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Non-discretionary imprecise data in efficiency measurement

***S. Razavyan*[1]*; G. Tohidi*[2*]**

[1]*Assistant Professor, Dep. of Mathematics, Islamic Azad University, South- Tehran Branch, Tehran, Iran*

[2]*Assistant Professor, Dep. of Mathematics, Islamic Azad University, Tehran-Central Branch, Tehran, Iran*

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Abstract: This paper introduces discretionary imprecise data in Data Envelopment Analysis (DEA) and discusses the efficiency evaluation of Decision Making Units (DMUs) with non-discretionary imprecise data. Then, suggests a method for evaluation the efficiency of DMUs with non-discretionary imprecise data. When some inputs and outputs are imprecise and non-discretionary, the DEA model becomes non-linear programming problem. By a Theorem, we use the translation of imprecise data into exact data and then use the standard linear DEA model for evaluating DMUs with non-discretionary and imprecise data, which is the generalized form of envelopment form in input oriented of CCR model. To illustrate the proposed method, a numerical example with non-discretionary imprecise data is solved.

Keywords: *Data Envelopment Analysis (DEA); Efficiency; Non-discretionary data; Imprecise data*

1. Introduction

Data Envelopment Analysis (DEA) is a mathematical programming technique for identifying efficient frontiers for peer Decision Making Units (DMUs). The original DEA models, say CCR model (Charnes *et al*., 1978), assume that inputs and outputs of DMUs are homogeneous and precise, i.e. they perform the same task with similar objective, consume similar inputs and produce similar outputs, and operate in similar operational environments (Golany and Roll, 1993); and inputs and outputs are measured by exact values (do not take into account non-discretionary or imprecise inputs and outputs). Often the assumption of homogeneous environments is violated and factors that describe the differences in the environments need to be included in the analysis. These factors, and other factors outside the control of the DMU, are frequently called non-discretionary factors, (Fired *et al.,* 1993; Ruggiero, 1996). On the other hand, the CCR model assumes that all inputs and outputs are known exactly. However this assumption may not be true, i.e. some or all of inputs and outputs may be imprecise. Imprecise data means that some data are known only to the extent that the true values lie within prescribed bounds while other data are known only to satisfy certain ordinal relations (Zhu, 2003). If we incorporate imprecise data into the standard linear CCR model, the resulting DEA model is a nonlinear and nonconvex program, and is called imprecise DEA (IDEA) (Zhu, 2003). Cooper *et al.* (1999) addressed the problem of imprecise data in DEA in its general form. Kim *et al.* (1999) discuss how to deal with bounded data, ordinal data, and ratio bounded data with an application to a set of telephone office.

In a recent study Zhu (2003), the nonlinear Imprecise DEA (IDEA) is solved in the standard linear CCR model and discusses the incorporation of weight restrictions in IDEA.

A number of different approaches have been developed to take into account non-discretionary inputs and outputs, when DMUs are evaluated. The first approach to account for difference in non-discretionary inputs and outputs was introduced by Banker and Morey (1986). Lovell (1994) and Ruggiero (1996) suggest an alternative approach. Golany and Roll (1993) generalize the approach introduced by Banker and Morey (1986) to account for both non-discretionary inputs and nondiscretionary outputs. Also, a number of multiple stage models have been suggested. Ray (1991) and Fried *et al.* (1993) introduced two-stage approaches. The first stage consists of a standard DEA with only discretionary factors. In the second stage, the efficiency score are corrected using regression analysis, in which non-discretionary factors are used as independent variables. Ruggiero (1998) and Fried *et al.* (1999) have further extended the two-stage approaches. But in this paper the researchers discuss the efficiency evaluation of DMUs with non-discretionary imprecise data in DEA and suggest a method for evaluation the efficiency of DMUs.

*Corresponding Author Email: ghatohidi@yahoo.com
Tel.: +98 2144811232

The rest of the paper is structured as follows: Section 2 briefly introduces the background of DEA and IDEA. In Section 3, the researchers introduce the non-discretionary imprecise data and propose a method for evaluating DMUs with non-discretionary imprecise data. In Section 4, the researchers obtain the efficiency of DMUs through an example. Conclusion is given in Section 5.

