MODAL IDENTIFICATION OF A TESTED STEEL FRAME USING LINEAR ARX MODEL STRUCTURE

Yavuz Kaya[1]

Department of Earthquake Engineering, Kandilli Observatory and Earthquake Research Institute, Bogzaici University, Istanbul, Turkey

This study contains the identification of modal dynamic properties of a 3-story large-scale steel test frame structure through shaking table measurements. Shaking table test is carried out to estimate the modal properties of the test frame such as natural frequencies, damping ratios and mode shapes. Among many different model structures, ARX (Auto Recursive Exogenous) model structure is used for modal identification of the frame structure system. The unknown parameters in the obtained ARX model structure are estimated by Least-Square method by minimizing the AIC criteria with the help of a program coded in advanced computing software MATLAB®. The adopted model structure is then tested out in time domain to verify the validity of the model with the selected model parameters. Then the modal characteristics of test frame and the story stiffness are estimated using the white noise shakings. An attempt is done to determine the change of modal characteristics and the story stiffness of test frame according to the velocity, which the test frame structure experienced during the shaking schedule and also during the input shaking of El Centro 1940 NS. Results shows that there is an increase in damping ratio and a decrease in both story stiffness and natural frequency for all modes when the damage forms at cementitious device and the test frame structure itself during the shaking schedule.

Keywords: modal identification, ARX model structure, modal dynamic properties, natural frequency, damping ratio, mode shape, and story stiffness

1. Introduction

In recent years, Structural Health Monitoring (SHM) has emerged as a new research area in civil engineering. SHM is of importance in order to reduce the maintenance cost of civil structures or

---

[1] Research Assistant

buildings. Visual inspection and evaluation of civil structures are usually done by means of site inspections, which sometimes cannot provide enough information for the maintenance due to the some undetectable structural damage, which can be hidden in the structure. With the help of rapid development in digital data acquisition and processing speed, and the availability of minicomputers and microprocessors, many different intelligent health monitoring methods using experimental data to identify the structures' modal characteristics have been presented (Doebling et al, 1996). Once the characteristics of the structures such as the modal frequencies, the modal damping ratio and the mode shapes are identified, the changes in these properties which may occurred during its functional life can be evaluated. In SHM, the true estimation of dynamic properties of structures plays an important role to detect and locate the damage. Therefore, it is obvious that there is a clear need for enhanced understanding of identifying the modal dynamic properties of structures during shaking or under ambient vibrations.

Modal identification allows us to build a mathematical model of a dynamic system based on measured input and output data. In other word, the modal identification is determination of the modal parameters of built structures from experimental data. In modal identification problem, the aim is always to find a model structure whose outputs are as close as possible to the true system outputs when same input is given to both systems. The modal parameters are natural frequencies (the resonance frequency), mode shapes (the way the structure moves at a certain resonance frequency) and damping ratio (the degree to which the structure itself is able to damp out vibrations). Thus it is clear that modal parameters are very important because they describe the inherent dynamic properties of structure. Since these dynamic properties are directly related to the mass and the stiffness, experimentally obtained modal parameters provide information about these two physical properties of a structure. Therefore, the intention of this study is drawn to the estimation of dynamic properties of structures and the estimation of story stiffness of structures as accurate as possible. To achieve this mission, 3-story large-scale steel test frame structure, which cementitious device is setup at the center of each story level as shown in Figure1 is used.

In order to perform the modal identification, first a mathematical model of the steel test frame structure must be formed by a model structure. The choice of model structure is one of the most important factors in the formulation of the system identification problem because it will affect the accuracy of the identified modal parameters. There are many model structures already developed to be used for the identification purpose. For the simplicity of problem, ARX dynamic model structure is utilized in this study to estimate the modal characteristics of the test frame through shaking table measurements. The parameters of obtained model structure are estimated by try and error method using AIC criteria as minimum. An attempt is made to minimize the error between the actual structural parameters and the obtained model structure's parameters. No matter how accurately the model structure's parameters are selected, there is always an error remains in the

obtained model structure due to the noise contamination in the input, system non-linearity and the uncertainty in the obtained model structure. Therefore, in order to minimize the error, among many methods, Least-Square approach is used in the estimation of model structure's coefficients. Then, in order to ensure that the obtained model structure is properly adjusted to actual structural model, a validation process is also performed in time domain additionally.

Once the actual steel test frame structure's mathematical model is formed using an ARX model structure whose coefficients are selected so that the error in the selected model is minimized by Least-Squares method, after that the modal characteristics of the steel test frame structure are estimated through selected ARX model structure. In addition to this, story stiffness of test frame structure is also estimated based on the identified modal characteristics.

During the shaking schedule, steel test frame structure is experienced different target level of velocity induced by El Centro 1940 NS. As a result of this, different level of damage is observed at both steel test frame and cementitious device as well. Therefore, modal properties of test frame are changed after each shaking due to the damage. White noise shakings scheduled in Table 2 are utilized in order to identify the change in modal properties of test frame. It is observed that the modal frequency decreases and damping ratio increases for all modes as the damage level increase at structure. As a result of that story stiffness is also decrease for all stories.

The experiment with El Centro 1940 NS (10cm/s^2) is selected to identify the change of modal characteristics of the test frame throughout the shaking period. To do that, recorded data is divided into 5-second lengths of waves in order to identify the change of modal characteristic at discreet time intervals.

2. Test Frame and Experiment

The shaking table tests are carried out at laboratory of National Research Institute for Earth Science and Disaster Prevention (NIED) in March 2002 in Japan. The dimension of shaking table is 15m by 14.5m. The table weight is 180,000kg. The maximum displacement capacity is ±23cm. Shaking table specifications are given in Table 1. The general view of steel test frame used in the shaking table test is shown in Figure 1(a).

