

Optimum Design of Reinforced Concrete Cantilever Retaining Walls by Cuckoo Optimization Algorithm (COA)

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Abstract

This paper presents an economical optimization for cost and weight of reinforcement cantilever concrete retaining walls using Cuckoo Optimization Algorithm (COA). The proposed optimization algorithm is inspired from the life of a bird family called cuckoo. The capability of this algorithm is compared with other optimization methods available in the literature including ant colony optimization (ACO), bacterial foraging optimization algorithm (BFOA), particle swarm optimization (PSO), accelerated particle swarm optimization (APSO), firefly algorithm (FA), and cuckoo search (CS). A computer program has been developed by using the COA method for optimizing retaining walls. Five types of retaining walls were considered and sensitivity analyses were performed to find out the role of important parameters such including stem height, surcharge, backfill slope, and backfill unit weight and friction angle. Also, Coulomb and Rankine methods are used to estimate lateral earth pressures. The results show that the COA can minimize retaining walls from both cost and weight viewpoints. In addition, the COA can achieve to better results than ACO, BFOA, PSO, APSO, FA, and CS. The performed sensitivity analysis illustrates that with increasing surcharge and stem height, the cost and weight of wall increase. Also, the cost and weight objective functions decrease with increasing the soil unit weight. In addition, the Coulomb method gives lower cost and weight quantities than the Rankine method.

Keywords: Retaining walls optimization; Sensitivity analysis; Cuckoo Optimization Algorithm, Objective function, Optimum design.

1- Introduction

Concrete cantilever retaining walls are widely used in civil engineering projects and thus must have sufficient safety against sliding, overturning, and structural and geotechnical requirement. In addition, they should have minimum cost and weight.

To have optimized retaining walls, various methods have been used. These include nonlinear programming (Sribas and Erbatur, 1996) [1], simulated annealing algorithm

(Ceranic and et al., 2001) [2], target reliability approach (Sivakumar and Munwar, 2008) [3], simulated annealing algorithm (Yepes et al., 2008) [4], ant colony algorithm (Ghazavi and Bazazzian, 2011) [5], foraging bacterial algorithm (Ghazavi and Salavati, 2011) [6], charged system search algorithm (Kaveh and Behnam, 2013) [7], gases brownian motion optimization algorithm (Shalchi et al., 2021) [13], firefly

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algorithm (Laali and Shalchi, 2018) [14], artificial bee colony algorithm (Shalchi and Laali, 2018) [15]. Also, Pei and Xia (2012) [8] presented three methods for optimization of retaining wall. They used genetic, simulated annealing and particle swarm optimization algorithms. Moreover, Gandomi et al. (2015) [9] investigated the wall optimization by swarm intelligence techniques and compared their results with four methods including Accelerated Particle Swarm Optimization (APSO), Firefly Algorithm (FA), Particle Swarm Optimization (PSO), and Cuckoo Search (CS). The wall optimization was performed for cost and weight of wall.

Sensitivity analyses have been performed in most of the abovementioned research. Saribas and Erbatur (1996) [1] investigated the variation of initial parameters on objective functions, including stem height, backfill slope, and surcharge. They found that costs and weights of wall increased with increasing the stem height and surcharge. In addition, the influence of these parameters on the cost objective function is more than the wall weight. The sensitivity analysis of backfill slope showed that the wall cost and weight first decreased with increasing this parameter from 0 to 20 degree and then both of them increased.

In order to optimize the retaining wall, a newly developed method is applied and the cost and weight of the wall will be minimized. The Cuckoo Optimization Algorithm (COA) is a new evolutionary algorithm that inspired from special lifestyle of cuckoo birds and their characteristics in egg laying and breeding. The basic of cuckoo algorithm is the effort to survive. The COA was presented by Rajabioun (2011) [10].

In this paper, the capability of the COA is investigated. For this purpose, the obtained results of COA methods are compared with

In optimization applications, the variables of the problem are considered as matrix which is defined by "habitat" for cuckoo algorithm. For the N_{var} -dimensional optimization problem, the current position of cuckoos is defined by a matrix of $1 \times N_{\text{var}}$ as:

$$\text{habitat} = [X_1, X_2, \dots, X_{N_{\text{var}}}] \quad (1)$$

nonlinear programming presented by Saribas and Erbatur (1996) [1]. In addition, the COA predictions will be compared with conventional method normally used by design engineers and other presented methods by Gandomi et al. (2015) [9]. In order to investigate the wall geometries effect on objective functions, the parametric studies are performed. Moreover, the influence of stem height, surcharge, backfill slope, and backfill unit weight on the wall cost and weight is determined by performing sensitivity analysis. Finally, the effect of computing lateral earth pressure method on objective functions for all types of wall is investigated.

2- Introduction of Cuckoo Optimization Algorithm (COA) [10]

In nature, there are a group of birds that they dispense with every convention of home making and parenthood. These birds called "brood parasites" can raise their families by cunning. These birds never build their own nests and they lay their eggs in nests of other species. In addition, they entrust the care of own young to parents of other species. The most famous of brood parasite is cuckoo. The mother picks up one egg laid in to the host mother. After that, she lays her own and flies off with the egg of host mother. This process is performed by cuckoo less than ten seconds. Cuckoos can accord the color and pattern of their own eggs with the eggs of hosts. Moreover, there are some birds that can recognize the cuckoo's broods and they throw out their eggs. In fact, the cuckoos continuously improve their imitation of the target nests eggs and also the host birds learn the methods of recognition of foreign eggs. This is a continuous process for survival between birds and cuckoos. The cuckoos' eggs hatch and grow up earlier than host eggs and throw out their eggs or broods. This is an instinctive event.

where variables $X_1, X_2, \dots, X_{Nvar}$ are defined by floating point number.

