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ORIGINAL RESEARCH

Residuals based process capability indices for two-stage processes

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Abstract The manufacturing operations often involve multistage processes where the process capability of each stage is affected by the process capability of its precedent processes. This property is known as the cascade property. The purpose of this paper is to estimate the process capability of the second stage of two-stage process while the cascade property impact is removed using residuals analysis. To this end, a method is proposed to determine the specification limits of the residuals based on the specification limits of the quality characteristics in the first and second stages. The C_p, C_{pk} and S_{pk} indices are used to calculate the capability of the second stage in the two-stage process. The results of simulation study show the satisfactory performance of the proposed method in estimating the pure process capability of the second stage.

Keywords Process capability index · Residuals · Two-stage process · Cascade property

Introduction

Process capability indices have been used in industries to measure the performance of the processes. A process is called capable if the product meets customer expectations. There are copious studies on process capability estimations. Kotz and Johnson (2002) reviewed the studies on process capability indices during the years 1992–2000. Yum and Kim (2010) provided a bibliography of approximately 530 journal papers and books published during the years 2000–2009. The process capability indices C_p, C_{pk} and C_{pm} are the most commonly used indices. The process capability index C_p was proposed by Juran (1974). This process capability index cannot evaluate the capability of the process properly when the process mean and target value are not equal. Kane (1986) introduced C_{pk} index to solve this problem. Hsiang and Taguchi (1985) proposed C_{pm} index. The C_{pm} process capability index considers the deviation of the process mean from its target value as well as the process variance. This index shows the ability of the process to manufacture products around the target value. Pearn et al. (1992) proposed the process capability index C_{pkm} that have a same link between C_p and C_{pk} with C_{pm}. Boyles (1994) proposed S_{pk} index based on the process yield of the process. Sharma and Rao (2013) employed define, measure, analysis, improvement and control (DMAIC) approach to reduce the variation of the engine connecting rod machining process and as a result to improve the capability of the process. Also, Sharma and Rao (2014) used DMAIC approach to improve the capability of the engine crankshaft manufacturing process.

The process capability indices are proposed under the assumption that each stage is independent from the other stages. While in the multistage processes, the process in each stage is affected by the processes in the previous stages. Therefore, the capability of the process in each stage is dependent on the capability of the processes in the previous stages and the process capability indices calculate the overall capability of the process in this stage. Most of the researches on multistage processes are related to control chart and multistage process capability analyses are not studied much. Zhang (1980) was the first who designed a

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control charts for multistage processes. The cause-selecting control charts are developed by Zhang (1984) for multistage processes. Zhang (1985a, b, 1989, 1990, 1992) developed the cause-selecting control chart. Zhang (1990) proposed two kinds of process capability indices in each stage of multistage processes including total and specific process capability indices. Total process capability index calculates the process capability in each stage when the quality characteristic in the current stage is affected by the quality characteristics of precedent processes. Specific process capability index calculates the capability of the process in the current stage when the effects of precedent processes are omitted. The total and specific process capability indices are calculated by Eq. (1).

$$C_{\mathrm{pt}} = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma_t}, \quad C_{\mathrm{ps}} = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma_s}, \tag{1}$$

where USL and LSL are the upper and lower specification limits, respectively, σ_t is the total standard deviation of the process in each stage and σ_s is the specific standard deviation of process that is obtained from cause-selecting control chart. Wade and Woodall (1993) proposed using prediction limits to improve the statistical performance of cause-selecting control charts. Yang (1997) proposed a new approach to compute the cost model for a two-stage process. Then, he designed economic X chart and cause-selecting control chart to monitor a two-stage process. Yang and Yang (2006) proposed an approach to monitor two-stage process when data are autocorrelated. Yang and Yeh (2011) proposed a cause-selecting control chart for the two-stage process with attribute data. Ghahyazi et al. (2014) examined the effects of cascade property in multistage process in monitoring simple linear profile in Phase II. Davoodi et al. (2015) monitored multistage Poisson count processes using exponentially weighted moving average (EWMA) and C control charts and proposed a changed point estimator to find the real time of a change in the process parameters. Pirhooshyaran and Niaki (2015) proposed a double-max multivariate exponentially weighted moving average (MEWMA) control chart to monitor the parameters of the multistage processes when data are multivariate and autocorrelated. Goodarzi et al. (2016) developed two cause-selecting control charts to monitor the censored lognormal reliability data in a three-stage process. Linn et al. (2002) proposed a method to determine the priority of the process variation reduction in multistage processes to improve the overall capability index. Chen et al. (2012) proposed a process capability index *CPMPCI* for the complex product machining process (multistage process) based on Taguchi loss function.

