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ORIGINAL RESEARCH

Stiffness Reduction Factor of Flat; Beam-Column and Waffle Slabs under Lateral Load

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Abstract:

Providing sufficient strength and stiffness is considered a primary principle for structural design. Roof flexural members have a remarkable impact on providing lateral and gravity stiffness. A vital issue in the analysis and stiffness assessment is to apply stiffness modification factor for the mentioned members in RC structures so that their impact in gravity and lateral load bearing could be changed. In the present study, to achieve an appropriate coefficient to decrease the stiffness of reinforced concrete slabs under simultaneous gravity and lateral loading, 20 structures ranging from 2 to 20-stories with various roofs including flat slab, beam-column slab, one-way and two-way waffle slabs were designed and analyzed, and then an equivalent overall coefficient for slabs stiffness decrease was obtained. The results indicate that the coefficient of 0.25 recommended by the building codes for flat slabs is approximately 60 percent conservative. In other categories of slabs, for which the available building codes have not recommended a determined coefficient, the coefficient of 0.45 to 0.5 is achieved based on the analysis of the current study.

Keywords: Slabs; Stiffness; Reduction factor; Lateral load.

Introduction

Slab roofs including flat slabs, waffle slabs, and beam-column slabs are categorized as the most applicable type of roofs. The early professional research work conducted on slabs returns to the early twentieth century so the first research is found between 1903 to 1910 (Brayton 1903; Macmillan 1910; Andrews 1909). In the past period of around a century, many researchers have done analytical and experimental research to obtain accurate and engineering approaches to determine moment distribution and slab deflection calculations (Slater et al. 1923; Morley 1966; Holmes and Majed 1972; Robertson 1997; ACI 435.6R-74 1989). Among the earlier and credible methods, the equivalent frame method, direct design method, and equivalent beam could be mentioned (ACI 435.6R-74 1989; ACI 421.3R-15 2015; Luo et al. 1994; Moehle 2015; Fintel 2004). Slabs inherit significant flexural stiffness due to the large width, and on the one hand, the structural design relies on the strength providing, stiffness, and ductility. Providing adequate stiffness is assessed using deflection control under gravity loads, and drift control under lateral loads.

In all available references, to control deflection and drift, due to the presence of creep and micro-cracks characteristics, the stiffness of flexural members is decreased (Luo et al. 1994; Moehle 2015; Fintel 2004; Concrete Reinforcing Steel Institute 2020; McCormac and Brown 2014; Seismology Committee Structural Engineers Association of California 2019; Hwang and Moehle 2000), so that the coefficient of less than 1 is multiplied by the stiffness of the beam, column, and slab. In many reinforced concrete building codes, the reduced stiffness is a function of the existing moment-to-crack ratio, which is based on Branson's research in 1965 (Branson 1965). Some of the references state that the slab stiffness modification coefficient is dependent on the slab's existing shear due to gravity loads, and some others relate it to the slab dimensions (Hwang et al. 2010; Han et al. 2009)). In some references including the European building code, the effective stiffness is calculated as a combination of concrete and steel stiffness (European Standard, BS EN 1992). Several research has also presented a consistent

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amount of 0.3 for slab stiffness reduction coefficient (Applied Technology Council 2010). Also, several researches have been done on the behavior of flat slabs or slab-column connection under gravity and lateral loads. Almeida et al (2016), Cho (2009), Coronelli et al (2021), Drakatos et al (2016), Michael (2022) Rha et al (2014) and Topuzi (2015) can be mentioned among these researches. The main approach of this research has been the behaviour and participation or non-participation of the slab in lateral loading. specially these effects on connection behaviour.

In most of the reinforced concrete building codes, stiffness modification coefficients for beams, columns, and flat slabs are 0.35, 0.7, and 0.25, respectively, while no coefficient is determined for other types of slabs. However, ACI 318, ACI 421, ACI 435, and other similar building codes emphasized that a comprehensive analytical approach could lead to obtaining the corresponding coefficient (ACI 435.6r-74 1989; ACI 421.3R-15 2015; ACI 318-19 2019; AS 3600 2019; NZS 3101 2006; Cement Association of Canada 2006; CSA Standard A23.3:19 2019; ACI 34OR-97 2001; ASCE 7-10 2010).

The selection of appropriate effective stiffness values for reinforced concrete frame members has dual purposes: 1) provide realistic estimates of lateral deflections, and 2) determine the distribution of forces and moments on the frame members. A detailed nonlinear analysis of the structure would adequately capture these two effects. An approximate method to estimate an equivalent nonlinear lateral deflection using linear analysis is to reduce the modeled stiffness of the concrete members in the structure.

On the other hand, in the seismic assessment of structures, slabs are not expected to perform ductile behavior and based on this logic, the provisions of part 18.3, ACI318-19 stated that the application of the slab-column system, which is a part of the lateral load bearing system, is only allowed in the seismic category of B for ordinary moment frame structures (ACI 318-19 2019). Additionally, in the provision of 12.2.1 in ASCE7-16, it is indicated that providing that the lateral load bearing system is moment frame, slabs are allowed to accompany in lateral load bearing system for ordinary moment frame structures in B seismic category, and for intermediate moment frame, in B and C seismic category (ASCE 7-10 2010). Then, a few questions are raised; firstly, what is the stiffness modification factor for deflection and drift control for slabs other than flat slabs including beam-column slabs, or one-way and two-way waffle slabs? Another question is, in the case that the building codes do not consider the application of slabs permitted as the seismic load-bearing element in intermediate and special moment frames, how much of the slab stiffness is contributed in lateral load bearing? provided that instead of involving slabs in lateral load bearing with appropriate stiffness factor, the whole slab function is neglected, how it would affect the slab design (what amount of internal efforts in slab design are neglected?). In better words, although the structurally designed and analyzed models were once investigated not considering the stiffness impact of slabs as the lateral load-bearing element, they were also studied thoroughly considering slab frame and shear wall (if any) together in the 3D structural design. The question is whether the stiffness factor should be assigned to the slab (as this question aforementioned, this is relevant to slabs other than flat slabs). Due to the absence of specific stiffness reduction factor in the design codes of reinforced concrete structures for all types of slabs on the one hand, and the absence of sufficient research regarding this coefficient for structures in which it is possible to use slabs or slab-beams on the other hand, the need to address this issue is deeply felt. This important matter is the basis of the present research.

In the present study, 20 structures between 2-story to 20-story structures with various slab types including flat slabs, beam-column slabs, and one-way and two-way waffle slabs were studied to respond abovementioned questions. The structures were initially investigated in 3D, and to apply seismic impacts, dynamic spectral analysis was conducted. Then, each element of the meshed slab was assessed in all load combinations, and for elements whose flexural moment is higher than the cracking limit, the stiffness modification factor was calculated and multiplied by their moment of inertia. Moreover, for other elements with flexural moments less than the cracking limit, the stiffness modification factor was considered 1. It is also noted that some references consider the whole slab under service loads not cracked, or with full stiffness (Ig) (ACI 421.3R-15). Based on the provisions of ACI421, a cracking factor of 1 could be assigned to not-cracked areas of the slab. Diverse references have also permitted comprehensive analytical methods to calculate the stiffness modification factors (ACI 421.3R-15). Ig may be used when the calculated tensile stress is less than the modulus of rupture. The calculation of the stiffness modification factor was done using moment-curvature curves for cracked elements (Park and Paulay 1975). Additionally, equivalent and identical stiffness modification factors obtained through trial and adjustment in an iterative process were assigned to the entire slab element. The iteration continued until the fundamental results including displacement and acceleration reach an acceptable accuracy with the mode in which exclusive slab modification factors were assigned to each slab element.

Methodology

As aforementioned in the introduction, in the present study, 20 structures between 2-stories and 20-stories are defined. In figures 1 to 20, the architectural plan of these structures with the placement of shear walls (in case of existence) is illustrated. In a general categorization, the case study structures in terms of lateral load-bearing systems are sorted as below:

- Type one: 2-story moment frame structures
- Type two: 5-story dual system of RC moment frame and RC shear walls
- Type three: 12-story dual system of RC moment frame and RC shear walls
- Type four: 15-story dual system of RC moment frame and RC shear walls
- Type five: 20-story dual system of RC moment frame and RC shear walls

Each type of mentioned structure was investigated in four roof systems including:

- Reinforced concrete structure with flat slab and no beams
- Reinforced concrete structure with beam-column slab system
- Reinforced concrete structure with beam-column slab including main and secondary beams
- Reinforced concrete structure with waffle (waffle) one-way and two-way slabs, with a secondary beam of different spacing.

