J Ind Eng Int (2016) 12:271–286
DOI 10.1007/s40092-016-0142-1

ORIGINAL RESEARCH

Cooperative vehicle routing problem: an opportunity for cost saving

Sedighe Zibaei[1] · Ashkan Hafezalkotob[1] · Seyed Sajad Ghashami[2]

Received: 28 May 2015 / Accepted: 19 January 2016 / Published online: 17 February 2016

Abstract In this paper, a novel methodology is proposed to solve a cooperative multi-depot vehicle routing problem. We establish a mathematical model for multi-owner VRP in which each owner (i.e. player) manages single or multiple depots. The basic idea consists of offering an option that owners cooperatively manage the VRP to save their costs. We present cooperative game theory techniques for cost saving allocations which are obtained from various coalitions of owners. The methodology is illustrated with a numerical example in which different coalitions of the players are evaluated along with the results of cooperation and cost saving allocation methods.

Keywords Multi depot vehicle routing problem · Cooperation · Coalition · Cooperative game theory · Cost saving allocation

✉ Ashkan Hafezalkotob
a_hafez@azad.ac.ir; hafezalkotob@iust.ac.ir

Sedighe Zibaei
hzibae@yahoo.com

Seyed Sajad Ghashami
Sajad_ghashami@yahoo.com

1 Industrial Engineering College, Islamic Azad University, South Tehran Branch, Entezari Alley, Oskoui Alley, Choobi Bridge, Tehran 11518-63411, Iran

2 Department of Manufacturing and Industrial Engineering, Faculty of Mechanical Engineering, University Teknologi Malaysia (UTM), 81310 Skudai, Malaysia

Introduction

Vehicle routing problem (VRP) is faced each day by thousands of companies and organizations which are engaged in delivery and collection of goods or people (Crevier et al. 2007). Basic components of the VRP are road networks, customers, depots and vehicles. The VRPs can be categorized, depending on the objectives and constraints occurring in many transportation logistics and distribution systems.

A basic version of the VRP is the capacitated VRP (CVRP) in which each vehicle has a known capacity and two other versions of CVRP are symmetric and asymmetric. Lots of new constraints are added on the route construction and made practical applications of the VRP (Toth and Vigo 2002). For example, Split-Delivery VRP (SDVRP) is a relaxation of the VRP in which the same customer can be served by different vehicles if it reduces the overall costs. It saves the total distance traveled as well as the number of required vehicles by allowing split deliveries (Jin et al. 2008). Hasani-Goodarzi and Tavakkoli-Moghaddam (2012) studied a split vehicle routing problem (SVRP) with capacity constraint for the multi-product cross-docks to determine the best vehicle routes and the optimal number of the utilized vehicles. By developing the CVRP, a VRP with Time Windows (VRPTW) can be obtained for which the service to each customer must start within a certain time window and the vehicle must remain at the customer's location throughout the service. Delivery of food or newspaper is a simple example for the VRPTW. Another extension of the CVRP is VRP with Backhauls (VRPB) in which the customers may demand or return some goods. The customers are divided into two groups: line haul and back haul customers. Each line haul customer requires a given quantity to be delivered while a given quantity of products must be picked up from back haul customers. The next VRP model is the VRP with Pickup and Delivery (VRPPD). The vehicles have the responsibility for delivering the goods to customers and picking the goods up at the customer locations. Another well-known generalization of the VRP is Multi-Depot VRP (MDVRP). In the MDVRP, there are several depots instead of just one depot and every customer is visited by a vehicle based at one of the several existing depots, while every vehicle route must start and end at the same depot. By presentation of the initial VRP and basic models introduced above, many researchers studied algorithms and models for different versions of the VRP.

With starting of economic depression and consequently adverse effects on global trade, the transportation markets particularly container sea transportation companies have been exposed to recession. The container sea transportation agents believe that these companies are able to integrate their resources to provide more services and get rid of this economic crisis (Sterzik et al. 2012). In this way, by expanding the VRP in transportation of containers (Vidović et al. 2011) the companies can cooperate with each other. One way for such a collaboration is when the companies are in the same port use the vehicles of each other; or the companies located in different ports use the capacity of vessels of each other. Therefore, the CoMDVRP can be used for this kind of problem that enables companies reduce the costs.