Profit function f_p at a habitat of $(X_1, X_2, \dots, X_{Nvar})$ is defined as:

$$\text{profit} = f_p(\text{habitat}) = f_p(X_1, X_2, \dots, X_{Nvar}) \tag{2}$$

The above algorithm is used for maximizing the problem of profit function. Therefore, in the cost minimization optimization problems, the profit function is defined by:

$$\text{profit} = -\text{cost}(\text{habitat}) = -f_c(X_1, X_2, \dots, X_{Nvar}) \tag{3}$$

At the first step of optimization, the habitat matrix of $N_{pop} \times N_{Nvar}$ is produced. After that, some randomly numbers of eggs are proposed for each of these initial cuckoo habitats. Each cuckoo lays from 5 to 20 eggs naturally. One of the cuckoo habit is that they lay eggs within a maximum distance from their habitat that it is called ELR (egg

laying radius). Each optimization problem has the higher and lower bound (var_{hi} , var_{low}) based on variables. A leg laying radius (ERL) for each cuckoo is proportional to the total number of eggs, the number of current cuckoo's eggs and variables limited, var_{hi} and var_{low} . The ELR is defined as:

$$\text{ELR} = \alpha \times \frac{\text{Number of current cuckoo's eggs}}{\text{Total number of eggs}} \times (\text{var}_{hi} - \text{var}_{low}) \tag{4}$$

where α is an integer that is used for maximum value calculation of ELR.

Each cuckoo starts the laying egg randomly in some of host birds' nests based on its own ELR, shown in Fig. 1. Therefore, all cuckoos' eggs are laid in the host bird nests. Some of them that they are less similar to host birds' own eggs, thrown out by host birds. For this reason, after laying egg, $p\%$ of all eggs will be killed. This number usually is about 10%. It is important to note that only one cuckoo can be grown in the nest of the host birds. When cuckoo egg hatches, she throws the eggs of host birds out of the nest. Moreover, if the eggs of host bird hatch earlier than cuckoo's egg, the cuckoo's chick eats most of the food host bird. Therefore, after some days, the chicks of the host bird die and the cuckoo chick survives in the nest. After young cuckoos grow up, they stay in their society and area for sometimes. When $\lambda \sim U(0,1)$

the time for egg laying approaches, they immigrate to new and better habitat. The host birds' eggs in the new place has more similarity with own eggs and also more food exist for new youngsters. Moreover, the recognition of belonging each cuckoo to its group in the case of mature cuckoos who live in all over the environment is so difficult. For this reason, the grouping of cuckoos is performed with k-means clustering method. In addition, the cuckoos do not fly all the way to the destination habitat. They only fly a part of way with a deviation. In other words, each of cuckoo flies $\lambda\%$ of way with deviation of φ radian (Fig. 2). The cuckoos can search more positions in environment with these parameters. For each cuckoo λ and φ are obtained using:

$$\varphi \sim U(-\omega, \omega) \tag{5}$$

where $\lambda \sim U(0,1)$ is a random number between 0 and 1 (with uniformly distributed) and ω constraints the values of deviation.

After the cuckoos immigrate to the new position, each cuckoo mature is given some eggs. Then, an ELR is calculated for each cuckoo based on eggs number of each bird. After determination of ELR, the new process for egg laying restarts.

In nature, there is always equilibrium in population of birds. Therefore, the maximum number of live cuckoos in environment can

be controlled by N_{max} . There is balance for food limitations, being killed by predators and inability to find proper nests for eggs. All cuckoos immigrate after some iterations. They immigrate to the best habitat that there are maximum similarity of eggs to the host birds and more food resources. Therefore, there are maximum profit and minimum losses of eggs in this habitat. If the more than 95% of all cuckoos convergence to the same habitat, the end of Cuckoo Optimization Algorithm (COA) is reached. The steps of

COA according to Rajabioun (2011) [10] are:

- 1- Initialize cuckoo habitats with some random points on the profit function.
- 2- Dedicate some eggs to each cuckoo.
- 3- Define ELR for each cuckoo.
- 4- Let cuckoos lay eggs inside their corresponding ELR.
- 5- Kill those eggs which are recognized by the host birds.
- 6- Let eggs hatch and chick grow.
- 7- Evaluate the habitat of each newly grown cuckoo.
- 8- Limit the maximum number of cuckoos in environment and kill those who live in worst habitat.
- 9- Cluster the cuckoos, find best group, and select goal habitat.
- 10- Let new cuckoo population immigrate toward the goal habitat.
- 11- If the stop condition is satisfied, stop, otherwise go to step 2.

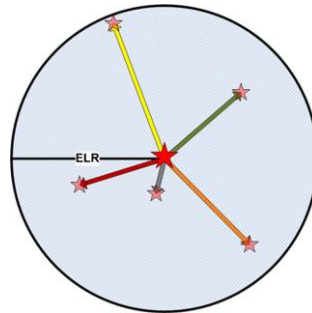


Fig. 1. Egg laying radius [10]

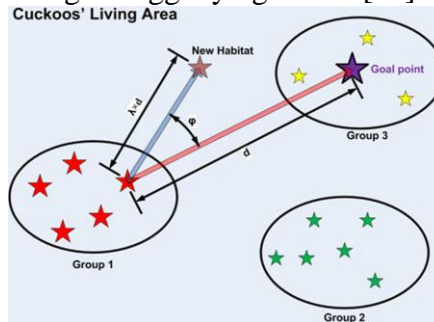


Fig. 2. Immigration of cuckoo to new habitat [10]

Many problems and defects have been dissolved in COA method. The COA can achieve to global optimum in lowest time and highest accuracy. It uses multiple operators and helps local search to reach the better results in global search. It has lots of superiority; faster convergence, higher speed and accuracy, reliability of local search besides of global search, lowest probability of catching in local optimum points, searching with variables population (because of population destruction in unsuitable area), general moving of population to better point by disappearing of unsuitable points and quick solving reliability of optimization problem with high dimension.

3- Details of Retaining Wall for Optimization

A typical retaining wall is shown in Fig. 3 for which the COA is used to minimize its cost and weight.

3-1- Design variables

Table 1 shows all parameters for design process. As seen, wall dimensions and required steel bars are defined as variables. In addition, required steel bar numbers for stem, toe, and heel are obtained from software outputs by considering maximum and minimum values based on the American Concrete Institute code (ACI-2008) [11].