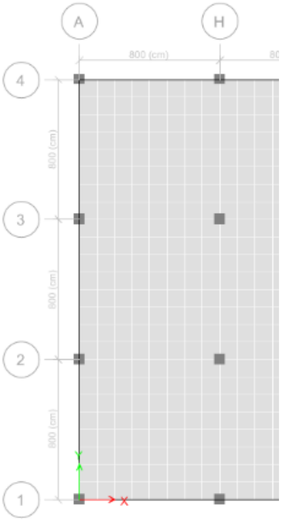

Figure1: 2NMBF

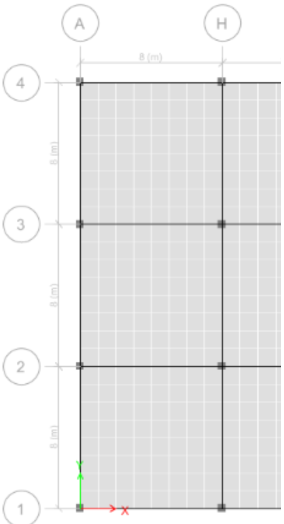

Figure2: 2MFSB

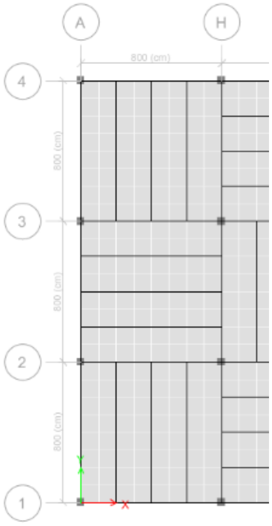

Figure3: 2MWS1

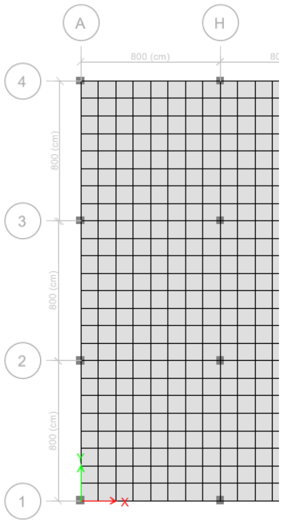

Figure4: 2MWS2

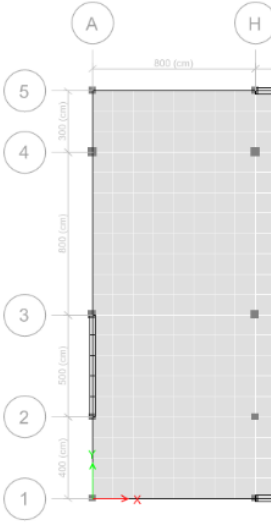

Figure5: 5DFSN

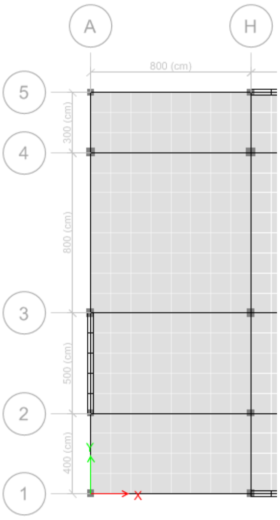

Figure6: 5DFSB

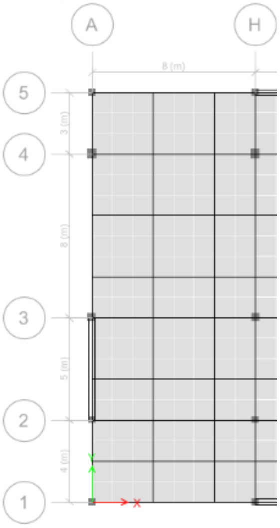

Figure7: 5DSMB

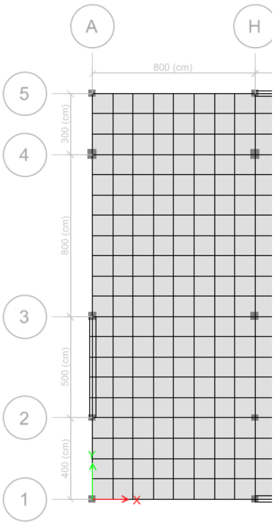

Figure8: 5DWS2

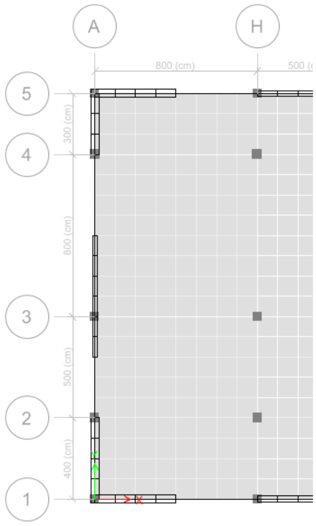

Figure9: 12DFSN

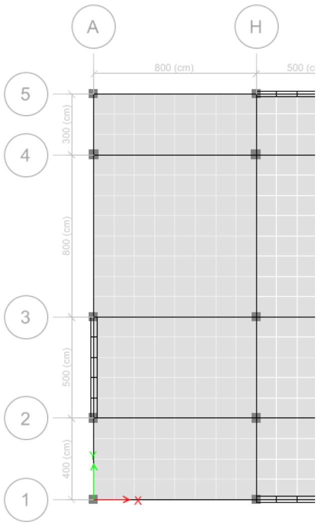

Figure10: 12DFSB

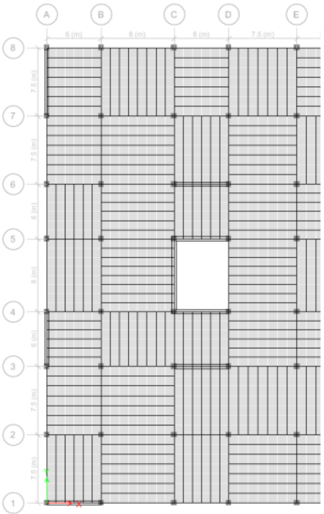

Figure15: 15DWS1

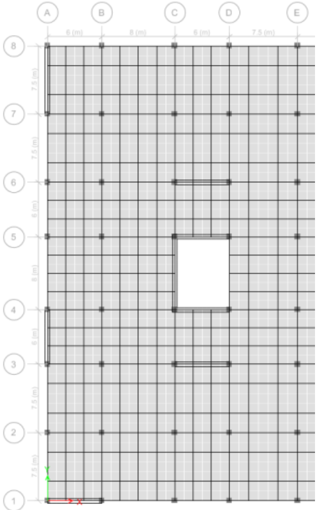

Figure16: 15DWS2

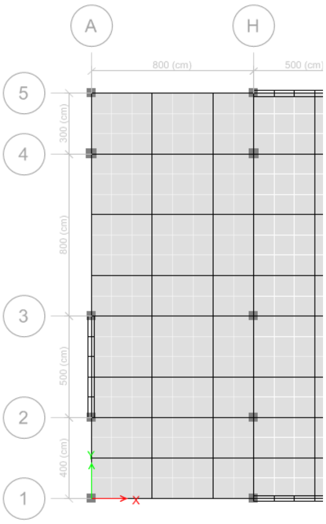

Figure11: 12DSMB

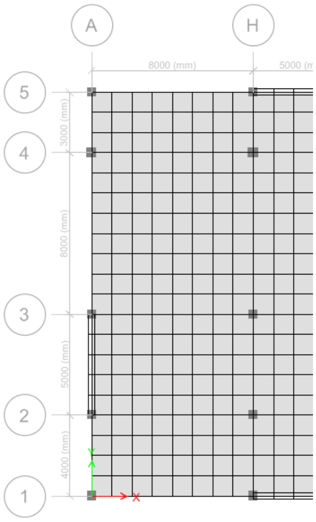

Figure12: 12DWS2

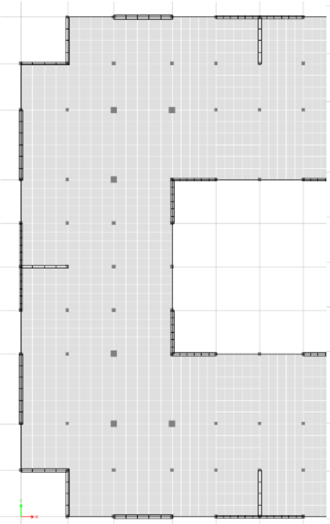

Figure17: 20DFSN

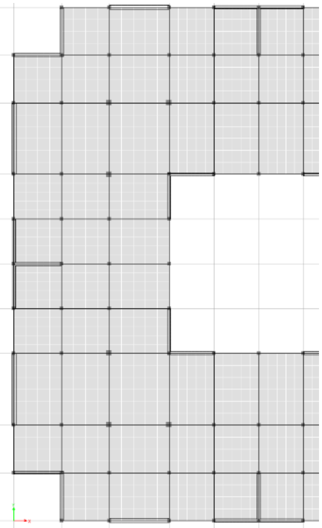

Figure18: 20DFSB

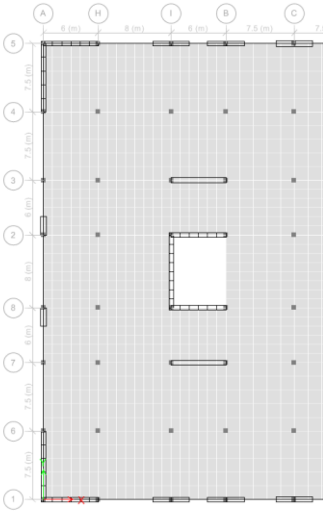

Figure13: 15DFSN

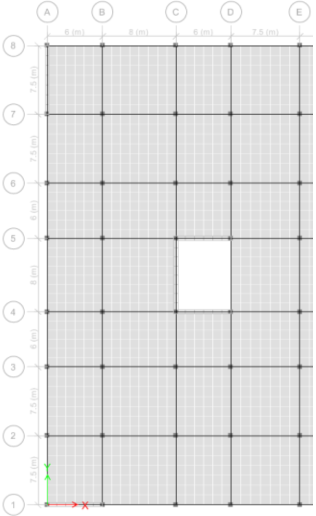

Figure14: 15DFSB

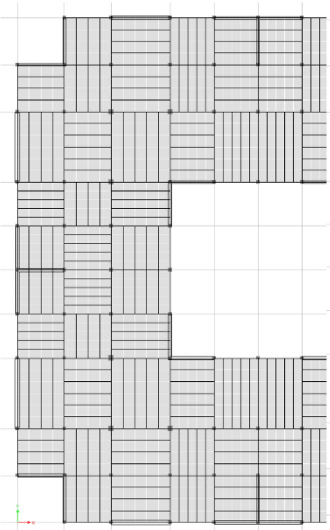

Figure19: 20DWS1

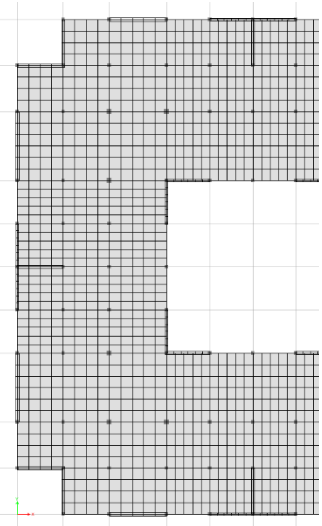

Figure20: 20DWS2

Name and description of each load-bearing system of 20 structures defined in table1. As indicated in the descriptions, for each elevation level of case study structures, various types of slabs including flat, beam-column, one-way, and two-way waffle were considered in this way, a relatively wide range of buildings can be analyzed and referenced. As shown in Table 1, the structures of 2, 15, and 20 floors, in addition to the flat slab roof and beam-column slab (with beams connected to the columns of each panel), with slab roof system with main beams and one-way middle (secondary) beams two-way secondary beams with various distances (2 to 3 meters) in each panel have been defined and analyzed.

However, in 5- and 12-story buildings, a flat slab system, beam-column slab, as well as a waffle slab system with beams at intervals of about one meter have been considered.

Table 1. Name and description of samples

No.	Sample name	No. of floors	Lateral load-bearing system	R
1	2NMBF	2	Flat slab and columns (Intermediate moment frame)	5
2	2MFSB		Intermediate reinforced concrete moment frame (beam-column slab roof)	5
3	2MWS1		One-way waffle slab and columns (Intermediate moment frame); Distance of middle beams = 2 meters	5
4	2MWS2		Two-way waffle slab and columns (Intermediate moment frame)); Distance of middle beams= 1 meter	5
5	5DFSN	5	Special reinforced concrete shear Wall (Flat slab roof)	6
6	5DFSB		Dual system: Special concrete shear walls, intermediate RC Moment Frames (Beam-Column slab roof)	6.5
7	5DSMB		Dual system: special concrete shear walls, middle RC moment frames (beam-column slab roof with secondary beams at almost 2 meters intervals)	6.5
8	5DWS2		Dual system: special concrete shear walls, intermediate RC moment frames (beam-column slab roof with secondary beams at almost 1-meter intervals)	6.5
9	12DFSN	12	Special reinforced concrete shear Wall (Flat slab roof)	6
10	12DFSB		Dual system: Special concrete shear walls, intermediate RC Moment Frames (Beam-Column slab roof)	6.5
11	12DSMB		Dual system: special concrete shear walls, intermediate RC moment frames (beam-column slab roof with secondary beams at almost 2 meters intervals)	6.5
12	12DWS2		Dual system: special concrete shear walls, intermediate RC moment frames (beam-column slab roof with secondary	6.5

			beams at almost 1-meter intervals)	
13	15DFSN	15	Special reinforced concrete shear Wall (Flat slab roof)	6
