

CFD and dimensionless parameter analysis of Froude number to determine the flow regime over ogee spillways

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ABSTRACT

In this research, the numerical method of computational fluid dynamics and dimensionless parameter of Froude number have been applied to determine the flow regime over ogee spillways. Froude number is a dimensionless dynamic parameter calculated using the square root of the ratio of inertia force and gravitational force. Considering that the study of the regimes of flow over hydraulic structures especially spillways is of great importance, and as the most applied spillway is ogee one, therefore it is required to study such structures. To study the sensitivity of the dimensionless parameters of Froude number and velocity using in the proper calculation of the profile of the flow over ogee spillway the software of fluent computational fluid dynamics software has been used. Based on Gauss–Seidel convergence condition applied for the convergence and control of equations, the step time of 0.01 second, quad pave meshing and RNG k-ε turbulence model have been used. The results of this research shows that the decrease of reference design head by 10, 20, and 30 percent leads to the decrease of reference average Froude number and reference average velocity of the flow over ogee spillway by 7.646, 15.2 18.835, and 6.519, 12.787, and 26.113 percent respectively. Moreover, the increase of the average Froude number of flow by 6.519, 12.787, and 26.113 percent, and average reference velocity by 13.933, 4.211, and 1.839 causes the reference design head to increase by 10, 20, and 30 percent. Whereas p-value of the profile of the flow in SPSS software is around 0.006 and very lower than 5%, it can be claimed that the numeral study is of high precision. In conclusion, the results obtained from numerical simulation have been provided and compared with the existing numerical and laboratory studies. The accuracy of this research is describable.

Keywords

Ogee spillway, numerical model, froude number, fluent software, velocity vector, flow profile, p-value

1. Introduction

Froude number is a dimensionless dynamic parameter calculated using the square root of the ration of inertia force to gravitational force. If the value of this parameter is bigger than one, the flow is critical and in case the

discharge is constant, the flow depth decreases and its velocity increases. If Froude number is less than one, the flow is subcritical and the same discharge causes that flow depth increases and velocity decreases. Criti-

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cal flow happens if Froude number is equal to one (Hosseini, Abrishami). Spillways are the most important members of big structures like dams. They are used to control floods and protect dams. Ogee spillways are located at the beginning of spillway and connected to the lake. Thereafter, at the end of the structure, chute is located, which attenuates energy. Ogee spillway is due to its unique hydraulic characteristics one of the hydraulic structures, which has been studied frequently (Savage, Johnson 2001) as follows:

In 1965, Cassidy used numerical model in a 2D space to determine the pressure on the crest of ogee spillway based on potential flow. In 1998, Olsen and Kjellesvig used Reynolds equations and standard $k-\varepsilon$ equation in finite volume method to analyze the flow over ogee spillway in 3D and 2D spaces. In 1999, Burgisser and Rutschmann used finite element method to analyze the vertical component of flow over the crest of spillway in 2D space supposing that there is incompressible and turbulent flow. Tufi and Wilson used in 2001 the finite difference method to analyze vertical flow over ogee spillway crest in a 2D space assuming that there is a potential flow and Neumann condition is imposed on the boundaries of flow field.

Bruce et al. conducted in 2001 a comprehensive study to compare the parameters of flow over standard crested ogee spillways using a physical model, numerical model and existing studies. Latif Bouhadji conducted in 2002 a numerical study on turbulent flow over spillways in a 3D space. In 2003, Chen et al. modeled turbulent flow over stepped spillway using finite volume method. In 2003, Ho et al. studied the maximum impacts of contingent floodwater over spillways. In 2004, Jean and Mazen modeled flow over ogee spillway numerically.

In 2005, Dae Geun Kim and Jae Hyun Park used the commercial numerical model of computational fluid dynamics of flow 3D software to study the properties of flow in-

cluding flow rate, water surface profile, pressure imposed on the crest of ogee spillway, pressure vertical distribution and speed based on the scale of model, the impact of surface roughness and details. In 2006, Bhajantri et al. studied the hydraulic model of flow over ogee spillway numerically considering downstream and the information obtained from two physical models have been compared with the results obtained from numerical study of two crested ogee spillways.

In 2009, Vosoughifar and Daneshkhah used numerical methods such as finite difference and finite volume to evaluate numerically the impacts and function of the surge tank of hydraulic hammer on the cover of water tunnels transferring water to concrete dam turbines. In 2009, Angela Ferrari simulated numerically the flow with free surface over a spillway with sharp crest. In this research, investigation of the sensitivity of design head is compared to calculate properly the profile of flow over ogee spillway using finite volume method.

