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Buckling of functionally graded stiffened cylindrical shells with isotropic rings and stringers

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Abstract

This paper is concerned with the elastic buckling of axially compressed stiffened circular cylindrical shells. Equilibrium and stability equations of a functionally graded stiffened cylindrical shell with isotropic rings and stringers under mechanical loading are derived. Assuming that the material properties vary linearly through the thickness direction and using the variational method, the system of fundamental partial differential equations is established. Then, buckling analysis of functionally graded stiffened cylindrical shells under external lateral pressure is carried out and the results are given in closed-form solutions. By equating power law index to zero, the results are derived for the buckling of homogeneous cylindrical shells which is available in the literature.

Keywords: Functionally graded material; Buckling; Stiffened Cylindrical shells

1- Introduction

In recent years studies on new performance materials have addressed new materials known as a functionally graded material (FGM). These are high performance, heat resistant materials able to withstand ultra high temperatures and extremely large thermal gradients used in aerospace industries. FGMs are microscopically inhomogeneous in which the mechanical properties vary smoothly and continuously from one surface to the other [1,2].

The elastic buckling problem of bars and columns was first solved by Euler (1774) and it continues to be a subject of intensive research for more than two centuries [3,4]. Using a shell theory, Lorenz [5] was probably the first researcher to solve the elastic buckling problem of cylindrical shells under compressive axial force. Ever since then, extensive studies have been conducted on the buckling of cylindrical shells. The large number of publications on shell buckling bears testimony to the research intensity in determining the buckling loads and behavior of cylindrical shells under different loads and boundary conditions, shapes, materials and theoretical models (see standard texts on elastic stability such as Timoshenko and Gere [3], the Structural Stability Handbook compiled by C.R.C.J [6], Timoshenko and Woinosky-Krieger [7], Brush and Almroth [8], Donnell [9], Calladine [10], Yamaki [11] and Bazant and Cedolin [12]).

Buckling analysis of an FGM structures are rare in the literature. Birman [13] studied the buckling problem of a functionally graded composite rectangular plate subjected to uniaxial compression. The buckling analysis of circular FGM plate is given by Najafizadeh and Eslami [14,15]. The thermal buckling of an FGM circular plate based on the first, third order shear deformation plate theory is studied by Najafizadeh and Hedayati [16]. HS Shen [17,18] studied the post buckling analysis of axially and pressure loaded functionally graded cylindrical shells in thermal

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environments. The buckling and post buckling of stiffened cylindrical shells under axial compression is investigated by HS Shen et al. [19]. Also, the buckling and post buckling of cylindrical shells under combined external pressure and axial compression is studied by HS Shen et al. [20]. Recently, Woo and Meguid [21] studied analytical solution for large deflection of thin FGM plates and shallow shells. The stabilization of a functionally graded cylindrical shell under axial harmonic loading is investigated by Ng et al. [22]. The general instability of stiffened cylindrical shell under axial compression load is investigated by Van Der Neut [23]. The instability analysis of stiffened cylindrical shells under hydrostatic pressure is given by Barush and Singer [24].

In this paper, buckling analysis of a functionally graded stiffened cylindrical shell is considered. The Donnell nonlinear strain-displacement relations are used and simply supported boundary conditions are assumed. The shell is under external lateral pressure load and the expression for mechanical load is obtained analytically. By equating power law index to zero, the results are compared with Barush and Singer [24].

2. Basic equations

Consider a stiffened cylindrical shell made of functionally graded material. The shell is assumed to be graded through the thickness direction. The constituent materials are assumed to be ceramic and metal. The volume fractions of ceramic V_c and metal V_m corresponding to the power law are expressed as [25]

$$V_c = \left(\frac{2z+h}{2h}\right)^k \quad , \quad V_m = 1 - V_c \tag{1}$$

where z is the thickness coordinate ($-h/2 \le z \le h/2$), h is the thickness of the shell , and k is the power law index that takes values greater than or equal to zero [25]. In this paper, it is assumed that $k=1$. The value of k equals to zero represents a fully ceramic shell. The mechanical properties of FGMs are determined from the volume fraction of the material constituents. We assume that the nonhomogeneous material properties such as the modulus of elasticity E and the coefficient of thermal expansion α change in the thickness direction z based on Voigt's rule over the whole range of volume fraction [25], while Poisson's ratio ν is assumed to be constant [13] as