14	15DFSB		Dual system: Special concrete shear walls, intermediate RC Moment Frames (Beam-Column slab roof)	6.5
15	15DWS1		Dual system: special concrete shear walls, special RC moment frames (beam-column slab roof with one-way secondary beams at almost 1-meter intervals)	6.5
16	15DWS2		Dual system: special concrete shear walls, special RC moment frames (beam-column slab roof with two-way secondary beams at a combination of almost 2 and 3 meters intervals)	6.5
17	20DFSN	20	Special reinforced concrete shear Wall (Flat slab roof)	6
18	20DFSB		Dual system: Special concrete shear walls, intermediate RC Moment Frames (Beam-Column slab roof)	6.5
19	20DWS1		Dual system: special concrete shear walls, special RC moment frames (beam-column slab roof with one-way secondary beams at almost 2 meters intervals)	6.5
20	20DWS2		Dual system: special concrete shear walls, special RC moment frames (beam-column slab roof with one-way secondary beams at almost 2 meters intervals)	6.5

After defining geometrical features and determining gravity and lateral load-bearing systems, each structure was modeled in 3D, and the loading and structural design was conducted based on the provisions of ASCE7-16 and ACI318-19. To consider the seismic effect, spectral analysis was employed. By applying the building code standard design spectrum, the spectral analysis was conducted, and the standard spectrum was scaled so that shear due to the dynamic analysis equals 100 percent of equivalent static shear. Fundamental parameters including drift, orthogonal effect, and adequacy of rebar reinforcement in beams, columns, and shear walls were taken into consideration, which accommodates that all the structures were safely and economically designed based on the provisions of ACI318-19. In all the structures, the slab thickness is determined so that firstly satisfies deflection, and secondly one-way and two-way shear strength is provided, then bending reinforcement and added required longitudinal rebars are designed and detailed. At the beginning of the study, the cracking factor was considered 0.25 and 0.35 for flat slabs and slabs with beams, respectively. These values were selected based on the available references mentioned in the introduction. After the analysis, the moment values in each element of meshed slabs in entire roof areas were observed in both X and Y directions (M_{xx} or the M_{11} in the element local coordinate system, and M_{yy} or M_{22} in the element local coordinate system). By observing the changes of the two mentioned flexural moments contour, it is achieved that in which areas of the slab, the moment obtained from the analysis (in various load combinations) is higher than the cracking moment (M_{cr}). It is also noted that the value of M_{cr} is obtained from classic reinforced concrete design relations (Eq.1), and by placing the maximum tensile stress in the transformed cross section equal to the modulus of rupture (Eq. 2).

$$Mcr = \frac{fr.Ig}{y_t}$$

(Eq. 1)

$$Fr = 0.6\sqrt{f'c}$$

(Eq. 2)

In the abovementioned relations, I_g and y_t are moments of inertia and the distance of the neutral axis from the farthest tensile axis of the transformed cross-section, which is achieved from solid mechanic principles for elastic cross-sections after the conversion of rebar reinforcement to concrete in converted cross-section. $F`_c$ and f_r are concrete compressive strength and modulus of rupture. The value of the cracking moment is exclusively obtained for each structure and each area of the slab about the slab design reinforcement. The negative flexural moment mostly exceeds the cracking moment near the structural axis (as indicated in fig. 21).

