International Journal of Advanced Structural Engineering, Vol. 1, No. 2, Pages 135-151, December 2009

Published online December 2009 at (http://journals.azad.ac.ir/IJASE)

RELIABILITY INDEX FOR REINFORCED CONCRETE FRAMES USING NONLINEAR PUSHOVER AND DYNAMIC ANALYSIS

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Received 27 September 2009
Revised 14 November 2009
Accepted 17 December 2009

In the conventional design and analysis methods affecting parameters (loads, materials' strength, etc) are not set as probable variables. Safety factors in the current Codes and Standards are usually obtained on the basis of judgment and experience, which may be improper or uneconomical. In technical literature, a method based on nonlinear static analysis is suggested to set Reliability Index on strength of structural systems. In this paper, a method based on Nonlinear Dynamic analysis with rising acceleration (or Incremental Dynamic Analysis) is introduced, the results of which are compared with those of the previous (Static Pushover Analysis) method and two concepts namely Redundancy Strength and Redundancy Variations are proposed as an index to these impacts. The Redundancy Variation Factor and Redundancy Strength Factor indices for reinforced concrete frames with varying number of bays and stories and different ductility potentials are computed and ultimately, Reliability Index is determined using these two indices.

*Key words: r*eliability index, ductility, concrete structures, redundancy strength index, redundancy variations index

1. Introduction

Performance of a structure is evaluated through safety, repairability and economy. Information relative to input variables is never absolute, accurate and complete. The source of uncertainties

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may be specified by: (1) Random intrinsic variables as physical uncertainties; (2) Information restriction as statistical uncertainties; (3) Inadequate knowledge as the model uncertainty; (4) Apparent errors; (5) Despite uncertainties, definite safety of a structure is impossible due to the following factors:

- Unpredictability- Including: (i) Loads on the structure during its life time; (ii) Material properties; and (iii) Human errors;
- Idealization of the structure towards formulation of its mathematical model;
- Numerical method restrictions.

The concept of reliability is applied for different purposes and is translated and interpreted by many methods. The common definition of reliability, generally accepted is the probability to formulate a target function in a given time under specific conditions. It has four important components: probability, target function, time and performance condition.

Probability is the first constituent component of reliability as far as uncertainty is concerned. The second component for a structure to be reliable, a definite safety function against stress, flexure, torsion, etc must be formulated for what ever designed. Reliability is associated with time, but in some certain state of structure, can be related to the lifespan of the structure. During the given lifespan of the structure, the attributed target function associated to the last component shall be formulated; these conditions in the form of loads, temperature, shocks, vibrations, wind and others may affect the structure. Nowak et al. (2000).

2. Redundancy index

Redundancy resistance index r_s represents the ability of a structural system in redistributing forces at the time of failure and the capability of a structure in transferring the forces of elements yielded to the elements with higher resistance. This index is a function of static redundancy, ductility, strain hardening and the average resistance of elements in a structural system. Second index having probability nature is an r_v redundancy variations index. This index measures the probability effect of elements resistance on structural system resistance. It is also a function of static redundancy in a structural system, and on the other hand is a function of statistical nature in ductility and structural element resistance. Following variables are used in computing above mentioned indices:

- Base shear in the beginning of yielding system;
- Ultimate base shear;
- The number of local failure or plastic hinges caused during ultimate failure of structure;
- The access of elements curvature to ultimate curvature. Husain et al. (2004).

2.1. Redundancy Resistance Index

Redundancy resistance index r_s is defined as the ratio between average ultimate resistance ($\bar{S}_u$) and the yielding resistance ($\bar{S}_y$). In which $\bar{S}_y$ is the average system resistance of the non redundant system.

$$r_s = \frac{\bar{S}_u}{\bar{S}_{nr}} = \frac{\bar{S}_u}{\bar{S}_y} \tag{1}$$

In this equation, both parameters $\bar{S}_u$ and $\bar{S}_y$ can be defined on the basis of the nonlinear static analysis curve, (Bertero et al., 1999), as can be seen in Figure 1.

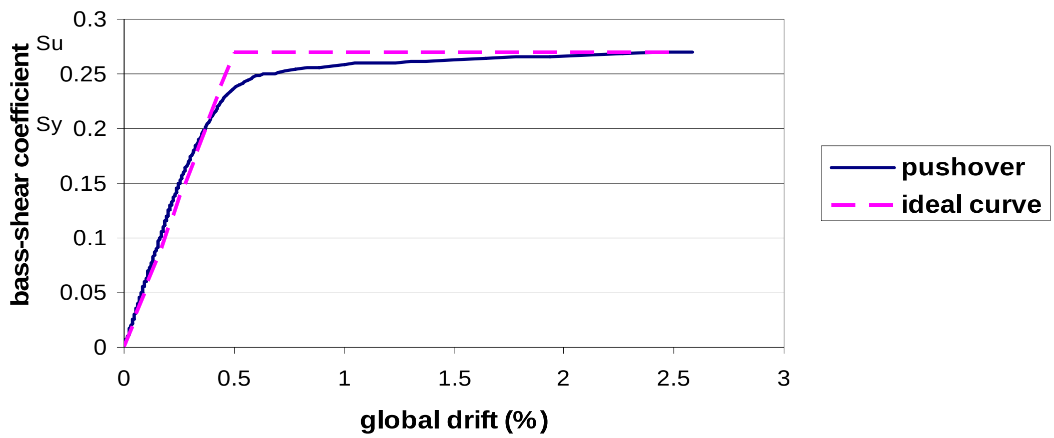

Figure1. Base-shear versus global drift ratio curve

In the method suggested in this paper, to study the effects of redundancy using nonlinear dynamic analysis with increased acceleration, the base shear during failure and yielding is considered. In previous studies, Masumi et al. (2004), this method was applied for studying the effects of overstrength. In this study, the system failure standards that will be considered in nonlinear static and dynamic analyses with increased acceleration are as follows:

(1) Limitations related to storey drift according to code for buildings in which the period lower than 0.7 second are limited to 2.5% and for structures with period more than 0.7 second are limited to 2%;

(2) The index of structure stability in a structure with high ductility is limited to 0.125 and in a structure with low ductility is limited to 0.25, (Iranian Building Code, 2005);

(3) The formation of failure mechanism in a structure and collapsing structure;

(4) Approaching of structure failure index to number one according to Park-Ang criterion, (Park et al., 1985).

