Journal of Industrial Engineering International *Islamic Azad University, South Tehran Branch*
July 2007, Vol. 3, No. 5, 19-26

Fuzzy model for risk analysis

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Abstract

The goal of this paper is to show how the concept of fuzzy logic can be used to establish a degree to which an investment project belongs to a class of risk. Also, the probability of the fuzzy event is presented and is applied to calculate the probability of the fuzzy event "the project X is a good investment". This process has to enable the decision maker to compare several alternative investments from the fuzzy logic perspective and, in this way, allows him to include the uncertainty that comes with the problem in reality. Moreover, by experiments with the proposed fuzzy model the user can obtain new knowledge for investment risk analysis.

Keywords: Fuzzy numbers; Fuzzy logic; Fuzzy probability; Risk analysis

1. Introduction

The investment selection process in practical application may sometimes appear to be disorderly and imprecise. Mathematical modelling plays an important role in developing our understanding of a problem and aids in the decision making process.

Rather than focusing primarily on automation of decision processes, the goal of mathematical modelling should be the humanization of decision processes. Humans can perform a wide variety of physical and mental tasks without any measurements and any conscious computations.

In Zadeh's view [5], this capability is essentially a matter of the brain's crucial ability to manipulate perceptions – perceptions of distance, size, weight, colour, speed, time, direction, force, number, truth, likelihood and other characteristics of physical and mental objects. He proposes computing with words as a foundation for a computational theory of perceptions. Developing such a theory can lead to great advances in the ability for understanding how humans make perception-based rational decisions in an environment of imprecision, uncertainty and partial truth.

The traditional approach used to model risky choice making situations is to describe choices involving risk in term of their underlying probability distributions and associated utilities [4].

However, more often the firm has little or no past experience to draw upon, particularly when investments in new products or processes are being considered, and therefore relevant historical data cannot found which can serve as a guide for decisions.

In general, uncertain feature of risk is relative to both randomness and fuzziness. In the process of risk evaluation, random is due to a large amount of unknown factors existing, and fuzziness is concerned with the terms of incomplete and imprecise knowledge about the possible results of different action courses. Intuitively, risk exists when loss is possible and its financial impact is significant. This linguistic definition captures a property of risk that eludes definition in terms of mathematical formulas. In general, risks are evaluated qualitatively rather than analysed quantitatively. In fact, as Jablonowski [2] stated, in the real world, the possibility and financial significance of loss cannot be defined with precision.

In this paper, it is assumed that decision-making in capital investment takes place in an uncertainty environment, that is to say, those situations in which a probability of occurrence is not known or cannot be assigned to the events being studied. In this sense, the fuzzy logic way of reasoning will be incorporated to Net Present Value (NPV) model for assessing investment projects.

The NPV of an opportunity is the sum, taking account of plus and minus sign, of each of the annual cash flows discounted according to how far into the future each of one will occur, i.e.

$$\mathrm{NPV} = \sum_{h=0}^{t} C_h / (1+a)^h \, , \qquad (1)$$

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where C_h is the annual mean of the net cash flow after h years, t is the life of the opportunity in years, a is the discount rate.

2. Fuzzy logic approach to risk definition

The concept of fuzzy logic was conceived by Zadeh [7] and presented as a way of processing data by allowing partial set membership rather the crisp set membership or non-membership. Fuzzy set theory uses linguistic variables to represent imprecise concepts. Below, some preliminary knowledge about the fuzzy set is presented.

Let X be a classical set of objects, called the universe. A fuzzy set $\tilde{A}$ in X is characterised by a membership function $\mu_{\tilde{A}}(x)$ that associates each element $x \in X$ with a real number in the interval [0,1].

A linguistic term $\tilde{B}$ on set R, denoted $(\underline{B}, B^m, \overline{B})$, is defined to be a fuzzy triangular number if its membership function $\mu_{\tilde{B}}(x): R \to [0,1]$ is:

$$\mu_{\tilde{B}}(x) = \begin{cases} \dfrac{x-\underline{B}}{B^m-\underline{B}}, \text{ for } x \in [\underline{B}, B^m] \\ \dfrac{x-\overline{B}}{B^m-\overline{B}}, \text{ for } x \in [B^m, \overline{B}] \\ 0, \quad \text{otherwise,} \end{cases} \qquad (2)$$

where $\underline{B} \le B^m \le \overline{B}$ and B^m is the modal value, $\underline{B}$ and $\overline{B}$ stand for the lower and upper values of the support of linguistic term $\tilde{B}$ respectively.

