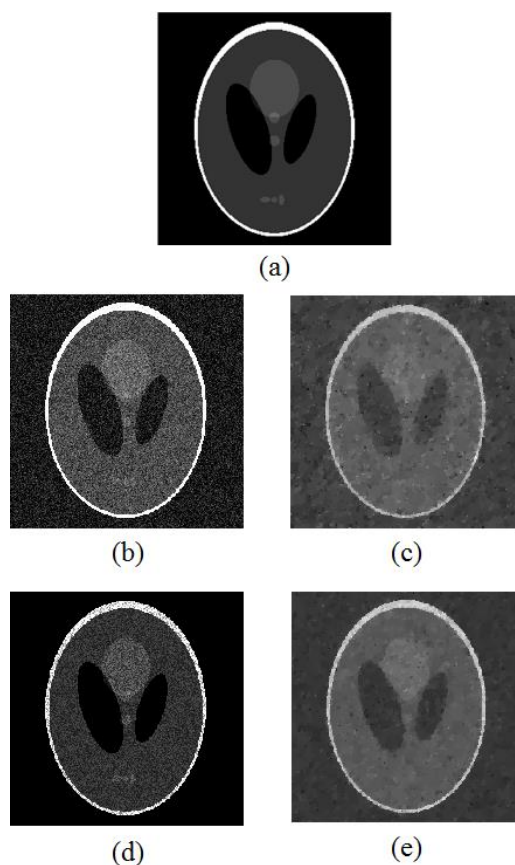


Fig. 5. (a) Original Phantom image with size 256×256 . (b), (d) corrupted images by Gaussian and speckle noise. (c), (e) the recovered image with only 5% samples.

Table 6. PSNR, SSIM of recovered noisy Phantom images.

Noise Type	Gaussian	Speckle
PSNR (dB)	37.66	37.67
SSIM	0.26	0.33

5. CONCLUSION

In this paper, the Noiselets and also structured random 0 and 1 measurement matrices are studied precisely and also compared with other well-known measurement matrices like Gaussian and Bernoulli in two point of views; matrix randomness by means of entropy and coherence with sparsifying matrices. Being

complex valued and also the need of large capacity to store is still the main drawback of using Noiselet in real applications. Whereas the structured random 0 and 1 measurement matrix showed a great potential to be used in CS framework as it is largely incoherent with Fourier basis, enables large scale data recovery by using ℓ_1 -norm algorithm, and enables the CS recovery algorithms to be

noise robust and also inherently remove noise.

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