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Portfolio selection through imprecise Goal Programming model: Integration of the manager's preferences

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Abstract

In the portfolio selection problem, the manager considers several objectives simultaneously such as the rate of return, the liquidity and the risk of portfolios. These objectives are conflicting and incommensurable. Moreover, the objectives can be imprecise. Generally, the portfolio manager seeks the best combination of the stocks that meets his investment objectives. The imprecise Goal Programming model will be utilized to build the most satisfactory portfolio. The concept of satisfaction functions will be utilized to integrate explicitly the preferences of the portfolio's manager. The developed model has been applied to portfolio selection within the Tunisian stock exchange market.

Keywords: Portfolio selection; Imprecise goal programming; Satisfaction function; Manager's preferences

1. Introduction

Markowitz [8] presents a bi-criterion portfolio selection model where the manager seeks to maximize the expected portfolio return and to minimize financial risk. In other words, we seek the portfolio that permits to increase investors' profits while minimizing the risk of financial losses. It is evident that these two criteria are conflicting and they cannot be optimized simultaneously. Thus, the manager has to make some compromises in order to find the most satisfactory portfolio. The literature review reveals that in practice these two criteria are the most popular. Elton and Gruber [6] present the various models of portfolio selection; stochastic dominance, multi-attribute utility models, discriminant analysis, heuristics, neurons networks, optimization models and multi-criteria analysis. Among these models, we find the Goal Programming model (GP). Lee and Chesser [7], Colson and Bruyn [5], Ballestero and Romero [3] and Arenas *et al*. [1,2] illustrate well the GP applications in the portfolio selection problem where they consider several objectives. However, these models do not explicitly take into account the preferences, the experience and the intuition of the portfolio manager.

The aim of this paper is to apply the imprecise GP model, where the goals associated to the different objectives are expressed through intervals. The proposed model integrates explicitly the preferences' structure of the portfolio manager by utilizing the satisfaction functions developed by Martel and Aouni [10]. The manager's preferences are revealed through a progressive and an evolutionary process. This process seeks to build the most satisfactory portfolio that meets the investor's aspiration levels. The considered criteria in our model are as follows: the return, the risk and the liquidity of portfolios. In order to deal with the imprecision related to the model parameters, we suggest to expressing the goals as intervals. The proposed model was applied to Tunisian stock exchange market.

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2. The imprecise GP model with the decision-maker's preferences

In their model, Martel and Aouni (1998) include explicitly the decision-maker's preferences while considering imprecise goals. The goal values are within a target interval $\left[g_i^l, g_i^u\right]$ where g_i^l and g_i^u represent respectively the lower and upper bounds of the goal g_i associated with the objective *i*. Indeed, the goals can be any unspecified point within the interval $\left[g_i^l, g_i^u\right]$, that $\zeta_i \in \left[g_i^l, g_i^u\right]$ where ζ_i is the goal for objective *i*.

For each objective *i*, we indicate respectively by, α_{id}^{+} and α_{id}^{-}, the indifference thresholds associated with the positive and negative deviations. These thresholds are given in the following expressions: $\alpha_{id}^{+} \geq g_i^u - \zeta_i$ and $\alpha_{id}^{-} \geq \zeta_i - g_i^l$. If the deviations are inside the intervals $\left[0, \alpha_{id}^{+}\right]$ or $\left[0, \alpha_{id}^{-}\right]$, the manager will be entirely satisfied.

These intervals indicate the indifference ranges. Within the indifference range the manager's satisfaction function is at its maximum level of 1. Outside these intervals, the satisfaction functions are monotonically decreasing in different forms. Moreover, each option or solution with a deviation larger than the veto threshold α_{iv} would be rejected by the portfolio's manager. The general form of the satisfaction functions is shown in Figure 1.

It should be noted that Martel and Aouni [9] deal with the imprecision related to the goals in a different manner compared to the imprecise and Fuzzy GP formulations. The main difference is regarding the way to handle the imprecise value of the goals.

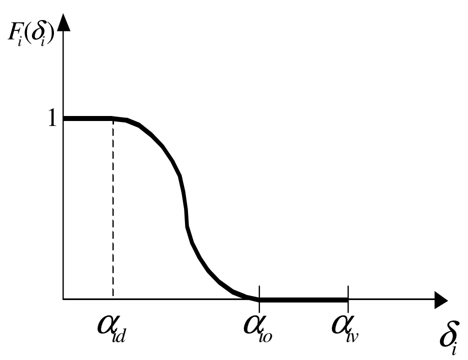

Figure 1. General form of the satisfaction functions.