Input waves of shaking are white noise, BCJ-L1 (simulation wave Level 1 of Building Center of Japan) and El Centro NS recorded in 1940 with different target velocities. Shaking schedule can be seen in Table 2. Sampling frequency of all input waves is 200Hz. Accelerometers are installed at each story level at the center of each floor in order to identify the dynamic characteristics of test frame in the x direction of excitation shown in Figure 1(b). Test frame is 3 by 4m in plan and has a total height of 5.4m with a 1.8m story height. The outline of test frame is given in Table 3.
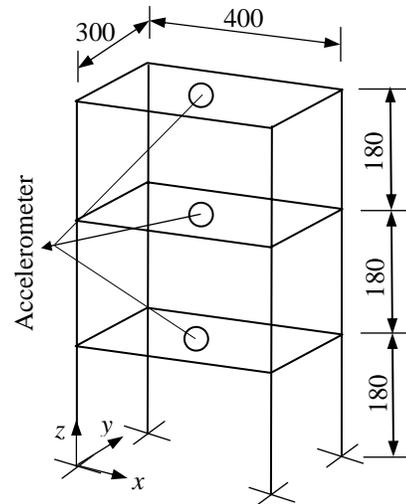
(a)   (b)

Figure 1. Test Frame: a) General View; b) Instrumentation layout (All dimensions in cm)

Table 1. Shaking table specifications

Table size	15m x 14.5 m
Power Output	3528kN (882kNx4)
Maximum Load Weight	500,000kg
Maximum Displacement	±23 cm
Maximum Velocity	90cm/sec
Maximum Acceleration	490cm/sec^2 (500,000kg) 921 cm/sec^2 (200,000kg)

Table 2. Excitation Function Program

No	Input Wave	Target Velocity
1	White noise	-
2	El Centro 1940 NS	5 cm/s
3	BCJ wave	2.5 cm/s
4	Hachinohe	5 cm/s
5	Kokuji-ha	2.5 cm/s
6	White noise	-
7	El Centro 1940 NS	10 cm/s
8	White noise	-
9	El Centro 1940 NS	15 cm/s
10	White noise	-
11	BCJ wave	22 cm/s
12	White noise	-
13	El Centro 1940 NS	30 cm/s
14	White noise	-
15	El Centro 1940 NS	40 cm/s
16	White noise	-
17	El Centro 1940 NS	50 cm/s
18	White noise	-
19	El Centro 1940 NS	60 cm/s
20	White noise	-

Table 3. Outline of test frame

Floor Height	1.8m
Total Height	5.4 m
Floor Plan	3m x 4m
Column	H148 x 100 x 6/9 (SM490)
Beam (cross direction of shaking)	H148 x 100 x 6/9 (SM490)
Beam (edge beam of shaking direction)	H150 x 150 x 7/10 (SM490)
Beam (center beam of shaking direction)	H300 x 150 x 6.5/9 (SM490)
Weight of floor	4000 kg

3. Modal Identification Method

A model of a system is a description of (some of) its properties, suitable for a certain purpose. The model need not be a true and accurate description of the system, nor need the used have to believe so, in order to serve its purpose (Ljung, 1987). System identification method used in this study consists of three main steps. The first step is qualitative operation, which defines the structure of the system for example, type and order (model parameters) of the differential equation relating the input to the output; it is known as characterization. This means selection of an appropriate model structure, e.g. auto- recursive with exogenous input (ARX), auto-recursive moving average with exogenous input (ARMAX). The second step is identification/estimation, which consists of determining the numerical values of the structural parameters (coefficients of the model structure) where the error between the system to be identified and its model structure is minimized. There are many estimation methods already developed, which minimize the error, some of which are Least-Squares (LS), Instrumental- Variable (IV), Maximum-Likelihood (MLE) and the Prediction-Error Method (PEM). This process can be called, in simple terms, a curve fitting exercise. The third step is verification where the system is related to the identified model responses in time or frequency domain to check the accuracy and quality of the extracted model. For time domain validation, data used in the estimation is selected and compared with model structure adopted in order to ensure that the model structure is properly adjusted to specific data records or input forms. On the other hand, for frequency validation, the power spectral density functions of the system and the model are compared. In this study only the time domain validation analysis is performed.

Among different model structures mentioned before, the most basic and the most commonly used parametric model, ARX dynamic model structure as shown in Figure 2 is adopted in this study to

define the model structure of the test frame system where the Least-Squares method is used to minimize the error in the input wave (Ljung, 1987).
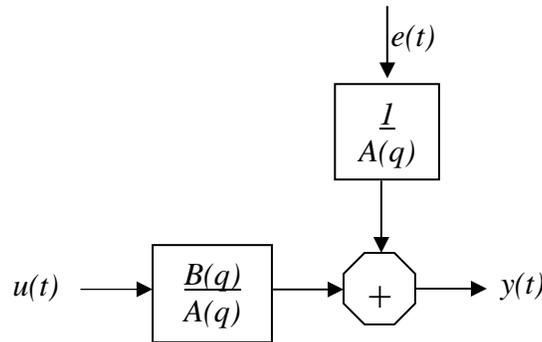
Figure 2. The ARX model structure

4. ARX Model Structure

ARX model structure is actually the simple linear difference equation where the current output $y(t)$ is considered to be a linear function of finite number of past values of both input $u(t-k)$ and output $y(t-k)$. Thus the ARX model takes the form of:

$$y(t)+a_1 y(t-1)+...+a_{na} y(t-na)=b_1 u(t-ns)+.........+n_{nb} u(t-ns-nb+1)+e(t) \qquad (1)$$

where a_j and b_j are the coefficients of the linear difference equation to be defined, $u(t)$ is the input, $y(t)$ is the output, n_a is equal to the number of poles, n_b is equal to the number of zeros and n_s is equal to the pure time delay (the dead-time) in the system.