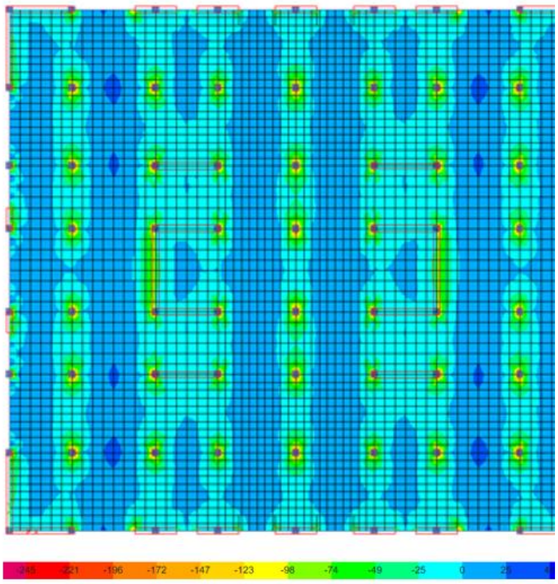

Figure21: M11 for one floor of 15DFSN (for example)

At the next stage, by using classical reinforced concrete relations and establishing balance and compatibility principles, for each cracked area of the slab, the moment-curvature curve is calculated. The slope of this curve in the area of rebar pre-yield is considered as the stiffness modification factor ($\frac{M_y}{\varphi_y} = \propto . EI$ ؛ M_y, and ϕ_y, which are moment and corresponding curvature for tensile rebar yield, E is the elastic modulus of concrete, I is moment inertia of gross cross-section and α which is stiffness modification factor). All the finite element analysis output values and calculated values are considered as yield and elastic capacity for a cross-section of the slab which owns the slab thickness and unit width.

Now, by owning the stiffness modification factor of each element of meshed slab of case study structures, the factor is assigned to the corresponding element, and other elements, in which the flexural moment of all load combinations is smaller than the cracking moment, the stiffness modification factor of 1 is assigned to; therefore, each structure is prepared to get analyzed again, which is named ("1&α") in the rest. Furthermore, in this stage, the structures are analyzed with the stiffness modification factors of 0.25 and 0.35 for all story slab elements. The structures are once analyzed considering the flexural stiffness modification factor of 0.01. The last case is considering simulating the modes in which the slab is not involved in seismic behavior; in other words, the situation that the building codes do not consider the slab permitted to contribute to the lateral load-bearing system such as moment frame. Once, all slab elements are the optional factor of less than 1 assigned (assigning equivalent general stiffness modification factor), next, the structure is analyzed and the fundamental parameter values of the period, acceleration, and lateral displacement are obtained and compared to the abovementioned modes. This equivalent general stiffness modification factor varies and the analysis is conducted repetitively so that the difference between the values of the accurate file ("1&α") and the file with equivalent stiffness becomes less than 10% (as an acceptable criterion); therefore, an equivalent stiffness modification factor is obtained for all of the structures in various modes.

Analysis results

As mentioned in the previous section, after the analysis of structures, the values of displacement, story center of mass acceleration, and vibration period of the first and second modes (absolute motion in X and Y directions) were obtained for each structure. The software employed to analyze and design structures is ETABS (2016). In figures 22 to 81, the mentioned results for each structure with various elevations are illustrated. Among the above-mentioned figures, diagrams related to the floor's displacement and acceleration are given in the text, and diagrams related to the period of structures are in Appendix 1. In each diagram, the corresponding values are shown for different cases of stiffness modification factor equal to 0.25, 0.01, and 1&alpha (accurate case) and overall equivalent modification factor achieved by try and adjustment. Also in the period diagrams in Appendix 1, the period of the first and second vibration modes of the buildings (which are related to the movement in the x and y directions), has shown for different stiffness modification factors mentioned above.

