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Simulation of water surface profile in vertically stratified rockfill dams

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ABSTRACT

Detention rockfill dams are accounted as economically efficient structures for flood control, river bed and banks protection, flow diversion, etc. As the hydraulic behavior of these structures, when are used for flood control, is affected by the depth of water in their porous media, there is interests to predict water surface profile through the body of these structures. In this research, we developed a numerical model for prediction of the water surface profile in heterogeneous (stratified) detention rockfill dams. The new model is a modified form of gradually varied flow (GVF) equation which has been solved by direct step method and can also be applied to the flood routing. To validate the numerical model, a set of laboratory experiments has been conducted and the results were compared with those provided using the numerical model. As the maximum mean error is determined as 17.6%, it is found that the introduced model gives satisfactory results and it can be used to determine the water surface profile, and consequently, computing flood routing.

Introduction

Detention rockfill dams are considered as efficient structures for multi purposes in river engineering issues. Rockfill dams are simply made of rocks without any impervious core, so they are economical where the rock materials are available near the dam site.

Rockfill dams are constructed in steep mountains, as single or successive structures, along rivers to reduce hydraulic gradient and consequently, decrease downstream erosion. Moreover, they are built in plain or moderately steep areas for flood control, flow diversion, river bed and banks protection, etc. So, they are considered as check structures to stabilize the river banks. The most advantage of these structures is to allow the sediments, both bed and suspended carrying by flow, are able to pass through the porous body of the dam which results in decreasing the impact of the structures on downstream; i.e. as there is no considerable change between the amount of the sediments transported by flow at upstream and downstream of the dam, there is not a considerable erosion at the dam

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downstream. However, the advantage gradually is disappearing when some of the sediment particles trapped among the dam materials results in the clogging of the pours so that a part of sediments come to rest at the upstream of the dam. As a result, the flow passing over the dam could cause erosion downstream to fulfill its sediment transport capacity.

Furthermore, rockfill dams provide a reservoir upstream to decrease the peak of the hydrograph as well. However, the volume of the reservoir is decreasing as the sediment start to be trapped inside and upstream outside of the dam results in increasing river bed elevation and vanishing dam reservoir. To avoid this and to increase the amount of the bed load transporting through dam body, a vertical stratification can be assumed by considering a coarser material for the bottom layer of the dam.

As flood control is one of the purposes of constructing of rockfill dams, the flood routing is accounted as an important issue in analyzing the effects of these structures on flood control. The flood routing computation, however, needs to understand the steady flow condition as initial condition. Hence, the water surface profile within the dam body is needed to start the computation of flood routing, as well as to perform the stability analysis of downstream slope of these structures.

From hydraulically point of view, the behavior of flow in coarse porous media is not restricted to rockfill dams, but it is observed in different applications; from gabion weirs to a long valley filled by stones caused by mining operations (Fig. 1). As a result, many researches have been carried out to investigate the water surface profile within the porous material at different conditions of flow depths at upstream and downstream of the structures.

Insert Fig.1 here

Hosseini and Joy (1997) developed an unsteady non-linear model to analyze the flow through coarse porous media. The developed model by Hosseini and Joy, however, was limited for relatively long rockfill structures and can only be applied for 1-D cases.

Bari and Hansen (2002) used a gradually-varied flow algorithm to simulate the water surface profile for 1-D non-Darcy flow through homogenous porous medium in buried streams. Having evaluated with laboratory experiments, Bari and Hansen (2002) found that the prediction of the model was reasonable. Moreover, they evaluated the model via three different friction-slope averaging methods; i.e. the arithmetic, geometric and harmonic averages. They found that all averaging methods result in a satisfactory flow profile (Bari and Hansen, 2002).

As stated above, a coarser layer of rocks can be laid or even deposited over the river bed to increase the conductivity of the dam material. However, such alteration in structure of dams prohibits the application of classic methods of flow analysis including water surface profile and semi-theoretical stage-discharge relations recommended by Shayan-nejad and Samani (2004). This issue makes great demand for developing appropriate models of water surface prediction for multi-layered rockfill dams.