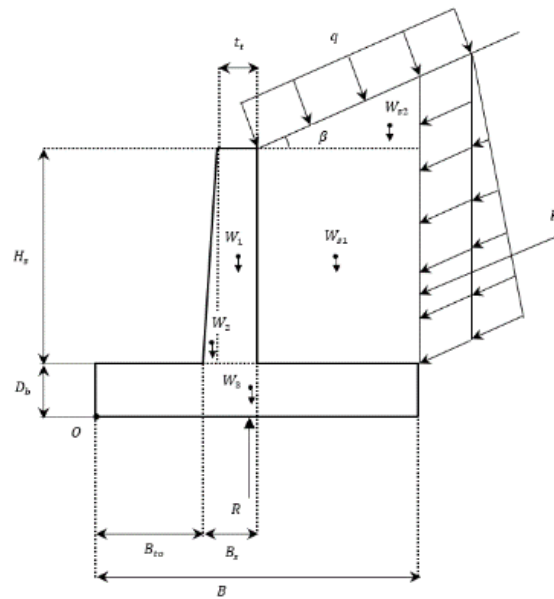


Fig. 3. Geometry of retaining wall

Table 1. Problem variables and software output for retaining wall design

Groups	Name	Unit	Symbol	Type
Variables of wall geometry	Total base width		B	
	Toe width		B_{to}	
	Stem thickness at bottom	m	B_s	Continuous
	Thickness of base		D_b	
	Stem thickness at top		t_t	
Variables of specification of used steels	Stem tensile steel area		A_{stS}	
	Toe tensile steel area		A_{stT}	
	Heel tensile steel area	(cm^2/m)	A_{stH}	Continuous
	Stem compressive steel area		A_{scS}	
	Toe compressive steel area		A_{scT}	
Software output	Heel compressive steel area		A_{scH}	
	Number of stem tensile steel		n_1	
	Number of toe tensile steel		n_2	
	Number of heel tensile steel		n_3	
	Number of stem compressive steel	—	n_4	Discrete
	Number of toe compressive steel		n_5	
	Number of heel compressive steel		n_6	

It is important to note that all continuous variables consist of upper and lower bounds. These values are shown in Table 2 where the stem height (H_s) is an initial and fixed parameter. The maximum and minimum

values for the upper and lower bounds of the wall dimensions are considered based on Saribas and Erbatur (1996) [1]. The maximum and minimum amount of steel is controlled by ACI-2008 [11] as constraints.

For this reason, the initial values are assumed for these parameters in the first step. Moreover, the compressive steel is defined as variables. Programming is performed such that the compressive steel will be obtained zero when the tensile steel

is enough for the applied moment. In other words, if the wall with maximum tensile steel cannot resist, compressive steel will be used. Otherwise, the compressive steel is zero.

Table 2. Upper and lower bounds of continuous variables

Variable name	Unit	Lower bound	Upper bound
Total base width	m	$(24 \times H_s)/55$	$(7 \times H_s)/9$
Toe width	m	$(8 \times H_s)/55$	$(7 \times H_s)/27$
Stem thickness at bottom	m	0.2	$H_s/9$
Thickness of base	m	$H_s/11$	$H_s/9$
Stem thickness at top	m	0.2	0.3
Area of tensile and compressive steel	(cm ² /m)	0	80

3-2- Objective functions

In this paper, the wall optimization is performed to minimize the cost and weight wall for which all variables defined in Table 1 are obtained based on structural and geotechnical constrains and minimizing the wall cost and weight. Moreover, the required development length of steel bars (l_{dh} , l_{dc}) are calculated according to the ACI code (2008) [11]. The cost and weight objective functions are defined as:

$$f(C) = C_s W_s + C_c V_c \quad (7)$$

$$f(W) = W_s + 100V_c \gamma_c \quad (8)$$

where C_s is the cost of steel unit (\$/kg), C_c is the cost of concrete unit (the selected value is considered for forming, concretion, vibration and work force cost) (\$/m³), W_{st} is the steel weight in the wall length unit (kg), V_c is the concrete volume in the wall length unit (m³), and γ_c is the weight of concrete unit (kN/m³). Moreover, the unit of $f(C)$ and $f(W)$ are \$ and kg per unit length of the wall, respectively.

3-3- Design constraints

The structural and geotechnical constraints ($g_i(x)$) are defined as:

$$g_i(x) \leq 0, \quad i = 1, 2, \dots, m \quad (9)$$

where m is the number of constraints.

All constraints for retaining wall optimization are shown in Table 3. It is should be noted that the methods of lateral earth pressure and bearing capacity calculation are Rankine and Hansen, respectively.

Table 3. Design constraints

Names of constraints	Unit	Names of constraints	Unit
Overturning stability	kN.m	Yielding of tensile steel	–
Sliding stability	kN	Yielding of compressive steel	–
No tension condition in foundation	m	Minimum footing depth	m
Bearing capacity	kPa	Stem slope control	–
Shear control	kN	Minimum distance of tensile steel	m
Moment control	kN.m	Minimum distance of compressive steel	m
Minimum of tensile steel	–	Maximum distance of tensile steel	m

Maximum of tensile steel	—	Maximum distance of	
Control of lower and upper bound of variables	—	compressive steel	m

4- Verification

In order to investigate the COA capability, its optimized predictions are compared with those given by SE (Saribas and Erbatur, 1996) [1], foraging bacterial algorithm (BFOA) (Ghazavi and Salavati, 2011) [6], ant colony algorithm (ACO) (Ghazavi and Bazzazian Bonab, 2011) [5]. As another verification, comparison is made between COA and conventional manual design method (Bowles, 1982) [12]. Furthermore, the obtained results from COA are compared with Gandomi and et al. (2015) [10] research.