This linear difference equation can be more conveniently expressed in shift operator form as follows:

$$A(q)y(t)= q^{-ns} B(q)u(t)+ e(t) \qquad (2)$$

where q is the shift operator.

$$y(t)= G(q)u(t)+ e(t)/A(q) \qquad (3)$$

$$G(q)= q^{-ns} \frac{B(q)}{A(q)} \qquad (4)$$

where $A(q)$ and $B(q)$ are defined as

$$A(q) = 1 + \sum_{j=1}^{na} a_j q^{-j}$$

(5)

$$B(q) = \sum_{j=1}^{nb} b_j q^{-j+1-ns}$$

(6)

$A(q)$ and $B(q)$ can also be expressed in explicit way as follows:

$$A(q) = 1 + a_1 q^{-1} + a_2 q^{-2} + \ldots\ldots + a_{na} q^{-na}$$

(7)

$$B(q) = b_1 + b_2 q^{-1} + \ldots\ldots\ldots + b_{nb} q^{-nb+1}$$

(8)

As can be seen easily from the Equation 5 and Equation 6, many different ARX model structure can be used for the model identification based only on the selection of n_a, n_b and n_s values. Therefore, in order to determine an ARX model whose outputs are as close as possible to the true system outputs when same input is given to both systems, n_a, n_b and n_s values are determined in which AIC is estimated as minimum where $n_b = n_a + 1$ whereas n_s is defined as zero (Akaike, 1973). More details on the general aspects of identification theory can be found in (Ljung, 1987). After having check the validity of ARX model structure in time domain to ensure that the adopted model structure's parameters are selected accurately, then the estimation of modal characteristics can be made.

As for the estimation of modal parameters, p_i is defined as the root of $A(q) = 0$ whereas r_i is defined as the residue of $G(q)$. Natural frequency f_i, damping ratio h_i and participation function $\beta\Phi_i$ are defined as follows:

$$f_i = \frac{\sqrt{(\log|p_i|)^2 + (\arg p_i)^2}}{2\pi\Delta t}$$

(9)

$$h_i = \frac{-\log|p_i|}{2\pi f_i \Delta t}$$

(10)

$$\beta\Phi_i = R\left[\frac{2 r_i \sqrt{1 - h_i^2}}{\Delta t\left(2\pi f_i h_i - i \operatorname{sign}\left[I(p_i)\right]2\pi f_i\left(1 - 2h_i^2\right)\right)}\right]$$

(11)

where $\beta\Phi_i$ is participation function, (β is the participation factor, Φ_i is the mode shape), R represents the real part of a complex number and I represents the imaginary part of a complex number. The parameters of ARX model structure used in this study is presented in Table 4.

Table 4. The parameters of ARX Model Structure

White-noise Number	n_a	n_b	n_s
1	22	23	0
12	22	23	0
20	28	29	0

5. Stiffness Matrix Calculation

Identified mode shapes are normalized with respect to mass matrix:

$$[\Phi]^T [M][\Phi] = [I] \tag{12}$$

As for the global story stiffness matrix $[K]$ calculation, equation of free vibration can be written as follows (Chopra, 1995):

$$[K]\{\phi\} = [\omega^2][M]\{\phi\} \tag{13}$$

where $[K]$, $\{\phi\}$, $[\omega^2]$ and $[M]$ are global stiffness matrix, mode shape vector, squared natural circular frequency matrix and mass matrix, respectively.

Multiplying the Equation 13 by $\{\phi\}^T$ from left will yield the following equation:

$$\{\phi\}^T [K]\{\phi\} = [\omega^2]\{\phi\}^T [M]\{\phi\} \tag{14}$$

The Equation 15 is obtained by taking advantage of the orthogonality feature of the mode shape (see Equation 10):

$$\{\phi\}^T [K]\{\phi\} = [\omega^2] \tag{15}$$

Driving this equation for $[K]$ will yield:

$$[K] = \left(\{\varphi\}^T\right)^{-1}[\omega^2]\{\phi\}^{-1} \tag{16}$$

Therefore, the stiffness matrix of structure can be estimated based only on the identified mode shape vector and the identified natural circular frequency matrix. Equation 16 indicates the influences of the various modal frequencies and mode shapes on the stiffness matrix $[K]$. Then story stiffness k_i can be calculated by multiplying the estimated global stiffness matrix $[K]$ with

a displacement vector where the relative story displacement above the story considered is unity and zero elsewhere.

6. Results of Modal Identification

6.1. White Noise function

The input wave number 1, 12 and 20 as scheduled in Table 2, are used in order to identify the modal properties of the test frame structure using the ARX model structure. Since the input wave number 1 is the first shaking table test, it represents the undamaged state of the steel test frame structure. On the other hand input wave number 12 and 20 represents the mid-damaged and the most damaged states of the steel test frame structure, respectively. As explained before, firstly the coefficients of the ARX model structure n_a, n_b and n_s have to be determined. These coefficients are very important because they play significant role in accurate determination of modal properties and later on the identification of story stiffness as well. Therefore, wrong selection of these coefficients would result in inaccurate estimation of modal characteristics of test frame. For that reason, special attention is given to the selection of n_a, n_b and n_s values. On the other hand it must also be noted that no matter how accurately these values are selected, there will always be an error remains in the results due to the noise contamination in the input, system non-linearity and the uncertainty in model structure.