2. Flow Dynamic Equation

quations describing the behavior of the flow in fine porous media, often neglect the effect of inertia; e.g. Richards 1931's equation that simplified to Laplace Equation by considering some assumptions. However, for flow through coarse materials, flow inertia can not be

negligible. Therefore, due to resemblance between the relatively fast flow within the coarse material and the flow in open channels, dynamic equation of gradually varied flow (GVF) is suggested by some researchers in order to analyzing the flow in porous media under non-Darcy condition (Parkin, 1991; Stephenson, 1979; Hansen, 2002).

The GVF equation in a channel having no lateral inflow or outflow is derived by the following assumptions (Chudhary, 2008):

1. The slope of channel bottom is small.

2. The channel is prismatic.

3. The pressure distribution is hydrostatic at all sections of channel.

4. The head loss in gradually-varied flow may be determined by using the equations used for uniform flows.

GVF equation can be written as below.

$$\frac{dy}{dx} = \frac{S_0(x) - S_f(x,y)}{1 - Fr_p^2(y)} \qquad (1)$$

Where x is distance along the channel (m); y is vertical depth of water (m); $S_0(x)$ is channel slope (-); $S_f(x,y)$ is friction slope or hydraulic gradient (-); and Fr_p is Pore Froude number (-). Owing to the stipulated assumptions, it should be considered that using this method for rockfill structures in steep-mountainous rivers (steepness more than 5 percent) may lead to discrepancy between computational and real profile. Fig 2 shows a schematic longitudinal section of a rockfill structure with two random streamlines manifested that the streamlines find their way between rocks and pass through a serpentine path. The frequent changes of direction by the streamlines violates the assumption of 1-D flow, and consequently deviate the

pressure distribution from hydrostatic state as the velocity head in vertical direction would be considerable in some points while it is negligible in some other points as well. This phenomenon is exacerbated by growth in size of the rocks and can be a major source of error.
Insert Fig. 2 here

Furthermore, the head losses in coarse porous media are no longer governed by the roughness of the stream bed, but by the characteristics of coarse porous media (Bari and Hansen 2002). As a result, the equations of Non-Darcy regime should be extended to analyze the various hydraulic features of flow; e.g. water surface profile.

Baska (1997) reviewed a number of studies reported in the soil science literature and developed a classification scheme for flow regimes, shown in Fig. 3 in its modified form. Baska (1997) identifies three main zones as pre-Darcy zone, where the increase of the flow velocity can be larger than proportional to the increase of fluid pressure gradient, Darcy zone, where fluid flow is laminar and Darcy's law holds its validity and the fluid velocity is directly proportional to the applied gradient, and finally, Non-Darcy zone, where the increase of fluid velocity is smaller than proportional to the increase of fluid pressure gradient (Mancini et al., 2011).
Insert Fig. 3 here

Many laboratory and numerical studies have been conducted for determination of the upper range of validity of Darcy's law. Customarily, this limit has been signified by means of a critical value for the Reynolds number (Re) beyond which the head gradient is no longer proportional to the flow velocity. Critical values of Re at the onset of nonlinear flow, according to most experiments, range between 1 and 15 (Hassanizadeh and Gray, 1987).

Since Reynolds number of flow in detention rockfill dams is extremely over 15, the flow would be certainly classified as Non-Darcy. So, it is necessary to find a substitute equation which sits in the GVF dynamic equation instead of friction slope (S_f), and describes the slope of the curvature of turbulent zone in Fig. 3.

