Analysis of Lateral Instability of Slender Comb Drive Fingers Considering Lateral and Rotational Stiffness of Elastic Suspension

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***Abstract*—** This paper presents the lateral electromechanical instability of individual comb-drive fingers. The model considers both lateral translation and rotational stiffness of elastic suspensions of comb-drive rotor. It is shown that slenderness of comb fingers causes non-negligible deflection that affects the side pull-in of comb-drive. In this work, the critical electromechanical state of individual comb fingers is analytically solved. Numerical comparison reveals that the critical voltage obtain from present model is always less than those obtain from the model that only considers deflection of the comb fingers. The analytical solution can be used to design comb-drives in which side pull-in of individual fingers is avoided.

***Keywords*—**Side pull-in, Comb drive

I. Introduction

ELECTROSTATIC comb-drive actuators have been developed and employed for many applications such as resonators [1–3], electromechanical filters [4], optical shutters [5–7], microgrippers [8] and voltmeters [9]. They have also been used as the driving element in, e.g. vibromotors [10] and micromechanical gears [11]. However, side stability (side pull-in) limits the actuator stroke constraining its applications. Hence, extending the stable traveling range is an important issue for designing electrostatic comb-drive actuators. In comb-drive actuators the fixed electrode (stator) and mobile electrode (rotor) are each shaped as a comb with parallel rectangular fingers. The fingers of the stator and rotor combs are intertwined and also separated by a free-space gap.

The axial motion of the rotor is parallel to the fingers so that the gap is unaffected by the motion. The well-known side pull-in occurs in comb-drives when the electrostatic stiffness transverse to the axial direction of motion exceeds the transverse mechanical stiffness of the suspension [12, 13]. This side pull-in may be avoided by increasing the transverse stiffness of the suspension [13–15]. Hirano et al. [16] studied the lateral instability of in-plane comb drive devices based on a one-dimensional mass-spring model considering a lateral translational stiffness. Bochobza-Degani et al. [17] proved that in general, electrostatic actuators under charge excitation would have a larger range of stability than under voltage excitation by analyzing the “pull-in” of systems with multiple degrees-of-freedom. Bochobza-Degani et al. [18] presented the “pull-in” voltage and critical angle in out-of-plane torsional actuators using polynomial algebraic equations. Pamidighantam et al. [19] studied the instability of electrostatically actuated comb fingers based on the linear beam theory. Zhou and Dowd [20] analyzed the side instability of comb drive actuators based on a one-dimensional model with the lateral translational stiffness of structure. Instead of focusing on analysis, other researchers have turned their attention towards devising control schemes to ensure device stabilization. Chu and Pister [21] proposed a voltage control algorithm for various features of MEMS devices, based on analysis of the spring-mass model. Seeger and Crary [22] designed a capacitive control scheme with an added capacitor in series to a MEMS device, which imposes an external control to the device for greater stability. So far, however, all analyses of the lateral instability of the in-plane comb drive MEMS actuators have been limited to the one-dimensional model with a lateral translational stiffness [23, 24]. Huang and Lu [25] analyzed lateral instability of in plan comb drive actuators based on two-dimensional model. They only considered lateral translational and rotational stiffness of rotor suspension in their model. Elata and Leuse [26] studied the side pull-in of comb-drive cause by flexibility of comb fingers. But in their work they have not considered the stiffness of suspension structure.

With the advancement of microfabrication technology, thinner fingers and smaller gaps can be micromachined. This can allow for a denser spacing of fingers and thus increase the power density of comb-drive actuators. However, slender comb fingers cannot be considered as rigid, and the flexibility of individual fingers can affect the side pull-in. Therefore, side pull-in instability resulted from the transverse flexibility of the suspension that supports the rotor and flexibility of each comb

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drive finger. The purpose of this study is to analyze the stability of single fingers in comb-drives. Specifically, the parameters that dominate the side pull-in of individual fingers are derived. These parameters are important for proper design of comb-drive actuators.

II. Theory

The present study aims to determine the side pull-in of the slender comb-drive fingers. Figure 1 shows the physical model used in our analysis for a single movable comb finger. It is assumed that the transverse stiffness of the suspension is not sufficiently high to prevent transverse motion of the rotor. Also it is assumed that the extreme fingers on the sides of the rotor are each confined between two stator fingers (figure 1). A single rotor finger confined between two neighboring stator fingers is presented in figure 1. The rotor finger is of length L, width w and thickness b, and the gap g between the rotor and stator fingers is assumed to be uniform. When a voltage difference V is applied between the rotor and stator, the movable comb finger is driven into the gap between two fixed ones. Normally, the movable finger is in the center of the gap (along the x axis). When a small disturbance occurs and the movable finger is deviated from the center, lateral electrostatic forces are pulled onto the finger. As both the lateral translation displacement y and the rotational θ are small, the comb fingers are approximately parallel. Neglecting the fringing effect and applying the parallel plate capacitor theory, transverse equilibrium of a single finger is governed:

$$E^{*}I\frac{d^4y}{d^4x}=\begin{cases}0 & 0\le x\le x_1\\ \frac{1}{2}\varepsilon_0 bV^2\left[\frac{1}{(g_0-y)^2}-\frac{1}{(g_0+y)^2}\right] & x_1\le x\le x_2\end{cases} \tag{1}$$