$$E(z) = E_c V_c + E_m (1 - V_c) \quad ,(2) \qquad \upsilon(z) = \upsilon_0 \; , \; \alpha(z) = \alpha_c V_c + \alpha_m (1 - V_c)$$

where subscripts m and c refer to the metal and ceramic constituents, respectively. By substituting volume fraction ratio from Eqs. (1) into Eqs. (2), materials properties of the FGM shell is determined, which are the same as the equations proposed by Praveen and Reddy [25]

$$E(z) = E_m + E_{cm}\left(\frac{2z+h}{2h}\right)^k , \; \alpha(z) = \alpha_m + \alpha_{cm}\left(\frac{2z+h}{2h}\right)^k , \; \upsilon(z) = \upsilon_0 \tag{3}$$

where

$$E_{cm} = E_c - E_m \; , \quad \alpha_{cm} = \alpha_c - \alpha_m \tag{4}$$

3. Equilibrium and Stability equations

Consider a thin cylindrical shell of mean radius a and thickness h with length L. The Geometry and coordinate system of a stiffened cylindrical shell is shown in Fig. 1. The Donnell form of the kinematic relations for cylindrical shells is are follows [8]

$$\varepsilon_x = u_{,x} + \frac{1}{2}\beta_x^2 \qquad k_x = \beta_{x,x}$$

$$\varepsilon_\theta = \frac{v_{,\theta} + w}{a} + \frac{1}{2}\beta_\theta^2 \qquad \beta_\theta = -\frac{w_{,\theta}}{a}$$

$$\gamma_{x\theta} = \left(\frac{u_{,\theta}}{a} + v_{,x}\right) + \beta_x \beta_\theta \quad)5(\qquad k_\theta = \frac{\beta_{\theta,\theta}}{a}$$

$$\beta_x = -w_{,x} \qquad k_{x\theta} = \frac{1}{2}\left(\frac{\beta_{x,\theta}}{a} + \beta_{\theta,x}\right)$$

where ε_x and ε_θ are the normal and circumferential strains and $\varepsilon_{x\theta}$ is the shear strain and k_x, k_θ, $k_{x\theta}$ are the curvatures. The indices x and θ refer to the axial and circumferential directions, respectively. (u, v, w) are the axial, circumferential, and lateral deflections of shell, respectively, and the subscript (,) indicates partial derivative.

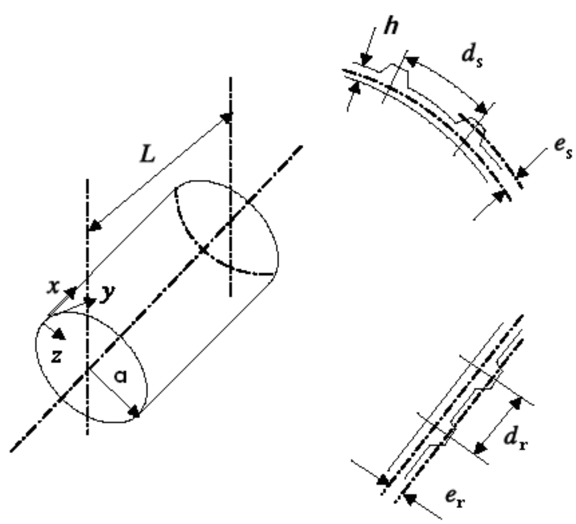

Fig.1. Geometry and coordinate system of a stiffened cylindrical shell.