Forchheimer (1901) was the first who proposed a quadratic equation to relate hydraulic gradient to the flow velocity. The quadratic equation is made up by adding a velocity squared term to the Darcy equation and usually written as follows (Mancini et al., 2011):

$$-gradP = \frac{\mu}{k}\vec{v} + \beta\rho\vec{v}^2 \qquad (2)$$

In which $-gradP$ is friction angle (-); μ is fluid dynamic viscosity (kg/m.s); k inherent permeability of the porous medium (m/s); $\vec{v}$ = flow velocity (m/sec); β = inertial factor depends on the characteristics of medium (-); and ρ = fluid density (kg/m^3).

Various studies attempted to formulate coefficients of velocity terms in Equation (2). One of the simple and widely used forms of Forchheimer 1901's equation has been presented by Stephenson (1979) as below:

$$s_f = \frac{800\upsilon}{gnd^2}v + \frac{k_t}{n^2gd}v^2 \qquad (3)$$

Where s_f is hydraulic gradient (-); υ is kinematic viscosity (m^2/s); g is gravitational acceleration (m/s^2); n is

porosity (-); d is mean diameter of particles (m); v is bulk velocity:(m/sec); and k_t is friction factor in the turbulent region of flow (ranging from 1 for polished spheres to 4 for angular and crushed stone).

By analogy to flow in conduits Stephenson 1997's equation can be rewritten as:

$$s_f = \frac{k_{st}v^2}{n^2gd} \qquad (4)$$

Where $k_{st} = \frac{800}{Re} + k_t$ and $Re = Pore$

Reynolds number $= \frac{vd}{n\upsilon}$

3. Development of the numerical model

The general form of the equation of GVF is a first-order ordinary differential equation while several algorithms to compute water surface profile have been introduced in literature. In this study, we utilized the classic direct step method in which the method starts the calculations using a control section, in which flow depth and other hydraulic specifications are known, and by assuming an optional but reasonable depth, the longitudinal distance between the control section and the location where the assumed depth is occurred, is calculated as follows. It should be noted that in the following algebraic statements, subscript 1 and 2 denote successive sections respectively.

$$\frac{y_2 - y_1}{\Delta x} = \frac{S_0 - \frac{1}{2}(S_{f1} + S_{f2})}{1 - \frac{1}{2}(Fr_{p1}^2 + Fr_{p2}^2)} \qquad (5)$$

Or

$$\frac{y_2 - y_1}{\Delta x} = \frac{S_0 - \frac{1}{2}((\frac{(\frac{800}{Re} + k_t)\frac{q^2}{y^2}}{n^2gd})_1 + (\frac{(\frac{800}{Re} + k_t)\frac{q^2}{y^2}}{n^2gd})_2)}{1 - \frac{1}{2}(Fr_{p1}^2 + Fr_{p2}^2)} \qquad (6)$$

Where Fr_{p1} and Fr_{p2} are pore Froude number in sections 1 and 2, respectively. From Eq. (6) it can be seen that the right hand side of the equation is a function of depth and the characteristics of the media in each section of 1 and 2. Moreover, as the characteristics of each section are a function of the characteristics of both bottom layer (denoted by ') and top layer (denoted by ″) as shown in fig 4, averaging is needed in each section so that Eq. (6) can be written as following equation:

Insert Fig. 4 here

$$\frac{y_2 - y_1}{\Delta x} = \frac{S_0 - \frac{1}{2}\left(\frac{\left(\left(\frac{800}{q\frac{d'y'+d''y''}{y}}{\upsilon y \frac{n'y'+n''y''}{y}}\right)+k_t\right)\frac{q^2}{y^2}}{g\frac{d'y'+d''y''}{y}\left(\frac{n'y'+n''y''}{y}\right)^2}\right)_1}{1 - \frac{1}{2}\left(\left(\frac{q}{\frac{n'y'+n''y''}{y}y\sqrt{gy}}\right)_1 + \left(\frac{q}{\frac{n'y'+n''y''}{y}y\sqrt{gy}}\right)_2\right)} -$$

$$\frac{\frac{1}{2}\left(\frac{\left(\left(\frac{800}{q\frac{d'y'+d''y''}{y}}{\upsilon y \frac{n'y'+n''y''}{y}}\right)+k_t\right)\frac{q^2}{y^2}}{g\frac{d'y'+d''y''}{y}\left(\frac{n'y'+n''y''}{y}\right)^2}\right)_2}{1 - \frac{1}{2}\left(\left(\frac{q}{\frac{n'y'+n''y''}{y}y\sqrt{gy}}\right)_1 + \left(\frac{q}{\frac{n'y'+n''y''}{y}y\sqrt{gy}}\right)_2\right)} \qquad (7)$$