In this section, these three ARX model parameters (n_a, n_b and n_s) are determined by try and error method with the help of a program coded in MATLAB® (MATLAB User's Manual, 2002). The system identification tools of MATLAB are utilized in the program. The coefficients of linear difference equation (ARX model structure) are estimated by least squares method where the error term $e(t)$ defined in Equation 2 is tried to be minimized. When forming the mathematical model of the test frame, several different, n_a (number of poles) values (from 10 to 50, only even numbers) are tried by try and error method in order to find the best fitted ARX model by taking into account the AIC criteria as minimum. The ARX model parameters adopted for the white noise number 1, 12 and 20 are given in Table 4. As explained before, $n_b = n_a + 1$ and $n_s = 0$.

After having determined the parameters of the model, a validation is done in time domain to test out how much the obtained ARX modal structure fits to the input. In the validation process, the current output considered is not taken into account in the validation process in order to make a stringent test for the adopted ARX model structure. Consequently, a zero-ahead prediction where the extracted model knows only what the inputs are (pure simulation) is done. The result of time domain validation is given in Table 5.

Table 5. Time domain validation (%)

Input-wave number	First story	Second story	Third story
1	71.23	79.51	81.40
12	58.13	69.30	71.50
20	58.33	63.79	69.51

Estimation of modal characteristic of 3-story steel test frame structure is done using the adopted model structure with the selected model parameters. Estimated natural frequencies and damping ratios can be seen in Table 6 and Table 7, respectively. The initial (undamaged) modal characteristics of test frame are estimated using the White noise number 1. The first modal natural frequency and damping ratio are 3.9436Hz and 0.56%, respectively. The initial mode shapes of the structure are shown in Figure 3.

After performing 7 different excitation inputs scheduled in Table 2, white noise number 12 is utilized to identify the change in modal characteristics. Due to the damage formed at test frame structure, it is observed that the first modal natural frequency decreased and damping ratio increased by 8.3% and 230%, respectively. Story stiffness is also decreases as seen in Figure 4.

White noise number 20 is the last shaking representing the most damaged state of the structure. Modal characteristics of the test frame are changed significantly. The change in first natural frequency is 28.2%. Change in mode shapes is shown clearly in Figure 3. As a result of change in modal characteristics, story stiffness is changed significantly too. The change in first story stiffness is about 66.7%, which shows that the damage is concentrated mostly on the first story.

Table 6. Estimated Modal Frequencies (Hz)

Mode Number	Input-wave number		
	1	12	20
1	0.56	1.85	4.35
2	0.66	1.82	3.15
3	0.70	2.13	2.12

Table 7. Estimated Modal Damping Ratios (%)

Mode Number	Input-wave number		
	1	12	20
1	3.9436	3.6172	2.8333
2	12.1389	10.8922	8.9944
3	20.1560	17.5541	14.3571

(a) (b) (c)
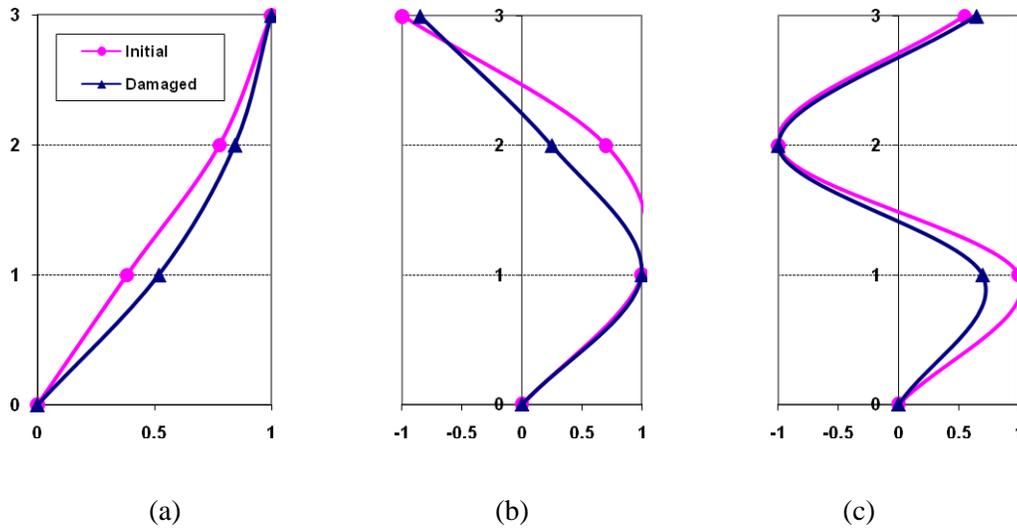
Figure 3. Estimated initial mode shapes: a) First mode; b) Second mode and c) Third mode
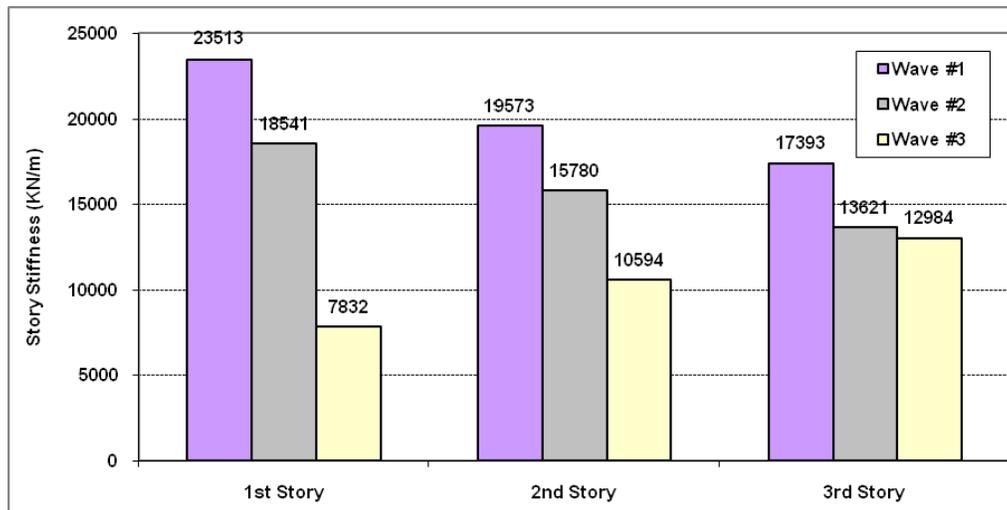
Figure 4. Change of story Stiffness