According to Martel and Aouni [9], since the decision-maker (DM) does not have accurate and precise information regarding the goal value that are expressed through an interval, hence any solution leading to an achievement level within the target interval $\left[g_i^l, g_i^u\right]$ will provide a total satisfaction to the DM.

It is also possible that the indifference range can be larger than or equal to the width of target interval $\left[g_i^l, g_i^u\right]$ (with $\alpha_{id}^{+} \geq g_i^u - \zeta_i$ and $\alpha_{id}^{-} \geq \zeta_i - g_i^l$). The mathematical reformulation of the imprecise GP model with the satisfaction functions, as proposed by Martel and Aouni [9], is as follows:

$$\text{Maximize} \quad Z = \sum_{i=1}^{p} \left(W_i^{+} F_i^{+}(\delta_i^{+}) + W_i^{-} F_i^{-}(\delta_i^{-}) \right)$$

Subject to :

$$\sum_{j=1}^{n} a_{ij} x_j - \delta_i^{+} + \delta_i^{-} = \zeta_i \quad (\text{for } i = 1, 2, \ldots, p),$$

$$x \in X,$$

$$\delta_i^{+} \text{ and } \delta_i^{-} \leq \alpha_{iv},$$

$$\zeta_i \in \left[g_i^l, g_i^u\right],$$

$$\delta_i^{+} \text{ and } \delta_i^{-} \geq 0 \qquad (\text{for } i = 1,2,\ldots,p),$$

$$x_j \geq 0 \qquad (\text{for } j = 1,2,\ldots,n).$$

where

- g_i The goal associated to the objective *i*,
- x An *n*-dimensional vector of decision variables that is $x = (x_1, x_2, \ldots, x_n)$,
- a_{ij} The technological parameters related to the system of constraints,
- W_i^{+} The importance coefficient associated with the positive deviation,
- W_i^{-} The importance coefficient associated with The negative deviation,
- δ_i^{+} The positives deviation of the objective *i*,
- δ_i^{-} The negative deviation of the objective *i*,
- $F_i^{+}(\delta_i^{+})$ The DM's satisfaction function associated with the positive deviations,

$F_i^-(\delta_i^-)$ The DM's satisfaction function associated with the negative deviations.

The indifference thresholds α_{id} ($\alpha_{id}^-, \alpha_{id}^+$) are used to characterize the imprecision related to the goal values. The thresholds will be established in such way to accurately reflect the portfolio manager's preferences. The satisfaction functions shape and the thresholds can be reviewed at any time during the decision making process. The portfolio manager intuition, experience and judgment will be expressed explicitly through the satisfaction functions.

3. Portfolio selection through the imprecise GP

The model presented in the previous section will be applied to select a financial portfolio within the Tunisian Stock Exchange market. The stock exchange data utilized in this study are related to thirty four (34) Tunisian listed companies during the period of January 1999 to December 2002. The companies of this sample are included in the permanent quotation and they have been chosen on the basis of the availability of the financial data from the time of their introduction into the Stock Exchange (Table 1).

First, we have defined a set of objectives related to the stocks that will be considered by the investor. Next, we will determine the target interval associated with each portfolio objective, then we will apply the proposed model as well as the satisfaction functions and we will finally present and discuss the results.

The portfolio selection process involves a set of objectives that are often conflicting. For example, Markowitz [8] considered two objectives: the return and the risk of portfolios. Lee and Chesser [7], Zopounidis and Doumpos [11] and Ben Abdelaziz *et al.* [4] suggest a set of objectives that the portfolio manager can consider to evaluate the stocks. In this study, we retained the following objectives:

a) The first objective is the rate of return that is calculated as $R_j = (P_{j,t} - P_{j,t-1} + D_{j,t}) / P_{j,t-1}$. This objective measures the profitability of each stock. Indeed, the manager invests with an aim of gaining higher future profits. It can also be considered as capital gain, dividend and financial growth. Here $P_{j,t}$ is the price of stock j at time t and $D_{j,t}$ is the dividend received during the period $[t-1;\ t]$. The rate of return R_j (j = 1, 2,…, 34) is to be maximized.