Fuzzy logic can be applied in the analysis of the criterion of net present value. For this, the annual net cash flow and discount rate will be estimated by fuzzy numbers of triangular form:

$$\tilde{C}_h = (\underline{C}_h, C_h^m, \overline{C}_h), \qquad (3)$$

where $\underline{C}_h$, C_h^m, $\overline{C}_h$ are the lower, modal and upper values of the annual cash flow C_h, and the fuzzy discount rate:

$$\tilde{a} = (\underline{a},\ a^m,\ \overline{a}). \qquad (4)$$

The uncertain net present value, that is $N\tilde{P}V = (\underline{NPV},\ NPV^m,\ \overline{NPV})$, is obtained as follows:

$$\underline{NPV} = \sum_{h=0}^{t} \underline{C}_h / (1+\overline{a})^h$$
$$NPV^m = \sum_{h=0}^{t} C_h^m / (1+a^m)^h. \qquad (5)$$
$$\overline{NPV} = \sum_{h=0}^{t} \overline{C}_h / (1+\underline{a})^h$$

Using formula (2), the membership function $\mu_{N\tilde{P}V}(x)$ can be obtained. As the risk exists when NPV is negative, the risk value R can be defined by the rate between the area given by the negative values of NPV and the total area described by the all values of NPV:

$$R = \frac{\int_{\underline{NPV}}^{0} \mu_{N\tilde{P}V}(x)dx}{\int_{\underline{NPV}}^{\overline{NPV}} \mu_{N\tilde{P}V}(x)dx}. \qquad (6)$$

Small variation in the risk value may not be significant when the time come to choose from among several alternative investments.

Ferrer *et. al.* [1] showed that, from the perspective of investment risk, it may be more appropriate to establish the possibility distribution of such risk by using a linguistic scale from Table 1. Then, the decision maker can consider that the investment is "good" up to a degree of acceptance (1-R).

This is more in line with human reasoning when data is uncertain, and is generally enough to help an investor to make a decision.

3. Risk of an investment project with uncertain life

In the previous section, the expression for the risk of an investment project has been determined, with the assumption that the annual net cash flow and discount rate are uncertain, but a unique value is given for the life t of the project. However, for a number of reasons, the life of an investment objective is uncertain, such that it can be represented by a fuzzy number with discrete support:

$$\tilde{t} = \{(t_1, \mu_{\tilde{t}}(t_1)), (t_2, \mu_{\tilde{t}}(t_2)), \ldots, (t_n, \mu_{\tilde{t}}(t_n))\}, \qquad (7)$$

where $t_1, t_2, \ldots, t_n$ are the possible values of the life of the project.

Table 1. Classes of risk and linguistic equivalences.

Risk value R	**R = 0**	**0<R≤0.2**	**0.2<R≤0.4**	**0.4<R<0.6**	**0.6≤R<0.8**	**0.8≤R<1**	**R=1**
Class of risk ω	1	2	3	4	5	6	7
Linguistic equivalence L^{ω}	Null risk (N)	Very small risk (VS)	Small risk (S)	Intermediate risk (I)	High risk (H)	Very high risk (VH)	Total risk (T)
Degree of acceptance $\mu_{\tilde{A}}(A^{\omega})$	1	0.9	0.7	0.5	0.3	0.1	0

In this case, the net present value will be fuzzy subset of order 2:

$$\tilde{\tilde{NPV}} = \{(N\tilde{P}V_1, \mu_{\tilde{t}}(t_1)), (N\tilde{P}V_2, \mu_{\tilde{t}}(t_2)) ,\ldots, (N\tilde{P}V_n, \mu_{\tilde{t}}(t_n))\}, \quad (8)$$

where every $N\tilde{P}V_k$, for k =1, …, n, is fuzzy net present value calculated for $t = t_k$.