In the multistage processes, the process capability indices are calculated when the process is in control. The conditions of the processes are determined using case-selecting control charts. As mentioned former, in a two-stage process, the quality characteristic in the first stage affects the quality characteristic in the second stage referred to as the cascade property. Therefore, using the traditional process capability index to measure the capability of the second stage may provide misleading results due to ignoring the cascade effect. In other words, if one is going to measure the capability of the second stage specifically, she cannot use the traditional process capability indices on the quality characteristic in the second stage and she must use a statistic independent from the quality characteristic in the first stage. This is the main point which is not considered in any studies so far.

In this paper, we propose a method to eliminate the effects of the first stage on the quality characteristic of the second stage and calculate the specific capability of the second stage in a two-stage process. In our method, the process capability indices are calculated for the residuals in the second stage and these indices represents the specific capability of that stage instead of the overall capability. The residuals in the second stage are independent from the quality characteristic in the first stage; therefore, the process capability indices for the residuals calculate the specific capability of the second stage. In addition, a method is proposed to estimate the specification limits of the residuals based on the specification limits of the quality characteristics in the first and second stages. The performance of the proposed method is evaluated for two-stage processes with different mean and standard deviation values of the quality characteristic in the first and second stages. The structure of the paper is as follows: The proposed method is outlined in the following section. A simulation study to evaluate the performance of the proposed method is presented in the next section which is followed by a section on a real case study and finally conclusion is presented in the last section.

Proposed method

In the multistage processes, the process capability indices in each stage are affected by the capability of the processes in the previous stages. Therefore, in some cases the incapability of the process is the effect of incapability of its preceding processes. Hence, we attempt to estimate the process capability of each process independent of the capability of its previous processes. In our method, the specific process capability indices for each stage are calculated by measuring the process capability indices for the residuals. In the two-stage process, assumed that the parts in stage 1 feed into stage 2 and the quality characteristic in stage 1 is x that follows a normal distribution $x \sim N(\mu_x, \sigma_x^2)$. The quality characteristic of the second stage is defined as:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \tag{2}$$

where ε_i follows normal distribution $\varepsilon_i \sim N(0,\sigma_\varepsilon^2)$ and the parameters β_0 and β_1 are estimated based on the analysis on historical data using Eqs. (3) and (4)

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}, \tag{3}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} y_i (x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}, \tag{4}$$

where $\bar{x}$, $\bar{y}$ are the mean values of the quality characteristics in the first and second stages, respectively.

The mean and variance of the quality characteristic in the second stage can be expressed as:

$$\mu_y = \hat{\beta}_0 + \hat{\beta}_1 \mu_x. \tag{5}$$

$$\sigma_y^2 = \hat{\beta}_1^2 \sigma_x^2 + \sigma_\varepsilon^2. \tag{6}$$

The Eqs. (5) and (6) show that the mean and variance of the quality characteristic in the second stage are affected by the mean and variance of the quality characteristics in the first stage. Therefore, the process capability indices for the quality characteristic in the second stage are affected by the quality characteristic in the first stage and calculate the overall capability of that stage. In the multistage processes, the residuals have been used to omit the effects of the previous stages. The residuals are not affected by the previous processes. Therefore, the process capability indices for the residuals calculate the specific process capability of the process in the second stage. The residuals are obtained by:

$$e_i = y_i - \hat{y}_i, \tag{7}$$

where $\hat{y}_i$ is the prediction value for the quality characteristic y_i and can be obtained as follows:

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i. \tag{8}$$

The specification limits are the allowable range of the quality characteristic that meet the customers' expectations. The lower and upper specification limits are LSL and USL, respectively, and determined by design engineers or customers. The specification limits for the quality characteristics of the stages 1 and 2 are [LSL_x USL_x] and [LSL_y USL_y], respectively. To estimate the process capability of the second stage based on the residuals, we require the specification limits for the residuals. We used the specification limits of the quality characteristics in the first and second stages to obtain the specification limits for residuals. In our method, the specification limits for the residuals are determined using the process yield. The process yield for the quality characteristic x can be obtained by:

$$P_x = \mathrm{pr}(\mathrm{LSL}_x \le x \le \mathrm{USL}_x) = \Phi\left(\frac{\mathrm{USL}_x - \mu_x}{\sigma_x}\right) - \Phi\left(\frac{\mathrm{LSL}_x - \mu_x}{\sigma_x}\right), \tag{9}$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution.

Similarly, the process yields of the quality characteristic y can be computed. The process yield of the quality characteristic y determines the process yield of the second stage when it is affected by the quality characteristic in the first stage. Since the residual and the quality characteristic x are independent, the process yield of the quality characteristic y can be calculated by Eq. (10) in the case that the specification limits of the residuals are known.

$$P_y = P_x P_e - \left[1 - \mathrm{pr}(\mathrm{LSL}_y \le \beta_0 + \beta_1 x + \varepsilon \le \mathrm{USL}_y | \mathrm{LSL}_x \le x \le \mathrm{USL}_x, \mathrm{LSL}_e \le \varepsilon \le \mathrm{USL}_e)\right], \tag{10}$$

where P_x and P_e are the process yields of the quality characteristic x and the residuals, respectively. The process yield of the residuals is calculated by Eq. (9).

In this paper, the specification limits of the residuals are determined under the following assumptions:

1. The mean values of the quality characteristics in the first and second stages are equal to target values.
2. The term $\mathrm{pr}(\mathrm{LSL}_y \le \beta_0 + \beta_1 x + \varepsilon \le \mathrm{USL}_y | \mathrm{LSL}_x \le x \le \mathrm{USL}_x, \mathrm{LSL}_e \le \varepsilon \le \mathrm{USL}_e)$ is equal to 1 when the process yields of the quality characteristic in the first stage and the residual are equal to 0.9973. Therefore, P_y can be obtained by:

$$P_y = P_x P_e. \tag{11}$$

The second assumption represents that the quality characteristic in the second stage should be located between its specification limits if the residuals and the quality characteristic in the first stage lie between their corresponding specification limits. The proposed method for determining the specification limits of the residuals consists of two steps. The first step is calculating the variance of the residuals under the assumption that the mean of the residuals is equal to the target value. The second step is determining the specification limits of the residuals.

The process capability indices C_p, C_pk and S_pk for the residuals can be obtained using Eqs. (12), (13) and (14), respectively

$$C_{\mathrm{pe}} = \frac{\mathrm{USL}_e - \mathrm{LSL}_e}{6\sigma_e}, \tag{12}$$

$$C_{\mathrm{pke}} = \min\left\{\frac{\mathrm{USL}_e - \mu_e}{3\sigma_e}, \frac{\mu_e - \mathrm{LSL}_e}{3\sigma_e}\right\}, \tag{13}$$

$$S_{\mathrm{pke}} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{\mathrm{USL}_e - \mu_e}{\sigma_e}\right) + \frac{1}{2}\Phi\left(\frac{\mu_e - \mathrm{LSL}_e}{\sigma_e}\right)\right\}, \tag{14}$$

where, USL_e and LSL_e are the specification limits of the residuals, μ_e and σ_e are the mean and standard deviation of the residuals, respectively. The variance of the residuals is calculated by

$$\sigma_e^2 = \frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)}{n-2} = \frac{\sum_{i=1}^{n} e_i^2}{n-2}. \tag{15}$$

Similarly, the C_{p}, C_{pk} and S_{pk} indices can be calculated for the quality characteristics x and y.