Where ε_0 is the permittivity of free space between fingers, $E^*=E/(1-\nu^2)$ is the effective elastic modulus in bending (assuming $b\gg w$), g_0 is the nominal one side air gap, E and ν are the Young modulus and Poisson ratio of the structure material, respectively, $I=bw^3/12$ is the second moment of the beam cross-section, b is finger thickness in z direction (out of plane). x_1 and x_2 are the x-coordinate of the overlap of the movable and fixed fingers.

This governing equation may be rewritten in the following normalized form:

$$E^{*}I\frac{d^4\tilde{y}}{d^4\tilde{x}}=\begin{cases}0 & 0\le\tilde{x}\le\gamma\\ \tilde{V}^2\frac{\tilde{y}}{(1-\tilde{y}^2)^2} & \gamma\le\tilde{x}\le1\end{cases} \tag{2}$$

Where

$$\tilde{y}=\frac{y}{g_0}\qquad \tilde{x}=\frac{x}{x_2}\qquad \gamma=\frac{x_1}{x_2} \tag{3}$$

$$\tilde{V}^2=\frac{24\varepsilon_0 x_2^4}{E^{*}w^3g_0^3}V^2$$

The individual finger is supported by K_y and K_θ at its base and is assumed to be free of loads at its free edge ($\tilde{x}=1$). Where K_y and K_θ are the global lateral and rotational stiffness distributed to a single movable finger, respectively. The origin of x-y coordinate system is located at the rotation center. Accordingly, the boundary conditions of the problem are:

$$at\ \tilde{x}=0\qquad \tilde{y}'''=\tilde{k}_y\tilde{y}\quad and\quad \tilde{y}''=\tilde{k}_\theta\tilde{y}' \tag{4a}$$

$$at\ \tilde{x}=1\qquad \tilde{y}''=\tilde{y}'''=0 \tag{4b}$$

Where $\tilde{k}_y$ and $\tilde{k}_\theta$ are normalized lateral and rotational stiffnesses, respectively

$$\tilde{k}_y=\frac{12K_yx_2^3}{E^{*}bw^3}\quad and\quad \tilde{k}_\theta=\frac{12K_\theta x_2}{E^{*}bw^3} \tag{5}$$

At the verge of side pull-in of the individual finger, the deflection $\tilde{y}$ is small and therefore the distributed electrostatic force on the right-hand side of (2) may be approximated by the Taylor expansion:

$$\tilde{V}^2\frac{\tilde{y}}{(1-\tilde{y}^2)^2}=\tilde{V}^2(\tilde{y}+2\tilde{y}^3+O(\tilde{y}^5)) \tag{6}$$

Omitting high-order terms, the governing equation reduces to:

$$E^{*}I\frac{d^4\tilde{y}}{d^4\tilde{x}}=\begin{cases}0 & 0\le\tilde{x}\le\gamma\\ \tilde{V}^2\tilde{y} & \gamma\le\tilde{x}\le1\end{cases} \tag{7}$$

This equation has two solutions. One solution is the trivial case of $\tilde{y}=0$. In this case no bending occurs and therefore both right and left hand side of (7) vanish. In the other solution, the distributed mechanical restoring forces are exactly balanced by the distributed electrostatic forces, for any arbitrary small deflection $\tilde{y}(\tilde{x})$. This means that for small deflections the stiffness of the system vanishes, and this is a bifurcation transition of the equilibrium state [10]. This bifurcation transition is related to the phenomenon of electromechanical buckling [11]. The nontrivial solution of (7) is given by:

$$\tilde{y}_1(\tilde{x})=a_0\tilde{x}^3+a_1\tilde{x}^2+a_2\tilde{x}+a_3\quad (0\le\tilde{x}\le\gamma) \tag{8}$$

$$\tilde{y}_2(\tilde{x})=b_0e^{\lambda\tilde{x}}+b_1e^{-\lambda\tilde{x}}+b_2\cos(\lambda\tilde{x})+b_3\sin(\lambda\tilde{x})\quad (\gamma\le\tilde{x}\le1) \tag{9}$$