2. DEA and imprecise data

To describe the DEA efficiency measurement, we assume that there are n DMUs to be evaluated, indexed by $j\,(j=1,\dots,n)$ and each DMU is assumed to produce s different outputs from m different inputs. Let the observed input and output vectors of DMU_j be $x_j=(x_{1j},\dots,x_{mj})$ and $y_j=(y_{1j},\dots,y_{sj})$ respectively, that all components of vectors x_j and y_j for all DMUs are non-negative and each DMU has at least one strictly positive input and output.

To obtain efficiency of DMU_o we use the CCR model, which is as follows:

$$\text{Max} \sum_{r=1}^{s} \mathrm{u}_{\mathrm{r}}\, y_{ro}$$

Subject to:

$$\sum_{r=1}^{m} v_i x_{io} = 1 \qquad (1)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j=1,..,n$$

$$u_r \ge 0, \qquad r=1,\dots,s$$

$$v_i \ge 0, \qquad i=1,\dots,m$$

where v_i and u_r are the weights associated, respectively, with input i and output r and $x_o=(x_{1o},\dots,x_{mo})$ and $y_o=(y_{1o},\dots,y_{so})$ are inputs and outputs of DMU_o, respectively. When x_{ij} (for some i) and y_{rj} (for some r) are imprecise and unknown decision variables such as bounded and ordinal data, Model (1) becomes a nonlinear programming and is called imprecise DEA (IDEA) (Cooper *et al.*, 1999). The bounded data can be expressed as:

$$\underline{y}_{rj} \le y_{rj} \le \overline{y}_{rj}, \qquad r \in BO$$

$$\underline{x}_{ij} \le x_{ij} \le \overline{x}_{ij}, \qquad i \in BI \qquad (2)$$

where $\overline{x}_{ij}$ and $\overline{y}_{rj}$ are upper bounds, $\underline{x}_{ij}$ and $\underline{y}_{rj}$ are lower bounds and BO and BI represent the associated sets containing bounded outputs and inputs, respectively. The weak ordinal data can be expressed as:

$$x_{ij} \le x_{io}$$

and

$$y_{rj} \le y_{ro} \qquad \forall j \ne o, i \in WI, r \in WO \qquad (3)$$

or to simplify the representation,

$$y_{r1} \le y_{r2} \le \dots \le y_{rk} \le \dots \le y_{rn}, \qquad r \in WO$$

$$x_{i1} \le x_{i2} \le \dots \le x_{ik} \le \dots \le x_{in}, \qquad i \in WI \qquad (4)$$

where WO and WI represent the associated sets containing weak ordinal outputs and inputs, respectively. The strong ordinal data can be expressed as:

$$y_{r1} \le y_{r2} \le \dots \le y_{rk} \le \dots \le y_{rn}, \qquad r \in SO$$

$$x_{i1} \le x_{i2} \le \dots \le x_{ik} \le \dots \le x_{in}, \qquad i \in SI \qquad (5)$$

where SO and SI represent the associated sets containing strong ordinal outputs and inputs, respectively.

If we incorporate Equations (2) to (5) into Model (1), we have:

$$\text{Max} \sum_{r=1}^{s} u_{\mathrm{r}}\, y_{ro}$$

Subject to:

$$\sum_{r=1}^{s} v_i x_{io} = 1 \qquad (6)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j=1,\dots,n$$

$$(x_{ij}) \in H_i^-, (y_{rj}) \in H_r^+$$

$$u_r \ge 0, \; r=1,\dots,s$$

$$v_i \ge 0, \; i=1,\dots,m$$

where $(x_{ij}) \in H_i^-$ and $(y_{rj}) \in H_r^+$ represent any of or all of (2)-(5). The following theorem provides the theoretical foundation to the approach developed by Zhu (2003) where the standard DEA model is used to solve the IDEA Model (6).

Theorem 1. Suppose H_i^- and H_r^+ are given by (2). Then for DMU_o the optimal value to (6) can be achieved at $x_{io} = \underline{x}_{io}$ and $y_{ro} = \overline{y}_{ro}$ for DMU_o and $x_{ij} = \overline{x}_{ij}$ and $y_{rj} = \underline{y}_{rj}$ for DMU_j $(j \neq o)$.

Proof. See Zhu (2003).

Theorem 1 shows that when DMU_o is under evaluation we can have a set of exact data via setting at $x_{io} = \underline{x}_{io}$ and $y_{ro} = \overline{y}_{ro}$ for DMU_o and $x_{ij} = \overline{x}_{ij}$ and $y_{rj} = \underline{y}_{rj}$ for DMU_j $(j \neq o)$.