Hook’s law for a functionally graded cylindrical shell at a distance z from the middle plane of the shell is defined as [25]

$$\sigma_x = \frac{E(z)}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_\theta), \sigma_\theta = \frac{E(z)}{1-\nu^2}(\varepsilon_\theta + \nu\varepsilon_x), \sigma_{x\theta} = \frac{E(z)}{2(1+\upsilon_0)}\varepsilon_{x\theta} \qquad (6)$$

The stress resultants N_i and M_i are expressed as

$$(N_i, M_i) = \int_{\frac{-h}{2}}^{\frac{h}{2}} \sigma_i (1, z)\, dz \qquad i = x, \theta, x\theta \tag{7}$$

Substituting Eqs. (5) into Eq. (6) and substituting Eq. (6) into Eq. (7) give the constitutive relation as

$$\begin{aligned}
N_x &= \frac{A}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_\theta) + \frac{C}{1-\nu^2}(k_x + \nu k_\theta) \\
N_\theta &= \frac{A}{1-\nu^2}(\varepsilon_\theta + \nu\varepsilon_x) + \frac{C}{1-\nu^2}(k_\theta + \nu k_x) \\
N_{x\theta} &= \frac{A}{2(1+\nu)}\gamma_{x\theta} + \frac{C}{1+\nu}k_{x\theta} \\
M_x &= \frac{C}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_\theta) + \frac{B}{1-\nu^2}(k_x + \nu k_\theta) \\
M_\theta &= \frac{C}{1-\nu^2}(\varepsilon_\theta + \nu\varepsilon_x) + \frac{B}{1-\nu^2}(k_\theta + \nu k_x) \\
M_{x\theta} &= \frac{C}{2(1+\nu)}\gamma_{x\theta} + \frac{B}{1+\nu}k_{x\theta}
\end{aligned} \tag{8}$$

where

$$(A, B, C) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, z^2) E(z) dz \tag{9}$$

The total potential energy V for the cylindrical shell under mechanical loading may be written as

$$V = \frac{1}{2} a \iiint (\sigma_x \varepsilon_x + \sigma_\theta \varepsilon_\theta + \tau_{x\theta}\gamma_{x\theta}) dx d\theta dz - pa \int w dx d\theta \tag{10}$$

where p is the external lateral pressure load. Upon substitution of Eqs. (5) and Eq. (6) into Eq. (10), and integrating respect to z (from $-h/2$ to $h/2$), the functional of potential energy is obtained. Using Euler equations, the equilibrium equations for a stiffened cylindrical shell composed of functionally graded material are given by

$$\begin{aligned}
aN_{x,x} + N_{x\theta,\theta} &= 0 \\
aN_{x\theta,x} + N_{\theta,\theta} &= 0
\end{aligned} \tag{11}$$

$$aM_{x,xx} + 2M_{x\theta,x\theta} + \frac{1}{a}M_{\theta\theta,\theta\theta} - N_\theta - aN_x\beta_{x,x} - N_{x\theta}(a\beta_{\theta,x} + \beta_{x,\theta}) - N_\theta\beta_{\theta,\theta} = -pa$$

The stability equations of cylindrical shell may be derived by the variational approach. If V is the total potential energy of the shell, the first variation δV is associated with the state of equilibrium. The stability of the original configuration of the shell in the neighborhood of the equilibrium state can be determined by the sign of second variation $\delta^2 V$. However, the condition of $\delta^2 V$ =0 is used to derive the stability equations of many practical problems on the buckling of shell. Thus, the stability equations are represented by the Euler equations for the integrand in the second variation expression [8]

$$
\begin{aligned}
&aN_{x1,x} + N_{x\theta1,\theta} = 0 \\
&aN_{x\theta1,x} + N_{\theta1,\theta} = 0 \\
&aM_{x1,xx} + 2M_{x\theta1,x\theta} + \frac{1}{a}M_{\theta1,\theta\theta} - N_{\theta1} - \left[aN_{x0}\beta_{x1,x} + N_{x\theta0}\left(a\beta_{\theta1,x} + \beta_{x1,\theta}\right) + N_{\theta0}\beta_{\theta1,\theta}\right] = 0 \qquad (12)
\end{aligned}
$$