In pushover static analyses performed in this study, it is assumed that lateral loads with reverse triangular distribution are inserted into the structure that is acceptable according to Iran 2800 standard earthquake force. In nonlinear dynamic analysis with increasing acceleration, the maximum acceleration of any record will be equaled by a primary number (here it is considered to be 0.02g) and in one stage it will be increased to 0.02g and the structure is analyzed in every step until one of the four above mentioned criteria is occurred. In this stage, the analysis is stopped and base shear is used during yielding and maximum base shear is used for measuring r_s redundancy resistance index.

2.2. Redundancy Variation Index

The relation between resistance of a structural system and the resistance of its components is obtained using plastic analysis of structure. In this case, the selection of failure mechanism is important because it can result in non-actual estimates from redundancy variation index. For the sake of computation, a sway mechanism is considered according to Figure 2. This mechanism is based on the "strong column" and "weak beam" assumption in which column resistance is at least 20% more than the resistance of beams (Husain et al., 2004).

The frame strength (base shear strength) for any failure mode could be represented by the following equation:

$$S = \sum_{i=1}^{n} C_i . M_i \tag{2}$$

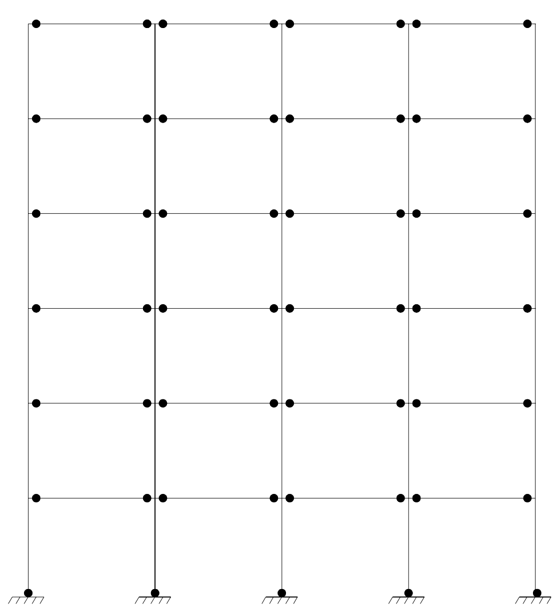

Figure 2. Sway type failure mode of a generic plane frame

where S= frame strength (base shear); n= the number of plastic hinges in the frame resulted from the particular failure mode or considered collapse mechanism; Mi= yield moment of the structural element where plastic hinge "i" is formed; and Ci = coefficient with length radian units that is a function of the plastic rotation and geometry of the structure. Equation (2) is the strength equation of a parallel system type.

The mean value of the frame strength can be derived from the following equation:

$$\overline{S} = \sum_{i=1}^{n} C_i . \overline{M}_i \tag{3}$$

where Mi = mean value of the strength of the structural element where plastic hinge "i" is formed. Accordingly, the standard deviation of the frame strength σ_f can be obtained from:

$$\sigma_f = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} C_i C_j \rho_{ij} \sigma_{Mi} \sigma_{Mj}} \tag{4}$$

where ρ_{ij} = correlation coefficient between the strengths Mi and Mj and σ_{Mi} = standard deviation of the yield moment Mi. Also ρ_{ij} = 1 for i=j. To simplify the deviation more, a regular multistory multi-bay frame with the following properties is considered.

(1) The frame is composed of elements with normally distributed identical strengths:

$$\overline{M}_i = \overline{M}_j = \overline{M}_e \tag{5}$$

$$\sigma_{Mi} = \sigma_{Mj} = \sigma_e \tag{6}$$

(2) The correlation coefficient between the strength of any two pairs of elements is the same.

$$\rho_{ij} = \rho_e \tag{7}$$

(3) The bays of the frame have identical spans and identical storey height which result in:

$$C_i = C_j = C \tag{8}$$

The mean value and the standard deviation of the frame strength become:

$$\overline{S} = n.c.\overline{M}_e \tag{9}$$

$$\sigma_f = C\sigma_e \sqrt{n + n(n-1)\rho_e} \tag{10}$$

The following relationship between the coefficient of variation (COV) of the frame strength v_f and the COV of the element strength v_e is calculated by dividing Equation (10) by Equation (9):

$$v_f = \frac{\sigma_f}{\overline{S}} = \frac{\sigma_e}{\overline{M}_e} \sqrt{\frac{1+(n-1)\rho_e}{n}} = v_e \sqrt{\frac{1+(n-1)\rho_e}{n}} \tag{11}$$

The redundancy variation index r_v is defined as the ratio between v_f and v_e :

$$r_v = \frac{v_f}{v_e} = \sqrt{\frac{1+(n-1)\rho_e}{n}} \tag{12}$$

For a parallel system with unequally correlated elements, ρ_e could be substituted with the average correlation coefficient ρ_e defined as: (Thoft-Christensen et al., 1982).

$$\overline{\rho} = \frac{1}{n(n-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \rho_{ij} \tag{13}$$

Therefore, Equation (12) can be modified using the average correlation coefficient of the strengths of the plastic hinges as follows:

$$r_v = \sqrt{\frac{1+(n-1)\overline{\rho}}{n}} \tag{14}$$

Hence the redundancy variation index r_v is a function of the number of plastic hinges “n” and their average correlation coefficient between their strengths, and represents a measure of the probabilistic effects of redundancy on the system strength. Its values range between 0 and 1.

For a building structure where a single plastic hinge causes collapse (n=1), r_v =1 and the structure under consideration is non redundant. The other extreme value r_v=0 indicates an infinitely redundant structural system and is reached either when an infinite number of plastic hinges are required to cause collapse (practically “n” attains large values) or when element strengths in a structure are uncorrelated (the average correlation coefficient in Equation (14) is zero, (Figure 3).