4-1- Comparison of the COA with Saribas and Erbatur (1996)

In this section, two different examples given by Saribas and Erbatur (1996) [1] are selected and results are compared with those given by COA. The initial parameters of these examples are given in Table 4. The retaining wall model is shown Fig. 3 and all variables and constraints for the first verification are presented in Table 5. Saribas and Erbatur (1996) [1] used a nonlinear programming by a specially prepared computer program, RETOPT [1]. The method of this research is differential manner and results are in lowest values. For this reason, the other methods like metaheuristic algorithms try to reach these results.

Table 4. Initial parameters for verification with data reported by Saribas and Erbatur (1996) [1]

Parameter	Symb ol	Unit	Example 1	Example 2
Height of stem	H_s	m	3	4.5
Stem thickness at top	t_t	m	0.2	0.25
Yield strength of reinforcing steel	F_y	MPa	400	400
Compressive strength of concrete	f'_c	MPa	21	21
Concrete cover	d_{co}	cm	7	7
Maximum steel percentage	ρ_{max}	—	0.016	0.016
Minimum steel percentage	ρ_{min}	—	0.00333	0.00333
Shrinkage and temporary reinforcement percent	ρ_{st}	—	0.002	0.002
Diameter of bar	ϕ_{bar}	cm	1.2	1.4
Surcharge	q	kPa	20	30
Backfill slope	β	degree	10	15
Internal friction angle of retained soil	ϕ	degree	36	36
Internal friction angle of foundation soil	ϕ	degree	0	34
Unit weight of retained soil	γ_s	kN/m ³	17.5	17.5
Unit weight of foundation soil	γ_s	kN/m ³	18.5	18.5
Unit weight of concrete	γ_c	kN/m ³	23.5	23.5
Cohesion of foundation soil	c	kPa	125	100
Depth of soil in front of wall	D_f	m	0.5	0.75
Cost of steel	C_s	\$/kg	0.4	0.4
Cost of concrete	C_c	\$/m ³	40	40
Factor of safety against sliding	SF_s	—	1.5	1.5
Factor of safety against overturning	SF_o	—	1.5	1.5
Factor of safety for bearing capacity	SF_b	—	3	3

Table 5. Variables and constraints for first verification [1]

Variables	Constraints
Total base width	Shear at bottom of stem
Toe width	Moment at bottom of stem
Stem thickness at the bottom	Overturning stability
Thickness of base	Sliding stability
Area of stem tensile steel	No tension condition in foundation
Area of toe tensile steel	Bearing capacity
Area of heel tensile steel	Toe shear
	Toe moment
	Heel shear
	Heel moment

Tables 6 and 7 compare results predicted by COA, SE [1], BFOA [6], and ACO [5] methods. As seen, there are small differences between objective functions and variables. In addition, the difference between COA and optimized data of SE method for cost and

weight objective function are %0 and %0.018, respectively. These values for other methods are slightly more than the COA. These comparisons show the capability and accuracy of the COA in wall optimization.

Table 6. Comparison of results between COA, SE [1], BFOA [6], ACO [5]

Objective function	Unit	SE minimum value (RETOPT) [1]	COA minimum value	BFOA minimum value [6]	ACO minimum value [5]	Difference	Difference	Difference
						between SE and COA minimum values	between SE and BFOA minimum values	between SE and ACO minimum values
Example 1								
Cost	\$/m	82.474	82.474	-	-	%0	-	-
Weight	kg/m	2498.7	2498.79	-	-	%0.003	-	-
Example 2								
Cost	\$/m	189.546	189.546	190.574	201.185	%0	%0.542	%6.140
Weight	kg/m	5280	5280.96	5343.221	5540.3	%0.018	%1.197	%4.929

Table 7. Values of variables for optimum points given by COA and SE [1]

Design variables	Unit	optimum values in cost minimum		optimum values in Weight minimum	
		SE (RETOPT)	COA	SE (RETOPT)	COA
Example 1					
X ₁ Total base width	m	1.578	1.578	1.574	1.574

X_2	Toe width	m	0.436	0.436	0.441	0.442
X_3	Stem thickness at the bottom	m	0.258	0.258	0.200	0.200
X_4	Thickness of base	m	0.273	0.273	0.273	0.273
X_5	Area of stem tensile steel	cm ² /m	12.574	12.573	21.072	21.072
X_6	Area of toe tensile steel	cm ² /m	6.551	6.551	6.551	6.551
X_7	Area of heel tensile steel	cm ² /m	6.551	6.551	6.681	6.686
Example 2						
X_1	Total base width	m	2.254	2.254	2.238	2.238
X_2	Toe width	m	0.655	0.655	0.655	0.655
X_3	Stem thickness at the bottom	m	0.417	0.418	0.300	0.300
X_4	Thickness of base	m	0.409	0.409	0.409	0.409
X_5	Area of stem tensile steel	cm ² /m	23.475	23.379	41.626	41.626
X_6	Area of toe tensile steel	cm ² /m	11.059	11.059	11.059	11.059
X_7	Area of heel tensile steel	cm ² /m	11.059	11.059	11.059	11.059

4-2- manual design

In this section, the retaining wall optimization is performed to the normal T-shape wall shown in Fig. 4. An example is considered from Bowles (1982) [12] who introduced conventional design procedure

for retaining walls. In order to show the capability of the COA method, the values of cost and weight objective functions from COA are compared with manual design. The initial parameters are presented in Table 8.