In order to get some insight respect to the buckling analysis, the stability equation must be writing down in term of displacement field. Thus, by substituting Eq. (5) into Eq. (8) and substituting Eq.(8) into Eq. (12) give the relation as

$$
\begin{aligned}
&\frac{aA}{1-\nu^2}\left(u_{1,xx} + \nu\left(\frac{v_{1,\theta x} + w_{1,x}}{a}\right)\right) + \frac{aC}{1-\nu^2}\left(k_{x1,x} + \nu k_{\theta1,x}\right) + \frac{A}{2(1+\nu)}\left(\frac{u_{1,\theta\theta}}{a} + v_{1,x\theta}\right) + \frac{C}{1+\nu}k_{x\theta1,\theta} = 0 \\
&\frac{A}{2(1+\nu)}\left(\frac{u_{1,\theta x}}{a} + v_{1,xx}\right) + \frac{C}{1+\nu}k_{x\theta1,x} + \frac{A}{1-\nu^2}\left(\left(\frac{v_{1,\theta\theta} + w_{1,\theta}}{a}\right) + \nu u_{1,x\theta}\right) + \frac{C}{1-\nu^2}\left(k_{\theta1,\theta} + \nu k_{x1,\theta}\right) = 0 \\
&\frac{aC}{1-\nu^2}\left(u_{1,xxx} + \nu\left(\frac{v_{1,\theta xx} + w_{1,xx}}{a}\right)\right) + \frac{aB}{1-\nu^2}\left(k_{x1,xx} + \nu k_{\theta1,xx}\right) + \frac{2B}{1+\nu}k_{x\theta1,x\theta} + \frac{C}{1+\nu}\gamma_{x\theta1,x\theta} \\
&+\frac{1}{a}\frac{C}{1-\nu^2}\left(\left(\frac{v_{1,\theta\theta\theta} + w_{1,\theta\theta}}{a}\right) + \nu u_{1,x\theta\theta}\right) + \frac{1}{a}\frac{B}{1-\nu^2}\left(k_{\theta1,\theta\theta} + \nu k_{x1,\theta\theta}\right) - \frac{A}{1-\nu^2}\left(\left(\frac{v_{1,\theta} + w_1}{a}\right) + \nu u_{1,x}\right) \qquad (13) \\
&-\frac{C}{1-\nu^2}\left(k_{\theta1} + \nu k_{x1}\right) - \left[aN_{x0}\beta_{x1,x} + N_{x\theta0}\left(a\beta_{\theta1,x} + \beta_{x1,\theta}\right) + N_{\theta0}\beta_{\theta1,\theta}\right] = 0
\end{aligned}
$$

In Eqs. (12) and Eqs (13), terms with the subscript 0 are related to the state of equilibrium and terms with the subscript 1 are those characterizing the state of stability. Also, the superscript 1 in the displacement component w_0^1 refers to the state of stability.

4. Generalized form of constitutive equations

For a shell-wall construction that is not symmetrical relative to the shell middle surface, there is a coupling between extensional forces and curvature change and between bending moments and extensional strain. To account for this coupling effect the constitutive equations for this construction may be generalized to the form

$$
\begin{aligned}
N_x &= C_{11}\varepsilon_x + C_{12}\varepsilon_\theta + C_{14}k_x + C_{15}k_\theta \\
N_\theta &= C_{12}\varepsilon_x + C_{22}\varepsilon_\theta + C_{24}k_x + C_{25}k_\theta \\
N_{x\theta} &= C_{33}\gamma_{x\theta} + C_{36}k_{x\theta} \\
M_x &= C_{14}\varepsilon_x + C_{24}\varepsilon_\theta + C_{44}k_x + C_{45}k_\theta \\
M_\theta &= C_{15}\varepsilon_x + C_{25}\varepsilon_\theta + C_{45}k_x + C_{55}k_\theta \\
M_{x\theta} &= C_{63}\gamma_{x\theta} + C_{66}k_{x\theta}
\end{aligned} \tag{14}
$$