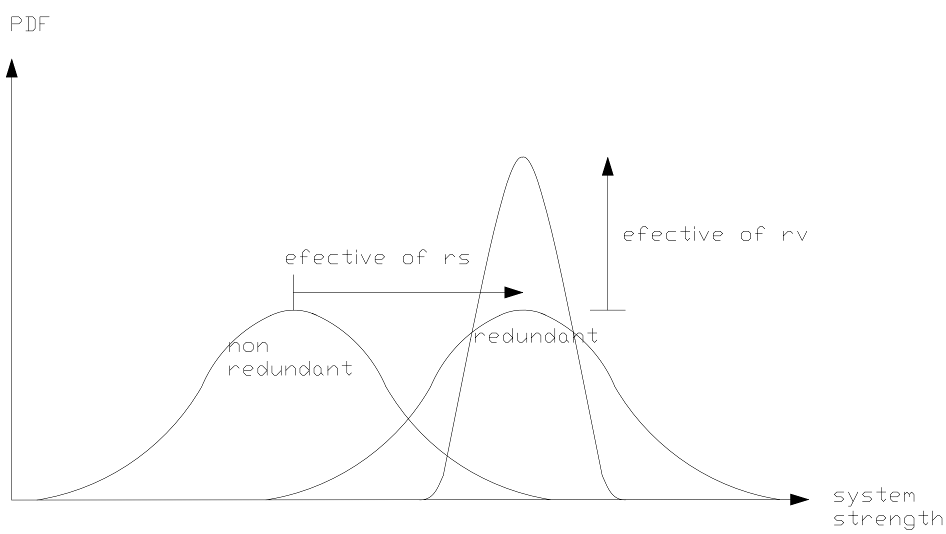

Figure 3. Effects of redundancy indices r_s and r_v on structural system strength
(PDF: Probability Density Function)

By using Equation (14), r_v can be estimated from a pushover or dynamic analysis and for a particular value of the average correlation coefficient of the structural member strength.

3. Computing Reliability Index (β) in Terms of r_s and r_v

It is tried hard to develop a safety index concept for application in different engineering issues. Accordingly, some structural reliability methods are founded.

Computation for reliability index in terms of r_s and r_v is carried out as following: (Wen et al., 2003 and Sophocleous et al., 2002).

Reliability index (β) indicates how far the original structure is away from its collapse position. It is a function of both structural strength and load parameters, which is given by the equation below: (Ranganatan, 1990).

$$\beta = \frac{\overline{S}_u - \overline{L}}{\sqrt{\sigma_f^2 + \sigma_l^2}} \tag{15}$$

where $\overline{S}_u$ is the average system strength, L is the average applied loads, σ_f is the standard deviation of system strength and σ_l is the standard deviation of applied loads. Substituting relation ($\overline{S}_u = r_s \overline{S}_{nr}$) and ($\sigma_f = r_v r_s v_e \overline{S}_{nr}$) in Equation (15) will result in following equation:

$$\beta = \frac{(r_s.\overline{S}_{nr} - \overline{L})}{\sqrt{v_f^2.r_s^2.\overline{S}_{nr}^2 + v_L^2 \overline{L}^2}} \tag{16}$$

where v_f is the strength variation factor for the frame and v_l is the applied load variation factor. Substituting relation (12) ($v_f = r_v.v_e$) in Equation (16) will result in:

$$\beta = \frac{(r_s - l)}{v_e\sqrt{r_v^2.r_s^2 + l^2.v^2}} \tag{17}$$

$v = \dfrac{v_l}{v_e}$ and $l = \dfrac{\overline{L}}{\overline{S}_{nr}}$ are thus accounted for.

Therefore, reliability index (β) is a function of: (1) System redundancy, by virtue of its two measures r_s and r_v; (2) Strength variation factor for the structural elements, v_e; (3) The deterministic ratio of the mean load to the mean non-redundant strength, l; (4) The ratio of the load variation factor to the element strength variation factor, v. Figure 4 shows (β) value relative to r_s and r_v for $v_e = 0.1$ and v=2 and l= 1.

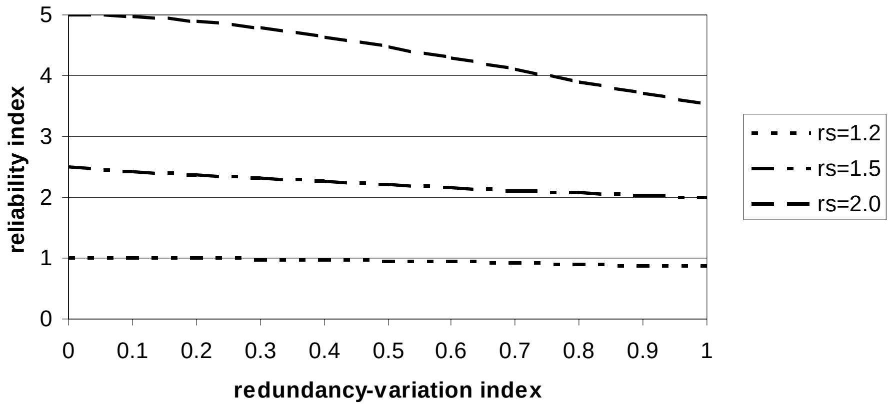

Figure 4. Variation of reliability index with respect to r_s and r_v for structural systems with(kv_e =0.2)

The reliability index is calculated for a ratio of mean load to mean strength level l=1, which means that safety, if it exists, is due to the redundancy of the system and not due to any additional strength. For a structure with load values v =2 and v_e =0.1, reliability index increases with a rise in redundancy (by increase in r_s value and decrease in r_v value). A 33% rise in r_s value (from 1.5 to 2) would lead to an increase in reliability index between 60 to 100 percent.

4. Case Study of Reliability Indices for Two-Dimensional Concrete Frames

To compute reliability indices, 16 samples of frames from 2 to 5 bays with two, four, six and ten stories were designed using SAP2000 software, (Habibullah, 2000), and were subjected to IDARC-Ver.5 software, (Valles, 1999), for nonlinear analysis. IDARC is used for nonlinear static and dynamic analysis of reinforced concrete frames. For designing and analyzing of two-dimensional concrete frames, 128 samples of frames are designed and analyzed under different loading conditions. The reason of selecting these models is to consider the impact of the number of bays and stories, bay length, bay height, intensity of the gravitational loading and ductility of flexural elements. In the first case, bay length is 4 meters and the story height is 3 meters. In the second case, story height is raised from 3 to 4 meters. In the third case, bay length is increased to 5 meters and finally in the forth case, intensity of the gravitational loading is raised up to 30%. The lateral load pattern applied to the structure is inverse triangular, similar to earthquake lateral forces in the standard 2800 of Iran (Iranian Building Code, 2005).