Variance of the residuals

The standard deviation values of the quality characteristics x and y are determined under the assumption that the corresponding mean values are equal to target values and the process yields of the quality characteristic in the first stage and the residuals are 0.9973. Therefore, the process yield of the quality characteristic in the second stage is calculated by Eq. (11). The standard deviation values of the quality characteristics are obtained by Eqs. (16) and (17), respectively

$$\sigma_x = \frac{\mathrm{USL}_x - T_x}{\Phi^{-1}(0.99865)} = \frac{\mathrm{LSL}_x - T_x}{\Phi^{-1}(0.00135)}. \tag{16}$$

$$\sigma_y = \frac{\mathrm{USL}_y - T_y}{\Phi^{-1}(0.9973)} = \frac{\mathrm{LSL}_y - T_y}{\Phi^{-1}(0.0027)}. \tag{17}$$

The variance of the residuals based on the standard deviations of the quality characteristics x and y that are calculated by Eqs. (16) and (17), respectively, and obtained as follows:

$$\sigma_e^2 = \sqrt{\sigma_y^2 - \hat{\beta}_1^2\sigma_x^2} = \sqrt{\left(\frac{\mathrm{USL}_y - \mathrm{LSL}_y}{\Phi^{-1}(0.9973)}\right)^2 - \hat{\beta}_1^2\left(\frac{\mathrm{USL}_x - \mathrm{LSL}_x}{\Phi^{-1}(0.99865)}\right)^2}. \tag{18}$$

The specification limits for the residuals

Under the assumptions that the mean of the residuals is equal to the target value and the process yield of the residuals is 0.9973, the specification limits of the residuals are obtained as follows:

$$\mathrm{USL}_e = \sigma_e\Phi^{-1}(0.99865). \tag{19}$$

$$\mathrm{USL}_e = \sigma_e\Phi^{-1}(0.00135). \tag{20}$$

The target value for the residuals is zero.

Simulation studies

In this section, simulation studies are carried out to check the accuracy of the proposed method to determine the specification limits of the residuals and the process capability indices calculated based on them. We considered the two-stage process, the quality characteristic for the stage 1 is x and generated from a normal distribution as $x \sim N(\mu_x, \sigma_x)$ and the quality characteristic for the second stage is y that is generated from the following equation:

$$y_i = 7.86 + 0.5x_i + \varepsilon_i,$$

where ε_i follows a normal distribution as $N(\mu_e, \sigma_e)$.

The specification limits for the quality characteristics x and y are set to [6.56, 19.73] and [9.46, 19.32], respectively. We assume that the process yields for the residuals and the quality characteristic x are 0.9973. Then, the process yield of the quality characteristic y is calculated from Eq. (11). The standard deviation values of the quality characteristics x and y are calculated as

$$\sigma_x = \frac{19.73 - 13.145}{\Phi^{-1}(0.99865)} = \frac{19.73 - 13.145}{2.99998} = 2.195$$

$$\sigma_y = \frac{19.32 - 14.39}{\Phi^{-1}(0.9973)} = \frac{19.32 - 14.39}{2.78215} = 1.772$$

The specification limits of the residuals are obtained by Eqs. (19) and (20). The lower and upper specification limits of the residuals are obtained as −4.1736 and 4.1736, respectively.

We calculate the C_{p}, C_{pk}, S_{pk} and the process yields for the residuals and the quality characteristics x and y under different sample sizes, mean and variance values of the residuals and the quality characteristic x. For each simulated case, the true values of the process capability indices are calculated. The true values of the process capability indices are calculated based on the true values of the mean and variance of the quality characteristics in stage 1, stage 2 and the residuals as shown in Tables 1, 2, and 3. The process capability indices for each case are calculated for different sample sizes $n = 25, 50, 100, 200$. The mean values and variances of the quality characteristics in the first and second stages and the residuals are estimated from the random samples and then the process capability indices are calculated. These simulations were repeated 10,000 times and the process capability indices and process yields were calculated in each replicate. The mean and the true values of the C_{p}, C_{pk} and S_{pk} indices for different cases under different sample sizes are shown in Tables 1, 2 and 3, respectively.