where the stiffness parameters C_{ij} are given by

$$
\begin{aligned}
C_{11} &= \frac{A}{1-\nu^2} + \frac{E_s h_s b_s}{d_s} & C_{33} &= \frac{A}{2(1+\nu)} \\
C_{12} &= \frac{\nu A}{1-\nu^2} & C_{36} &= \frac{C}{1+\nu} \\
C_{14} &= \frac{C}{1-\nu^2} & C_{44} &= \left(\frac{B}{1-\nu^2} + \frac{E_s I_s}{d_s}\right) \\
C_{15} &= \frac{\nu C}{1-\nu^2} & C_{45} &= \frac{\nu B}{1-\nu^2} \\
C_{22} &= \frac{A}{1-\nu^2} + \frac{b_r}{d_r} h_r E_r & C_{55} &= \frac{B}{1-\nu^2} + \frac{E_r I_r}{d_r} \\
C_{24} &= \frac{\nu C}{1-\nu^2} & C_{63} &= \frac{C}{2(1+\nu)} \\
C_{25} &= \frac{C}{1-\nu^2} & C_{66} &= \frac{B}{1+\nu} + \frac{1}{2}\left(\frac{G_s j_s}{d_s} + \frac{G_r j_r}{d_r}\right)
\end{aligned} \tag{15}
$$

where

$$
A = \frac{h}{2}(E_m + E_c) \quad , \quad B = \frac{h^3}{24}(E_m + E_c) \quad , \quad C = \frac{h^2}{12}(E_c - E_m)
$$

Here E_r and E_s are the Young΄s modulus for rings and stringers respectively. G_r and G_s are the shear modulus for rings and stringers respectively. The stiffened cylindrical shell parameters are as follow

$$
\begin{aligned}
a &= 0.24m \\
l &= 8.58m \\
h &= 7.19\times10^{-4}m \\
h_s &= h_r = 7.62\times10^{-3}m \\
b_s &= b_r = 2.46\times10^{-3}m \\
d_s &= d_r = 2.54\times10^{-2}m
\end{aligned}
$$

Here a is the cylinder radius, l the length of the cylinder, h the cylindrical shell thickness, h_s and h_r are the thicknesses of the stringers and rings respectively, b_s and b_r are the widths of the stringers and rings respectively, d_s and d_r the distances between two stringers and rings respectively and the eccentricities e_s and e_r represent the distance from the shell middle surface to the centroid of the stiffener cross section.

5. Buckling analysis

Consider a FGM stiffened cylindrical shell with simply supported edge conditions and subjected to uniform external lateral pressure p. To find the critical buckling lateral pressure p, the prebuckling forces should be found from the equilibrium equations and then substituted into the stability equations for the buckling analysis. Solving the membrane form of equilibrium equations, and using the method developed by Myers [26] in conjunction with Galerkin's formulation, gives the prebuckling force resultants

$$N_{x\theta 0}=0 \quad , \quad N_{\theta 0}=0 \quad , \quad N_{x0}=-\frac{p}{2\pi a} \tag{16}$$

The simply supported boundary condition is defined as [11]

$$w_1=w_{1,xx}=v_1=u_{1,x}=0 \qquad x=0,l \tag{17}$$

To solve the system of Eqs. (13), with the consideration of the boundary conditions (17), the approximate solutions are assumed as

$$\begin{aligned} u_1 &= A_1 Cos\overline{m}xSinn\theta \\ v_1 &= B_1 Sin\overline{m}xCosn\theta \\ w_1 &= C_1 Sin\overline{m}xSinn\theta \end{aligned} \tag{18}$$

where (A_1, B_1, C_1) are constant coefficients and $\overline{m}=m\pi/l$, where $m=1,2,3,\ldots$and $n=1,2,3,\ldots$. Substituting relations (18) into the stability Eqs. (13) yield a system of three homogeneous equations for (A_1, B_1, C_1) that is,

$$\begin{aligned} a_{11}A_1+a_{12}B_1+a_{13}C_1&=0 \\ a_{12}A_1+a_{22}B_1+a_{23}C_1&=0 \\ a_{13}A_1+a_{23}B_1+\left(a_{33}-\overline{p}\right)C_1&=0 \end{aligned} \tag{19}$$