Response curves are computed in terms of displacement at the top of structure (Δ_{tar}) versus base shear divided by the structure weight (C_b). Base shear coefficient at yielding moment and also maximum base shear coefficient are two important values on curve. The r_s index is obtained by dividing maximum base shear coefficient by the base shear coefficient at the yielding moment. By using maximum number of plastic hinges formed in nonlinear static analysis, r_v index can also be obtained. Ultimately, having these two indices, reliability index may be obtained in accordance with relations in the third section. Figure 5 shows r_s variations with respect to different number of bays and stories for high ductility and figure 6 shows same values for low ductility. Accordingly, Figures 7 and 8 also show r_v variations for the first case. Finally, Figure 9 shows β variations for high ductility and Figure 10 shows β charges for low ductility.

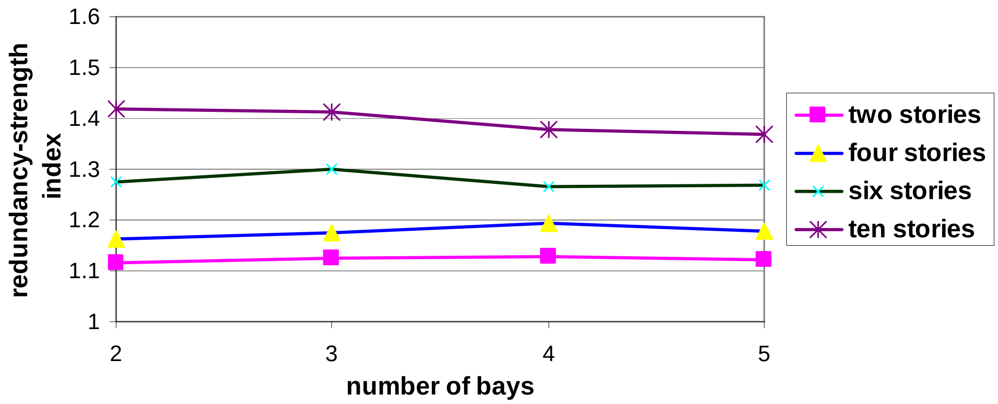

Figure 5. Variation of redundancy-strength index with number of bays and stories for pushover analysis and high ductility

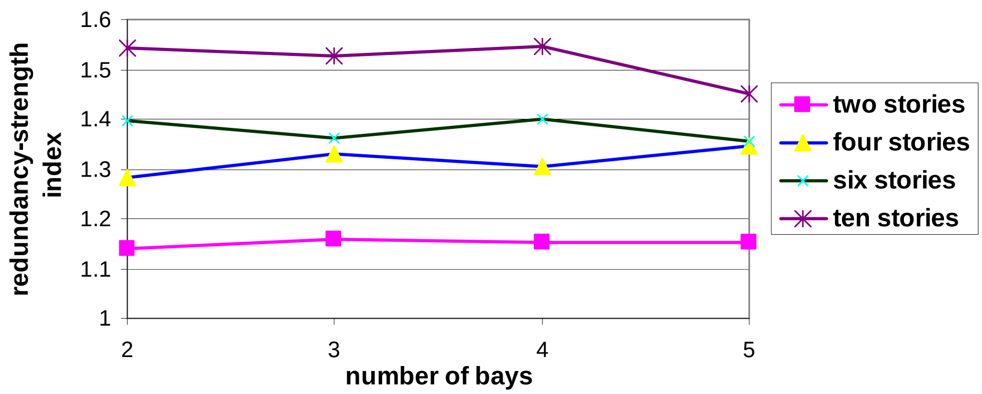

Figure 6. Variation of redundancy-strength index with number of bays and stories for pushover analysis and low ductility

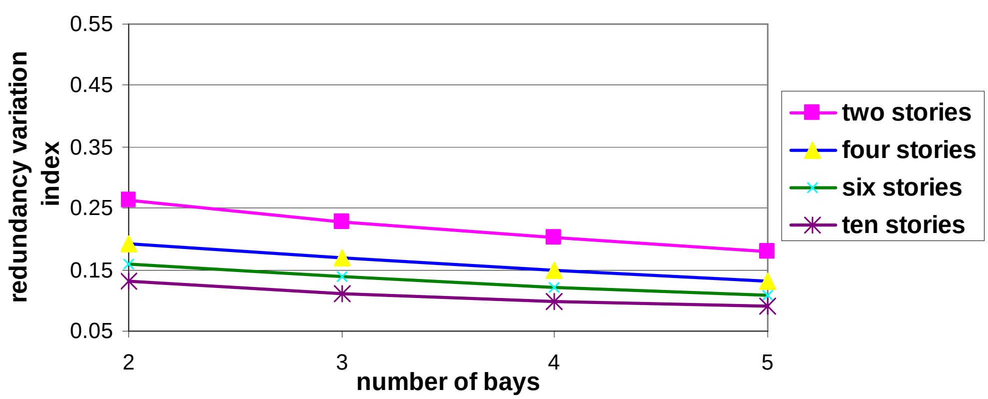

Figure 7. Variation of redundancy-variation index with number of bays and stories for pushover analysis and high ductility

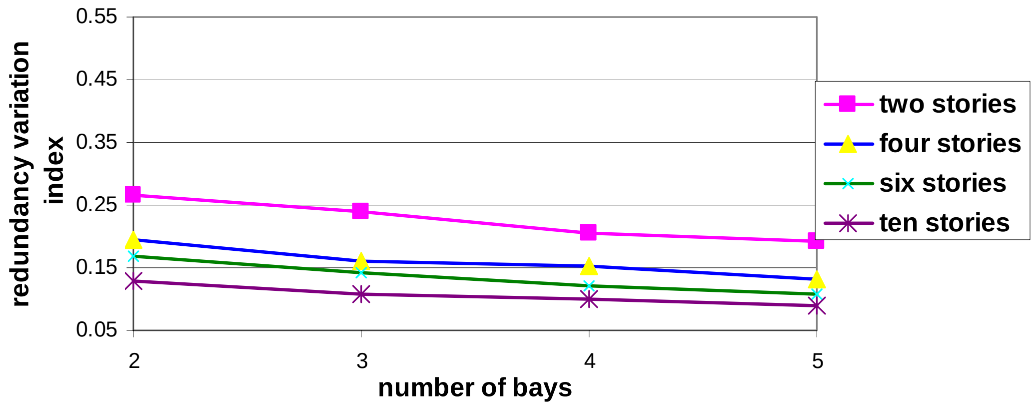

Figure 8. Variation of redundancy-variation index with number of bays and stories for pushover analysis and low ductility