Equation (7) can be used to estimate the amount of longitudinal distance (Δx) between two successive sections in which the depth and other relevant parameters should be known at one section (known as control section at the starting reach) while a reasonable depth should be assumed for the other section so that by computing the required parameters, Δx can be determined. The key point in this procedure is to find the control section to start the calculation, so that many studies have been carried out in this regard.

It should be noted that for the case of rockfill dams, due to formation of a reservoir upstream of the dam structure, the approaching flow enters the porous medium is subcritical, so it is controlled by downstream. Moreover, discharge and flow depth have a special relation in control section.

Solvik (1966) and Leps (1973) based on observations of model test concluded that the exit depth occurred where the slope of the energy grade line is equal to the slope of the downstream face; i.e.

$$s_f = \tan(\beta) \qquad (8)$$

And regarding Eq. (4) it can be written as:

$$\frac{k_{st}v^2}{n^2 gd} = \tan(\beta) \qquad (9)$$

Where β is angle of the downstream slope. Parkin (1991) proposed a slight

modification to Leps (1973) model, and proposed that the tangent of the downstream slope could be replaced by the sine of the downstream slope. Stephenson (1978) assumed that the critical-flow condition exists at the exit point so the equations relevant to critical condition can be used for this point. Hansen (1992) based on results from extensive flume studies which conducted on different rockfills, concluded that the considering critical depth at the exit section by Stephenson (1978) gives better results than other available models.

Sedghi asl et al. (2010) carried out a series of experiments on rock drains of quasi-spherical and angular rocks. They found a considerable discrepancy between the exit depth and critical depth (Fig. 5) therefore they proposed empirical stage-discharge equations for prediction of exit depth.

Insert Fig. 5 here

To test the accuracy of Eq. (7), we constructed a physical model of rockfill dams in a laboratory flume and performed a series of tests. In the following section, the specifications of the physical model and the tests to validate the equation are described.

4. Experimental setup

To perform the tests, we used a glass-walled flume with a length of 5 m and width of 0.5 m at the Research Center of Water, Department of Irrigation and Reclamation Engineering, University of Tehran. We constructed four models of rockfill dams (Table 1) and applied 3 different discharges of 2.06, 2.73, and 4.84 lit/sec over each model. To observe the water depth through the porous medium of the models, 13 piezometers installed under nearly 1.00 m length of the dam in approximate distances of 0.09m. In

Fig. 6, a schematic view of the experimental setup is shown.
Insert Table 1 here
Insert Fig. 6 here

All rocks used for the experiments were fluvial rocks with round corners. A wire-frame net has been embedded between two layers to prevent vertical migration of finer rocks to the relatively big pores of the coarse sub-layer.

5. Model performance, observations and discussion
A) Exit depth
In Fig. 7 the observed magnitude of exit depth versus discharge are shown. Moreover, the amount of critical depth is shown in Fig. 7 as dashed line. As shown in Fig. 7, the magnitude of exit depth depends on the specifications of the rock materials and there are some disagreements between the exit and critical depths for the same amount of discharge. These results are different from those reported by previous researches which concluded that the exit depth was independent from the porous media characteristics. Moreover, based on previous studies, the exit depths are always bigger than critical depth while according to observation at the present study, super critical out-flow occurred during some tests and hence, the exit depth is less than critical depth. All exit depths provided from tests in this research, have been used as control depths (boundary condition) to start the calculation of water surface profile through the rockfill dams.
Insert Fig. 7 here