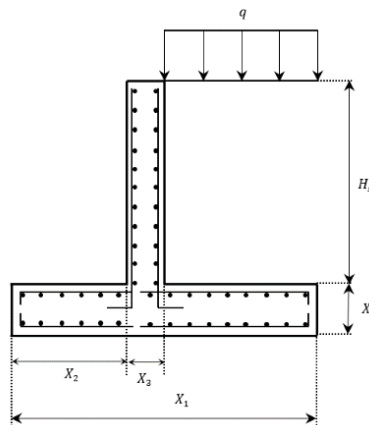


Fig. 4. Wall model in second verification

Table 8. Design parameter for second verification case

Parameter	Symbol	Unit	Value
Height of stem	H_s	m	2.44
Concrete cover	d_{co}	cm	5
Shrinkage and temporary reinforcement percent	ρ_{st}	—	0.0018
Diameter of bars	Φ_{bar}	cm	2
Surcharge	q	kPa	12
Backfill slope	β	degree	0

Internal friction angle of retained soil	ϕ	degree	36
Internal friction angle of base soil	ϕ	degree	0
Unit weight of retained soil	γ_s	kN/m ³	18.86
Unit weight of base concrete	γ_c	kN/m ³	23.6
Unit weight of base soil	γ_s	kN/m ³	17.3
Cohesion of base soil	c	kPa	120
Depth of soil in front of wall	D_f	m	1.22
Factor of safety for bearing capacity	SF_b	–	3
Factor of safety against sliding	SF_s	–	1.5
Factor of safety against overturning	SF_o	–	1.5
Yield strength of reinforcing steel	F_y	MPa	400
Compressive strength of concrete	f_c	MPa	21

The objective functions given by COA and manual design are presented in Table 9. As seen, the cost and weight of the wall decrease to values of %46.58 and %45.61 by using the COA, respectively. The values of variables

at optimum point of COA are presented in Table 10. As seen, the COA can significantly reduce the cost and weight of the wall compared with conventional design procedure.

Table 9. Optimum values for cost and weight objective functions in second verification case

Method	Objective function	unit	value
Bowles (manual design) [12]	Cost	\$/m	86.7692
	Weight	kg/m	3525.3
COA	Cost	\$/m	46.346
	weight	kg/m	1917.251

Table 10. Optimum variable values for cost and weight functions in verification with Bowles (1982) [12]

Design parameters	Unit	Optimum value	
		Cost	Weight
X_1 Total base width	m	1.3934	1.3934
X_2 Toe width	m	0.4702	0.4763
X_3 Stem thickness at the bottom	m	0.200	0.200
X_4 Thickness of base	m	0.2218	0.2218
X_5 Stem tensile steel area	cm ² /m	6.2833	6.2834
X_6 Toe tensile steel area	cm ² /m	6.2838	6.2894
X_7 Heel tensile steel area	cm ² /m	6.2832	6.2843
X_8 Stem compressive steel area	cm ² /m	0	0
X_9 Toe compressive steel area	cm ² /m	0	0
X_{10} Heel compressive steel area	cm ² /m	0	0
X_{11} Number of stem tensile steel	–	3	3
X_{12} Number of toe tensile steel	–	3	3
X_{13} Number of heel tensile steel	–	3	3
X_{14} Number of stem compressive steel	–	0	0
X_{15} Number of toe compressive steel	–	0	0
X_{16} Number of heel compressive steel	–	0	0

4-3- Comparison of the COA with Gandomi and et al. (2015)

Gandomi and et al. (2015) [10] presented three examples for retaining wall optimization using PSO, APSO, FA, and CS. The wall shapes of first and second example for this research are shown in Fig. 3. Also, the objective functions are according to Eqs. (7) and (8). The initial parameters with a small change are similar to this study as presented in Table 4. The lower and upper limits for variables and also the number of constraints are considered based on Gandomi and et al. (2015) [10]. According to Gandomi et al. (2015), the diameter of bars for these examples are not the same. But they are approximately the same and 10 mm. For this reason, in this section, the diameter

of bar is considered 10 mm. Also, the cohesion of soil for example 2 is zero. In this section, the results obtained the COA method are compared with those given by Gandomi et al. (2015) [10] for two examples. Table 11 shows the constraints considered by Gandomi and et al. (2015) [10]. The COA results are shown in Table 12. The best results of Gandomi and et al. (2015) [10] were obtained by 100 runs, whereas, the COA gives them in only one run. This means that the COA operates faster with more accuracy than PSO, APSO, FA and CS methods.

Table 11. Design constraints considered by Gandomi and et al. (2015) [10]

Names of constraints	Unit
Overtuning stability	kN. m
Sliding stability	kN
No tension condition in foundation	m
Bearing capacity	kPa
Shear control	kN
Moment control	kN. m
Minimum of tensile steel	–
Maximum of tensile steel	–
Control of lower and upper bound of variables	–

Table 12. Comparison of results between Gandomi and et al. (2015) (PSO, APSO, FA and CS) [10] and COA method

Objective function	Unit	PSO minimum values	APSO minimum values	FA minimum values	CS minimum values	COA minimum values	Difference between COA minimum values and best results from Gandomi and et al. (2015)
Example 1							
Cost	\$/m	73.06	73.06	73.16	73.06	67.92	%7.03
Weight	kg/m	2665.8	2668	2666.5	2665.8	2494.95	%6.4
Example 2							
Cost	\$/m	162.37	162.64	162.8	162.42	158.2	%2.57
Weight	kg/m	5550.3	5552	5566.3	5550.4	5525.72	%0.44

5- Parametric Studies

In this section, five different types of retaining walls are considered and the wall geometric influence on the cost and weight objective functions are investigated (Fig. 5). The required initial parameters are according

to Table 4 and used in Example 2. Moreover, all constraints mentioned in Table 3 are applied to all wall types. The optimum values of objective functions are given in Table 13.

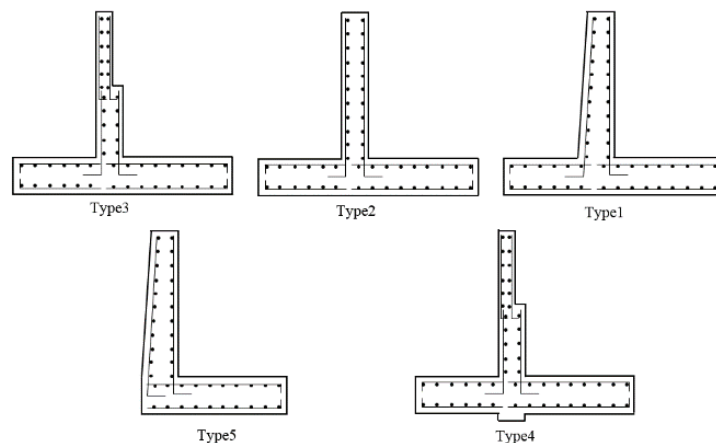


Fig. 5. Wall types considered for parametric studies

Table 13. Objective functions values for parametric studies

Type of wall	Objective function	
	Cost (\$/m)	Weight (kg/m)
Type1	144.14	4927.4
Type2	161.17	5373.54
Type3	134.57	4732.93
Type4	140.18	4875.57
Type5	144.61	5007.76