3. Non-discretionary imprecise data

For simplicity and without loss of generality, all the models presented are formulated in an input oriented form and without non-discretionary imprecise outputs. These assumptions can be relaxed. Suppose $x_j \in R^k$ and $z_j \in R^l$ present the discretionary and non-discretionary inputs, respectively, where $m = k + l$. When x_{ij} (for some *i*) and z_{kj} (for some *k*) are imprecise and nondiscretionary imprecise inputs and unknown decision variables such as bounded and ordinal data, Model (1) becomes a nonlinear programming and is called non-discretionary imprecise DEA (ND-IDEA). The discretionary and non-discretionary bounded inputs can be introduced as:

$$\underline{x}_{ij} \le x_{ij} \le \overline{x}_{ij}, \qquad i \in DBI$$
$$\underline{z}_{kj} \le z_{rj} \le \overline{z}_{kj}, \qquad K \in NDBI \tag{7}$$

where, $\overline{x}_{ij}$ and $\overline{z}_{kj}$ are upper bounds, $\underline{x}_{ij}$ and $\underline{z}_{kj}$ are lower bounds and *DBI* and *NDBI* represent the associated sets containing bounded imprecise and non-discretionary imprecise inputs, respectively. The discretionary and nondiscretionary weak ordinal data can be introduced as:

$$x_{ij} \le x_{io}, \quad z_{kj} \le z_{ko}, j \neq o, i \in WDI, K \in WNDI \tag{8}$$

or to simplify the representation,

$$x_{i1} \le x_{i2} \le \ldots \le x_{ip} \le \ldots \le x_{in}, \qquad i \in WDI$$
$$z_{k1} \le z_{k2} \le \ldots \le z_{kp} \le \ldots \le z_{kn}, \quad k \in WNDI \tag{9}$$

where *WDI* and *WNDI* represent the associated sets containing weak ordinal imprecise and non-discretionary imprecise inputs, respectively. The discretionary and non-discretionary strong ordinal data can be expressed as:

$$x_{i1} < x_{i2} < \ldots < x_{ip} < \ldots < x_{in}, \qquad i \in SDI$$
$$z_{k1} < z_{k2} < \ldots < z_{kp} < \ldots < z_{kn}, \quad k \in SNDI \tag{10}$$

where *SDI* and *SNDI* represent the associated sets containing strong ordinal imprecise and non-discretionary imprecise inputs, respectively.

Banker and Morey (1986) suggested the following model for evaluation DMUs with non-discretionary inputs:

$$MB = \text{Min}\,\theta$$

Subject to:

$$\sum_{j=1}^{n} \lambda_j y_j \ge y_o \tag{11}$$
$$-\sum_{j=1}^{n} \lambda_j x_j + \theta x_o \ge 0$$
$$-\sum_{j=1}^{n} \lambda_j z_j + z_o \sum_{j=1}^{n} \lambda_j \ge 0$$
$$\lambda_j \ge 0, \quad j = 1, \ldots, n.$$

The dual of the model (11) is as follows:

$$\text{Max } u y_o$$

Subject to:

$$v^1 x_o = 1 \tag{12}$$
$$u y_j - v^1 x_j - v^2 (z_j - z_o) \le 0, \quad j = 1, \ldots, n$$
$$u \ge 0,\ v^1 \ge 0,\ v^2 \ge 0$$

Now suppose some of the inputs are imprecise data in the forms of bounded data, ordinal data and ratio bounded (discretionary and non-discretionary imprecise inputs). If we incorporate (2)-(6) and (7)-(10) into Model (12), we have:

$$\text{Max } \sum_{r=1}^{s} u_r \; y_{ro}$$

Subject to:

$$\sum_{i\in I_1} v_i^1 \, x_{io} = 1 \tag{13}$$

$$\sum_{r=1}^{s} u_r \; y_{rj} - \sum_{i\in I_2} v_i^1 x_{ij}$$

$$- \sum_{k\in I_2}^{s} v_i^2 (z_{kj} - z_{ko}) \le 0$$

$$j = 1,...,n, \; (x_{ij}) \in H_i^-, (z_{kj}) \in H_k^-, \; (y_{rj}) \in H_r^+$$

$$u_r \ge 0, \;\; v_i^1 \ge 0, \; v_k^2 \ge 0$$

$$r = 1,...,s, \forall i \in I_1, \forall k \in I_2$$

where, I_1 and I_2 are the index sets of discretionary and non-discretionary inputs, respectively. The above model is nonlinear, but its ratio form is as follows:

$$\text{Max } \sum_{r=1}^{s} \mu_r \, y_{ro} \, / \sum_{i\in I_1} v_i^1 x_{io}$$

Subject to: (14)

$$\sum_{r=1}^{s} \mu_r \, y_{rj} \, / (\sum_{i\in I_1} v_i^1 x_{ij} + \sum_{k\in I_2} v_k^2 (z_{kj} - z_{ko})) \le 1$$

$$j = 1,...,n, \; (x_{ij}) \in H_i^-, \; (z_{kj}) \in H_k^-, \;\; (y_{rj}) \in H_r^+$$

$$\mu_r \ge 0, \; v_i^1 \ge 0, \; v_k^2 \ge 0, r = 1,...,s,$$

$$\forall i \in I_1, \; \forall k \in I_2$$

The Models (13) and (14) are equivalent. By Theorem (2), Model (13) can be converted a linear programming problem.

The following theorem provides the theoretical foundation to the approach developed by Zhu (2003), where the DEA Model (12) is used to solve the ND-DEA model.

Theorem 2. Suppose H_i^- and H_r^+ are given by (2) and (7). Then for DMU$_o$ the optimal value to (13) can be achieved at

$$x_{io} = \underline{x}_{io}, i \in I_i, \;\; z_{ko} = \underline{z}_{ko}, k \in I_2$$

and $y_{ro} = \overline{y}_{ro}$ for DMU$_o$ and $x_{ij} = \overline{x}_{ij}$, $z_{kj} = \overline{z}_{kj}, k \in I_2$ and $y_{rj} = \underline{y}_{rj}$ for DMU$_j$ $(j \ne o)$.

Proof. Suppose the optimal value to (13) or (14) is achieved at x_{ij}^*, y_{rj}^* and z_{ij}^* for DMU$_j$ $(j \ne o)$ such that:

$$\underline{x}_{ij} \le x_{ij}^* \le \overline{x}_{ij}, \, i \in DBI$$

$$\underline{z}_{io} \le z_{io}^* \le \overline{z}_{io}, \, i \in ND - BI$$

$$\underline{y}_{rj} \le y_{rj}^* \le \overline{y}_{rj}, \, r \in BO \tag{15}$$

where *DBI* and *ND-BI* are discretionary bound inputs and non-discretionary inputs, respectively and for $r \notin BO, y_{rj}$ is exact output. So, for all $j (j = 1,...,n)$ we have:

$$(\sum_{r\in Bo} \mu_r^* \underline{y}_{rj} + \sum_{r\notin Bo} \mu_r^* y_{rj})$$

$$/ (\sum_{r\in DBI} v_i^{1*} \overline{x}_{ij} + \sum_{k\in ND-Bi} v_k^{2*} (\overline{z}_{kj} - \underline{z}_{ko})$$

$$+ \sum_{i\notin DBI} v_i^{1*} x_{ij} + \sum_{k\notin ND-Bi} v_k^{2*} (z_{kj} - z_{ko}))$$

$$< (\sum_{r\in Bo} \mu_r^* \underline{y}_{rj} + \sum_{r\notin Bo} \mu_r^* y_{rj})$$

$$/ (\sum_{r\in DBI} v_i^{1*} x + \sum_{k\in ND-Bi} v_k^{2*} (\overline{z}_{kj} - \underline{z}_{ko})$$

$$+ \sum_{i\notin DBI} v_i^{1*} x_{ij} + \sum_{k\notin ND-Bi} v_k^{2*} (z_{kj} - z_{ko})) \tag{16}$$

So,

$$(\mu_r^*, \forall r, v_i^{1*}, i \in DBI, v_k^{2*}, \, k \in ND - BI,$$

$$v_i^{1*}, i \notin DBI, v_k^{2*}, k \notin ND - BI,$$

$$x_{ij} = \bar{x}_{ij},\ i \in DBI, z_{kj} = \bar{z}_{kj},$$

$$k \in ND - Bi,\ y_{rj} = \underline{y}_{rj},\ r \in BO) \tag{17}$$

is a feasible solution to (14). Therefore, the optimal value to (14) can be achieved at $x_{ij} = \bar{x}_{ij}$, and $y_{rj} = \underline{y}_{rj}$ for DMU$_j$ $(j \neq o)$.