Logistic costs and especially transportation costs often constitute a large part of the operational costs in companies. One solution to reduce these costs is cooperation among the logistic companies. The cooperative game theory (CGT) can be adopted for modeling the cooperation among the companies. Cooperative games are concerned with distribution of the cooperation benefits when the players cooperate. Most applications of the CGT are in scheduling, cost saving, negotiation and bargaining (Barron 2013). A cooperative game actually considers that the players may choose to cooperate by forming some coalitions. In the coalitions, the players might be lucky to receive greater benefits than they could gain individually on their own. Then, the players should allocate the benefits among each other. Now the key question is how the total extra benefits (or cost saving) should be fairly distributed among them. We adopt some well-known CGT methods such as Shapley value, τ value, and least core to address this question.

The outline of this paper is as follows. The related literature is reviewed in "Literature review" section. "Prerequisites and assumptions of MDVRP" section defines the prerequisites and assumptions of the proposed model. The model formulation of the MDVRP is discussed in "Model formulation for MDVRP (non-cooperation situation)" section. Then, in "Formulation of vehicle routing problem for coalition of players" section the mathematical method of the CoMDVRP is introduced. Afterwards, cooperative techniques for cost saving allocation are described in "Cooperative techniques for cost saving allocation" section. A numerical example is also shown in "Numerical Example for Cost saving methods in cooperative VRP" section and finally, some concluding remarks are made in "Conclusion and further research" section.

Literature review

Multi-depot vehicle routing problem

A basic version of the vehicle routing problem that is defined under capacity and route length restrictions is called capacitated VRP (CVRP). In this problem, each vehicle has a capacity that is known, so loading the vehicle more than its capacity is not allowed.

Variant model of VRP obtained by generalization of the classical VRP (Cordeau et al. 2007). An extension of VRP is MDVRP that is studied in this paper. MDVRP deals with routing of several vehicles from different depots. Mathematical programming models of the MDVRP have been developed by several researches. Crevier et al. (2007) presented an extension of the MDVRP that vehicles may use the intermediate depots along their route to become full. Ray et al. (2014) worked out a new integer linear programming model, called multi-depot split-delivery vehicle routing problem (MDSDVRP) which allows establishing depot locations and routes for serving the customer demands within the same objective function. Aras et al. (2011) proposed two mixed-integer linear programming formulations for the selective MDVRP that are extensions of the classical MDVRP in which each visit to a broker is associated with a gross profit and a purchase price to be paid to take the cores back. Wasner and Zapfel (2004) investigated the necessity of developing a VRP with considering the number and locations of hubs and depots and their assigned service areas. Karakati and Podgorelec (2014) optimized an extension of the classical VRP by adding multiple depots. Kang et al. (2000) designed a least costly schedule for MDVRP to minimize transportation costs.

Ho et al. (2008) introduced the MDVRP as an NP-hard problem and developed two hybrid genetic algorithms (HGAs) for solving it. Contardo and Martinelli (2014) formulated the MDVRP using a vehicle-flow and a set-partitioning formulation presented a new exact method for the MDVRP under capacity and route length constraints. Ray et al. (2014) stated a multi-depot logistics delivery problem including the depot selection and shared commodity delivery. They mentioned that the MDSDVRP is suitable for multi-depot, multi-vehicle and split delivery problem. Liu et al. (2010) proposed a mathematical programming model of the multi-depot CVRP with the objective of minimizing movements of empty vehicles. They focused on full truckloads in transportation. Surekha et al. (2011) presented a formulation and solution for MDVRP using Genetic Algorithms. Salhi et al. (2014) proposed an MILP formulation for the MDVRP with heterogeneous vehicle fleet and designed a variable neighborhood search (VNS) algorithm for the problem. Prescott-Gagnon et al. (2014) developed different heuristics for MDVRP in an oil delivery industry. Montoya et al. (2014) presented a complete review on scientific literature of MDVRP. They comprehensively analyzed the single-objective and multi-objective MDVRPs and their solution algorithms. In this paper, we establish a mathematical programming model for the MDVRP when the depot owners cooperate with each other.