2. Research Method

In this research, the numerical method of computational fluid dynamic has been used to solve second-order nonlinear partial differential equations. Fluent software has been applied to analyze computational fluid dynamic equations using finite volume method. At first, it is required that GAMBIT software is used to prepare the geometry and meshing of the model for solving flow field. Considering the impact of turbulence, VOF (volume of fluid) method introduced by Nichols and Hirt was applied for determining the profile of flow free surface. For each part of the cell located in free surface, the values between zero to one has been taken into consideration, in case the cell of fluid is full, the volume fraction of cell is one and in case it is empty, the volume fraction of cell is zero. In this research, 0.09 portions of cell is water and 0.01

portions is air. The equations governing flow in this software include continuity equation (law of mass conservation), and Navier-Stokes equation (momentum conservation law). In this research, Reynolds-Average method was used to solve Navier-Stokes equation, if flow is turbulent. As a result, continuity equations and momentum are determined. Continuity equation:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad (1)$$

Momentum equation:

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + g_{x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \bar{u}_i}{\partial x_j} - \overline{u'_i u'_j} \right) \quad (2)$$

Eq.(2) is known as Reynolds Averaged Navier Stokes Equation. This equation is just like Navier-Stokes equation, and the only additional term of these equations is the parameter of $(-\rho \overline{u'_i u'_j})$, which is called Reynolds stress and it applied to the fluid and causes the impact of turbulent vortex to be applied to the fluid (Fluent Inc 2006). Different models of turbulence have been applied to analyze Reynolds stress.

Turbulent models have been classified based on the application of their design and number of differential equations to create relation between turbulence stresses and averaged rates or their gradients. These models include zero equation models, one equation model (Spalart–Allmaras Model), two equation models, algebraic stress model, Reynolds stress model, Reynolds stress models (five equation model).

Among these models, two-equation model has been studied due to the satisfactory results and simplicity of application for ogee spillways (Savage, Johnson 2001). This model has also been divided into the following classes: standard k– ω model (shear stress

transport (SST) k– ω model) and k– ϵ model (standard k– ϵ model – realizable k– ϵ model – renormalization group (RNG) k– ϵ model) (Wilcox 1993).

2.1 Two Dimensional Discretization of Equation

For the convergence of numerical modeling, it was required to control the equations and their related relations in order to define temporal steps for the software. For this purpose, the temporal steps shall be applied based on the meshing measure to achieve sustainability. The discrete differential equation of nonpermanent fluid flow is as follows:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + S \quad (3)$$

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + a_T T_T + a_B T_B + b \quad (4)$$

There are different iteration methods for solving algebraic equations, among which the Gauss-Seidel Point-by-Point Method was used. In this method, the variable values of each node have been calculated based on specified instruction.

$$a_T T_P = \sum a_{nb} T_{nb} + b \quad (5)$$

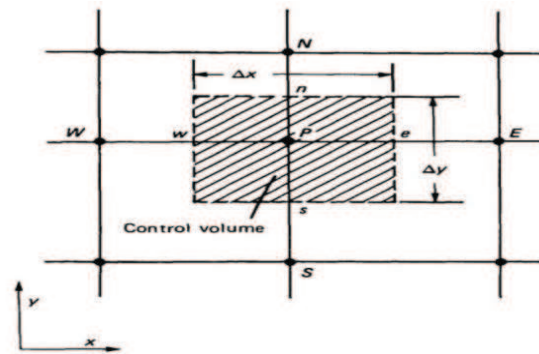


Fig.1: Control Volume for 2D Situation

Discrete equation is as follows:

Where, nb indicates neighbor node, and T_p is calculated using the following formula.

$$T_p = \frac{\sum a_{nb} T_{nb}^* + b}{a_p} \quad (6)$$

In the above-mentioned equation, T_{nb}^* indicates the value of neighbor node in the memory of the computer. For the neighbors considered previously during the current iteration, the value of T_{nb}^* is related to the previous iteration. Anyhow, T_{nb}^* is the last accessible value for the temperature of neighbor node. After all nodes are taken into consideration, one iteration is completed using the method of Gauss – Seidel Point-by-Point. The said method is not always convergent. Indeed, whenever Scarborough criterion (1985) is true, the convergence of Gauss-Seidel method is guaranteed. This is as follows:

$$\frac{\sum |a_{nb}|}{|a_p|} \begin{cases} \leq 1 & \text{for all equations} \\ < 1 & \text{Atleast for one equation} \end{cases} \quad (7)$$

In case the interval times are chosen properly, the above-mentioned convergence relation will be acceptable (Patankar 1980). The time interval used in this research is equal to 0.01 second, which covers the convergence condition of Gauss–Seidel very well.