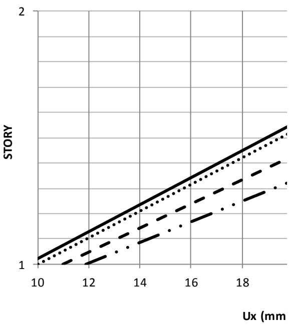

Figure 22: Floor displacements of 2NMBF

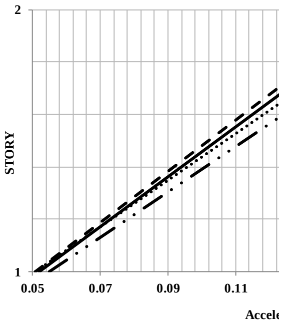

Figure 23: Floor acceleration of 2NMBF

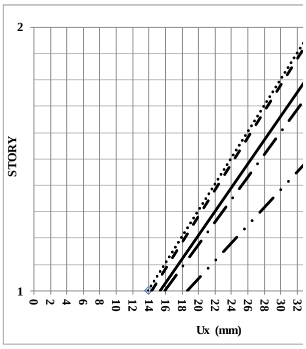

Figure 25: Floor displacements of 2MFSB

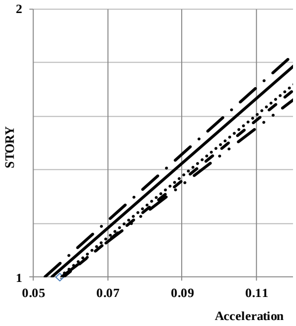

Figure 26: Floor acceleration of 2MFSB

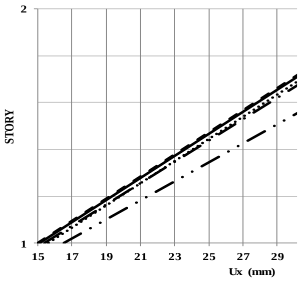

Figure 28: Floor displacements 2MWS1

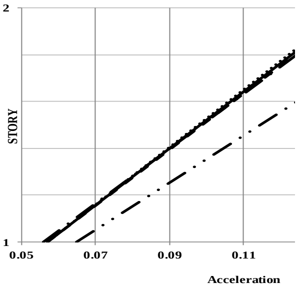

Figure 29: Floor acceleration of 2MWS1

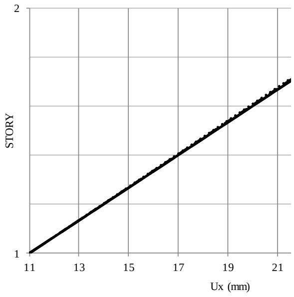

Figure 31: Floor displacements of 2MWS2

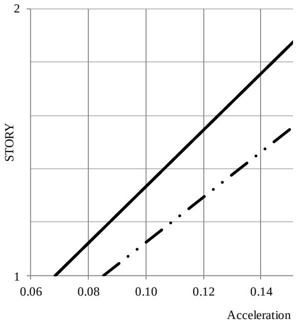

Figure 32: Floor acceleration of 2MWS2

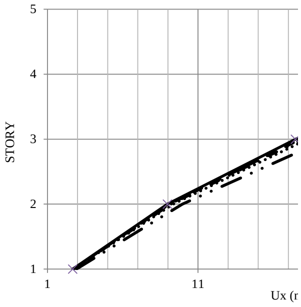

Figure 34: Floor displacement of 5DFSN

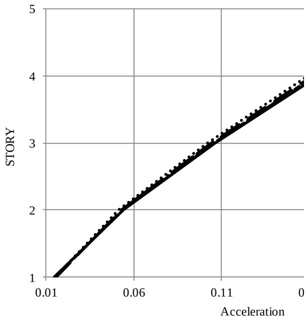

Figure 35: Floor acceleration of 5DFSN

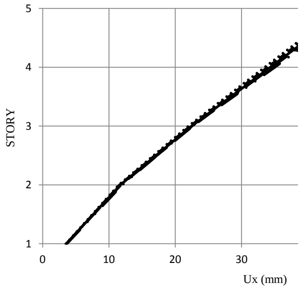

Figure 37: Floor displacement of 5DFSB

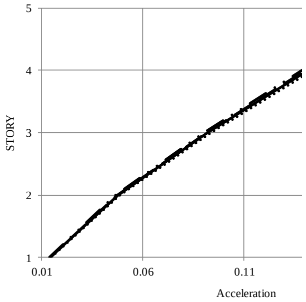

Figure 38: Floor acceleration of 5DFSB

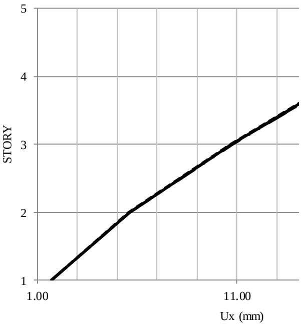

Figure 40: Floor displacement of 5DSMB

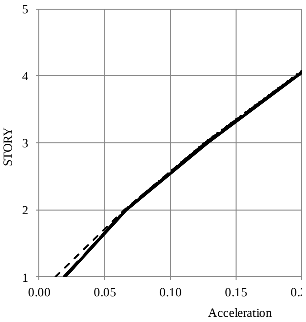

Figure 41: Floor acceleration of 5DSMB

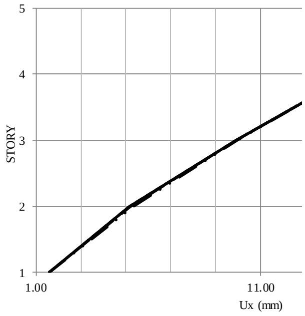

Figure 43: Floor displacement of 5DWS2

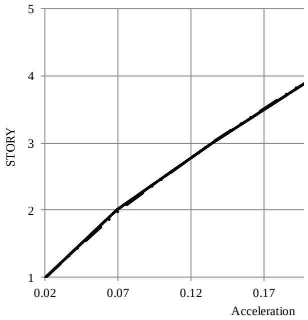

Figure 44: Floor acceleration 5DWS2

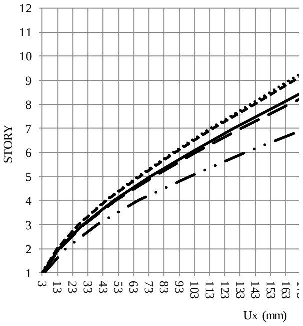

Figure 46: Floor displacement of 12DFSN

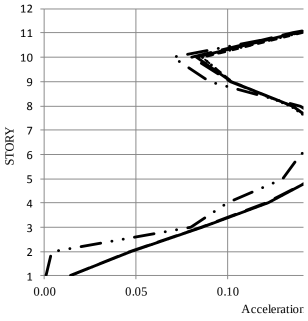

Figure 47: Floor acceleration of 12DFSN

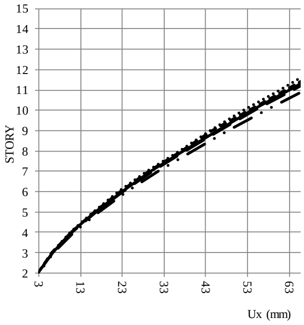

Figure 58: Floor displacement 15DFSN

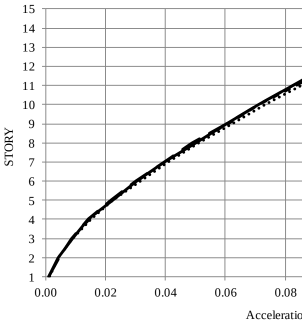

Figure 59: Floor acceleration of 15DFSN

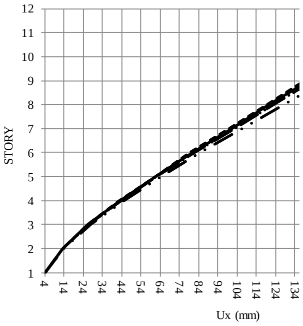

Figure 49: Floor displacements of 12DFSB

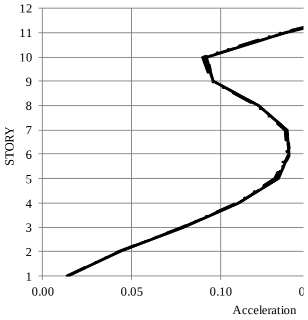

Figure 50: Floor acceleration of 12DFSB

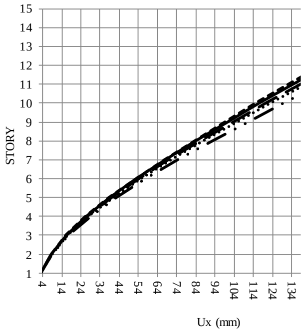

Figure ٦1: Floor displacement of 15DFSB

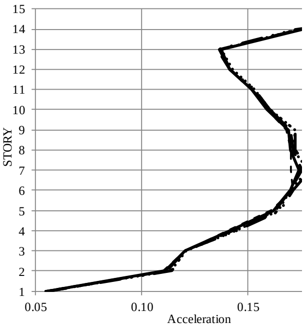

Figure ٦2: Floor acceleration of 15DFSB

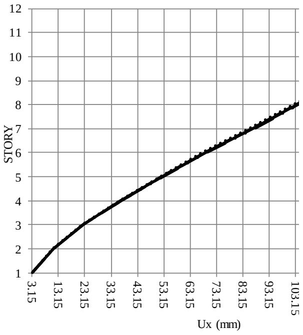

Figure 52: Floor displacements of 12DSMB

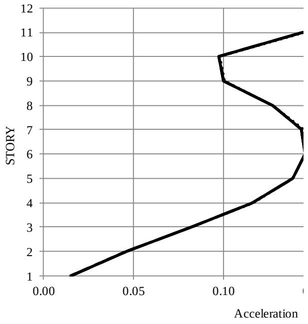

Figure 53: Floor acceleration of 12DSMB

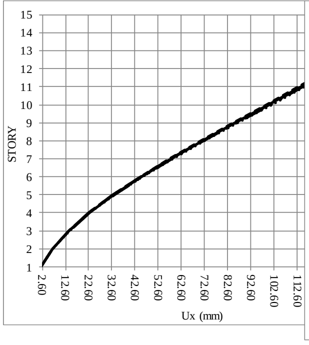

Figure 64: Floor displacement of 15DWS1

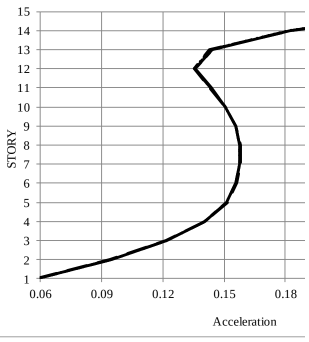

Figure ٦5: Floor acceleration 15DWS1

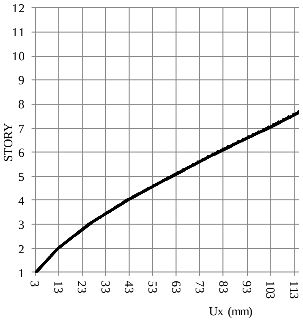

Figure 55: Floor displacements of 12DWS2

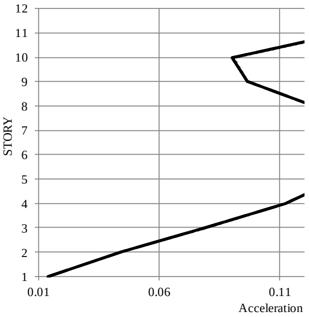

Figure 56: Floor acceleration of 12DWS2

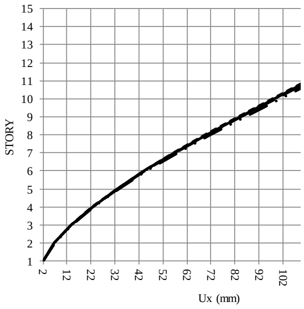

Figure ۶٧: Floor displacement of 15DWS2

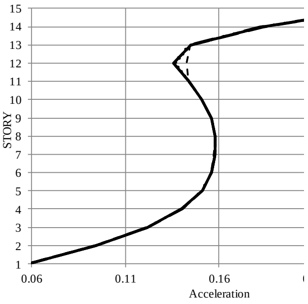

Figure ٦٨: Floor acceleration of 15DWS2

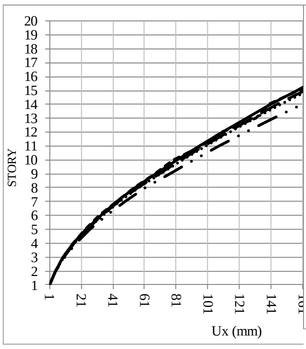

Figure 70: Floor displacement of 20DFSN

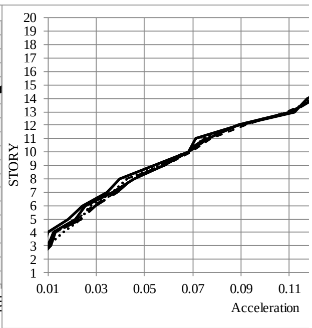

Figure 71: Floor acceleration 20DFSN

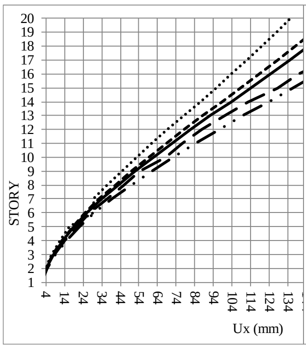

Figure 73: Floor displacement 20DFSB

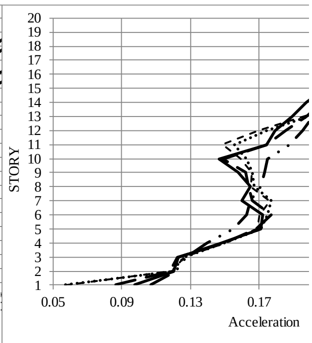

Figure 74: Floor acceleration of 20DFSB

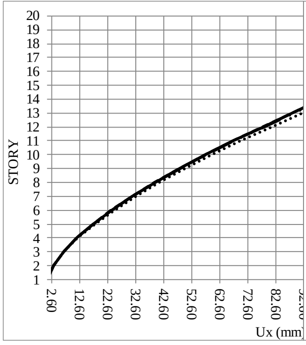

Figure 76: Floor displacement of 20DWS1

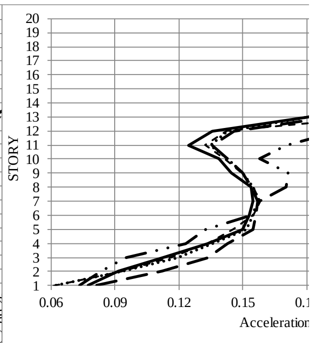

Figure 77: Floor acceleration of 20DWS1

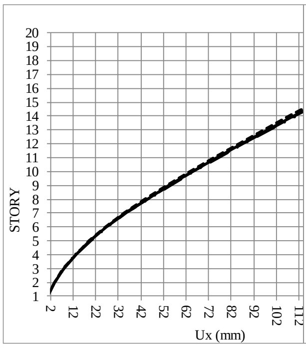

Figure 79: Floor displacement 20DWS2

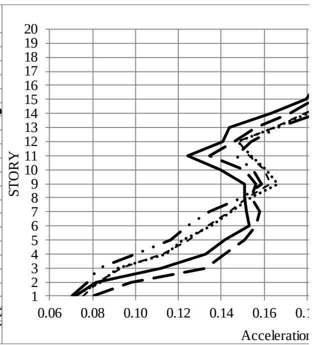

Figure 80: Floor acceleration 20DWS2

It is also noted that the curves for each structure are extracted for the below-mentioned forms:

1. Assigning the stiffness modification factor of 0.01 (which accommodates that the slab is modeled as membrane and is not involved in lateral load bearing)
2. Assigning the slab stiffness modification factor of 0.25
3. Assigning the slab stiffness modification factor of 0.35
4. Assigning the slab stiffness modification factor of 1 for not-cracked areas and assigning the slab stiffness modification factor for cracked areas using the moment-curvature curve
5. Assigning a general slab modification stiffness factor to all story slab elements, which is obtained from trial and adjustment through an iterative process so that the responses of structure (displacement and acceleration) are approximately identical with ("1&α") accurate file.

In addition to figures 22 to 81, the results are numerically presented in table 3 and table 4, which is the readout of the curves.

Table 3. Results of
roofs with a diff

PERIOD				
mode1				mode 2
Flat Slab	Beam-Column slab	Beam-Column slab with secondary beams at different	Beam-Column slab with closer secondary beams	Flat Slab
0.79	0.86	0.88	0.77	0.66
0.83	0.87	0.86	0.77	0.82
0.79	0.91	0.89	0.78	0.66
1.99	1.01	0.94	0.78	1.98
0.78	0.94	0.65	0.63	0.78
0.79	0.93	0.65	0.63	0.78
0.78	0.95	0.65	0.64	0.77
1.74	0.96	0.65	0.63	1.73
2.19	2.13	2.09	2.17	2.11
2.18	2.17	2.08	2.16	2.07
2.38	2.17	2.11	2.17	2.28
2.77	2.22	2.12	2.17	2.65
1.55	1.99	1.83	1.82	1.29
1.53	2.05	1.85	1.83	1.29
1.57	2.02	1.84	1.83	1.3
1.83	1.81	1.84	1.85	1.8
2.55	2.2	2.24	2.33	2.47
2.66	2.1	2.3	2.33	2.56