By contradiction suppose that for DMU$_o$ the optimality is achieved at $\underline{x}_{io} < x^*_{io} \leq \bar{x}_{io}, i \in DBI$, $\underline{y}_{ro} \leq y^*_{ro} < \bar{y}_{ro}, r \in BO$ and $\underline{z}_{ko} < z^*_{ko} \leq \bar{z}_{ko}$, $k \in ND - BI$ with:

$$\frac{\sum_{r \in BO} \mu^*_r y^*_{ro} + \sum_{r \notin BO} \mu^*_r y_{ro}}{\sum_{i \in DBI} v^{1*}_i x^*_{io} + \sum_{i \notin DBI} v^{1*}_i x_{io}} < \frac{\sum_{r \in BO} \mu^*_r \bar{y}_{ro} + \sum_{r \notin BO} \mu^*_r y_{ro}}{\sum_{i \in DBI} v^{1*}_i \underline{x}_{io} + \sum_{i \notin DBI} v^{1*}_i x_{io}} \tag{18}$$

which is a contradiction. □

Theorem 2 asserts that for obtaining efficiency score of DMU$_o$ we can have a set of exact data via setting at $x_{io} = \underline{x}_{io}, i \in I_i$, $z_{ko} = \underline{z}_{ko}, k \in I_2$ and $y_{ro} = \bar{y}_{ro}$ for DMU$_o$ and $x_{ij} = \bar{x}_{ij}$, $z_{kj} = \bar{z}_{kj}, k \in I_2$ and $y_{rj} = \underline{y}_{rj}, r \in BO$ for DMU$_j$ $(j \neq o)$, while the Model (13) maintains the efficiency score rating for DMU$_o$. In this case the Model (13) is converted to the following linear programming model:

$$\varphi^*_o = \text{Max} \sum_{r \in BO} \mu_r \bar{y}_{ro} + \sum_{r \notin BO} \mu_r y_{ro}$$

Subject to:

$$\sum_{r \in BO} \mu_r \underline{y}_{rj} + \sum_{r \notin BO} \mu_r y_{rj} \tag{19}$$

$$- \sum_{i \in DBI} \omega^1_i \bar{x}_{ij} - \sum_{k \in ND-BI} \omega^2_k (\bar{z}_{kj} - \underline{z}_{ko}) - \sum_{i \notin DBI} \omega^1_i x_{ij}$$

$$- \sum_{k \notin ND-BI} \omega^2_k (z_{kj} - z_{ko}) \leq 0, \qquad j = 1,\dots,n, j \neq 0$$

$$\sum_{r \in BO} \mu_r \bar{y}_{ro} + \sum_{r \notin BO} \mu_r y_{ro} - \sum_{i \in DBI} \omega^1_i \underline{x}_{io} - \sum_{i \notin DBI} \omega^1_i \underline{x}_{io} \leq 0$$

$$\sum_{i \in DBI} \omega^1_i \underline{x}_{io} + \sum_{i \notin DBI} \omega^2_i x_{io} = 1$$

$$\omega^1_i, \omega^2_k, \mu_r, \forall_i, k, r.$$

The dual of the Model (19), which is used for evaluating of DMUs, is as follows:

Min θ_0

Subject to:

$$\sum_{j=1, j\neq 0}^{n} \lambda_j \bar{x}_{ij} + \lambda_o \underline{x}_{io} \leq \theta_o \underline{x}_{io},\ i \in DBI$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o x_{io},\ i \in DBI \tag{20}$$

$$\sum_{j=1, j\neq 0}^{n} \lambda_j \underline{y}_{rj} + \lambda_o \underline{y}_{ro} \geq \bar{y}_{ro},\ r \in BO$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro},\ r \notin BO$$

$$\sum_{j=1, j\neq 0}^{n} \lambda_j (\bar{z}_{kj} - \underline{z}_{ko}) \leq 0,\ k \in ND - BI$$

$$\sum_{j=1, j\neq 0}^{n} \lambda_j (z_{kj} - z_{ko}) \leq 0,\ k \notin ND - BI$$

$$\lambda_j \geq 0,\ j = 1,\dots,n,\ j \neq 0.$$

Theorem 3. The Problem (20) is feasible and bounded.