where

$$\begin{aligned} a_{11}&=C_{11}\overline{m}^2+C_{33}n^2 \qquad a_{12}=\left(C_{12}+C_{33}\right)\overline{m}n \\ a_{13}&=\left(C_{12}\overline{m}-\frac{C_{14}}{a}\overline{m}^3-\frac{1}{a}\left(C_{15}+C_{36}\right)\overline{m}n^2\right) \\ a_{22}&=C_{33}\overline{m}^2+C_{22}n^2 \end{aligned} \tag{20}$$

$$a_{23}=\left(C_{22}n-C_{25}\frac{n^3}{a}-\frac{1}{a}(C_{36}+C_{24})\overline{m}^2 n\right)$$

$$a_{33}=C_{44}\frac{\overline{m}^4}{a^2}+\frac{2}{a^2}(C_{45}+C_{66})\overline{m}^2 n^2+\frac{1}{a^2}C_{55}n^4+C_{22}-2C_{25}\frac{n^2}{a}-\frac{C_{24}}{a}\overline{m}^2-C_{24}\overline{m}^2$$

which a_{ij} is a symmetric matrix. By setting $|a_{ij}|=0$ to obtain the nonzero solution, the value of p is found.

$$\frac{p}{2\pi a}=\frac{a_{33}}{\overline{m}^2}+\frac{2a_{12}a_{23}a_{13}-a_{22}a_{13}^2-a_{11}a_{23}^2}{(a_{11}a_{22}-a_{12}^2)\overline{m}^2} \quad (21)$$

The critical buckling load is obtained by minimizing p with respect to m and n, the number of longitudinal and circumferential buckling waves .

6. Results and discussions

This paper deal with the buckling analysis of functionally graded stiffened cylindrical shell with isotropic rings and stringers .A ceramic-metal functionally graded cylindrical shell with isotropic rings and stringers is considered. The combination of materials consists of steel and alumina. The Young̊ s modulus for the steel and alumina are: $E_m=68.948$ GPa and $E_c=374.2911$ GPa, respectively. The Poisson's ratio is chosen to be 0.3 for steel and alumina.
Comparisons of the critical buckling loads for the isotropic cylindrical shell and isotropic stiffened cylindrical shell are presented in Table (1). In this table for the case of isotropic cylindrical shell, it is assumed that $k=0$. Table 1 shows that the buckling pressure increases when the cylindrical shell stiffened and increases by the increasing of the thickness h. Comparisons of the critical buckling loads for the functionally graded cylindrical shell and functionally graded stiffened cylindrical shell with isotropic rings and stringers are presented in Tables (2). Table 2 shows that the buckling pressure increases when the cylindrical shell stiffened and increases by the increasing of the thickness h. Comparisons of the critical buckling loads for the functionally graded cylindrical shell and isotropic stiffened cylindrical shell with isotropic rings and stringers are presented in Tables (3). Table 2 shows that the buckling pressure for functionally graded stiffened cylindrical shell is generally upper than the corresponding value for the isotropic stiffened cylindrical shell.

Table 1
The Effect of rings and stringers on the critical buckling load of isotropic cylindrical shell

h ($\times 10^{-3}$m)	Stiffened with rings and stringers ($\times 10^9$N)	without rings and stringers ($\times 10^9$N)
0.1	2.40	0.013
0.2	3.35	0.016
0.4	4.81	0.065
0.6	6.068	0.15
0.7	6.76	0.21
0.8	7.22	0.26
1	8.26	0.40

1.2	8.76	0.58
2	10.92	1.62
4	18.15	6.80
6	29.52	14.71
8	46.82	28.05
10	63.96	48.62

Table 2
The Effect of rings and stringers on the critical buckling load of functionally graded cylindrical shell

h ($\times10^{-3}$m)	Stiffened with rings and stringers ($\times10^9$N)	without rings and stringers ($\times10^9$N)
0.1	4.25	0.078
0.2	6.29	0.22
0.4	8.66	0.59
0.6	10.62	1.02
0.7	11.97	1.31
0.8	12.56	1.53
1	13.67	2.15
2	21.41	7.48
4	55.04	35.78
6	299.72	101.13
8	252.41	219.72
10	412.78	405