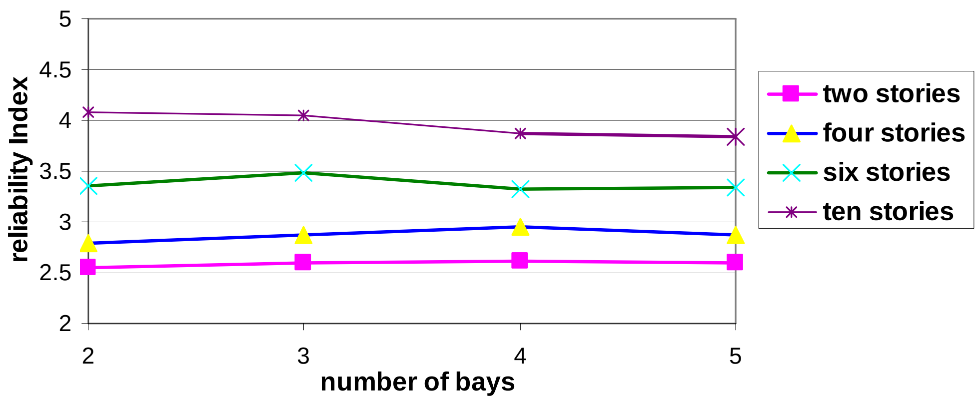

Figure 9. Variation of reliability index with number of bays and stories for pushover analysis and high ductility

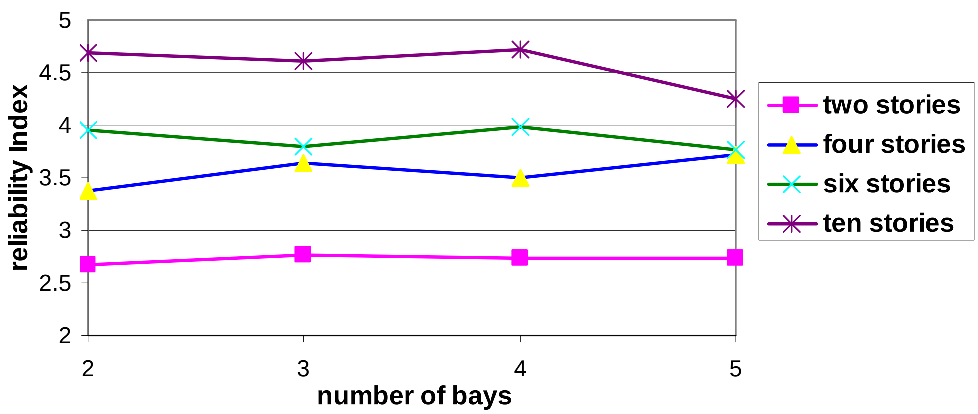

Figure 10. Variation of reliability index with number of bays and stories for pushover analysis and low ductility

As it is shown by the figures, redundancy strength factor is not very much sensitive to the number of bays in both high and low ductility, but it is increased with a rise in the number of stories. Moreover, increasing the number of bays and stories would reduce redundancy variation factor. As a result, reliability index is increased by rising the number of stories and is not sensitive to the number of bays. Therefore, to compute reliability index, the values of four bays is averaged.

To carry out nonlinear dynamic analysis with rising acceleration using IDARC, since it was required to input a new PGA value at each step, a special software was developed to do the operations automatically. The software begins to analyze with a primary PGA value and it continuously increments by 0.02g at each time interval until one of the five failure conditions is achieved. In this case, the base shear coefficient value at yield moment and maximum base shear coefficient is applied for computing r_s, but for computing r_v index, the number of plastic hinges formed at failure moment is used. The computation for 16 frames with high ductility and 16 frames with low ductility under eight seismic records are done in a manner similar to that of nonlinear static analysis in 4 different conditions. Finally, frames were analyzed for different conditions and the values of base shear coefficient at the moment of forming the first plastic hinge, maximum base shear coefficient, and number of plastic hinges at failure were used as parameters required for computing r_s, r_v and β indices. It shall be noted that the average values obtained by eight records have been the basis for computing the above indices. Figure 11 shows r_s values for frames with different number of bays and stories for high ductility and Figure 12 shows r_s values for low ductility. Similarly, Figures 13 and 14 show r_v values for high and low ductility cases and Figures 15 and 16 show β values in terms of different number of bays and stories for high and low ductility, respectively.

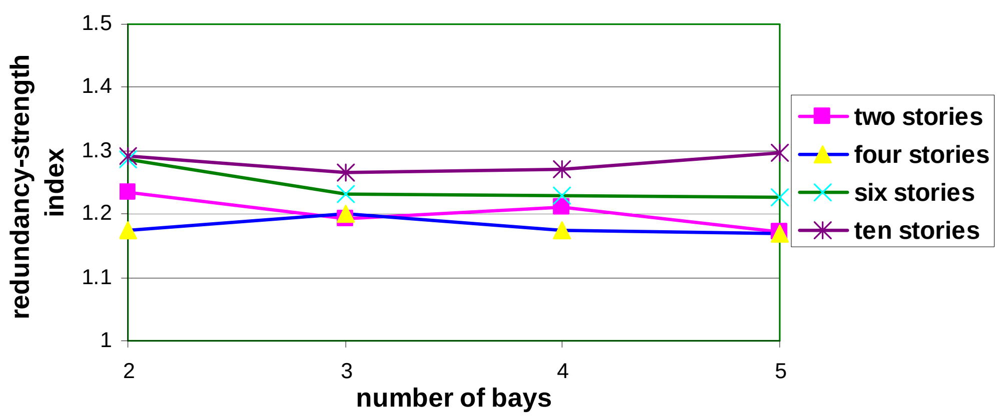

Figure 11. Variation of redundancy-strength index with number of bays and stories for dynamic analysis and high ductility