Proof. It can be seen that $(\theta_o = 1, \lambda_o = 1, \lambda_j = 0,$ $j = 1,\cdots,n, j \neq o)$ is a feasible solution of the Model (20) and $\theta^*_o \leq 1$. Therefore, the Model (20) is feasible and bounded.□

Zhu (2003) proposed a procedure to convert the weak and strong ordinal data, into a set of exact data. In this paper, we apply Zhu's method, Zhu (2003), for converting the discretionary and non-discretionary weak and strong ordinal data, into a set of exact data.

4. Numerical example

The researchers provide a numerical example via Table 1. This Table portrays 14 DMUs that produce 4 outputs using 4 inputs. Table 1 reports the data x_1, x_2, x_3 and x_4 as the four inputs and y_1, y_2, y_3 and y_4 as the four outputs. Note that x_4 and y_4 are ordinal inputs and outputs, respectively. Suppose that the relations between x_4 and y_4 are weak ordinal and factor of x_4 is outside of the control of the DMU and is non-discretionary imprecise input and have 4 ordinal scales, i.e. "4" for the worst and "1" for the best. y_4 have 14 ordinal ranks. The current paper reports y_4 differently with "1" for the worst and "14" for the best. Since small input values and larger output values are preferred in DEA. y_4 imprecise and x_4 non-discretionary imprecise and they hold in (5) and (9), respectively. Therefore, we can convert y_4 and x_4 into bounded data by Zhu's method.

Then by Theorem 1 and Model (20), the envelopment form in input oriented, we obtain the efficiency of DMUs. To illustrate the proposed method we consider z_4. From column 10 of Table 1 we have:

$$z_{10,4}(=1) \le z_{9,4} \le z_{8,4} \le z_{3,4} \le z_{2,4} \le z_{5,4} \le \cdots \le z_{11,4} \le z_{1,4}(=14)$$

Now, for the input z_4 in weak ordinal relations, we set up the following intervals, Zhu (2003):

$$z_{kj} \in [0,1], \quad (j=10)$$

$$z_{kj} \in [1,14], \; (j=1,2,...,9,11,...,14) \qquad (21)$$

Suppose DMU_{10} is under evaluation by Model (20). Based upon Theorem 1, we know that optimal value of Model (20), θ^*_{10}, remains the same and (21) is satisfied if $\underline{z}_{10,4}=0$ (lower bound) for DMU_{10} and $\bar{z}_{4j}=14, (j \ne 10)$ (upper bound for other DMUs). A similar illustration holds about y_4. The column 10 of Table 1 shows the efficiency scores under the assumption of weak ordinal relations when they evaluated by Model (20). For $o \in \Lambda = \{2,5,6,10,12,14\}$ using Model (20) DMU_o is efficient and other DMUs are inefficient. Finally, the researchers obtained a model, Model (20), which is the generalized form of envelopment form of CCR model in input oriented. The proposed method can be generalized to DMUs with variable retunes to scale.

5. Conclusion

In some situations, some inputs and outputs of DMUs are non-discretionary and imprecise. In this paper, the researchers introduced these data and proposed a method, by CCR model, for efficiency evaluation of DMUs with non-discretionary imprecise data. The CCR model with non-discretionary imprecise data is a non-linear problem. By the second Theorem, the researchers convert resulted non-linear problem to a linear problem. Therefore, the presented procedure uses the linear programming problem for efficiency evaluation, which is always feasible and bounded.

Table 1: Data for the 14-DMUs and efficiency scores with weak ordinal relations.

DMU	x_1	x_2	x_3	$x_4=z_4$	y_1	y_2	y_3	y_4	Efficiency
1	217	4.11	131	14	11.39	4.38	29.41	2	0.8226
2	441	7.71	214	5	25.59	33.01	61.2	3	1
3	204	3.64	163	4	9.57	3.56	32.27	4	0.7894
4	216	3.24	154	9	11.46	9.02	32.81	2	0.8573
5	242	5.12	270	6	24.57	20.72	65.06	1	1
6	234	2.52	126	11	8.55	7.27	31.55	2	1
7	204	4.24	174	8	11.15	2.95	32.47	3	0.7336
8	356	7.95	299	3	22.25	14.9	66.04	2	0.8527
9	292	4.52	236	2	14.77	16.35	49.97	3	0.8725
10	141	5.21	63	1	9.76	16.26	21.48	2	1
11	220	6.09	179	13	17.25	22.09	47.94	2	0.9853
12	239	7.03	158	12	16.67	34.04	47.1	2	1
13	261	3.94	163	10	14.11	19.97	37.47	3	0.9904
14	170	2.1	90	7	6.8	12.64	20.7	3	1

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