Table 3
Comparisons of the critical buckling loads for the functionally graded stiffened cylindrical shell and isotropic stiffened cylindrical shell [24]

h ($\times10^{-3}$m)	isotropic stiffened cylindrical shell ($\times10^9$N)	functionally graded stiffened cylindrical shell ($\times10^9$N)
0.1	2.40	4.25
0.2	3.35	6.29
0.4	4.81	8.66
0.6	6.068	10.62
0.7	6.76	11.97
0.8	7.22	12.56
1	8.26	13.67
2	10.92	21.41
4	18.15	55.04
6	29.52	299.72
8	46.82	252.41
10	63.96	412.78

The critical buckling load of cylindrical shell and stiffened cylindrical shell for isotropic and functionally graded materials is shown in Fig. 2. This Figure shows that the critical buckling load increases when the cylindrical shell or stiffened cylindrical shell is made of functionally graded materials. Also, this figure shows that the critical buckling load increases when the functionally graded or isotropic cylindrical shell is stiffened with rings and stringers.

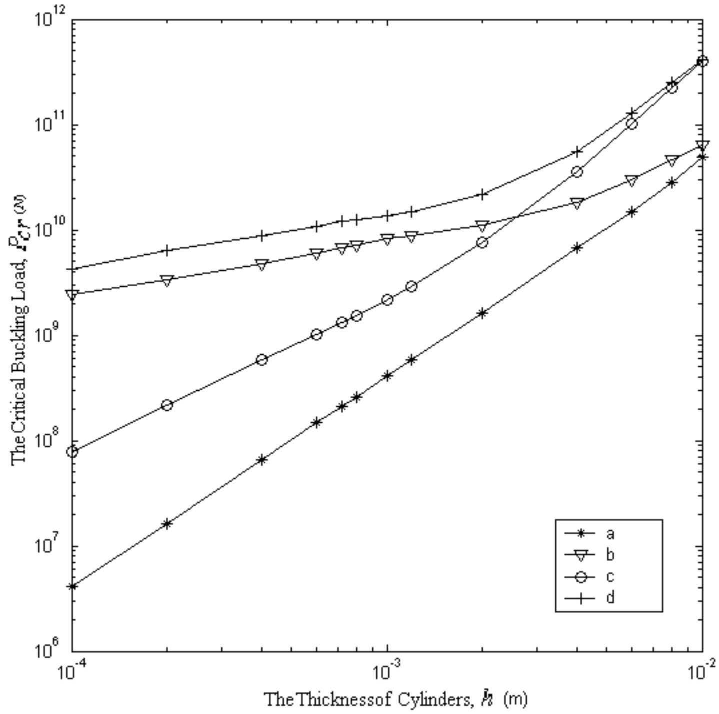

Fig. 2. Critical buckling load for cylinders subjected to external lateral pressure load versus to thickness of cylinders (h) a) Isotropic cylindrical shell; b)Isotropic stiffened cylindrical shell with isotropic rings and stringers; c) FGM cylindrical shell; d) FGM stiffened cylindrical shell with isotropic rings and stringers.

7. Conclusions

In the present paper, equilibrium and stability equations for simply supported functionally graded stiffened cylindrical shells are obtained. Then, the buckling analysis of functionally graded cylindrical shells under uniform external lateral pressure is investigated. It is conclude that:

1. The critical buckling pressure for functionally graded stiffened cylindrical shell is generally upper than the corresponding value for the isotropic cylindrical shell.
2. The critical buckling pressure for stiffened functionally graded cylindrical shell is generally upper than the corresponding value for the functionally graded cylindrical shell.
3. The critical buckling pressure for stiffened isotropic cylindrical shell is generally upper than the corresponding value for the isotropic cylindrical shell.
4. The critical buckling pressure for functionally graded stiffened cylindrical shell is increased by increasing the thickness h.