Where a_0, a_1, a_2, a_3, b_0, b_1, b_2 and b_3 are constant parameters and the eigenvalue λ is given by

$$\lambda^4=\tilde{V}^2 \tag{10}$$

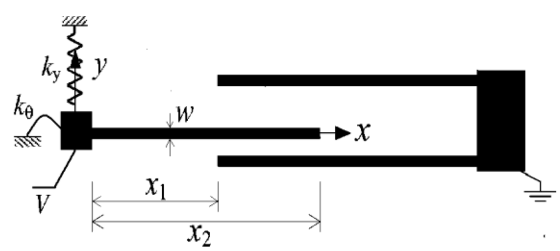

Fig. 1 A single rotor finger between two stator fingers

From the base conditions (4a), by substituting (8), we have:

$$6a_0=\tilde{k}_y(a_3) \tag{11a}$$

$$2a_1=\tilde{k}_\theta(a_2) \tag{11b}$$

And for the free end of finger (4b), by substituting (9), we have:

$$b_0\lambda^2e^{\lambda}+b_1\lambda^2e^{-\lambda}-b_2\lambda^2\cos(\lambda)-b_3\lambda^2\sin(\lambda)=0 \quad (12a)$$

$$b_0\lambda^3e^{\lambda}-b_1\lambda^3e^{-\lambda}+b_2\lambda^3\sin(\lambda)-b_3\lambda^3\cos(\lambda)=0 \quad (12b)$$

The compatibility of the two domains requires that the deflection, angle, bending moment and shear force be continuous at $\tilde{x}=\gamma$

$$\tilde{y}_1=\tilde{y}_2 \quad (13a)$$

$$\tilde{y}_1'=\tilde{y}_2' \quad (13b)$$

$$\tilde{y}_1''=\tilde{y}_2'' \quad (13c)$$

$$\tilde{y}_1'''=\tilde{y}_2''' \quad (13d)$$

One can rewrite (11), (12) and (13) in the following matrix form:

$$\begin{bmatrix} 6 & 0 & 0 & -\tilde{k}_y & 0 & 0 & 0 & 0 \\ 0 & 2 & -\tilde{k}_\theta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda^2e^{\lambda} & \lambda^2e^{-\lambda} & -\lambda^2\cos(\lambda) & -\lambda^2\sin(\lambda) \\ 0 & 0 & 0 & 0 & \lambda^3e^{\lambda} & -\lambda^3e^{-\lambda} & \lambda^3\sin(\lambda) & -\lambda^3\cos(\lambda) \\ \gamma^3 & \gamma^2 & \gamma & 1 & -e^{\lambda\gamma} & -e^{-\lambda\gamma} & -\cos(\lambda\gamma) & -\sin(\lambda\gamma) \\ 3\gamma^2 & 2\gamma & 1 & 0 & -\lambda e^{\lambda\gamma} & \lambda e^{-\lambda\gamma} & \lambda\sin(\lambda\gamma) & -\lambda\cos(\lambda\gamma) \\ 6\gamma & 2 & 0 & 0 & -\lambda^2e^{\lambda\gamma} & -\lambda^2e^{-\lambda\gamma} & \lambda^2\cos(\lambda\gamma) & \lambda^2\sin(\lambda\gamma) \\ 6 & 0 & 0 & 0 & -\lambda^3e^{\lambda\gamma} & \lambda^3e^{-\lambda\gamma} & -\lambda^3\sin(\lambda\gamma) & \lambda^3\cos(\lambda\gamma) \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (14)\ ()\ ()$$

The nontrivial solution is given by the first nonzero root of the determinant of the matrix.

In the case that fingers have complete overlap (i.e., γ = 0) (7) reduced to:

$$E^*I\frac{d^4\tilde{y}}{d^4\tilde{x}}=\tilde{V}^2\tilde{y} \qquad 0\le\tilde{x}\le 1 \quad (15)$$

From the base conditions (4a) we have:

$$b_0(\lambda^3-\tilde{k}_y)-b_1(\lambda^3+\tilde{k}_y)-b_2\tilde{k}_y-b_3\lambda^3=0 \quad (16a)$$

$$b_0(\lambda^2-\lambda\tilde{k}_\theta)+b_1(\lambda^2+\lambda\tilde{k}_\theta)-b_2\lambda^2-b_3\lambda\tilde{k}_\theta=0 \quad (16b)$$

And it should be mentioned that for the free end of finger, (12) is still valid.