Tables 1, 2 and 3 show that when the sample size increases the mean values of process capability indices approach to the corresponding true values. Tables 1, 2 and 3 show the result of using the proposed method for different simulation cases. In the first two-stage process, the processes in the first and second stages are capable. In this condition, the process capability indices for residuals is greater than 1, therefore, the process in the second stage is capable. In the second process, the variance of the quality characteristic in the first stage is large and the process in the first stage is not capable and it affects the capability of the process in the second stage and makes it incapable. While the process capability indices for the residuals are greater than 1. It shows that the incapability of the process in the second stage is the effects of the incapability of the process in the first stage. In the third case, the process in the first stage is capable but the process in the second stage is not capable. The process capability indices for the residuals show that the process in the second stage is not capable. In the fourth process, the deviation of the mean value of the process in the first stage from the target value is large, therefore, the C_{pk} index for the x and y quality characteristics show that the process in the first stage and the process in the second stage are not capable. Moreover, C_{pk} index for the residuals are greater than 1. The other simulation cases can be interpreted similarly. Table 4 shows the accuracy of the second assumption.

Table 4 shows that the process yield of the quality characteristic y, that is obtained using Eq. (11), accurately estimates the true value of the process yield of the quality characteristic y properly when the process in the first stage and the residuals are capable. When the sample size increases, the gap between the estimated value of the process yield of the quality characteristic y decreases for all cases.

A real case

In this section, the proposed method is used to calculate the specific capability of the process in the second stage of the automobile brake system measurement process. This case

Table 1 C_{p} index for the residuals and the quality characteristics x and y under different sample sizes and simulated case

Simulated case				n	True value of C_{px}	Mean of C_{px}	True value of C_{px}	Mean of C_{pe}	True value of C_{px}	Mean of C_{py}
μ_x	σ_x^2	μ_e	σ_e^2							
13.6	4	0	1.21	25	1.0975	1.1345	1.2647	1.2499	1.1054	1.1440
				50		1.1155		1.2564		1.1221
				100		1.1063		1.2621		1.1152
				200		1.1019		1.2632		1.1100
13.6	6.76	0	1.21	25	0.8442	0.8721	1.2647	1.2492	0.9650	0.9953
				50		0.8568		1.2594		0.9787
				100		0.8503		1.2623		0.9724
				200		0.8466		1.2632		0.9684
13.6	4	0	2.25	25	1.0975	1.1340	0.9272	0.9192	0.9116	0.9428
				50		1.1140		0.9240		0.9261
				100		1.1062		0.9255		0.9190
				200		1.1018		0.9262		0.9149
14.42	4	0	1.21	25	1.0975	1.1342	1.2647	1.2513	1.1054	1.1404
				50		1.1141		1.2581		1.1213
				100		1.1065		1.2609		1.1137
				200		1.1026		1.2631		1.1107
14.05	4.84	0	1.21	25	0.9977	1.0306	1.2647	1.2526	1.0564	1.0919
				50		1.0126		1.2581		1.0722
				100		1.0058		1.2615		1.0652
				200		1.0014		1.2639		1.0603
13.6	4.84	0	1.96	25	0.9977	1.0319	0.9935	0.9819	0.9230	0.9521
				50		1.0134		0.9889		0.9366
				100		1.0060		0.9917		0.9299
				200		1.0012		0.9925		0.9271

Table 2 C_{pk} index for the residuals and the quality characteristics x and y under different sample sizes and simulated case