As shown in Table 13, the minimum cost and weight values are obtained for T-shape wall with two stem thicknesses (wall type 3). In addition, the maximum cost and weight of the wall are obtained for normal T-shape wall (wall type 2). Moreover, the second, third and fourth rank for two objective functions are T-shape wall with two thicknesses in stem and shear key (wall type 4), T-shape wall with variables thickness (Type 1) and L-shape wall (Type 5), respectively. The obtained results show that the T-shape wall with two thicknesses in stem can reduce %16.5 cost and %11.92 weight of wall compared with normal T-shape. In addition, the shear key increases %4 and %2.9 the wall cost and weight, respectively. In contrast, the results indicate that the T-shape wall with two thickness in stem and shear key is better than the normal T-shape, T-shape with variable thickness and L-shape wall. Moreover, the values of wall cost in L-shape and T-shape wall with

variables thickness are approximately similar. However, the wall weight of L-shape is %1.63 more than T-shape wall with variables thickness. Furthermore, the results of wall fourth and fifth rank illustrate that the normal T-shape has bad performance compared with type 4 with %10.56 and %8.3 differences for cost and weight of wall, respectively. It is important to note that the all values are in per unit of wall and it is obvious that the %1 reduction is significant in design.

6- Sensitivity Analysis

The initial parameter variations affect significantly objective functions and optimized points. In order to observe their influence, sensitivity analyses are performed. In this section, the influence on cost and weight objective functions on the stem height, backfill unit weight, surcharge, and backfill slope are investigated. Other required initial parameters are selected

according to Section 5 and Table 4. It is mentioned that the backfill unit weight is changed based on internal friction angle. In other words, the linear connection is assumed between two parameters as shown in Table 14.

Table 14. Parameters values of sensitivity analyses

Parameter	Unit	Symbol	Value
Backfill slope	degree	β	0-10-20-30
Stem height	m	H_s	3-4-5-6
Surcharge	kPa	q	0-10-20-30-40
Backfill unit weight	kN/m^3	γ_s	15-16-17-18*
Backfill internal friction angle	degree	ϕ	30-33.4-36.7-40*

(*Note: Values of $\gamma_s = 15-16-17-18$ correspond to $\phi = 30-33.33-36.66-40$, respectively)

6-1- Effect of Backfill slope

The effect of backfill slope on objective functions is shown in Fig. 6. As seen, these functions initially decrease for all wall types with increasing the β value from zero to 20° and then both functions increase. The minimum and maximum values of both cost and weight objective functions for all wall types are obtained for $\beta = 20^\circ$ and $\beta = 0^\circ$, respectively. It is also noted that the maximum value of the weight objective function for normal T-shape wall is obtained for $\beta = 30^\circ$. The reason of objective functions variations is that the shear control constraint governs the wall toe change significantly by increasing β . The variation of this constraint is similar to variations of objective functions. To find out the reason of this event, the variations of parameters are investigated by increasing β value. It was found that the applied force to the wall toe, the minimum and maximum pressures (q_{\max} , q_{\min}) change with increasing β . It is noted that with increasing β the value of q_{\max} decreases and q_{\min} increases. In addition, the tension control constraint in foundation changes by changing q_{\max} and q_{\min} . For more clarification, values of constraints corresponding to two optimum points of $\beta = 10^\circ$ and $\beta = 20^\circ$ are determined and compared. The obtained variables for $\beta = 20^\circ$ are used for constraints of $\beta = 10^\circ$. The results show that these constraints for

$\beta = 10^\circ$ cannot gratify. Accordingly, the tension control constraint is affected by the backfill slope variation. Fig. 6 shows that in all types of wall except L-shape, cost and weight functions gradually decrease up to $\beta = 20^\circ$ and then slightly increase. Though the L-shape wall from high point plunges and then go up slightly. Furthermore, the cost and weight of normal T-shape wall reach a peak at $\beta = 30^\circ$. This peak for the weight objective function of normal T-shape is obtained suddenly. As seen, both objective functions for L-shape wall for $\beta = 0^\circ$ are similar to those of normal T-shape wall. This indicates that L-shape wall has bad performance for zero angle of backfill slope.

6-2- Effect of wall stem height

Fig. 7 shows the effect of wall stem height on objective functions. As seen, with increasing the stem height, both functions soar for all wall types. In addition, with increasing the stem height from 3 m to 4 m, the values of both functions are greater than when the stem height increases from 4 m to 5 m. Moreover, both functions for normal T-shape wall increase more quickly than these functions for other wall types. Also, with increasing the wall stem height, the cost objective function grows more rapidly than the weight objective function.