2.2 Statistical Analysis Method with SPSS

The Statistical analysis method (p-value) with SPSS software has been used to compare the parameters of flow over ogee spillway. Statistical analysis method is used to confirm null hypothesis (meaningful difference) or reject null hypothesis (considerable difference). The correctness of null hypothesis (H_0) has been always emphasized, unless something contrary is proved. Significance level (α) is the error level designated by the researcher as a criteria to reject null hypothe-

sis (H_0) (usually 5%). P-value, which is called decision criterion, is level of error for rejecting null hypothesis and it is shown in form of *Sig (2-tailed)*. If the P-value is lesser that Alpha, null hypothesis can be rejected more easily. In this research, the lesser amount of this value proves the reality of numerical solution (Mirzadeh 2009).

2.3 Case Study

2.3.1 Physical Model

The physical model of the case study of ogee spillway is made of Plexiglas prepared at Utah Water Research Laboratory (UWRL) based on standard mold. The dimensions of this model are as follows, Fig.2 (Savage, Johnson 2001):

- Design head (H_d) = 0.301 m
- Width = 1.83 m
- Height of spillway crest (P)= 0.8127 cm

2.3.2 Boundary Conditions

According to Fig.3, the original boundary conditions of flow over ogee spillway include inlet (speed inlet, and pressure inlet), outlet (pressure outlet), stationary wall (wall) and free surface (pressure inlet).

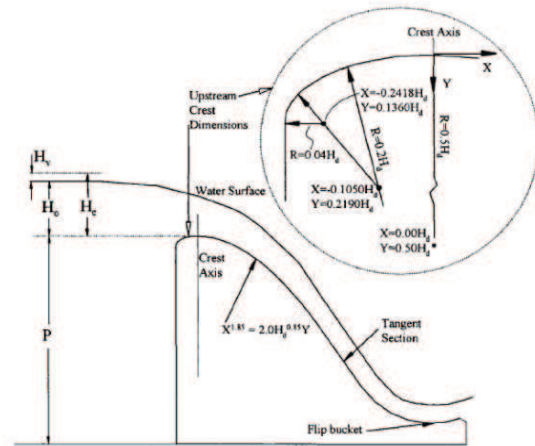


Fig.2: The Dimensions of Ogee Spillway and Flow Parameters

2.3.3. Analysis Method

According to Fig.4, it can be claimed that the change in the geometry of ogee spillway from upstream quadrant to downstream equation of spillway, depends on design head (H_d) (Hydraulic Design Criteria).

At first, it is required to verify the accuracy of meshing, correctness of the choice of turbulence model and ensure that they have no effect on the results. Considering that ogee spillway is curved in form, it is recommended that quad-pave meshing to be used to minimize the relative error of discharge parameter and optimize the time of numerical solution. The measure of meshing, time and stem time has been chosen based on Gauss-Seidel condition equal to 0.0065 m, 70 seconds, and 0.01 seconds, respectively (Daneshkhah, Vosoughifar 2011). Based on the study of different turbulence models under similar conditions, RNG turbulence model has been used in accordance with Fig.5, due to its ability to simulate exactly the profile of flow surface and minimize the relative error of flow discharge parameter (Daneshkhah, Sabbagh-Yazdi 2011).

Velocity vector in different points of the profile of the flow over ogee spillway has been shown in the Fig.6.

The volume of air-phase cell is zero and the value of one indicates that the cell is full with water. Considering the compression of velocity vectors in upstream input and increase in the velocity vectors in downstream of ogee spillway, it can be claimed that the velocity field of the flow over spillway has been modeled properly.

To model the dimensionless parameter of Froude number of flow over ogee spillway, finite-volume method has been used, and power law scheme has been applied to discretize the value. Moreover, simple iteration algorithm has been used to couple the terms of velocity and pressure. The solution of flow field is continued until the residue reaches 10^{-4} . Moreover, the plan is continued with the