2.64	2.44	2.26	2.37	2.55

ACCELERATION ROOF

UX ROOF

	MEMBRANE	2.78	2.52	2.27	2.4	2.61	2.45	2.21	2.33	0.18	0.26	0.26	0.27

Table 4 - The ratio of each analysis results in a similar value in 1 & alpha mode

No. of floors	COEFFICIENTS	PERIOD								ACCELERATION			
		mode1				mode 2				ROOF			
		Flat Slab	Beam-Column slab	Beam-Column slab with secondary beams at different	Beam-Column slab with closer secondary beams	Flat Slab	Beam-Column slab	Beam-Column slab with secondary beams at different	Beam-Column slab with closer secondary beams	Flat Slab	Beam-Column slab	Beam-Column slab with secondary beams at different	Beam-Column slab with closer secondary beams
2	Equivalent overall modifier	0.95	0.99	1.02	1.00	0.80	0.99	1.01	1.00	0.94	1.07	1.00	1.00
	"1&α"	---	---	---	---	---	---	---	---	---	---	---	---
	0.25	0.95	1.05	1.03	1.01	0.80	1.05	1.03	1.01	0.94	0.93	1.00	1.00
	MEMBERANE	2.40	1.16	1.09	1.01	2.41	1.16	1.08	1.01	1.06	7.50	1.14	1.19
5	Equivalent overall modifier	0.99	1.01	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00
	"1&α"	---	---	---	---	---	---	---	---	---	---	---	---
	0.25	0.99	1.02	1.00	1.02	0.99	1.02	1.00	1.00	1.00	1.00	1.00	1.00
	MEMBERANE	2.20	1.03	1.00	1.00	2.22	1.03	1.00	1.00	1.00	1.00	1.00	1.00
12	Equivalent overall modifier	1.00	0.98	1.00	1.00	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	"1&α"	---	---	---	---	---	---	---	---	---	---	---	---
	0.25	1.09	1.00	1.01	1.00	1.10	1.01	1.01	1.00	1.00	1.00	1.00	1.00
	MEMBRANE	1.27	1.02	1.02	1.00	1.28	1.03	1.02	1.00	1.00	1.00	1.00	1.00
15	Equivalent overall modifier	1.01	0.97	0.99	0.99	1.00	0.98	0.99	0.99	1.00	1.00	0.96	1.00
	"1&α"	---	---	---	---	---	---	---	---	---	---	---	---
	0.25	1.03	0.99	0.99	1.00	1.01	0.99	0.99	1.01	1.00	1.00	1.00	1.00
	MEMBRANE	1.20	0.88	0.99	1.01	1.40	1.04	1.00	1.01	1.00	1.00	0.96	1.00
20	Equivalent overall modifier	0.96	1.05	0.97	1.00	0.96	1.03	0.98	1.00	1.00	1.00	1.00	0.96

--	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---
.99	1.16	0.98	1.02	1.00	1.17	0.98	1.02	1.00	1.00	1.00	1.00	0.98	1.39	0.96	1.02
.05	1.20	0.99	1.03	1.02	1.20	0.99	1.04	1.00	1.00	1.00	1.00	1.10	1.49	0.97	1.03

As shown, the value for displacement in structures with flat slabs and columns owns the largest difference in various modes of stiffness compared to each other, which is evident in 2, 5, 12, 15, and 20-story structures. According to the present study, the distance between the curves reaches the maximum value when the slab is under lateral loading with the stiffness modification factor of 0.01, which means the flexural (out of plane) stiffness of the flat slab is ignored. It should be noted that the application of flat slabs with column (alone) as the lateral load bearing system is not permitted in structures with more than 3 stories, but, in the present study, to evaluate the impact of the slab in lateral load bearing and also achieving a stiffness modification factor in case of application, these structures were defined and analyzed in the elevation levels of 5, 12, 15 and 20 stories with the shear wall as a lateral bearing system. In the modes that other values are assigned to stiffness modification factor, the difference of displacement values reaches less than 10 percent compared to the "1&α" model. Reinforced concrete slabs, due to the out-of-plane stiffness the same as other structural elements, absorb a portion of lateral loads, therefore, even when the slab is under lateral loading with the lowest stiffness factor, contribute significantly to stiffness increase. The observations of the displacement results prove that in 2 to 20-story structures, with beam-slab systems, due to the presence of main flexural beams, the distance between curves due to assigning the stiffness modification factor varies. Moreover, in another mode including slabs with main and secondary beams, waffle slabs with one-way beams, and waffle slabs with two-way beams, respectively, it is indicated that the distance between curves of stiffness modification factors reaches the least, so that by raising the height of the structures and increasing the number of secondary beams of slabs, the curves in various stiffness modes become approximately identical, and it could be stated that assigning or not assigning the stiffness modification factors of 0.01 to 0.5 would be ineffective due to the softness of the structure and also the contribution of secondary beams in lateral load bearing. A similar situation is seen for acceleration and period curves, so that in structures with flat slabs, the period and acceleration reach their maximum value with the stiffness factor of 0.01, however, by assigning other values, no remarkable change is obtained. It is also noted that the additional slab beams which are created due to roof system change, lead to an increase in the stiffness of the lateral load-bearing system so that the stiffness leaves an impact on period reduction and acceleration.