Simulated case				n	True value of C_{pkx}	Mean of C_{pkx}	True value of C_{pke}	Mean of C_{pke}	True value of C_{pky}	Mean of C_{pky}
μ_x	σ_x^2	μ_e	σ_e^2							
13.6	4	0	1.21	25	1.0217	1.0467	1.2647	1.1976	1.0449	1.0674
				50		1.0363		1.2193		1.0560
				100		1.0295		1.2358		1.0536
				200		1.0258		1.2446		1.0494
13.6	6.76	0	1.21	25	0.7859	0.7978	1.2647	1.1976	0.9121	0.9234
				50		0.7925		1.2222		0.9184
				100		0.7907		1.2360		0.9174
				200		0.7878		1.2445		0.9150
13.6	4	0	2.25	25	1.0217	1.0472	0.9272	0.8676	0.8616	0.8735
				50		1.0340		0.8872		0.8684
				100		1.0291		0.8992		0.8666
				200		1.0256		0.9262		0.8644
14.42	4	0	1.21	25	0.8850	0.9146	1.2647	1.1996	0.9530	0.9831
				50		0.8995		1.2211		0.9674
				100		0.8917		1.2344		0.9596
				200		0.8895		1.2441		0.9579
14.05	4.84	0	1.21	25	0.8606	0.8886	1.2647	1.2011	0.9503	0.9796
				50		0.8729		1.2204		0.9636
				100		0.8674		1.2349		0.9578
				200		0.8643		1.2454		0.9544
13.6	4.84	0	1.96	25	0.9288	0.9492	0.9935	0.9305	0.8724	0.9492
				50		0.9395		0.9517		0.8774
				100		0.9355		0.9654		0.8771
				200		0.9321		0.9738		0.8761

Table 3 S_{pk} index for the residuals and the quality characteristics x and y under different sample sizes and simulated case

Simulated case				n	True value of S_{pkx}	Mean of S_{pkx}	True value of S_{pke}	Mean of S_{pke}	True value of S_{pky}	Mean of S_{pky}
μ_x	σ_x^2	μ_e	σ_e^2							
13.6	4	0	1.21	25	1.0717	1.0922	1.2647	1.2305	1.0884	1.1093
				50		1.0819		1.2457		1.0962
				100		1.0766		1.2563		1.0940
				200		1.0742		1.2602		1.0909
13.6	6.76	0	1.21	25	0.8318	0.8447	1.2647	1.2302	0.9533	0.9666
				50		0.8367		1.2486		0.9585
				100		0.8343		1.2566		0.9565
				200		0.8322		1.2602		0.9546
13.6	4	0	2.25	25	1.0717	1.0926	0.9275	0.9043	0.9017	0.9168
				50		1.0799		0.9158		0.9081
				100		1.0763		0.9212		0.9049
				200		1.0740		0.9240		0.9029
14.42	4	0	1.21	25	0.9592	0.9860	1.2647	1.2322	1.0203	1.0453
				50		0.9723		1.2473		1.0322
				100		0.9653		1.2550		1.0258
				200		0.9633		1.2600		1.0246
14.05	4.84	0	1.21	25	0.9305	0.9529	1.2647	1.2825	1.0057	1.0330
				50		0.9409		1.2726		1.0214
				100		0.9351		1.2682		1.0161
				200		0.9332		1.2667		1.0134
13.6	4.84	0	1.96	25	0.9778	0.9954	0.9934	0.9663	0.9127	0.9252
				50		0.9853		0.9800		0.9177
				100		0.9819		0.9871		0.9155
				200		0.9793		0.9901		0.9147

Table 4 Process yield of the residuals and the quality characteristics x and y under different sample sizes and simulated case