b) The second objective is the risk coefficient β (where $\beta_j = COV(R_j, R_m)/VAR(R_m)$. Here R_j is the rate of return of stock j ; j =1, 2, …, 34, and R_m is the market rate of return. This objective measures the correlation of stock's return with the market return. Lower correlation with the market indicates the stock performance on its own rather than by the movements of the market. In order to select a portfolio as risky as the market, we propose, as Lee and Chesser [7] did to set a goal of 1 for this objective. Moreover, this coefficient permits the diversification of portfolio. This objective is to be minimized.

c) The third objective is the exchange flow ratio that is calculated as L_j = treated capitals / stock exchange capitalization. This objective measures the security liquidity degree. The higher the ratio the more liquid the stock is. This objective is to be maximized.

The investor is neither able to establish precisely the exact goal values associated with the above considered objectives, nor can precisely estimate the technological coefficients a_{ij} in the constraints related to the portfolio return. For modeling this imprecision, we will express the fuzzy parameters through intervals. Indeed, the goals are defined by intervals which have a lower (g_i^l) and an upper (g_i^u) limits. In the same way, the technological coefficients within the constraint of the portfolio return are within two limits lower a_{ij}^l (lower) and a_{ij}^u (upper). The intervals related to the portfolio return, risk, and liquidity are summarized in Table 2.

Besides the objective and goals constraints, we will consider a set of additional constraints (system constraints) as follows:

- The fixing of an upper limit of investment in each stock in order to diversify the portfolio; $x_j \le 0.1$, for $j = 1,\dots,n$, where the x_j is the proportion to be invested in stock j,
- The sum of the proportions invested in stocks is equal to 1; $\sum_{j=1}^{n} x_j = 1$;
- In order to diversify the selected portfolios, we propose to invest less than 40% in each of finan-

cial, industrial and service sectors: $\sum_{s=1}^{S_e} x_s \leq 0.4$ (for $e = 1, 2, 3$), where the x_s denotes the proportion of investment in finance ($e = 1$), industry ($e = 2$) and service ($e = 3$).

These constraints were used to determine the optimal values (see Table 2) of each objective *i*, (for *i*=1, 2 and 3) by solving the following model:

$$\text{Optimize } Z = \sum_{j=1}^{n} c_j x_j$$

Subject to:

$$\sum_{j=1}^{n} x_j = 1,$$

$$0 \leq x_j \leq 0.1 \text{ (for } j = 1, 2, ..., n),$$

$$\sum_{s=1}^{S_e} x_s \leq 0.4 \text{ (for } e = 1, 2, 3),$$

where x_j, for $j = 1, 2, ..., 34$, the proportion to be invested in the stock *j*.

In this study, we have considered two distinct situations where the values of the technological parameters in constraints related to the return of portfolio and of the goal values are fixed as follows: a) to the central values of the respective intervals (situation 1) ; and b) to the upper bounds of the intervals (situation 2).

The portfolio manager's objective is to establish (or select) a profitable, safe and liquid portfolio. This portfolio must maximize the manager's satisfaction degree. In this study we have utilized following satisfaction functions:

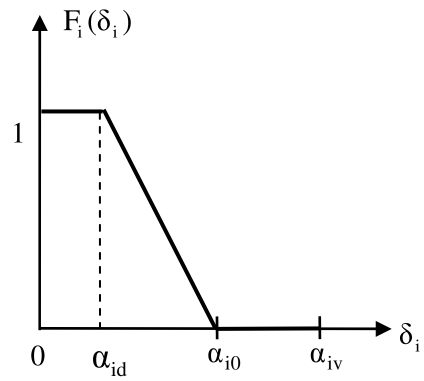

Figure 2. Form of the retained satisfaction function.

These satisfaction functions reach their maximum when the objective achievement levels are within the target interval of the goals. Moreover, they require the determination of a set of thresholds as indicated in Figure 2.

The indifference thresholds α_{id}, the null satisfaction threshold α_{i0}, the veto threshold α_{iv}, the value of the goals g_i (for *i* = 1, 2 and 3) of this study are provided in Table 3.