For every possible value t_k of the life of the project can be applied the formula (6) to calculate risk value:

$$R_k = \frac{\int_{\underline{NPV_k}}^{0} \mu_{N\tilde{P}V_k}(x)dx}{\int_{\underline{NPV_k}}^{\overline{NPV_k}} \mu_{N\tilde{P}V_k}(x)dx}, \quad k = 1,\ldots,n. \quad (9)$$

The investment risk can be interpreted as a fuzzy subset:

$$\tilde{R} = \{(R_1, \mu_{\tilde{R}}(R_1)), (R_2, \mu_{\tilde{R}}(R_2)) ,\ldots, (R_n, \mu_{\tilde{R}}(R_n))\}, \quad (10)$$

with $\mu_{\tilde{R}}(R_k) = \mu_{\tilde{t}}(t_k)$, for $k = 1,\ldots,n.$

For each value R_k, the decision maker establishes the linguistic equivalence $L_k{}^{\omega}$ and the pairs of the form $(L_k{}^{\omega}, \mu_{\tilde{R}}(R_k{}^{\omega}))$, for $k = 1,\ldots,n$, for risk class $\omega \in \{1,2,3,4,5,6,7\}$.

Let's consider an investment project X for which was obtained the following fuzzy risk:

$$\begin{aligned}\tilde{R} = \{&(N, \mu_{\tilde{R}}(R_k{}^1), \text{ for } k \in \{1,2,\ldots,n\}),\\ &(VS, \mu_{\tilde{R}}(R_k{}^2), \text{ for } k \in \{1,2,\ldots,n\}),\\ &(S, \mu_{\tilde{R}}(R_k{}^3), \text{ for } k \in \{1,2,\ldots,n\}),\\ &(I, \mu_{\tilde{R}}(R_k{}^4), \text{ for } k \in \{1,2,\ldots,n\}),\\ &(H, \mu_{\tilde{R}}(R_k{}^5), \text{ for } k \in \{1,2,\ldots,n\}),\\ &(VH, \mu_{\tilde{R}}(R_k{}^6), \text{ for } k \in \{1,2,\ldots,n\}),\\ &(T, \mu_{\tilde{R}}(R_k{}^7), \text{ for } k \in \{1,2,\ldots,n\})\}. \end{aligned} \quad (11)$$

The fuzzy event $\tilde{X}$ = "the X project is a good investment" can be defined by the risk values according to the degree of acceptance from Table 1:

$$\begin{aligned}\tilde{X} = \{&(N, \mu_{\tilde{A}}(A^1)), (VS, \mu_{\tilde{A}}(A^2)), (S, \mu_{\tilde{A}}(A^3)),\\ &(I, \mu_{\tilde{A}}(A^4)), (H, \mu_{\tilde{A}}(A^5)), (VH, \mu_{\tilde{A}}(A^6)),\\ &(T, \mu_{\tilde{A}}(A^7))\} = \{(N,1), (VS,0.9), (S,0.7),\\ &(I,0.5), (H,0.3), (VH,0.1), (T,0)\}. \end{aligned} \quad (12)$$

4. Probability of the fuzzy event "the X project is a good investment"

According to Zadeh [6], the probability of a fuzzy event $\tilde{X}$ has the value:

$$P(\tilde{X}) = \sum_{\omega=1}^{m} p_{\omega} \mu_{\tilde{A}}(A^{\omega}), \tag{13}$$

where,

$$p_{\omega} = \frac{\mu_{\tilde{R}}(R_k{}^{\omega})}{\sum_{\omega=1}^{m} (\mu_{\tilde{R}}(R_k{}^{\omega}))}.$$

If the investment project X has the risk null then $P(\tilde{X}) = 1$ and in the case that risk is total, $P(\tilde{X}) = 0$. Thus, this probability can be interpreted as a measure of acceptability for a particular project, because it provides a value which expresses the level in which the project is viable.

5. Numerical results

Let's consider a knowledge management project X, for which the annual net cash flows and discount rate were estimated by fuzzy numbers. In Table 2, the net present values $\underline{NPV}$, NPV^m, $\overline{NPV}$ are calculated with (5), for the discount rate expressed as a fuzzy number $\tilde{a} = (0.15, 0.18, 0.2)$, such that $\underline{a} = 0.15$, $a^m = 0.18$ and $\overline{a} = 0.2$.

For the investment objective with a certain life of 6 years, in Table 3, are given the membership degrees determined with (2). In order to apply (9), we have to calculate two areas: for the negative values of $N\tilde{P}V_6$ and the total area (see Figure 1).