(12) and (16) can be rewritten in matrix form:

$$\begin{bmatrix} \lambda^3-\tilde{k}_y & -(\lambda^3+\tilde{k}_y) & -\tilde{k}_y & -\lambda^3 \\ \lambda^2-\lambda\tilde{k}_\theta & \lambda^2+\lambda\tilde{k}_\theta & -\lambda^2 & -\lambda\tilde{k}_\theta \\ \lambda^2e^{\lambda} & \lambda^2e^{-\lambda} & -\lambda^2\cos(\lambda) & -\lambda^2\sin(\lambda) \\ \lambda^3e^{\lambda} & -\lambda^3e^{-\lambda} & \lambda^3\sin(\lambda) & -\lambda^3\cos(\lambda) \end{bmatrix} \times \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

For the nontrivial solution the determinant of the above matrix should be zero.

Solution of this equation yields the normalized pull-in voltage in the case of complete overlap.

III. Result and Dicussions

In the following sections, the variations of the normalized pull-in voltage with the overlap length of the movable and fixed fingers are analyzed. For increasing value of γ, the determinant of the matrix in (14) can be solved iteratively. The first root of this determinant $\lambda_i(\gamma)$ is an increasing function of γ. To ensure that the solution yields the first root $\lambda_1(\gamma)$ for $\gamma>0$, γ is increased from $\gamma=0$ in small increments and $\lambda_1(\gamma_{i+1})$ is computed using Newton method beginning with the initial guess $\lambda_1(\gamma_i)$ (with $\gamma_{i+1}>\gamma_i$).

Figure 2 illustrate the normalized side pull-in voltage as a function of the γ whit various k_y $(0.1,...,0.5)$. It can be seen that $\tilde{V}_{PI}$ is increased by increasing $\tilde{k}_y$. In these cases $\tilde{k}_\theta$ is assumed 50.

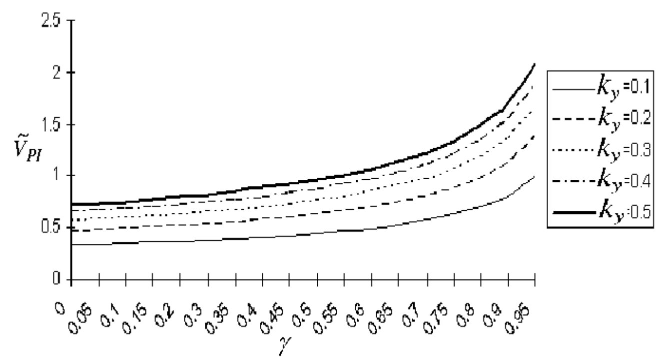

Fig. 2 The normalized pull-in voltage as a function of γ whit various k_y $(0.1,...,0.5)$

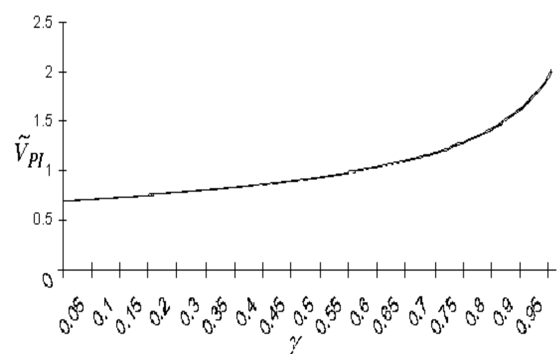

Fig. 3 The normalized pull-in voltage as a function of γ whit various k_θ $(10,....,50)$

Figure 3 illustrate the normalized side pull-in voltage as a function of the γ whit various k_θ $(10,....,50)$. It can be seen that the $\tilde{V}_{PI}$ is not sensitive to different values of $\tilde{k}_\theta$. Also $\tilde{k}_y$ is adjusted 0.5.

It can be found from figures (2) and (3) by increasing γ that means reducing overlap, the lateral stability is enhanced. On the other hand, device functions may require large engagement range, which implies large overlap lengths when voltage applied.

IV. Conclusion

In this work, lateral electromechanical instability of individual comb-drive fingers is considered. A model considering both lateral translational and rotational stiffness and also flexibility is established to analyze the lateral "pull-in" instability of an in-plane comb drive MEMS device. The critical state in which this instability is occurred is solved analytically. It is shown that if the comb fingers are very slender, the flexibility of each finger can affect the side pull-in instability. The result of present work can be used in design of comb-drive with a denser spacing of fingers by thin fingers and small gap. It is noted that the fringing fields in the third (out-of-plane) dimension are not considered in the analytical solutions. These fringing fields depend on specific geometrical parameters of the design (i.e. b / g_0) and can increase the electrostatic forces by a factor of up to 25% for typical cases [9]. Therefore, it is essential that this effect is considered in the design process, after the fabrication technology (and hence the device layer thickness b and the minimal trench size g_0) has been chosen. The effect of lateral and angular offset on critical voltage can also be investigated in future. Experimental proof of the proposed model remains to be done.

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