References

[1] M. Yamanouchi, M. Koizumi, I. Shiota, Proceeding of the first international symposium on functionally gradient materials, Sendai, Japan, 1990.
[2] S. Suresh, A. Mortensen, Fundamentals of functionally graded materials. London,Barnes and Noble Pub, 1998.
[3] S.P. Timoshenko, J.M. Gere, Theory of elastic stability ,New York,McGraw-Hill,1961.
[4] P.S. Bulson, The stability of flat plates, London, Chatto and Windus, 1970.
[5] R. Lorenz, Achsensymmetrishce Verzerrungen in dunnwandingen Hohlzylindern, Zeitschrift des Vereines Deutscher Ingenieure 52 (1908) 1706-13.
[6] Column Research Committee of Japan, Handbook of structural stability, Tokyo, Corona, 1971.
[7] S.P. Timoshenko, S. Woinowsky-Krieger, Theory of Plates and shells, New York, McGraw-Hill, 1959.
[8] D.O. Brush, B.O. Almroth, , Buckling of Bars,plates and shells ,1975.
[9] L.H. Donnell, Beams, Plates and shells, New York, McGraw-Hill, 1976.
[10] C.R. Calladine, Theory of shell structures, London,Cambridge University Press,1983.
[11] N. Yamaki, Elastic stability of circular cylindrical shells, Amesterdam, North Holland, 1984.
[12] Z.P. Bazant, Cedolin L, Stability of structures, New York, Oxford University Press, 1991.
[13] V. Birman,. Buckling of functionally graded hybrid composite plates. Proceeding of the 10th Conference on Engineering Mechanics 2 (1995) 1199-1202.
[14] M. M. Najafizadeh, M. R. Eslami,. Buckling analysis of circular plates of functionally graded materials under uniform redial compression. Journal of Mechanical. Science 4 (2002a) 2479-2493.
[15] M. M. Najafizadeh, M. R. Eslami, First-order-theory-based thermoelastic stability of functionally graded material circular plates. AIAA Journal 40 (2002b) 1444-1450.
[16] M. M. Najafizadeh, B. Hedayati, Refined theory for thermoelastic stability of functionally graded circular plates. Journal of Thermal Stresses 27 (2004) 857-880.
[17] HS Shen, Postbuckling analysis of axially-loaded functionally graded cylindrical shells in thermal environments. Composite Science and Technology 62 (2002) 977-987.
[18] HS Shen, Postbuckling analysis of pressure-loaded functionally graded cylindrical shells in thermal environments. Engineering Structure 25 (2003) 487-497.
[19] HS Shen, PZhou and TY Chen, Buckling and Postbuckling of Stiffened Cylindrical shells under axial compression. Appl. Math Mech. 12, 1195 (1991)
[20] HS Shen, TY Chen, Buckling and Postbuckling of cylindrical shells under combined external pressure and axial compression. Thin Walled Structures 12,321 (1991).
[21] J. Woo, S.A. Meguid, Nonlinear analysis of functionally graded plates and shallow shells. International Journal of Solids and Structures 38 (2001) 7409-7421.
[22] T.Y. Ng, Y.K. Lam, K.M. Liew, J.N. Reddy, Dynamic stability analysis of functionally graded cylindrical shell under periodic axial loading. International Journal of Solids and Structures 38 (2001) 1295-1300.
[23] A. Van Der Neut, ,General instability of stiffened cylindrical shell under axial compression, National Luchtraart-Laboratorium, Amsterdam,Report S- 314, REP Trans 13 (1947) 57-84.
[24] M. Barush, Singer Effect of Eccentricity of Stiffeners on the General Instability of Stiffened Cylindrical Shells under Hydrostatic Pressure. Journal of Mechanical Engineering science 5 (1963) 23-27.

[25] J.N. Reddy, G.N. Praveen, Nonlinear Transient thermoelastic analysis of functionally graded ceramic-metal plates, *In. J.* Solids and Struct 35 (1998) 4467-4476.

[26] C. A. Meyers, M. W. Hyer, Thermal buckling and postbuckling of symmetrically laminated composite plates. Journal of Thermal Stresses 14 (1991) 519-540.