Cooperative game theory

The cooperative game theory is defined as "A theory which is concerned primarily with coalitions of groups of players who coordinate their actions to reach more benefits." (Branzei et al. 2008). Several researchers have adopted CGT for modeling cooperation of logistic companies. Lozano et al. (2013) introduced a mathematical programming model to measure the benefits of merging the transportation demands from different companies. The joint transportation costs of the companies are reduced because of using larger vehicles and increased number of the connected trips. They illustrated the model with an example in which different cooperative game solution concepts are used. Frisk et al. (2010) studied the collaboration between logistic companies in the forest industries. They investigated a number of sharing mechanisms including Shapley value, nucleolus, separable and non-separable costs, shadow prices and volume weights. Hafezalkotob and Makui (2015) introduced a stochastic mathematical programming model for a multiple-owner graph problem. They used methods based on the CGT to show that the collaboration among independent owners of a logistic network can maintain a reliable maximum flow. McCain (2008) focused on the cooperative games in collaborating organizations and analyzed how these games expand the organization and increase its profit.

The methods of CGT may be adopted for allocating the cost saving to cooperating companies. Using CGT, Charles and Hansen (2008) suggested a theoretical cost saving framework for global cost minimization and cost saving assignment in an enterprise network. They showed that the proposed cost allocations obtained via the activity based costing technique is rational and stable. Vanovermeire and Sorensen (2014) dealt with the cooperation among shippers and stated that the cooperation between shippers is a proper way to increase the performance. They pointed out that the cooperation reduces the costs of distribution and delivery but this reduction will depend on the flexibility of the companies for delivery of goods. They used the Nucleolus and the Shapley value methods for this purpose. Lehoux et al. (2009) have worked on a variety of cooperation techniques in logistic networks including the Shapley value; Nucleolus and shadow prices. Engevall et al. (1998) investigated cost allocation methods for a traveling salesman game according concept of traveling salesman problem.

The current paper is closely related to those of Wang and Kopfer (2015), and Lozano et al. (2013). Wang and Kopfer (2015) considered horizontal coalitions among freight forwarders to enhance operational efficiency. The proposed collaborative transportation planning enable forwarders to fulfill customers' needs with lower costs. However, they studied neither VRP nor CGT methods. Lozano et al. (2013) investigated the cost savings of different logistic companies that may be achieved when they merge their transportation requirements using the CGT. In a MDVRP, we consider that the depots are managed by a set of owners. The owners of each depot are regarded as players, who like to coordinate with other players (owners) to reduce their transportation costs. The minimum transportation cost is obtained in different coalitions by solving the CoMDVRP for the coalitions. We calculate the cost saving and synergy of each coalition and then we use methods of CGT for cost saving allocation. In summary, we investigate how the coordination among the players (owners of depots) in the MDVRP gives them this opportunity to minimize the total transportation cost and how they can fairly share the cost saving of cooperation among themselves using the CGT techniques.

Research gap

To the best of authors' knowledge, no research was found that considers the cooperative VRP among different players. Therefore, there are two main contributions in this study with regard to MDVRP. First, we develop a cooperative approach for MDVRP. We study how the cooperation among the multiple owners of depots gives them the opportunity to reduce the costs of transportation. The cost savings indicate effectiveness (synergy) of owners' cooperation. The cost saving of cooperation is quantified by a new mathematical programming model for coalitions of owners. Second, we propose several methods of CGT to address the problem of allocating coalition cost saving to the cooperating owners.

Prerequisites and assumptions of MDVRP

In a classical VRP, there is only one depot. However, MDVRP is a problem for specifying the routes in which a set of customers are served by several vehicles from multiple depots. As shown in Fig. 1, in a non-cooperative MDVRP, each depot should serve its own customers. However, in cooperative situation, depots can serve the customers of partners. The cooperation among the depots reduces the route length of the vehicles. Consequently the total transportation cost may decrease as a result of the cooperation. The problem of each coalition can be analyzed by the CoMDVRP.

Assumptions

In a MDVRP, the number and location of the depots are predetermined. The location of each customer is known and the demands are deterministic. The demand of each customer should be fulfilled. Each player owns one depot only with only one vehicle The vehicles are not necessarily identical and each vehicle starts and finishes at the same depot, while each route begins and ends at the same depot. The capacity restrictions for the vehicles are considered. The total demand on each route is smaller than or equal to the capacity of the vehicle assigned to that route. Each customer is served by exactly one vehicle. Most remarkable is that the players are rational. Note that the aim of routing is to minimize the number of routes without desecrating the capacity constraints.