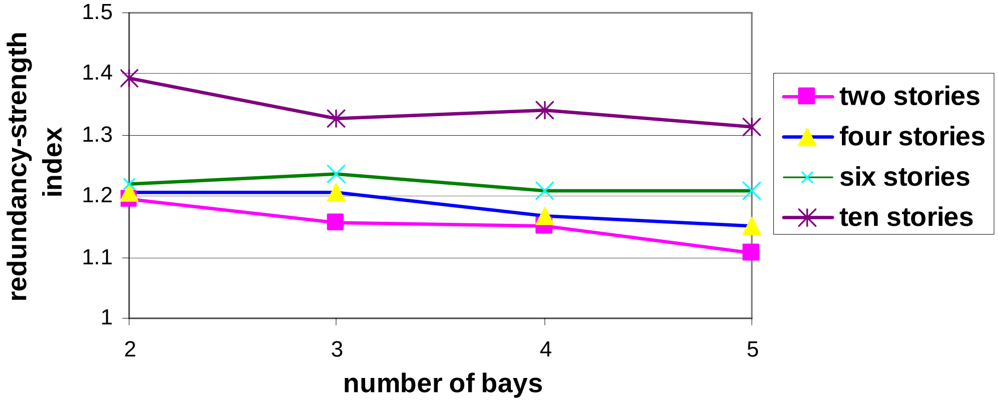

Figure 12. Variation of redundancy-strength index with number of bays and stories for dynamic analysis and low ductility

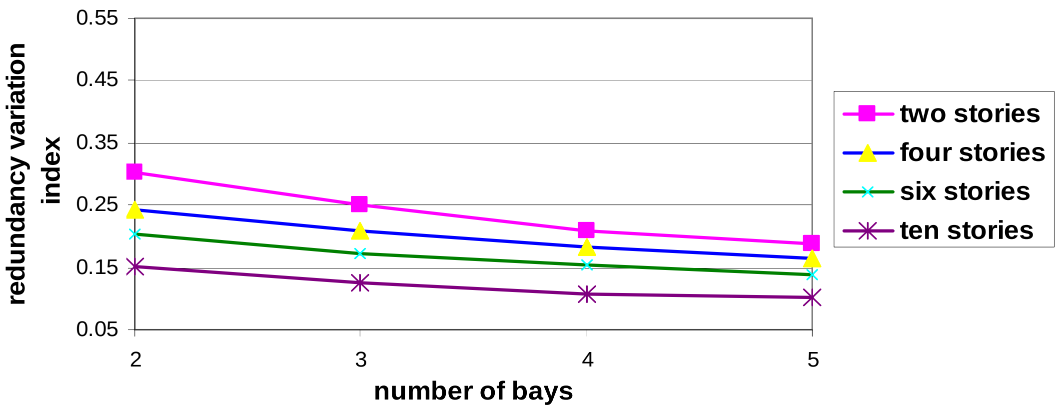

Figure 13. Variation of redundancy-variation index with number of bays and stories for dynamic analysis and high ductility

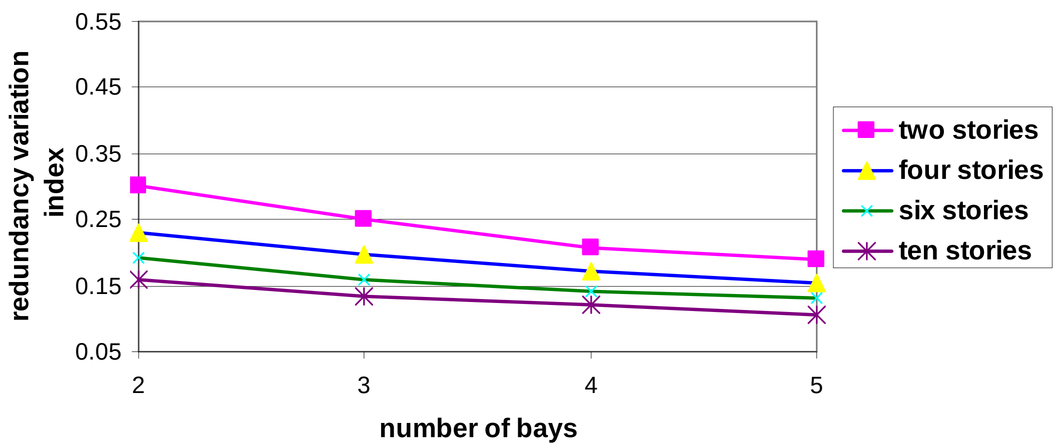

Figure 14. Variation of redundancy-variation index with number of bays and stories for dynamic analysis and low ductility

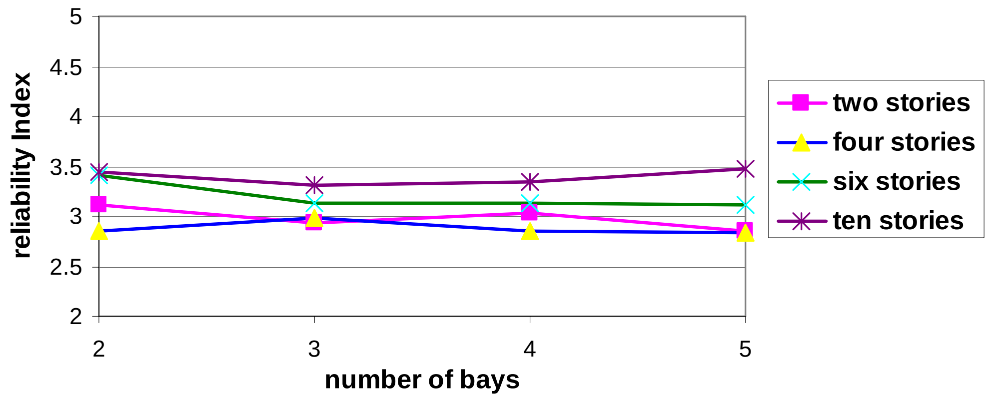

Figure 15. Variation of reliability index with number of bays and stories for dynamic analysis and high ductility

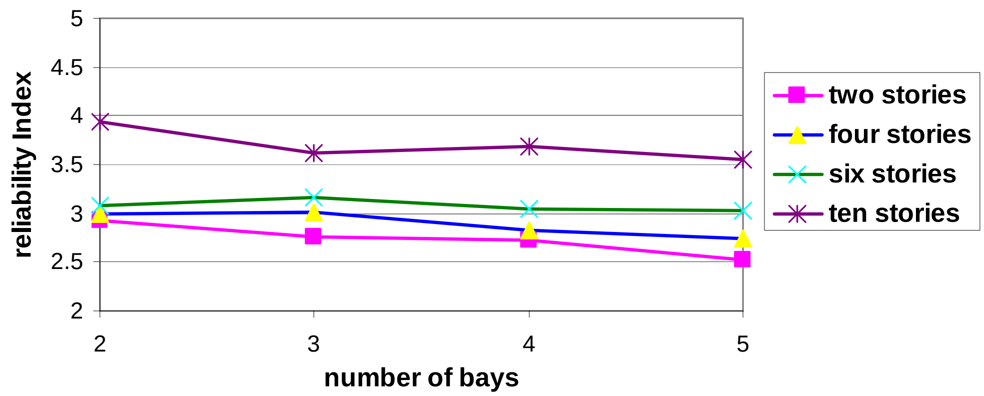

Figure 16. Variation of reliability index with number of bays and stories for dynamic analysis and low ductility