B) Simulated and observed profiles
To validate the improved numerical model (Eq. 7) for simulation of water surface profile in multi-layered porous media with

short length where the flow is 2D, the observed and computed water surface profiles are shown in Fig. 8. Note that the magnitude of Pore Reynolds Number (PRN) at all sections with known water depth were computed and found as varies from 963 to 29447 indicating the flow is non-Darcy. Moreover, as the magnitude of PRNs are too much bigger than the value corresponds to start the nonlinear flow, the probable difference between the observed and computed results can not be attributed to the lack of fully developed turbulent flow.

As shown in Fig. 8, depth of flow is generally decreasing toward the exit section in porous media. Furthermore, laboratory observations show that the depth profile intersects the critical depth, in some cases, and becomes less than critical depth when intersects the downstream slope. So, in these cases control section does not exit and to start the surface water profile calculations, a reasonable depth such as critical depth have to be assumed which in turn may cause some errors on predicting water surface profile.

The flow outgoing the rockfill dams is not usually free, it should be noted. If the normal depth of downstream channel exceeds the out-flow depth, the exit section of the rockfill dam would be submerged and hence, the normal depth of downstream flow is accounted as the control depth (or boundary condition) and water surface profile computations begin from this section toward upstream.
Insert Fig. 8 here

Fig. 8 also shows that there is a reasonable agreement between predicted and observed water surface profiles. The difference between two sets of data, however, is because of the point that the critical depth, which is more than real

depth at exit section in most of experiments, is assumed as the control depth. This assumption resulted in higher water surface profiles provided by Eq. (8) than those observed from laboratory experiments. To indicate the agreement quantitatively, we used index of Mean Relative Error (MRE) defined as:

$$MRE = 100(\frac{\sum_{i=1}^{n}(\frac{y_{comp} - y_{exp}}{y_{exp}})_i}{n}) \qquad (10)$$

Where is number of data; y_{comp} is computational depth; and y_{exp} is observed depth in the laboratory. In Table 2 the magnitude of MRE for all codes are indicated with the maximum of 17.6% demonstrates the point that the numerical model is able to predict water surface profile for flow passing through rockfill dams.
Insert table 2 here

Nomenclature

X	distance along the channel (m)
Y	vertical depth of water (m)
$S_0(x)$	channel slope
$S_f(x,y)$	friction slope
K	inherent permeability of the porous medium (m/s)
$\vec{v}$	flow velocity (m/s)
s_f	hydraulic gradient
υ	kinematic viscosity (m²/s)
g	gravitational acceleration (m/s²)
n	porosity
d	mean diameter of particles (m)
v	bulk velocity:(m/s)
k_t	action factor in the turbulent region of flow
β	inertial factor
ρ	fluid density (kg/m³).
Fr_p	pore Froude number
$gradP$	friction angle
μ	fluid dynamic viscosity (kg/m.s)

y_{comp} computational depth (m)

y_{exp} observed depth in the laboratory (m)

MRE   Mean Relative Error

6. Limitations

In this research the flow passing over the dam is not considered so the numerical model can not be used when the flow passes as overflow from dam crest and also through the dam body.

7. Conclusion

Detention rockfill dams are used for multi purposes from which flood control is accounted as one the main objects since by storage the water upstream, they reduce the pick of hydrograph and decrease the damages caused by flooding. However, to increase the time life of these structures, we used a coarser layer at the bottom part of the dam. To determine the water surface profile of the flow passing through a multi-layered dam, an equation based on assumption of gradually varied flow is introduced. To validate this equation, a series of experiments, including three different discharges applied on four different porous media, have been carried out. The observed data were compared with those obtained from the introduced equation. Using index of Mean Relative Error, it is found that the maximum MRE is 17.6% indicating that the numerical model gives satisfactory results so that it can be used as a useful model to determine the water surface profile for the procedure of flood control computation.