6.2. Change of Modal Parameters and Story Stiffness According to Experienced Velocity

As it is shown in Table 2, there are 20 different experiments that had been carried out. Every shaking has different target velocity induced by mainly El Centro 1940 NS. At each shaking, steel test frame structure experienced a different target velocity, which causes different level of damage at both steel test frame and cementitious device as well. Therefore, to understand the change of modal characteristics due to that damage, white noises scheduled in Table 2 are utilized. Each white noise is performed after a damage state. Estimation of modal characteristics and story stiffness of test frame structure are done based on the procedure explained before. ARX model structure is used again for the modal identification. The validation in time domain is done

for each white noise shaking. After having checked out the validity of the adopted ARX model, modal identification is performed for each white noise shaking scheduled in Table 2. The Results of modal identification with respect to experienced velocity are shown in Figure 5.

(a) First mode

(b) Second mode

(c) Third mode
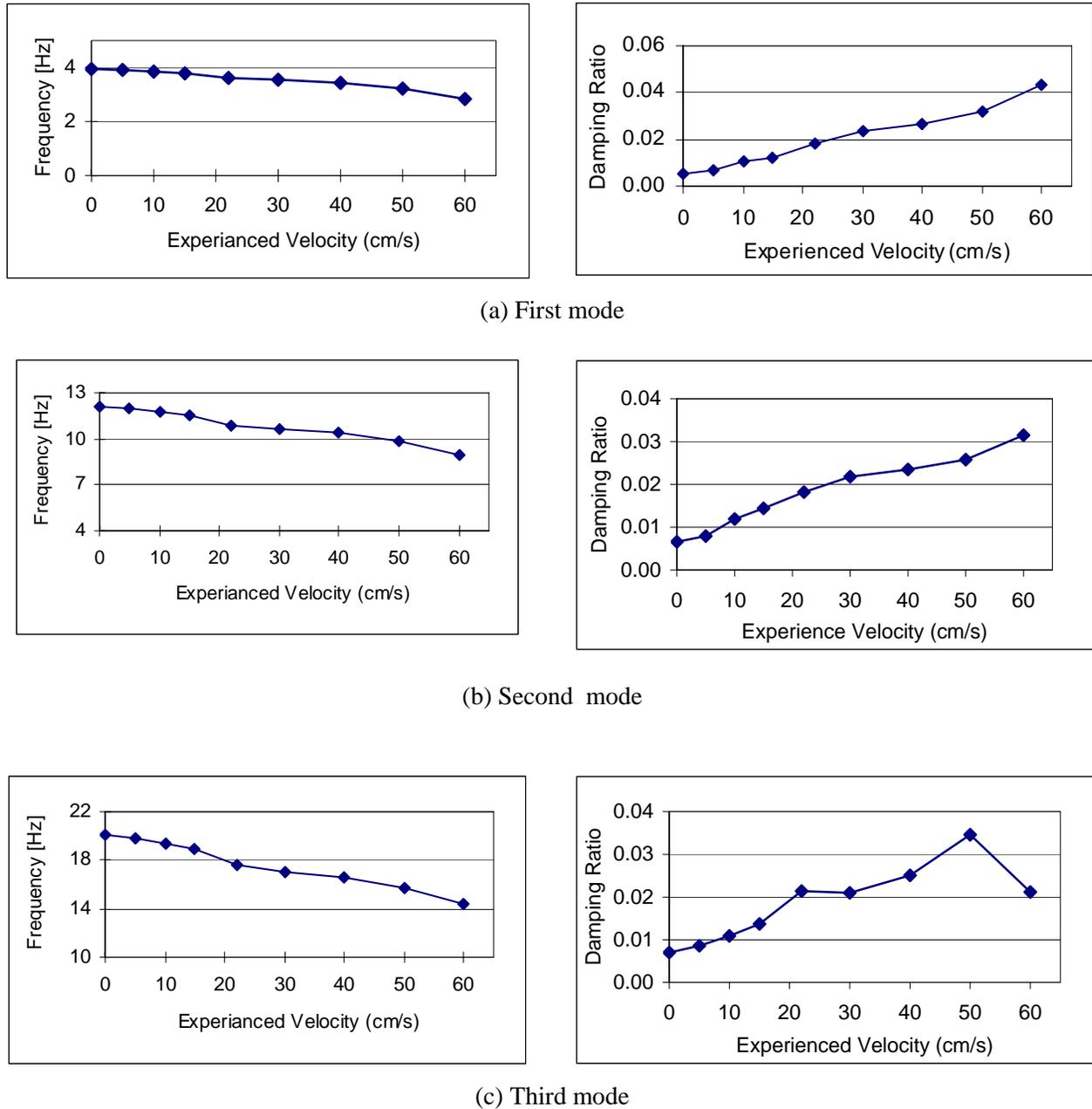
Figure 5. Change of modal frequency and modal damping according to experienced velocity

As it is shown in Figure 5, damping ratio increase as the experience velocity increase for all modes, which indicates that the more energy is damped out as the structure experience the

damage. On the other hand, natural frequency is decreasing for all modes due to the fact that different level of damage is formed at both steel frame structure and cementitious devices as well during the shaking experiment schedule. As a result of this damage, it is also observed that the story stiffness is decreased for all stories as shown in Figure 7.