The negative area is equal to:

(0-(-1023.47))*0.45/2 =231.23.

The total area is equal to:

(4137.93-(-1023.47))*1/2= 2580.70.

It follows that the risk R_6 of the project with 6 years of life is equal to 0.09 ∈ (0,0.2]. This is a very small risk (Table 1). If the life of 6 years of the investment objective X is certainly, the X project is a good investment with 0.9 degree of acceptance.

5.1. Uncertain life

Let's consider that the life of the project X, is an uncertain magnitude that can be estimated through the following fuzzy number of discrete form:

$$\tilde{t} = \{(2,0.2),(3,0.4),(4,1),(5,0.8),(6,0.6)\}. \tag{14}$$

From Table 2, it results that for an investment project with the life of 2 years or of 3 years, all values of NPV are negative such that the negative area is equal to the total area. The risk of the project is equal to 1, with the linguistic equivalence of total risk. Therefore, the degree of acceptance of 2 or 3 years investment project is zero.

In Table 4, the net present values $N\tilde{P}V_4$ are presented for the investment project with the life of 4 years.

It results that the negative area is of 1616.90, total area is of 1826.85 (see Figure 2). The project risk is equal to 0.89 ∈ (0.8, 1]. The project with the life of 4 years has a very high risk and the degree of acceptance of 0.1.

Table 5 presents the net present values $N\tilde{P}V_5$ for the investment project with the life of 5 years.

It results that the negative area is of 1014.08179, total area is of 2218.39199 (see Figure 3). The risk associated of this project is equal to 0.45712471 ≈ 0.46 ∈ (0.4, 0.6). The project has an intermediate risk and the degree of acceptance of 0.3.

Fuzzy set which describes the risk of the investment project with uncertain life is:

$$\tilde{R} = \{(1,0.2),(1,0.4),(0.89,1),(0.46,0.8),(0.09,0.6)\}. \tag{15}$$

In this fuzzy set, there are two different membership degrees for the value 1 of the project risk. Therefore, the membership degree for the risk with the value 1 will be equal to max from {0.2, 0.4} = 0.4.

Using the linguistic equivalences from Table 1, we obtain:

$$\text{Fuzzy risk} = \{(\text{Very small},0.6),(\text{Intermediate},0.8),(\text{Very high},1),(\text{Total},0.4)\}, \tag{16}$$

$$\text{Fuzzy acceptance} = \{(0.9,0.6),(0.5,0.8),(0.1,1),(0,0.4)\}. \tag{17}$$

With (13), the probability of a fuzzy event $\tilde{X}$ = "X is a good investment" is:

$$P(\tilde{X}) = \frac{0.9*0.6+0.5*0.8+0.1*1+0*0.4}{0.6+0.8+1+0.4} = 0.37143. \tag{18}$$

It results that is more false than true that the project is a good investment.

Table 2. Determination of the uncertain net present values (in monetary units).

	Values						
Year (h)	0	1	2	3	4	5	6
$\underline{C}_h$	-11000	3000	3000	3000	3000	3000	3000
C_h^m	-11000	3500	3500	3500	3500	3500	3500
$\overline{C}_h$	-11000	4000	4000	4000	4000	4000	4000
$(1+\underline{a})^h$	1	1.15	1.3225	1.52088	1.74901	2.01136	2.31306
$(1+a^m)^h$	1	1.18	1.3924	1.64303	1.93878	2.28776	2.69955
$(1+\overline{a})^h$	1	1.2	1.44	1.728	2.0736	2.48832	2.98598
$\underline{C}_h/(1+\overline{a})^h$	-11000	2500	2083.33	1736.11	1446.76	1205.63	1004.69
$C_h^m/(1+a^m)^h$	-11000	2966.1	2513.65	2130.21	1805.26	1529.88	1296.51
$\overline{C}_h/(1+\underline{a})^h$	-11000	3478.26	3024.57	2630.06	2287.01	1988.71	1729.31
		$N\tilde{P}V_1$	$N\tilde{P}V_2$	$N\tilde{P}V_3$	$N\tilde{P}V_4$	$N\tilde{P}V_5$	$N\tilde{P}V_6$
$\underline{NPV}$		-8500.00	-6416.67	-4680.56	-3233.80	-2028.16	-1023.47
NPV^m		-8033.90	-5520.25	-3390.04	-1584.78	-54.90	1241.61
$\overline{NPV}$		-7521.74	-4497.16	-1867.10	419.91	2408.62	4137.93