As it is shown by the figures, redundancy strength factor is not much sensitive to the number of bays in nonlinear dynamic method as in nonlinear static method and in both high and low ductility conditions, but it would rise with an increase in the number of stories. Accordingly, redundancy variation factor is also reduced with an increase in the number of bays and stories. As a result, reliability index increases with the increase to in the number of stories, but is not sensitive to the number of bays. Thus for computing redundancy modification factor, four bay values are averaged.

5. Conclusions

By comparing the values obtained from the analyses to computed reliability index β, the following conclusions were obtained:

- Figures 17 and 18 show redundancy index variations for nonlinear dynamic and static methods for both high and low ductility, respectively. As it is presented in the figures, in nonlinear dynamic analysis method with increasing acceleration (IDA), the β factor has increased in most conditions with reducing ductility and shows that structures designed with low ductility have higher β as compared to high ductility. In the Static Pushover Analysis method (SPO), similar to Incremental Dynamic Analysis method (IDA), the value of β is increased in most conditions with lowering ductility.

- Figures 19 and 20 show the reliability index variations to nonlinear dynamic and static methods in four different conditions and in high ductility, respectively. As it is observed, in Incremental Dynamic analysis (IDA), β factor in most conditions is increased with adding the number of stories for structures having high ductility, but, in Static Pushover Analysis method (SPO), β factor increases slightly by adding the number of stories for first and second conditions and is decreased slightly for the third case, but is not much different for the forth condition.

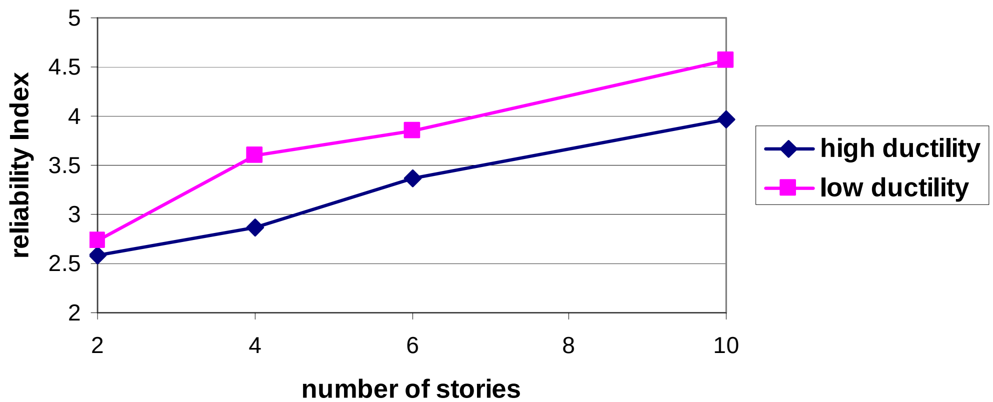

Figure 17. Variation of reliability index with number of stories for dynamic analysis (case1)

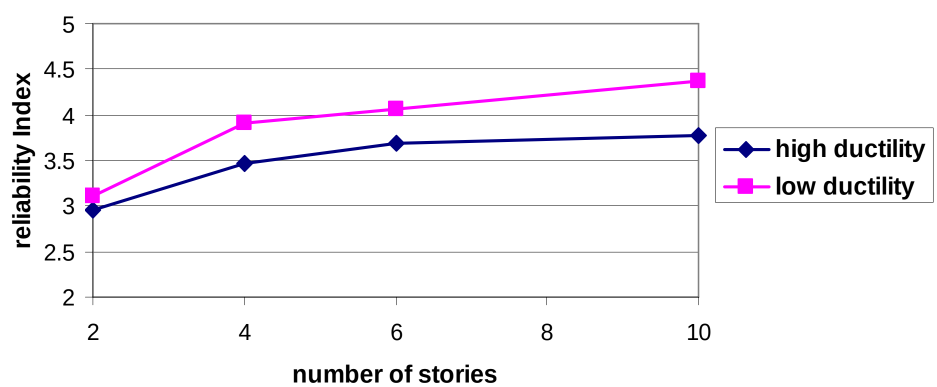

Figure 18. Variation of reliability index with number of stories for pushover analysis (case1)

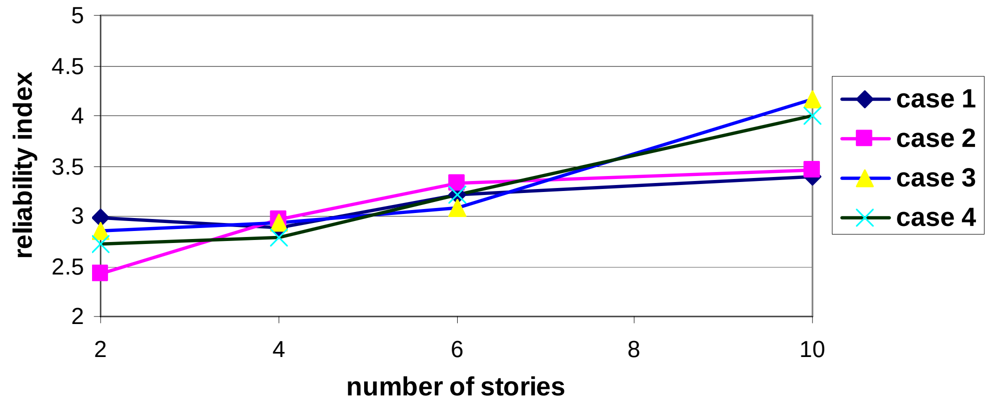

Figure 19. Variation of reliability index with number stories for dynamic analysis and high ductility