The values presented in tables 3 and 4 are illustrated as bar charts in Figs. 82 and 83. These two figs. show the curves for roof displacement and period for four roof systems including flat slabs, beam-column slabs, slabs with one-way middle beams (one-way waffle), and two-way waffle slabs with four modes of cracking factors. E.S.M., COR., GEN. And MEMB. These two diagrams are related to the mode of equivalent total stiffness modification factor, exact calculation method ("1&α"), application of 0.25 or 0.35 coefficient to all slab elements, and slab modeling in the membrane, respectively.

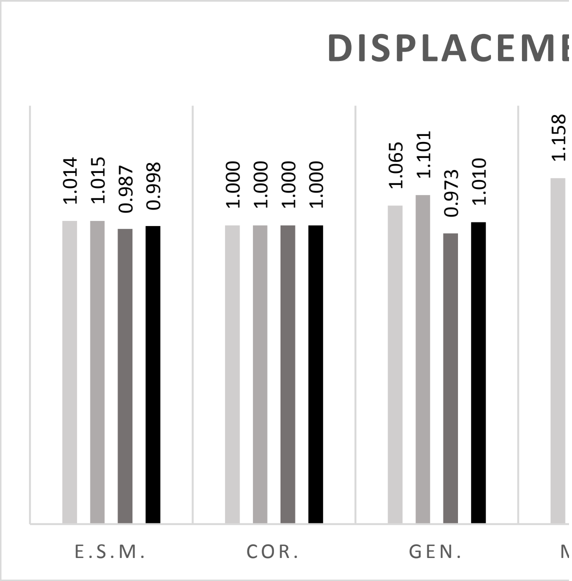

Figure 82 Ratio of displacement in 4 types of slabs with each reduction factor type to "1&α" form.

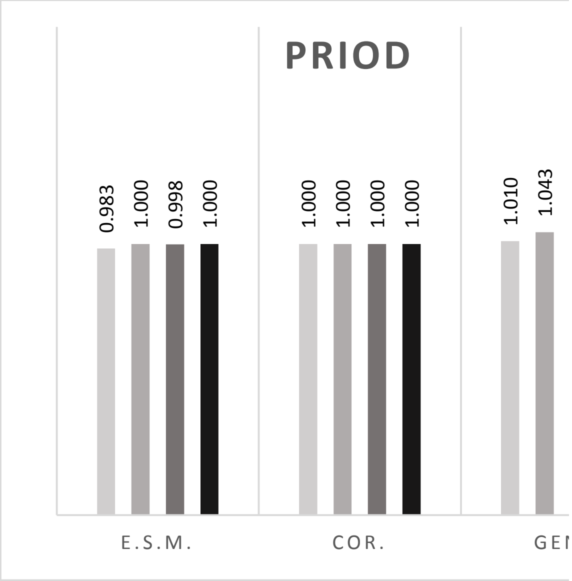

Figure 83 Ratio of the period in 4 types of slabs with each reduction factor type to "1&α" form.

The values of period and displacement in each mode (each roof system for each mode of stiffness modification factor) are average between the structures with various elevation levels and shown in the curves. Additionally, displacement and period for the motion in X and Y direction is individually investigated, and in regards to the symmetricity of both directions and the proximity of the results, one of them is employed to draw the curves. In these two curves, it is indicated that employing membrane mode in slab modeling to not involve the slab in lateral load bearing, could only lead the flat slab to develop a higher than 10% difference in the swing period. This difference, as evident in table 3, is seen in 2 to 20-story structures. This is also seen in structures with beam-column slabs in a lesser amount, though, however, in structures with waffle roof systems, no difference is seen in various modes. The obvious reason is the decrease in the impact of the slab stiffness due to the presence of shear walls, moment frames, and secondary beams. In the case of displacement, as shown in tables 3 and 4 and fig. 82, slab with flexural stiffness of "0.01" (membrane mode), only in flat slabs and beam-column slabs, could lead to a higher than 10% difference between the analysis results with the mode an accurate stiffness modification factor is assigned ("1&α"). The comparison of both results shows that the application of an equivalent general stiffness modification factor which is pointed out in section 2, could estimate the period and displacement with high accuracy in most cases with comparing to the accurate state of ("1&α"). Moreover, in the calculation of lateral displacement, providing the slab is not involved, approximately the displacement increases by 21% (bar chart presented in Fig. 82), while the period meets a 6-percent increase. This approach is stated in some refs. (Cement Association of Canada 2006), that in case the period of structures is not changed over 15% using slab, the application of it is permitted.

Stiffness modification factor in different modes

In this section, the most significant question of the study is responded, that is presenting an equivalent stiffness modification factor in different modes with regards to elevation. The equivalent stiffness modification factors for different modes are presented in table 5. The factors are obtained using an iterative process (try and adjustment) presented in section 2.

Table 5. Resultant stiffness modification factors

Flat slab	Beam-Column slab	Beam-column slab with secondary beams	Waffle slab with closer secondary beams
0.41	0.51	0.45	0.47
Total average	0.46		

The values presented in table 5 for each system are the average value between the structures with various stories having the corresponding roof system. As indicated in the table, this coefficient varies between 0.4 to 0.5 for flat slabs to beam-column slabs, of which 0.46 is considered the average. It is noted that in building codes such as ACI318, this coefficient is stated as 0.5, at most. Additionally, it is remarkable to mention that the reinforced concrete building codes recommended no stiffness modification factor for slab roofs except the flat slabs.

Effect of the earthquake on slab moment demand

The results indicate the contribution of slab stiffness in lateral loading, now the fundamental question is what impact the contribution or not the contribution of the slab in lateral load bearing (in modeling, analysis, and structural design) leaves on moment demand (design moments). In other words, what amount of slab design moments are missed by modeling the slab as a membrane? This question is responded to in table 6 based on the present study investigations. The values presented are the result of the division of slab moment values with the stiffness factor of 0.01 to the accurate mode. The moments due to the gravity load combination (1.2DL+1.6LL) are stated and then divided into the moment due to load combinations of lateral loads and gravity loads. Moreover, the values for each roof system among the entire 20 case study structures are averaged in then presented in this table. Table 6 shows that if the flat slab is modeled with a coefficient of 0.01 or the same membrane in the structure, about twenty percent and in all cases 12% the effect of lateral load on the structure is ignored and the computational bending moment for slab design - which only It is caused by gravity - the lower hand will be estimated. Accordingly, this would affect slab design, and especially, slab-column connection design, thus, the direct effect of moment ignoring would be seen in flexural rebar design.

Table 6. The bending moments of the slab neglected in membrane mode

	R_{M11}	R_{M22}	Max (R_{M11} & R_{M22} %)
Flat slab	0.80	1.00	20
Beam-column slab	0.89	0.93	11
Beam-column slab with secondary beams	0.90	1.00	10
Waffle slab with closer	0.91	1.00	9

secondary beams			
Total average	0.90	0.98	12.5

Conclusion

In the present study, 20 case study structures from 2 to 20-story with various roof systems consisting of flat slabs, beam-column slabs, and one-way and two-way waffle slabs were analyzed, and the determination of stiffness modification factor of reinforced concrete slabs was investigated. All structures were analyzed in 5 situations including assigning a stiffness modification factor of 0.25 to all slab elements, assigning a stiffness modification factor of 0.35 to all slab elements, assigning a stiffness modification factor appropriate to each cracked element using moment-curvature curves, and modeling slab roof as membrane; and assigning an equivalent overall modification factor obtained from an iterative procedure of the current study.