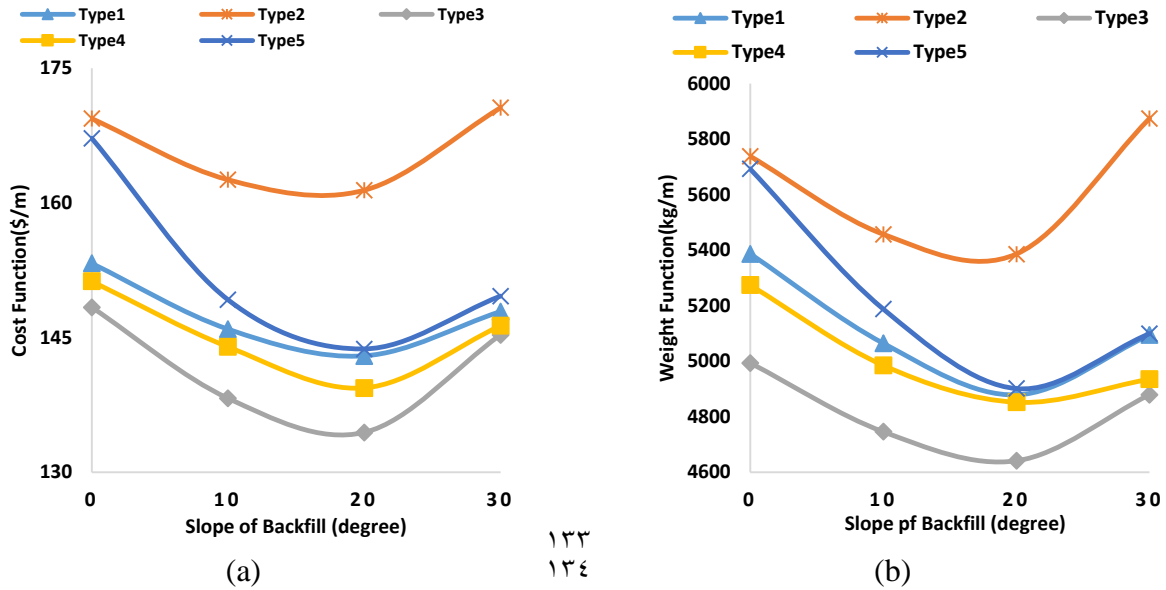


Fig. 6. Effect of backfill slope angle (\square) on values of: (a) cost objective function; (b) weight objective function

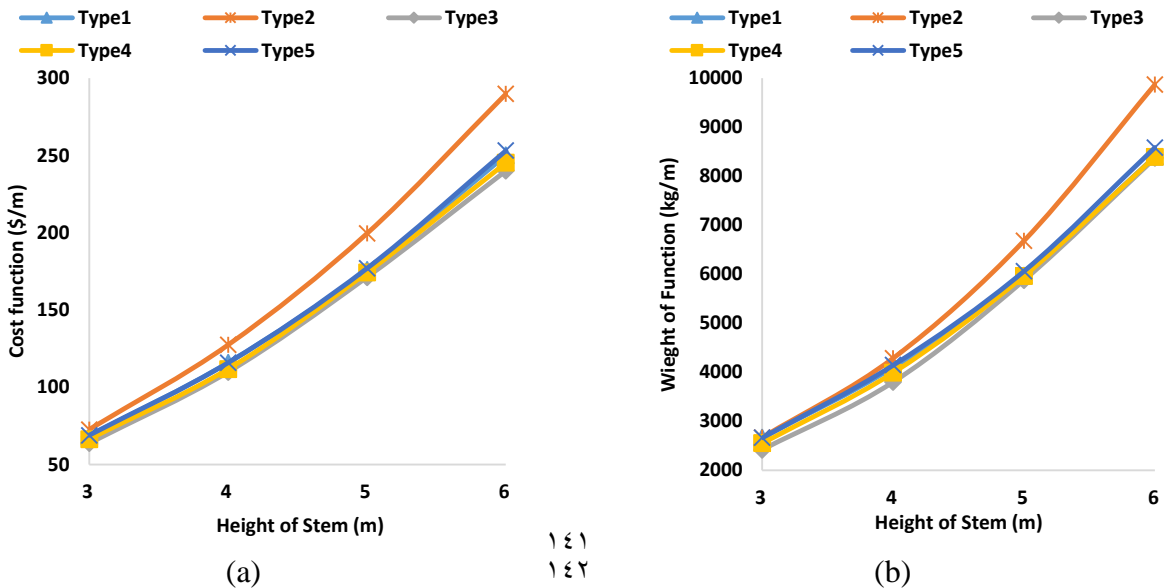


Fig. 7. Effect of wall height on objective functions; (a) cost, (b) weight

140 **6-3- Effect of surcharge**

146 Fig. 8 illustrates values of objective
 147 functions versus surcharge. As seen, both
 148 cost and weight objective functions
 149 increase obviously with increasing the
 150 surcharge values for all wall types. In
 151 addition, the increase of the cost objective
 152 function is more than the weight objective

153 function for all wall types. For example in
 154 the first wall type, the rates of increase for
 155 the cost and weight objective functions are
 156 %24.03 and %8.42, respectively. Saribas an
 157 Erbatur (1996) [1] reached the same
 158 finding. Moreover, both functions for T-
 159 shape wall grow more rapidly than other
 160 wall types.