6.3. Change of Modal Parameters and Story Stiffness during Shaking

An attempt is made to estimate the test frame structure's modal characteristics and story stiffness at discrete time intervals of 5 second. The input wave of El Centro 1940 NS with a target velocity of 10cm/s as shown in Table 2 (No:7) is used for the identification purpose. The response of the third story of steel test frame is amplified by about 5 times comparing to input at base level. The response of test frame structure is illustrated in Figure 6. As it is shown, maximum input acceleration is concentrated within the first 35 seconds. Therefore, the damage at cementitious devices and at test frame itself is expected to occur at this period of time.

(a)   (b)
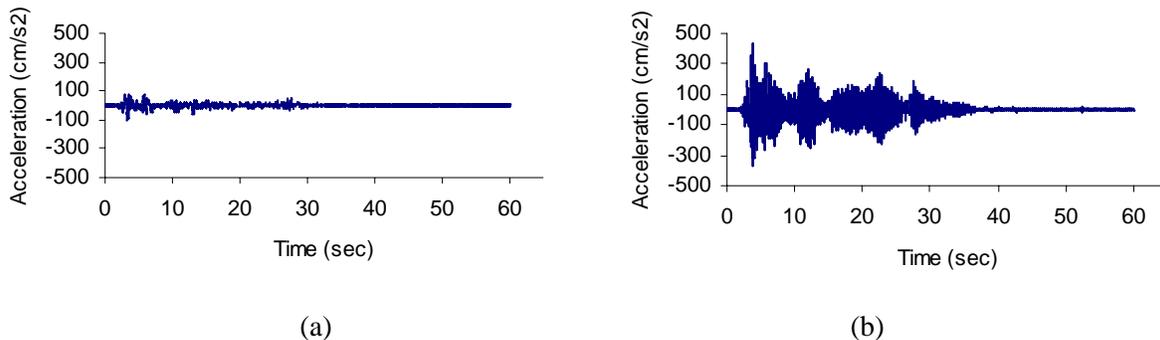
Figure 6. El Centro 1940 NS record with a velocity target of story stiffness: a) Input wave at base level; b) Measured response at third story level

Measured data is divided into 5-second lengths of waves so that the modal characteristics of test frame and later the stiffness of each story can be estimated at discrete time intervals during the input wave shaking. An attempt is done to find the modal characteristics at smaller time intervals such as 4-second or 2-second, but the results obtained were not reliable due to the difficulty in selecting the parameters of liner difference equation (ARX model structure). Eventually, the time interval of 5-second is decided to use.

The same procedure explained before is followed. ARX modal structure is used for the modal identification where n_a, n_b and n_s values are determined as 34, 35 and 0, respectively by try and

error method satisfying the AIC criteria as minimum for each 5-second discrete time intervals. Model validation is also done in time domain. An accuracy of 80.1% fit to the input is achieved.
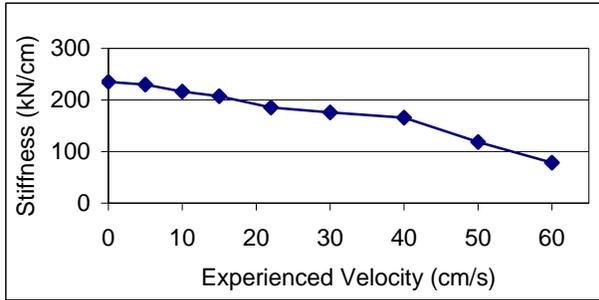
(a) First mode
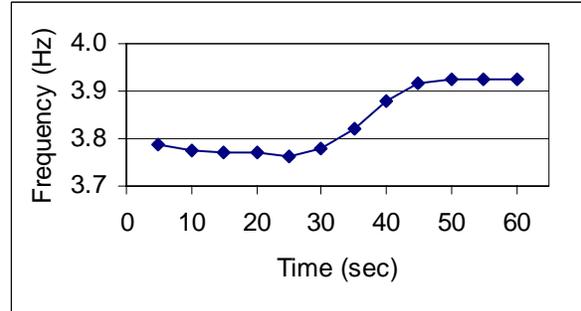
(a) First mode
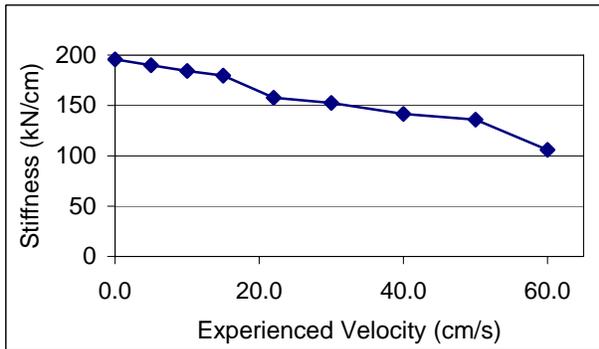
(b) Second mode
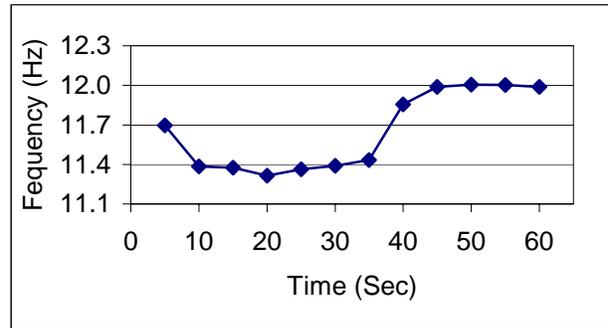
(b) Second mode
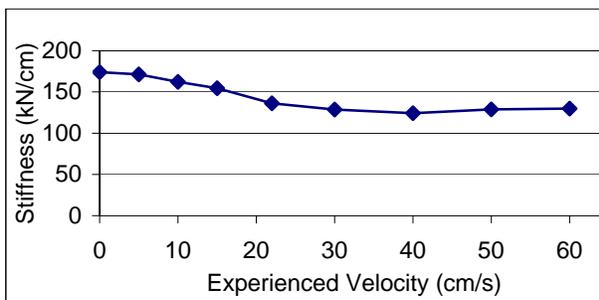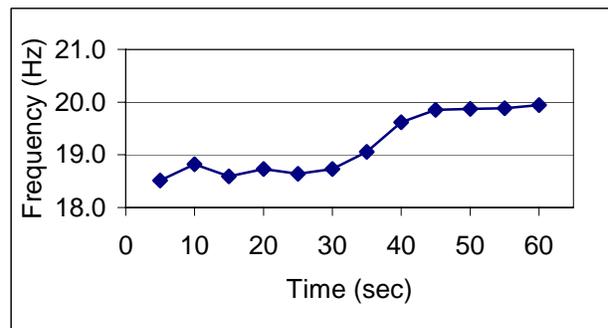
(c) Third mode