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List of tables

Table 1

Code no.	Dam type	Sub-layer rock diam. (cm)	Top-layer rock diam. (cm)	Sub-layer height (cm)	Top-layer height (cm)
1	Homogeneous	3	3	10	30
2	Heterogeneous	5.33	3	10	30
3	Heterogeneous	5.33	2.06	10	30
4	Heterogeneous	3	2.06	10	30

Table 2

Dam Code	Q (lit/sec)		
	4.84	2.73	2.06
1	10.3%	11.8%	12.2%
2	15.6%	16.6%	17.6%
3	4.3%	3.9%	4.8%
4	12.6%	11.1%	11.6%

List of fig.s

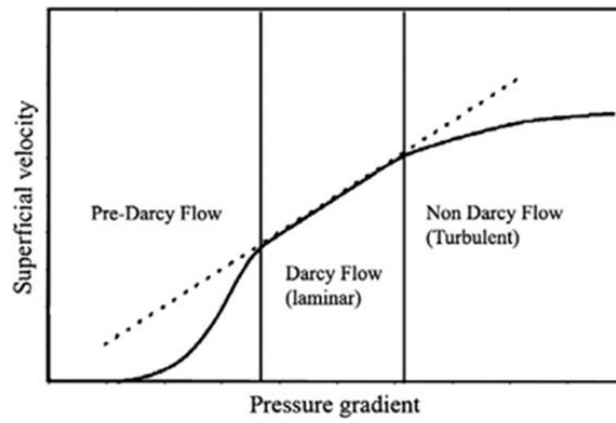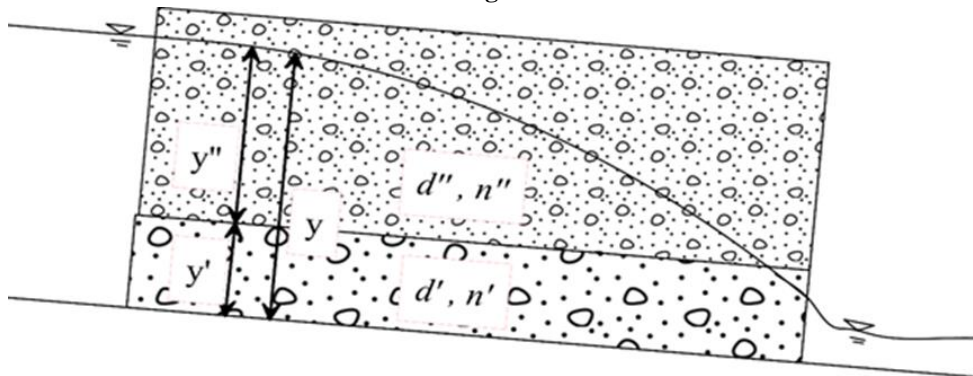a) Approximate length: 2500m）   (b) Approximate length: 2m

Fig. 1.
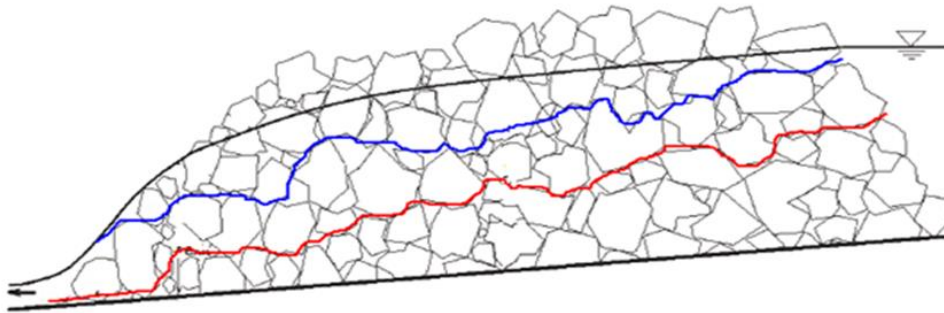
Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.