Simulated case				n	$\bar{P}_x$	$\bar{P}_e$	$\bar{P}_x.\bar{P}_e$	Mean of P_y
μ_x	σ_x^2	μ_e	σ_e^2					
13.6	4	0	1.21	25	0.9974	0.9991	0.9965	0.9977
				50	0.9981	0.9996	0.9976	0.9983
				100	0.9984	0.9997	0.9981	0.9986
				200	0.9985	0.9998	0.9983	0.9988
13.6	6.76	0	1.21	25	0.9833	0.9991	0.9825	0.9932
				50	0.9851	0.9996	0.9844	0.9947
				100	0.9863	0.9997	0.9860	0.9951
				200	0.9868	0.9998	0.9866	0.9954
13.6	4	0	2.25	25	0.9974	0.9890	0.9864	0.9901
				50	0.9980	0.9920	0.9900	0.9914
				100	0.9984	0.9933	0.9917	0.9923
				200	0.9985	0.9940	0.9925	0.9927
14.42	4	0	1.21	25	0.9940	0.9992	0.9932	0.9963
				50	0.9950	0.9996	0.9946	0.9970
				100	0.9955	0.9997	0.9952	0.9974
				200	0.9958	0.9998	0.9956	0.9976
14.05	4.84	0	1.21	25	0.9924	0.9992	0.9916	0.9958
				50	0.9934	0.9996	0.9930	0.9967
				100	0.9941	0.9997	0.9939	0.9972
				200	0.9945	0.9998	0.9943	0.9974
13.6	4.84	0	1.96	25	0.9944	0.9932	0.9876	0.9907
				50	0.9955	0.9953	0.9908	0.9922
				100	0.9961	0.9963	0.9924	0.9930
				200	0.9964	0.9967	0.9931	0.9934

Table 5 The mean and standard deviation values for the residuals and the quality characteristics in the first and second stages

	Residuals	Quality characteristic ROLLWT	Quality characteristic BAKEWT
Mean	−0.0003	210.24	201.131
Standard deviation	1.685	2.618524	2.072036

Table 6 The process capability indices C_p, C_{pk} and S_{pk} for the residuals and the quality characteristics in the first and second stages

	Residuals	Quality characteristic ROLLWT	Quality characteristic BAKEWT
C_p	1.1217	1.0006	1.0006
C_{pk}	1.1216	1.0006	1.0005
S_{pk}	1.1168	1.0006	1.0006

was discussed by Constable et al. (1988) and Linn et al. (2002). In this case, the quality characteristics for the first and second stages are ROLLWT and BAKEWT, respectively. The sample size is 70 and the first 45 observations were used to estimate the parameters $\hat{\beta}_0$ and $\hat{\beta}_1$ using Eqs. (2) and (3), respectively. Based on the parameters $\hat{\beta}_0$ and $\hat{\beta}_1$, the relationship between the quality characteristics in the first and second stage is defined as:

$$\hat{y} = 104.325 + 0.460456x,$$

where, x = RollWT and y = BAKEWT.

The specification limits for the quality characteristics in the first and second stages are [202.38, 218.10] and [194.91, 207.35], respectively. The mean and standard deviation values for the residuals and the quality characteristics in the first and second stages based on the last 25 observations are shown in Table 5.

We assumed that the process yields of the residuals and the quality characteristic x are 0.9973. Then, the process yield of the quality characteristic y is calculated by Eq. (11). The lower and upper specification limits of the residuals were obtained using Eqs. (19) and (20), respectively. The specification limits of the residuals are [−5.67, 5.67]. The process capability indices C_{p}, C_{pk} and S_{pk} for the residuals and the quality characteristics in the first and second stages are shown in Table 6.

Table 6 shows that the processes in the first stage and whole process are capable. The process capability indices for the residuals show the specific capability of the process in the second stage. It shows that the process in the first stage is less capable than the process in the second stage. This is an import result, as the manufacturer knows which process should be invested and improved.

Conclusion and future researches

In this paper, a method was proposed to calculate the specific capability indices for the process in the second stage of a two-stage process. The process in the second stage is affected by the process in the first stage; therefore, sometimes the incapability in the second stage is a result of the incapability of the first stage. We omitted the effect of the quality characteristic of the first stage on the quality characteristic of process in the second stage using the residuals. The process capability for the residuals shows the specific process capability of the process in the second stage. In addition, we proposed a method to determine the specification limits of the residuals based on the specification limits of the quality characteristics in the first and second stages. The simulation studies and the real case study showed that the process capability indices for the residuals calculate the specific process capability of the process in the second stage properly. This method can be extended to be used for the multistage processes with more than two stages. Future studies may also include calculating specific process capability indices for the multistage process when each stage has several correlated quality characteristics. The linear relation between quality characteristics is considered in this paper. Therefore, calculating specific process capability indices when the relations among quality characteristics are nonlinear can be considered as future research.

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