(c) Third mode

Figure 7. Change of story stiffness according to experienced velocity

Figure 8. Change of natural frequency during El Centro 1940 NS (V=10cm/s)
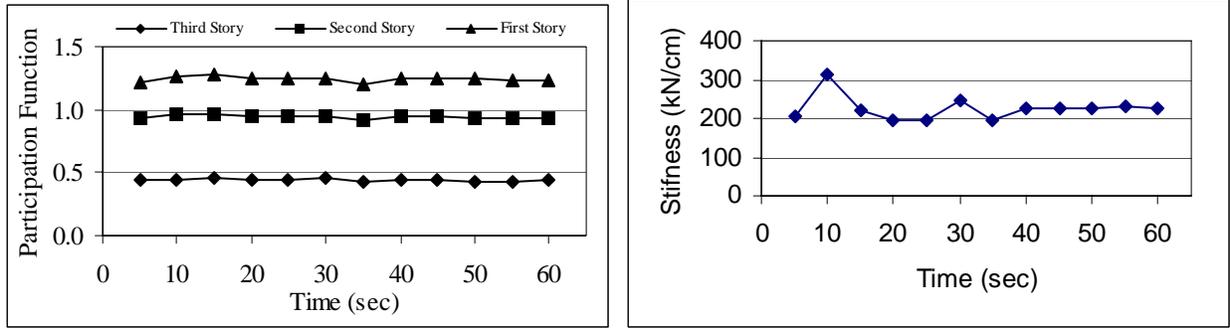
(a) First mode
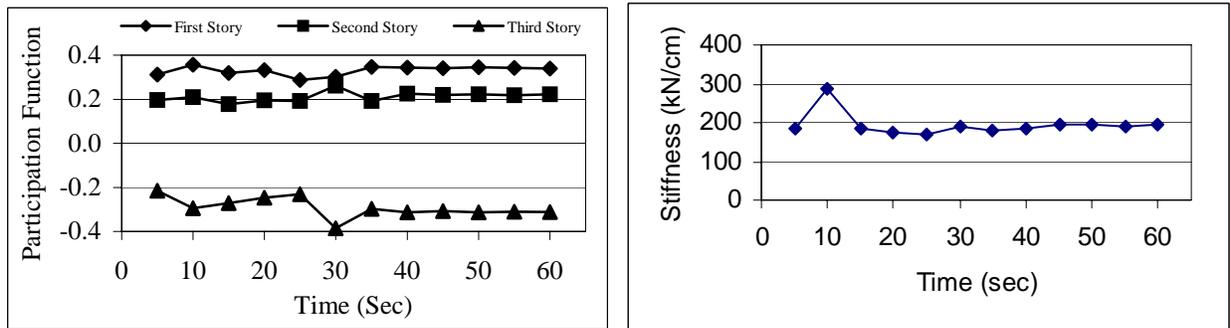
(b) Second mode
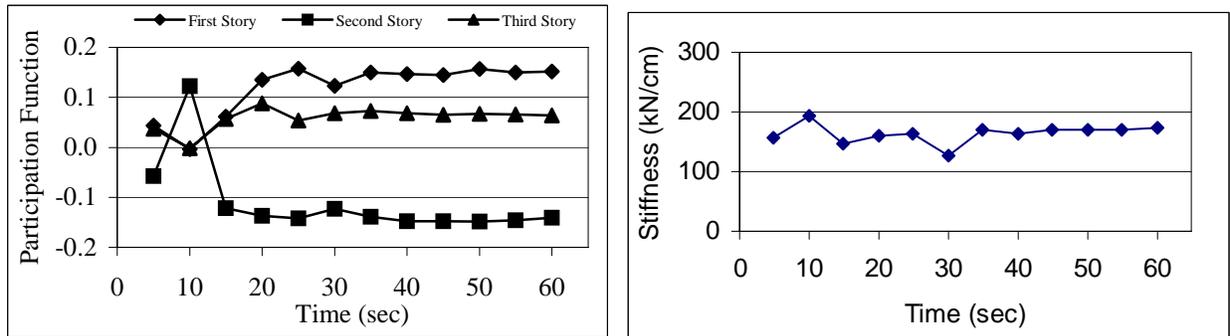
(c) Third mode