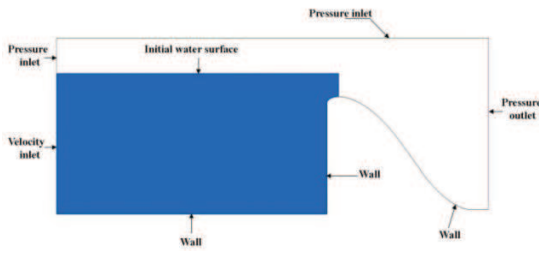


Fig.3: Original Boundary Conditions of the Field of Flow over Ogee Spillway

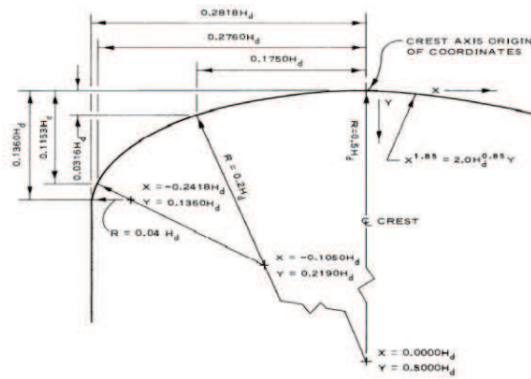


Fig.4: The Geometry of Ogee Spillway based on Design Head

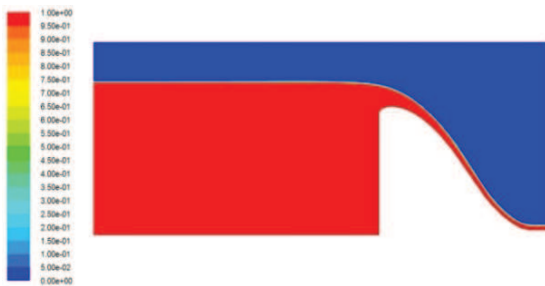


Fig.5: Flow Profile in RNG $k-\epsilon$ Turbulence Model

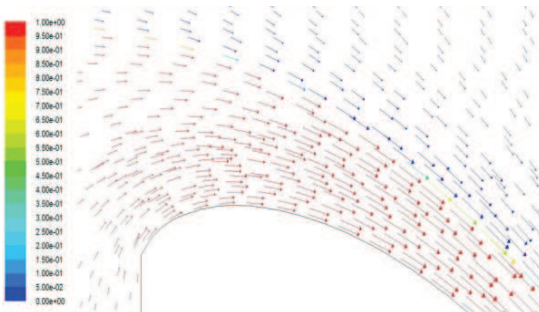


Fig.6: Velocity Field of Flow over Ogee Spillway

unsteady flow field to change it to a steady mode. In this research, we change the reference design head to study the changes in the parameters of Froude number and velocity. For this purpose, the design head is increased and decreased by 10, 20, and 30 percent more and less than reference design head to compare the velocity and dimensionless parameter of the flow over ogee spillway is.

3. Results

Based on the study of different modes of design head, and table 1, it can be claimed that the reduction of reference design head by 10, 20, and 30 percent leads to the decrease of reference average Froude number by 7.646, 15.2 and 18.835, and the increase of the reference average velocity of the flow over ogee spillway by 6.263, 12.787, and 26.113 percent causes the reference design

head to increase by 10, 20, and 30 percent.

Based on Fig. 7, the decrease of design head leads to higher ogee spillway analysis with higher flow depth value and lower Froude number. In Fig. 8 the flow Froude number with the design head, which is 30% greater than the reference design head (design head is equal to 0.3913 meters), is in the following form.

Based on Fig. 8, it is cleared that any increase in the design head decreases the depth of the flow over ogee spillway and increases the Froude number, which leads to a bigger ogee spillway. The change in the design head changes the velocity of the flow over ogee spillway. The Fig. 9 shows the changes in flow velocity with the changes in the reference design head.

Based on table 1 and Fig. 9, it can be claimed that the increase in the reference design head by 10, 20 and 30 percent, the

Table 1. The Change in Froude number and velocity of Flow over Ogee Spillway due to the Change in Design Head

Design Head	Average of Froude number	Percentage of averaged froude number tolerance	Average of flow velocity	Percentage of averaged tolerance
0.2107	1.382	-18.835	1.205	-13.047
0.2408	1.444	-15.200	1.215	-12.295
0.2709	1.573	-7.646	1.295	-6.519
0.301	1.703	0	1.385	0
0.3311	1.810	6.263	1.411	1.839
0.3612	1.921	12.787	1.444	4.211
0.3913	2.148	26.113	1.578	13.933