Table 3. The values of membership degrees associated to the net present values $N\tilde{P}V_6$.

x	Membership degree $\mu_{N\tilde{P}V_6}(x)$
$\underline{NPV}_6$ = -1023.47	0.00
$NPV_6\ null$ = 0.00	$\mu_{N\tilde{P}V_6}(x) = (0 - \underline{NPV}_6)/(NPV_6{}^m - \underline{NPV}_6)$ = 0.45
$NPV_6{}^m$ = 1241.61	1.00
$\overline{NPV}_6$ = 4137.93	0.00

Table 4. The values of membership degrees associated to the net present values $\tilde{NPV}_4$.

x	Membership degree $\mu_{\tilde{NPV}_4}(x)$
$\underline{NPV}_4$ = -3233.796	0.00
NPV_4^m = -1584.78	1.00
$NPV_4\ null$ = 0.00	0.21
$\overline{NPV}_4$ = 419.91	0.00

Table 5. The values of membership degrees associated to the net present values $\tilde{NPV}_5$.

x	Membership degree $\mu_{\tilde{NPV}_4}(x)$
$\underline{NPV}_5$ = -2028.16	0.00
NPV_5^m = -54.9	1.00
$NPV_5\ null$ = 0.00	0.97771
$\overline{NPV}_5$ = 2408.62	0.00

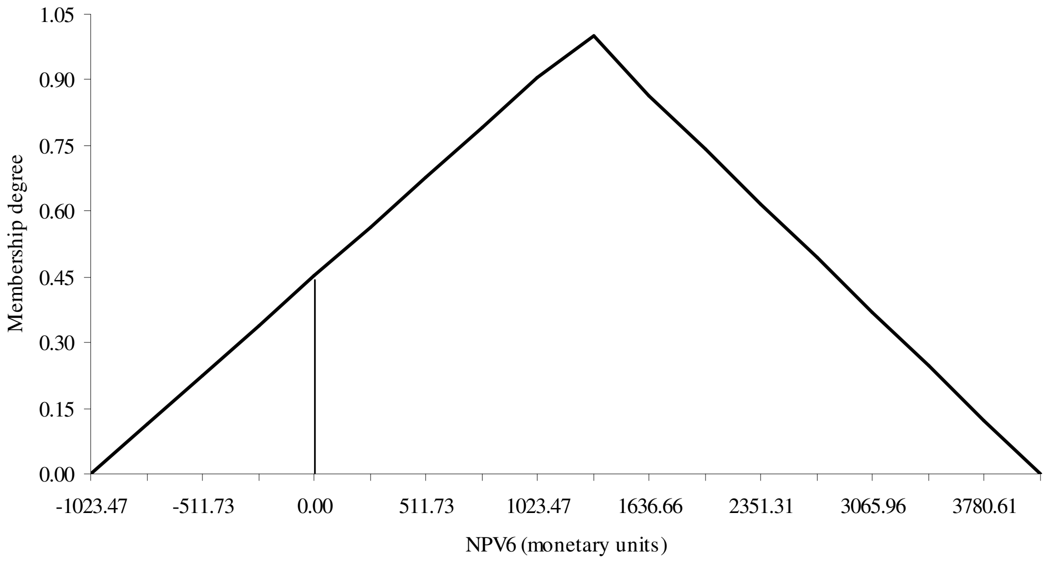

Figure 1. The negative area (on the left) and total area for the values of $\tilde{NPV}_6$.