Based on these data (Tables 1, 2 and 3), we will formulate the mathematical program that provides the best and the most satisfactory portfolio for the manager, as follows:

$$\text{Maximize } \mathrm{Z} = \sum_{i=1}^{3} \left(\mathrm{W}_i^+ \mathrm{F}_i^+ (\delta_i^+) + \mathrm{W}_i^- \mathrm{F}_i^- (\delta_i^-) \right)$$

Subject to:

$$\sum_{j=1}^{34} \tilde{R}_j x_j - \delta_1^+ + \delta_1^- = \zeta_1,$$

$$\sum_{j=1}^{34} \beta_j x_j - \delta_2^+ + \delta_2^- = \zeta_2,$$

$$\sum_{j=1}^{34} L_j x_j - \delta_3^+ + \delta_3^- = \zeta_3,$$

$$\sum_{j=1}^{34} x_j = 1,$$

$$x_j \leq 0.1 \text{ (for } j = 1, 2, ..., 34),$$

$$\sum_{s=1}^{S_e} x_s \leq 0.4 \text{ (for } e = 1, 2, 3),$$

$$\tilde{R}_j \in \left[R_j^l, R_j^u \right] \text{ (for } j = 1, 2, ..., 34),$$

$$\zeta_i \in \left[g_i^l, g_i^u \right] \text{ (for } i = 1, 2, 3),$$

$$\delta_i^+ \text{ and } \delta_i^- \geq 0 \text{ (for } i = 1,2,3),$$

$$x_j \geq 0 \text{ (for } j = 1,2,...,34).$$

The solutions for the two situations computed by the Lingo package was used to solve this program and the following results were obtained (Tables 4 and 5).

Table 1. Stocks' financial data.

Stocks	Minimal rate of return (R_j^l)	Maximal rate of return (R_j^u)	Central rate of return (R_j^c)	Risk (β_j)	Liquidity (L_j)
Amen Bank	-0.09	0.16	0.03	0.74	0.04
ATB	-0.07	0.09	0.01	0.56	0.03
BH	-0.10	0.15	0.03	0.99	0.09
BIAT	-0.09	0.16	0.03	0.85	0.14
BNA	-0.08	0.14	0.03	0.72	0.07
BS	-0.09	0.13	0.02	0.59	0.09
BT	-0.03	0.09	0.03	0.17	0.11
STB	-0.06	0.11	0.03	0.76	0.07
UBCI	-0.10	0.16	0.03	1.12	0.04
UIB	-0.08	0.12	0.02	0.75	0.07
BTEI	-0.06	0.08	0.01	0.19	0.30
CARTE	-0.17	0.15	-0.01	-0.17	0.11
STAR	-0.13	0.11	-0.01	0.40	0.03
TL	-0.10	0.12	0.01	0.75	0.42
Tuninvest	-0.17	0.20	0.01	0.45	0.23
ASTREE	-0.08	0.09	0.01	-0.07	0.01
CIL	-0.11	0.15	0.02	0.93	0.30
Plac de Tunisie	-0.18	0.11	-0.04	0.20	0.04
SPDIT	-0.08	0.17	0.04	1.36	0.13
ATL	-0.11	0.12	0.01	1.23	0.30
Amen Lease	-0.15	0.15	0.00	0.75	0.07
SOTETEL	-0.18	0.23	0.02	2.44	0.56
ALKIMIA	-0.11	0.11	0.00	-0.10	0.03
AMS	-0.15	0.16	0.00	-0.17	0.10
ICF	-0.12	0.07	-0.02	-0.01	0.02
SFBT	-0.11	0.19	0.04	2.20	0.20
Air liquide	-0.07	0.16	0.04	0.32	0.01
STIL	-0.24	0.32	0.04	0.64	0.01
TUNISIE LAIT	-0.33	0.24	-0.04	0.00	0.00
EL MAZRAA	-0.14	0.10	-0.02	0.02	0.09
MONOPRIX	-0.08	0.13	0.03	0.25	0.17
PBHT ADP	-0.17	0.16	0.00	0.31	0.14
TUNISAIR	-0.16	0.21	0.03	1.48	0.14
SIMPAR	-0.11	0.13	0.01	0.15	0.04

Table 2. Intervals related to the goals.