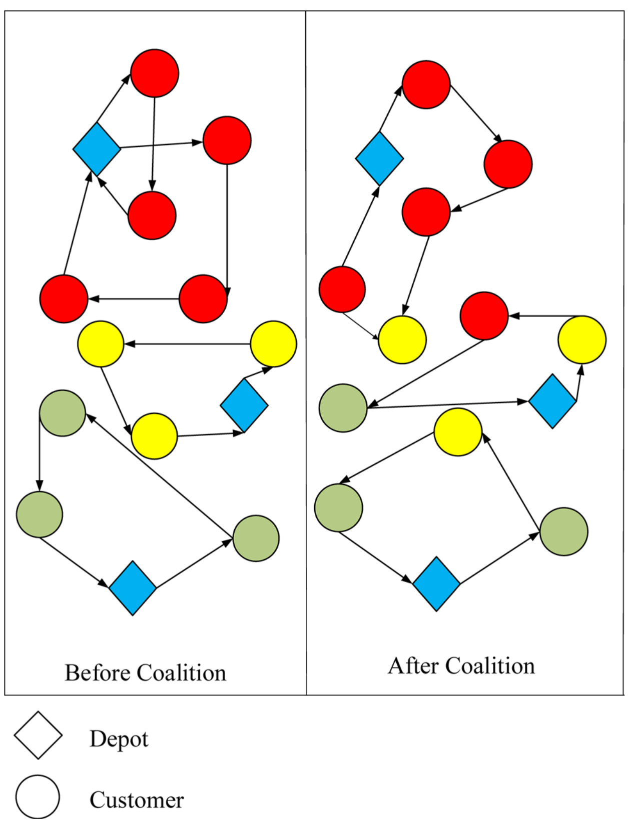

Fig. 1 Framework of CoMDVRP

Notations

Before description of the objective functions and model, the sets, indices, parameters and decision variables are explained.

Sets

I The set of all depots;
J The set of all customers;
K The set of all vehicles;
P The set of all players (i.e. the owners of depots).

Indices

i The index of depots;
j The index of customers;
k The index of routs;
p The index of players (i.e. owners of depots).

Parameters

N The number of customers;
C_{ij} The travel cost spent to go from point i to j, $i, j \in I \cup J$;
V_i The maximum throughput at depot i;
d_j The demand of customer j;
Q_k The capacity of vehicle (route) k.

Decision variables

$$x_{ijk}: \begin{cases} 1, & \text{if i immediately preceeds } j \text{ on route } k \ \ \forall i \in I, j \in J \\ 0, & \text{otherwise} \end{cases}$$

$$z_{ij}: \begin{cases} 1, & \text{if customer } j \text{ is served by depot } i \\ 0, & \text{otherwise} \end{cases}$$

U_{lk} the auxiliary variable for sub-tour elimination constraint in route k and $l \in J$.

Model formulation for MDVRP (non-cooperation situation)

This section presents a linear model for the MDVRP with the mentioned notations. Each player (owner) $p \in P$ minimizes total transportation cost of all their vehicles.

$$\text{Min TC}(\{p\}) = \sum_{i \in I \cup J} \sum_{j \in I \cup J} \sum_{k \in K} C_{ij} x_{ijk} \tag{1}$$

Subject to:

$$\sum_{k \in K} \sum_{i \in I \cup J} x_{ijk} = 1, \quad \forall j \in J \tag{2}$$

$$\sum_{j \in J} d_j \sum_{i \in I \cup J} x_{ijk} \leq Q_k, \quad \forall k \in K \tag{3}$$

$$U_{lk} - U_{jk} + N x_{ijk} \leq N - 1, \quad \forall\, l, j \in J,\, k \in K \tag{4}$$

$$\sum_{i \in I \cup J} x_{ijk} - \sum_{i \in I \cup J} x_{jik} = 0, \quad \forall\, k \in K,\, i \in I \cup J \tag{5}$$

$$\sum_{i \in I} \sum_{j \in J} x_{ijk} \leq 1, \quad \forall\, k \in K, \tag{6}$$