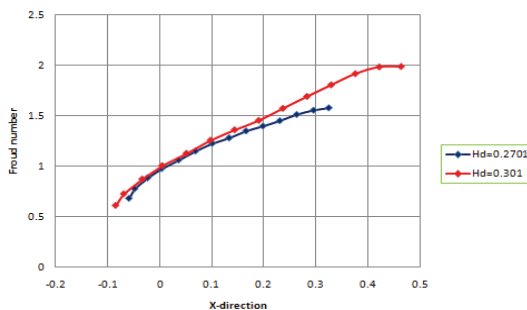


Fig.7: The Change in Froude number due to the decrease in Design Head

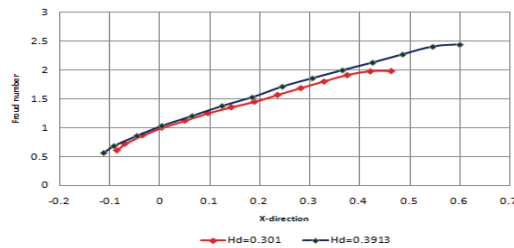


Fig.8: The Change in Froude number due to the increase in Design Head

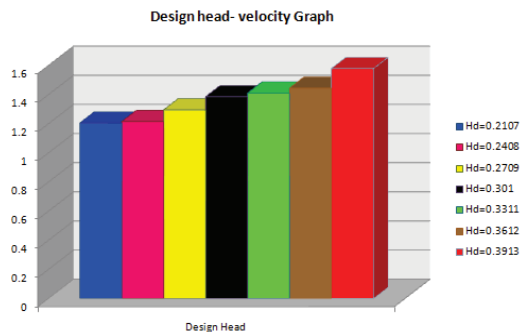


Fig.9: The Change in the Flow Velocity with the Change in the Design Depth

average velocity of the flow over ogee Fig. 7 the flow Froude number with the design depth of 0.2107 meters (30% less than reference head).

Spillway increases by 1.839, 4.211 and 13.933. Moreover, the reduction of average velocity of the flow over ogee spillway by 6.519, 12.295, and 13.047 reduces the design head by 10, 20, and 30 percent in comparison with the reference design head. In the following section, the most important results of this research have been listed:

1. Based on table 1, the flow regime of the upper stream flow over ogee spillway is subcritical and of the downstream flow over spillway super critical. Hence, for the simulation of the flow over ogee spillway, it is required that the both regimes can be simulated numerically.
2. The increase in the design head causes the cross-section of the spillway to increase and the volume of concrete works increases, which may lead to high expenses of construction. Moreover, the decrease of design head causes the cross-section of the spillway to decrease and the spillway to be thinner, which reduces the volume of concreting and cost saving.
3. Based on Fig. 7, the reduction of design head cause the Froude number, and the average velocity of the flow over ogee spillway to be reduced.
4. Based on Fig. 8, any increase in design head causes Froude number and average velocity of the flow over ogee spillway to be increased.
5. Based on table 1, the changes of design head and average Froude number of the flow over ogee spillway have no linear relation.
6. Based on Fig. 9 and table 1, the changes of average velocity of the flow over ogee spillway have not linear relation with the linear changes of design head.
7. The average p-value of Froude number of the flow in SPSS software is about 0.006,

which is less than 0.05. Therefore, it can be claimed that this numerical study is of high accuracy.

4. Discussion

In 2009, Ferrari studied the impact of viscosity on the profile of flow over spillway and he found that the impact of viscosity is of minor significance, which can be ignored. In this research, the viscosity of fluid is deemed constant. In 1980, Rodi used the standard k- ϵ turbulence model to analyze flow over ogee spillway, and he found that this model is highly accurate (Savage , Johnson 2001). Moreover, Ho et al. (2001), Kim (2003) and Savage et al (2001) used Flow 3D software and standard k- ϵ turbulence model to model the profile of the flow over ogee spillway, and found that the numerical model and laboratory sample are matched very well (Kim et al. 2005). In this research, Fluent Software and RNG k- ϵ turbulence model has been applied, whose accuracy is higher than the other models significantly.

The discharge rate obtained from the numerical solution of the Fluent Software has been equal to 0.1175781 m³/s , while the discharge rate obtained by Bruce et al. using Flow 3D software is equal to 0.125208 m³/s (Savage , Johnson 2001). The USBR graphs show the discharge equal to 0.119944 m³/s and USACE graphs show the discharge rate equal to 0.1222 m³/s.

Considering the results of the abovementioned methods, it can be observed that the results of this research are very similar to the results of USBR graphs, and are of high precision (Hydraulic Design Criteria). Decision criterion (P-value) obtained from numerical study is about 0.000025, which is in comparison with the results of the study of Bruce et al. conducted using flow 3D software, of high accuracy (Savage , Johnson 2001).

The decision criterion (P-value) obtained from numerical method of Fluent software with USBR graphs is approximately equal to

0.001037 and with USACE graphs is equal to 0.000138, which are lower than 0.05, and it proves the high accuracy of the numerical method used in this research (Hydraulic Design Criteria). Based on the above-mentioned facts, and this research to analyze the sensitivity of meshing, it can be claimed that RNG $k-\epsilon$ turbulence model is on great precision.

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