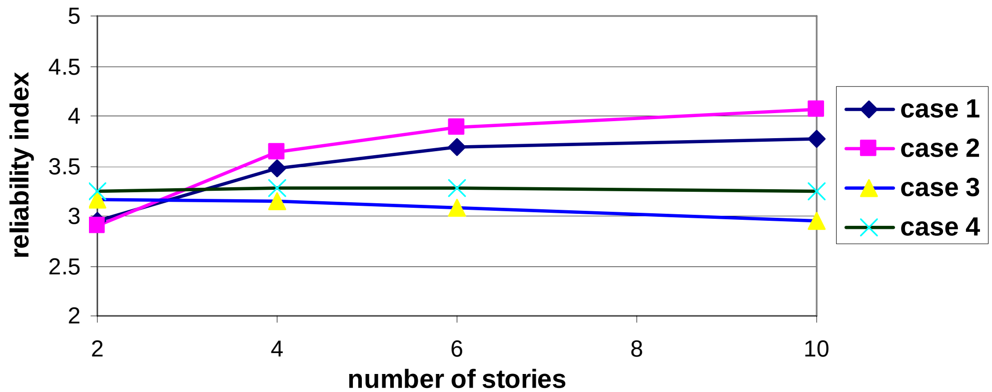

Figure 20. Variation of reliability index with number stories for pushover analysis and high ductility

- Figure 21 shows redundancy index variations for nonlinear dynamic and static method with high ductility. By comparing the responses obtained from Static Pushover Analysis method (SPO) to those of nonlinear dynamic method with Incremental Dynamic Analysis (IDA), it is concluded that in most conditions, β factor obtained from static method is larger than dynamic coefficients, but the difference is maximally 13%. According to the Figure 21, it can be concluded that the results obtained from nonlinear static method are in good agreement with results obtained by nonlinear time history method and may be used as a reliable method.

It is should be emphasized that the results presented in this paper are only acceptable for frames modeled in this study and might not be generalized to all other structural models.

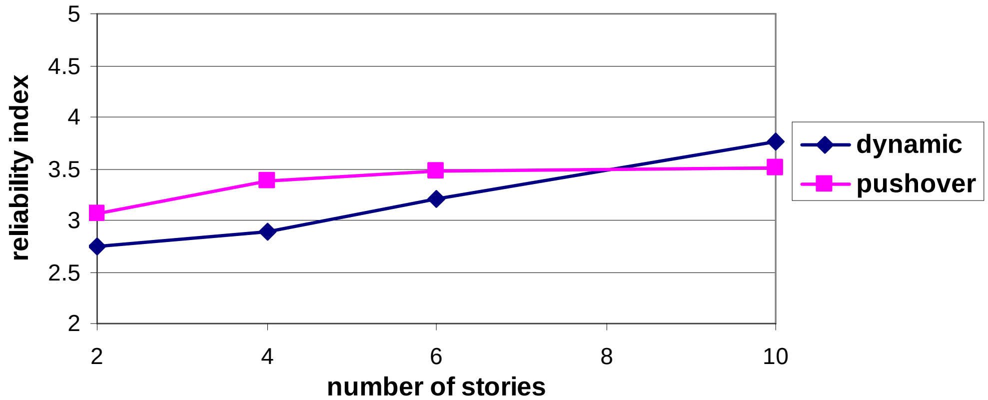

Figure 21. Comparing the reliability index for pushover and dynamic analysis with high ductility

References

Bertero, R.D. and Bertero, V.V. (1999), "Redundancy in earthquake-resistant design", *Journal of Structural Engineering*, Vol. 125, No. 1, Pages 81-88.

Committee of Revising the Code of Practice for Seismic Resistant Design of Buildings (2005), “Iranian Code of Practice for Seismic Resistant Design of Buildings (Standard 2800)”, 3rd Edition, *Building and Housing Research Center (BHRC)*, Tehran, Iran.

Husain, M. and Tsopelas, P. (2004), "Measures of structural redundancy in reinforced concrete buildings-I: Redundancy indices", *Journal of Structural Engineering*, Vol. 130, No.11, Pages 1651-1658.

Habibullah, A. and Wilson, E. (1997), "SAP2000 Version 9.02: A Computer Program for Integrated Finite Element Analysis and Design of Structures", *Computers and Structures Inc.*, University of California, Berkeley, CA. USA.

Masumi, A. and Tasnimi, A.A. (2004), "Determining the behavior coefficient of reinforced bending frames with emphasis on additional resistance and redundancy degree", *Ph.D. dissertation*, Faculty of Technical and Engineering, Tarbiat Modares University, Tehran, Iran.

Nowak, A.S. and Collins, K.R. (2000), “Reliability of Structures”, *McGraw-Hill Inc.*, New York, NY., USA.

Park, Y.J. and Ang, A.H.S. (1985), "Mechanistic seismic damage model for reinforced concrete", *Journal of Structural Engineering*, Vol. 111, No. 4, Pages 722-739.

Ranganatan, R. (1990), “Reliability Analysis and Design of Structures”, Tata *McGraw-Hill Publishing Company Limited*, New Delhi, India.

Sophocleous, A.A. and Syrmakezis, C.A. (2002), "The influence of overstrenght to the structural reliability of structures”, *12th European Conference on Earthquake Engineering*, Geneva, Switzerland, Paper No. 689.

Thoft-Christensen, P. and Sorensen, J.D. (1982), “Reliability of structural systems with correlated elements", *Applied Mathematical Modelling*, Vol. 6, No. 6, Pages 171-178.

Tsoplas, P. and Husain, M. (2004), "Measures of structural redundancy in reinforced concrete buildings-II: Redundancy response modification factor", *Journal of Structural Engineering*, Vol. 130, No. 11, Pages 1659-1666.

Valles, R.E., Reinhorn, A.M., Kunnath, S.K.C. and Madan, A. (1999), "IDARC-2D, Version 5.0: A computer program for inelastic damage analysis of building", *Technical Report NCEER-96-0010*, State University of New York, Buffalo, NY., USA.

Wen, Y.k, and Song, S.H. (2003), "Structural reliability/redundancy under earthquakes", *Journal of Structural Engineering*, Vol. 12, No. 1, Pages 56-66.

Fallah, A.A., Sarvghad-Moghadam, A. and Massumi, A. (2009), "A nonlinear dynamic based redundancy index for reinforced concrete frames", *Journal of Applied Science*, Vol. 9, No. 6, Pages 1065-1073.