The results obtained from this study are listed below:

1. In the application of flat slabs as the structural roof, as well as beam-column slabs, roof displacement in membrane mode is 16 % and 20 % higher, respectively, compared to assigning a stiffness modification factor of 0.25 or assigning an overall stiffness modification factor obtained in this study. In two other roof types, one-way and two-way waffle, the stiffness modification factor left only a 3-percent impact on the roof displacement of structures.
2. Aforementioned results about the period of structures are only 60% higher in flat slab application, however, in other types of roofs, the stiffness modification factor left no impact on the period of structures.
3. The average stiffness modification factor achieved in this study for flat slabs is approximately 0.41.
4. The average stiffness modification factor obtained for beam-column slabs is around 0.51.
5. The average stiffness modification factor for slabs with main and secondary beams is roughly 0.45.
6. The average stiffness modification factor for two-way waffle slabs is around 0.47.
7. In the design of ordinary structures, the moment due to the combination of gravity loads to the lateral load combination, is achieved 13% less for various types of slabs, on average. Thus, providing the slab roof is considered membrane, 9 to 20 percent of the moment is ignored in the design procedure, to which flat slabs are dedicated the most value.

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Appendix 1. Mode1 & mode2 period of analyzed structures with any stiffness reduction factor

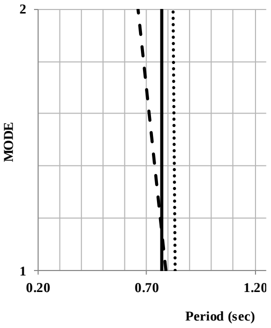


Figure 24: Period of 2NMBF

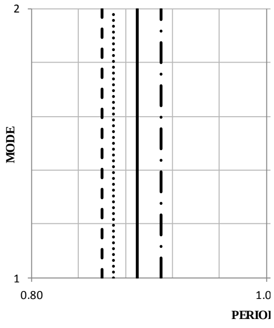


Figure 27: Period in a 2MFSB

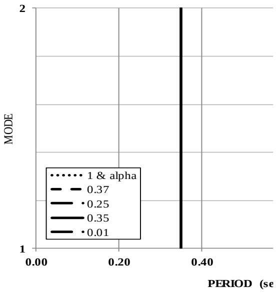


Figure 30: Period of 2MWS1

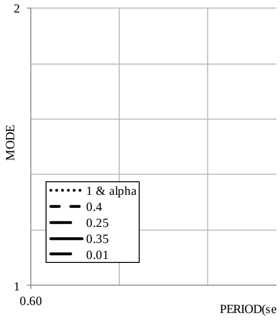


Figure 33: Period of 2MWS2

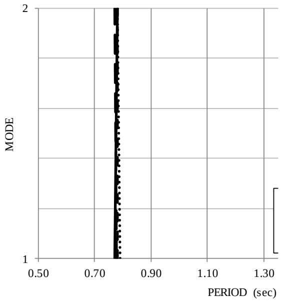


Figure 36: Period 5DFSN

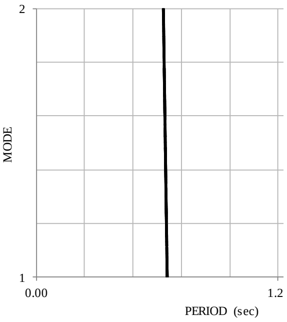
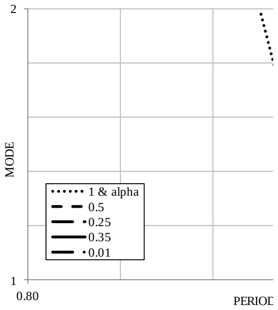


Figure 42: Period of 5DSMB



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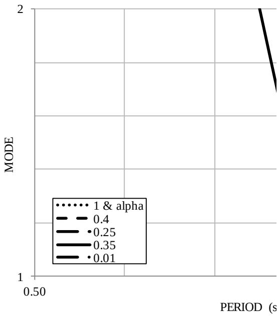

Figure 45: Period of 5DWS2

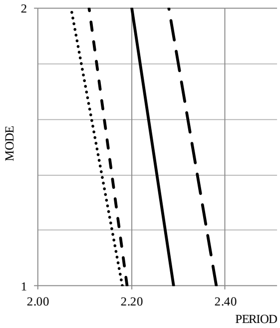

Figure 48: Period of 12DFSN

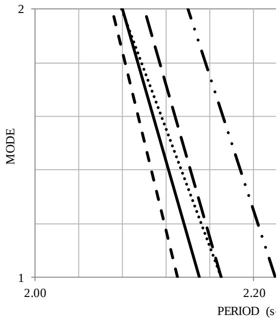

Figure 51: Period of 12DFSB

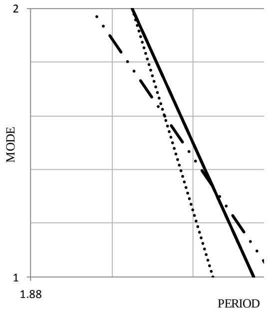

Figure 54: Period of 12DSMB

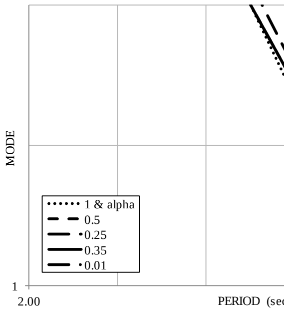

Figure 57: Period of 12DWS2

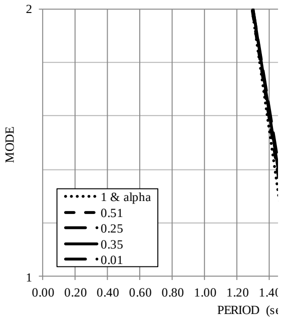

Figure ٦٥: Period of fifteen-story 15DFSN

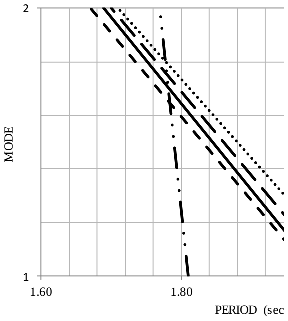

Figure ٦٣: Period of 15DFSB

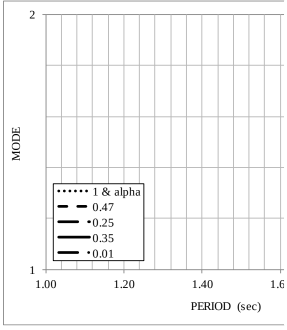

Figure 66: Period of 15DWS1

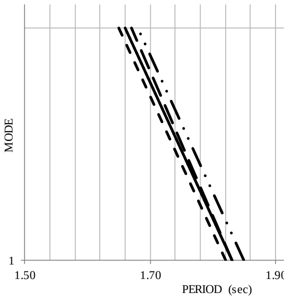

Figure 69: Period of 15DWS2

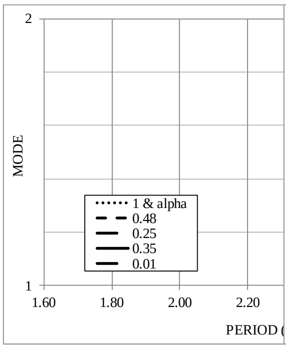

Figure 72: Period of 20DFSN

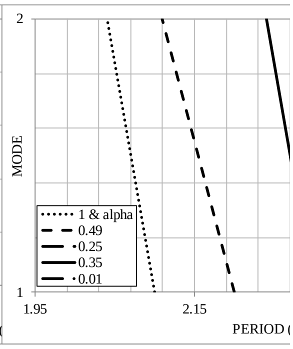

Figure 75: Period of 20DFSB

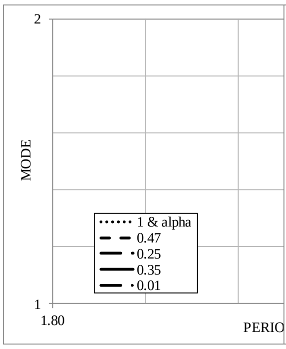

Figure 78: Period of 20DWS1

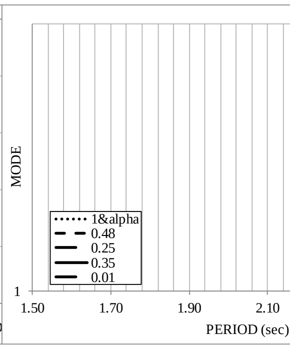

Figure 81: period of 20DWS2