Figure 9. Change of participation function and stiffness during El Centro 1940 NS (V=10cm/s)

As expected, cracks at cementitious devices are observed during El Centro 1940 NS (10cm/s) and main reinforcements yield during El Centro 1940 NS (30cm/s^2) shaking (Morita et al, 2005). Identified values for story stiffness according to experienced velocity, natural frequency, participation function, story stiffness and damping ratio can be seen in Figure 7, Figure 8, Figure 9 and Figure 10, respectively. As it is shown in Figure 8, after a slight decreasing in natural frequencies, it is followed by an increasing in the natural frequency of test frame for all modes after 35 second. The decrease in the natural frequency may result from the slight cracks occurred at cementitious device. The damping ratio for the first mode decreases gradually throughout the

input wave whereas second and third damping ratios are unstable for the first 40 seconds and keeps decreasing after 40 second. The 1st and the second mode participation function of test frame do not change much throughout the input wave indicating that the slight damage at cementitious device has no much influence on first and second mode on the other hand third mode shape change during first 30 second. Story stiffness is also stable except at 10 second. This sudden increasing in story stiffness at 10 second is maybe because of difficulty in selecting the ARX model properties at this specific period of time where the maximum input acceleration shows itself.
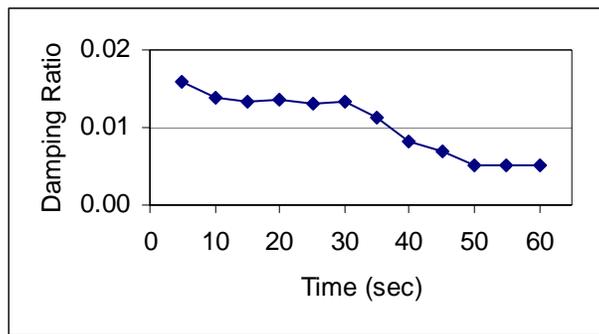
(a) First mode
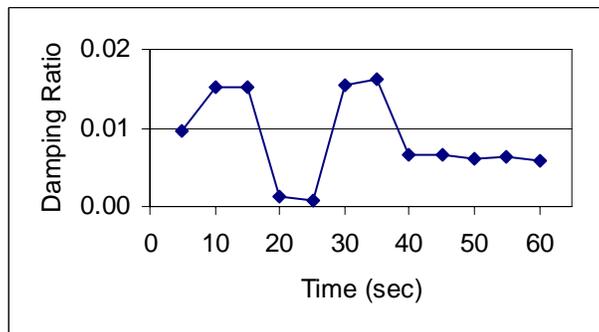
(b) Second mode
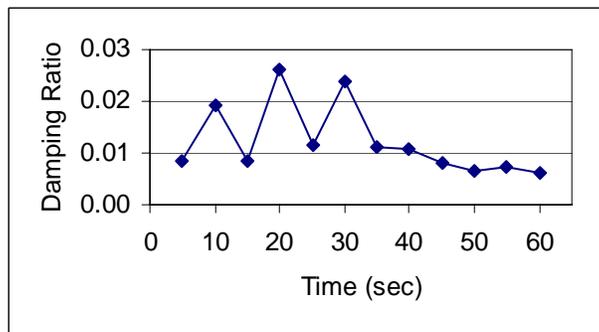
(c) Third mode

Figure 10. Change of damping ratio during El Centro 1940 NS (V=10cm/s)

7. Conclusions

Shaking table tests are carried out in order to identify the modal characteristics of 3-story steel test frame structure and the story stiffness as well. First, initial modal characteristics of the test frame is estimated using the white noise number 1 and a comparison is made with the results obtained from white noise number 12 and number 20 representing the mid-damaged and the most damaged state of frame structure, respectively. Then the change of modal properties according to experienced velocity induced by different input motions scheduled in Table 2 is studied using the white noises only. Finally, the change of modal characteristics during input motion of El Centro (V=10cm/s) is studied. ARX model structure is used for all identification. Results obtained from experiment are as follows:

1- White noise number 1, 12 and 20 are used in order to identify the change in modal properties. Modal characteristics changed when the damage formed at structure. Natural frequency for all modes decreased whereas the damping ratio for all modes increased. Significant change is observed in mode shapes too. As a result of these changes, story stiffness is also decreased for all stories.

2- Nine different white noise records are used in order to identify the change in modal characteristics of test frame throughout the shaking schedule. Modal frequency for all modes decreased and the modal damping ratio increased for all modes as the target velocity of the input wave increased. Significant change is also observed at mode shapes. Consequently, story stiffness for all stories is decreased for all stories.

References

Akaike, H. (1973), "Information theory and an extension of the maximum likelihood principle, in B.N. Petrov and F. Csaki (eds.) ", 2nd *International Symposium on Information Theory*, Akademiai Kiado, Budapest, Hungry, Pages 267–281.

Chopra, A.K. (1995), "Dynamics of Structures: Theory and Applications to Earthquake Engineering", *Prentice Hall*, Englewood Cliffs, USA.

Ljung, L. (1987), "System Identification: theory for the users", *Prentice Hall*, London, UK.

Doebling, S.W., Farrar, C.R., Prime, M.B. and Shevitz, D.W. (1996), "Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in Their Vibration Characteristics: A literature review", *Los Alamos National Laboratory Report*, LA-13070-MS, New Mexico, USA.

MATLAB User's Manual (2002), *The Math Works, Inc.*, Natick, Massachusetts, USA.

Morita, K., Teshigawara, M. and Hamamoto, T. (2005), "Detection and estimation of damage to steel frames through shaking table tests", *Structural Control and Health Monitoring*, Vol. 12, No. 3-4, Pages 357 – 380.