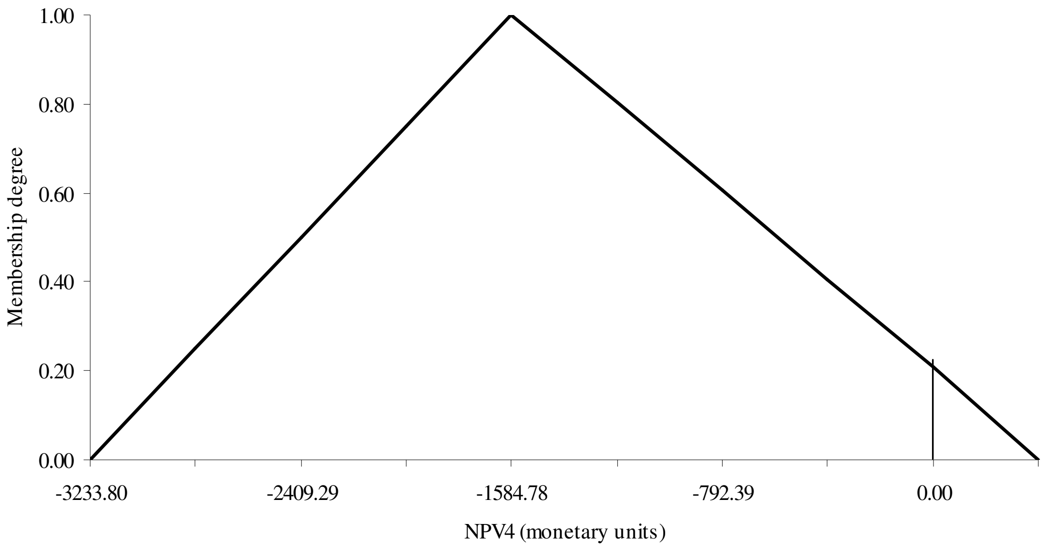

Figure 2. The negative area (on the left) and total area for the values of $N\tilde{P}V_4$.

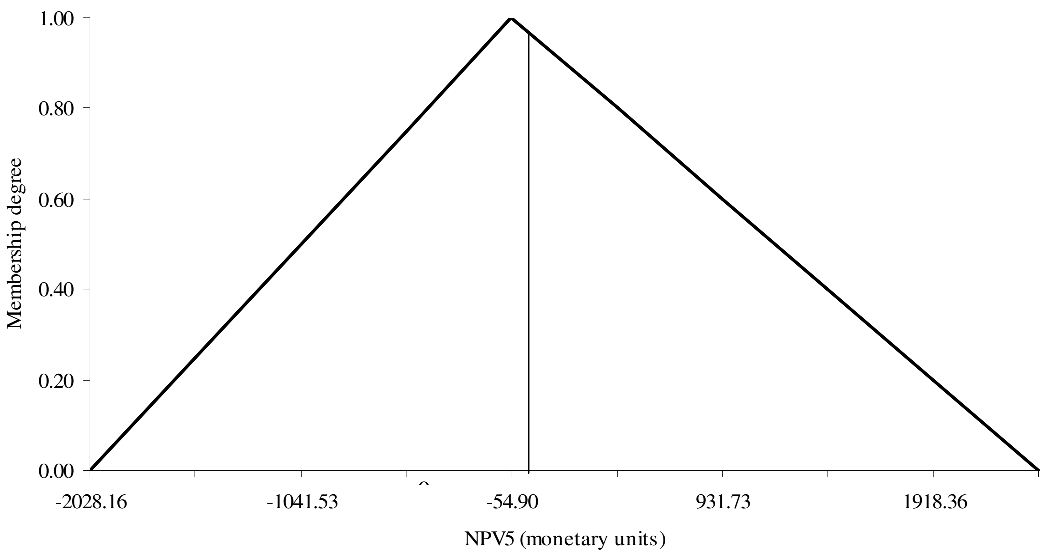

Figure 3. The negative area (on the left) and total area for the values of $N\tilde{P}V_5$.

6. Conclusion

In the paper, the concept of fuzzy probability event is used to establish a degree to which an investment project has an acceptable risk, from the point of view of uncertainty minimization. By this means, the investment risk R can be interpreted as a fuzzy set. This approach can be considered to be more in line with human reasoning when data are uncertain and it can help an investor to make a decision.

The results presented in this paper may be used to measure knowledge management project success. In [3], it is shown that since knowledge management is a fuzzy area and serious anecdotal evidence is chiefly used to measure knowledge management project success, fuzzy logic concepts could be applied to generate a fuzzified set of metrics.

Acknowledgements

This paper was supported from the research program, AA1/GR107/8.05.2006, type A, Theme 12, CNCSIS Code 1133 of the Academy of Economic Studies from Bucharest, Romania

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