Objectives	Minimal value	Maximal value	Central value	Optimal value
Risk (β_P)	0.60	1.4	1.0	1
Rate of return (R_P)	0.01	0.11	0.06	0.033
Liquidity (L_P)	0.10	0.33	0.22	0.22

Table 3. Thresholds and parameters related to satisfaction functions.

	Rate of return		Portfolio risk		Portfolio liquidity	
	Situation 1	**Situation 2**	**Situation 1**	**Situation 2**	**Situation 1**	**Situation 2**
- Goal's Value	0.06	0.11	1.0	1.4	0.22	0.33
Positive Deviations						
- Indifference	0.007	0.01	0.1	0.4	0.06	0.09
- Null satisfaction	0.017	0.02	0.3	0.6	0.12	0.13
- Veto threshold	0.030	0.03	0.5	0.8	0.15	0.20
Negative deviations						
- Indifference	0.007	0.01	0.1	0.4	0.06	0.09
- Null satisfaction	0.170	0.02	0.3	0.6	0.12	0.13
- Veto threshold	0.030	0.03	0.5	0.8	0.15	0.20

Table 4. Composition of the selected portfolios.

	Portfolio 1	Portfolio 2
Amen Bank	0.000	0.000
ATB	0.000	0.000
BH	0.000	0.000
BIAT	0.100	0.000
BNA	0.000	0.000
BS	0.000	0.000
BT	0.055	0.000
STB	0.000	0.000
UBCI	0.000	0.000
UIB	0.000	0.000
BTEI	0.000	0.000
CARTE	0.000	0.000
STAR	0.000	0.000
TL	0.000	0.100
Tuninvest	0.000	0.000
ASTREE	0.000	0.000
CIL	0.045	0.100
Plac de Tunisie	0.000	0.000
SPDIT	0.100	0.000
ATL	0.000	0.100
Amen Lease	0.000	0.000
SOTETEL	0.100	0.100
ALKIMIA	0.000	0.000
AMS	0.000	0.069
ICF	0.000	0.000
SFBT	0.100	0.100
Air liquide	0.100	0.017
STIL	0.100	0.014
TUNISIE LAIT	0.000	0.000
EL MAZRAA	0.000	0.100
MONOPRIX	0.100	0.100
PBHT ADP	0.000	0.100
TUNISAIR	0.100	0.100
SIMPAR	0.100	0.000

Table 5. Objectives attained by the portfolios.

Objectives	Portfolio 1	Portfolio 2
Return rate (R_p)	0.031	0.160
Risk (β_p)	1.021	0.965
Liquidity (L_p)	0.160	0.240
Satisfaction level	2.000	2.800

In table 4, we find the proportions (x_j) to be invested in each type of stock for both portfolios. The Table 5 presents the objectives achieved by the two portfolios. On the basis of these results, we notice that portfolio 2 is more profitable, less risky, have more liquidity and has a higher satisfaction level comparatively to portfolio 1. We also notice that portfolio 2 provides a satisfaction degree higher than portfolio 1. This is due to the fact that the imprecise parameters in the model leading to portfolio 1 are fixed to central values of the intervals and those of the model resulting in portfolio 2 are fixed at the upper bounds of the target intervals. Based on the satisfaction degree, it is recommended to the portfolio manager to adopt portfolio 2. In both portfolios, the three criteria (return rate, risk and liquidity) where considered imprecise and expressed through an interval.

The composition of the two portfolios has been obtained by incorporating explicitly the manager's preferences. We would like to highlight the fact that the satisfaction functions thresholds play double roles: a) to express the manager's preferences regarding the deviation between the achievement and the aspirations levels of each objective, and b) to characterize the imprecision related to goals. Moreover, the satisfaction functions allow the portfolio's manager to express, reveal and incorporate his/her experience, judgment and intuition to select the best and satisfactory portfolio. This approach is different from the existing models used in the portfolio selection literature.

4. Conclusion

The aim of this paper was to develop a model for portfolio selection problem within a decision-making environment characterized by imperfection of the information. The proposed model seeks to integrate explicitly the portfolio manager's intuition, experience and judgment. The proposed formulation is based on the imprecise GP model and the concept of satisfaction functions. This model has been utilized for selecting portfolios within a set of thirty four companies registered in the Tunisian Stock Exchange market. However, this model can be applied for cases with large size portfolio selection problems.

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