6-4- Effect of unit weight and internal friction angle of soil

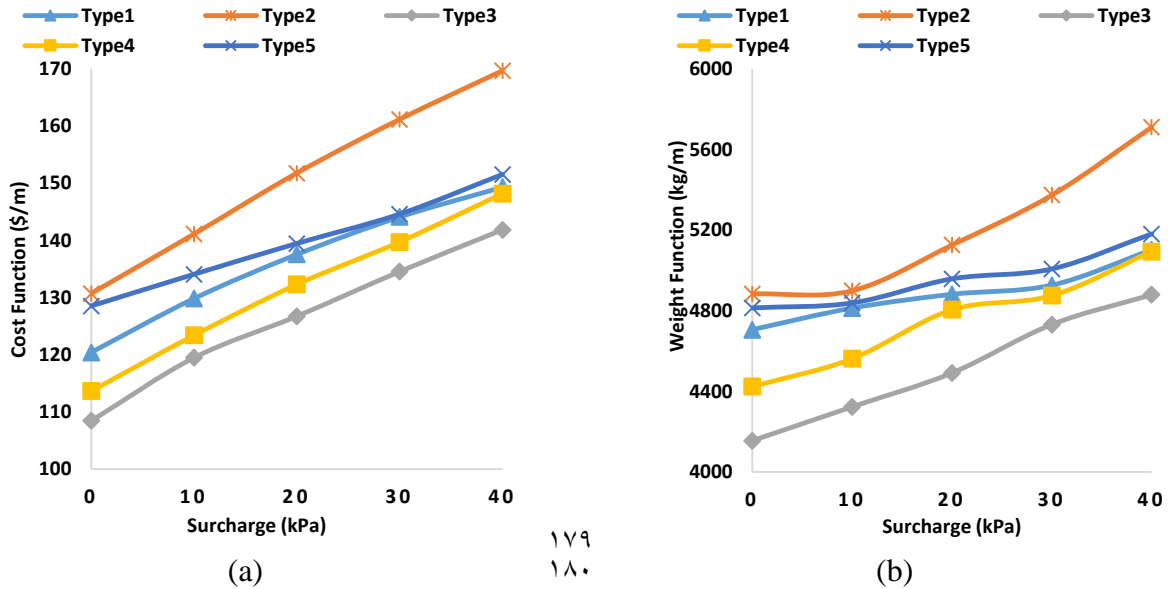
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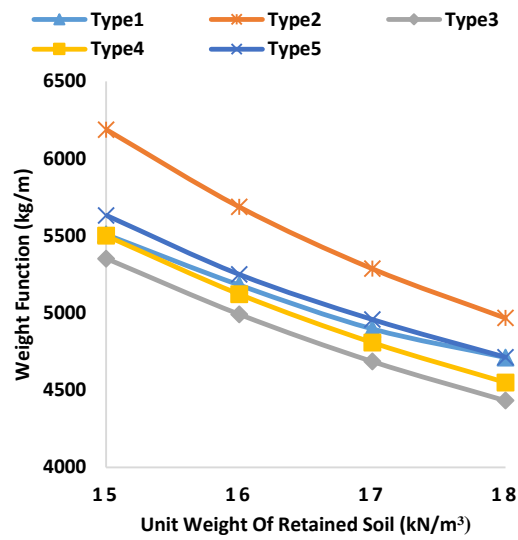
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۱۶۳ The influence of unit weight and internal
 ۱۶۴ friction angle of soil on the cost and weight
 ۱۶۵ objective functions are shown in Fig. 9. As
 ۱۶۶ observed, with increasing ϕ and α , both
 ۱۶۷ cost and weight objective functions fall for
 ۱۶۸ all wall types. For all walls, the reduction
 ۱۶۹ rate of cost and weight objective functions

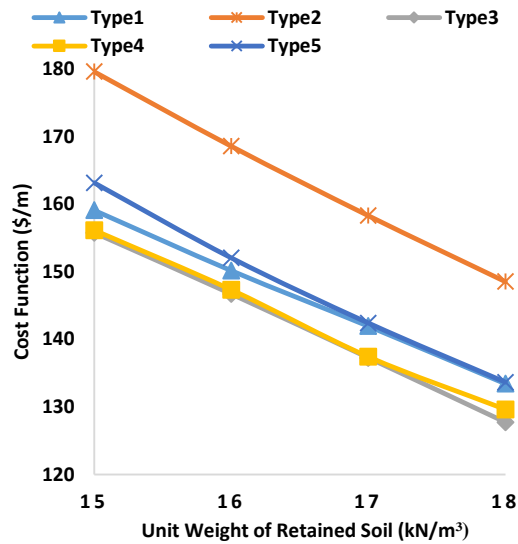
۱۷۰ varies between %15 and %20. This means
 ۱۷۱ that in all wall types, cost and weight
 ۱۷۲ functions decrease with increasing the
 ۱۷۳ backfill unit weight from 15 to 18 kN/m³.
 ۱۷۴ All curves in Fig. 9 almost have close
 ۱۷۵ decreasing slopes.
 ۱۷۶



۱۷۹
 ۱۸۰ Fig. 8. Effect of increasing the surcharge load on the objective function values; (a) cost, (b)
 ۱۷۷
 ۱۷۸
 ۱۸۱
 ۱۸۲
 ۱۸۳ weight



(b)



(a)

Fig. 9. Effect of increasing the unit weight of soil on objective functions; (a) cost, (b) weight

7- Investigation of Rankine and Coulomb Method Influence

In order to investigate the lateral earth pressure calculation method effect, objective functions for all wall types are calculated based on Rankine and Coulomb methods, using the initial parameters given in Table 4. The values of two objective functions for Rankine method are shown in Table 15. It should be noted that the Rankine results are picked up from section 5. According to Table

15, the use of Coulomb method leads to lower objective functions than Rankine method. In all walls types, cost decreases more than %5 except in wall type 3 for which, the cost reduction is about %3.7. In addition, the use of Coulomb method has more influence on wall weight reduction than cost wall.

Table 15. Objective functions values obtained from Rankine and Coulomb methods

Type of wall	Method	Objective function	
		Cost (\$/m)	Weight (kg/m)
Type1	Rankine	144.14	4927.4
	Coulomb	135.74	4651.48
Type2	Rankine	161.17	5373.54
	Coulomb	152.06	5036.53
Type3	Rankine	134.57	4732.93
	Coulomb	129.52	4447.14
Type4	Rankine	140.18	4875.57
	Coulomb	132.45	4577.11
Type5	Rankine	144.61	5007.76
	Coulomb	136.01	4655.96

8- Conclusions

In this paper, the optimization of retaining wall is performed by using Cuckoo optimization algorithm (COA). In addition, the influence of the wall geometries on its cost and weight are investigated. The main results of parametric studies and sensitivity analysis are:

- ◆ The COA method is more efficient than ACO, BFOA, PSO, APSO, FA and CS algorithms due to its lowest run time and highest accuracy.
- ◆ To have an optimum T-shape retaining wall from cost and weight viewpoints, it is suggested to design two thicknesses for wall stem.
- ◆ Among 5 wall types considered in the current study, normal T-shape walls have greater cost and weight objective functions.
- ◆ With increasing the backfill slope from zero to 20°, the cost and weight objective functions decrease and for $\square > 20^\circ$, objective functions increase. In addition, with increasing stem

height and surcharge, cost and weight of walls increase. Furthermore, cost and weight objective functions decrease with increasing the backfill unit weight and internal friction angle.

- ◆ The use of Coulomb method for lateral earth pressure calculation leads to reducing cost and weight of retaining walls more than about %5 compared with the Rankine method.

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