$$\sum_{j \in J} d_i z_{ij} \leq V_i, \quad \forall i \in I \tag{7}$$

$$-z_{ij} + \sum_{i=1}^{n} (x_{iuk} + x_{ujk}) \leq 1, \quad \forall i \in I, j \in J,\, k \in K \tag{8}$$

$$x_{ijk} \in \{0, 1\}, \quad \forall i \in I, j \in J,\, k \in K \tag{9}$$

$$z_{ij} \in \{0, 1\}, \quad \forall\, i \in I, j \in J \tag{10}$$

$$U_{lk} \geq 0, \quad \forall\, l \in J,\, k \in K \tag{11}$$

The objective function (1) is to minimize sum of the total transportation cost for the vehicles. Constraint set (2) guarantees that each customer will be visited exactly once, while constraint set (3) states the capacity for a set of vehicles. Constraint set (4) is the sub-tour elimination condition, where the flow conservation is expressed by Constraint set (5). Constraint set (6) means that each route can be served at most once. Constraint set (7) ensures the capacity of the depots and constraint set (8) specifies that a customer can be assigned to a depot only if there is a route from that depot going through that customer. Constraint sets (9) and (10) represent the binary requirements on the decision variables. The auxiliary variables U_{lk} taking positive values are declared in Constraint set (11).

Formulation of vehicle routing problem for coalition of players

A new method is proposed here that minimizes sum of the total transportation cost for the vehicles of a coalition. We assume that there are multiple players (owners of depots) that manage some depots. The players cooperate using the vehicles of each other (coalition members) with the aim of minimizing their transportation cost. Thus, by solving the CoMDVRP for a coalition of players, the assignment of customers to depots of the coalition are planned such that the total transportation cost of the coalition is minimized. Coalition m (from of $2^N - 1$ possible coalitions which N is the number of player) is denoted by $S_m (\subseteq P)$. All sets, indices and parameters in CoMDVRP are defined in the coalitional modes.

Notations

Sets

$I_{[S_m]}$ The set of all depots for coalition S_m;
$J_{[S_m]}$ The set of all customers for coalition S_m;
$K_{[S_m]}$ The set of all vehicles for coalition S_m.

Indices

i The depot index;
j The customer index;
k The route index;
m The coalition index.

Parameters

$N_{[S_m]}$ The number of customers in coalition S_m;
$C_{ij[S_m]}$ The travel cost spent to go from point i to$j$ under coalition S_m situation $i, j \in I \cup J$;
V_i The maximum throughput at depot i;
d_j The demand of customer j;
$Q_{k[S_m]}$ The maximum capacity of vehicle (route) k.

Decision variables

$$x_{ijk[S_m]} : \begin{cases} 1, \text{ if } i \text{ immediately preceeds } j \text{ on route } k \text{ in coalition } S_m; \\ 0, \text{ otherwise ;} \end{cases}$$

$z_{ij[S_m]}$: $\begin{cases} 1, & \text{if customer } j \text{ is served by depot } i \text{ in coalition } S_m; \\ 0, & \text{otherwise ;} \end{cases}$

$U_{lk[S_m]}$ the auxiliary variable for sub-tour elimination constraint in route k for coalition S_m and $l \in J_{[s_m]}$.

Model formulation for CoMDVRP

This section presents a model for the CoMDVRP considering interactions among owners as follows:

$$\text{Min } \mathrm{TC}(S_m) = \sum_{i \in I_{[S_m]} \cup J_{[S_m]}} \sum_{j \in I_{[S_m]} \cup J_{[S_m]}} \sum_{k \in K_{[S_m]}} C_{ij[S_m]} x_{ijk[S_m]} \quad (12)$$

Subject to

$$\sum_{k \in K_{[S_m]}} \sum_{i \in I_{[S_m]} \cup J_{[S_m]}} x_{ijk[S_m]} = 1, \quad \forall j \in J_{[S_m]} \quad (13)$$

$$\sum_{j \in J_{[S_m]}} d_j \sum_{i \in I_{[S_m]} \cup J_{[S_m]}} x_{ijk_{[S_m]}} \le Q_{k_{[S_m]}}, \quad \forall k \in K_{[S_m]} \quad (14)$$

$$U_{lk_{[S_m]}} - U_{jk_{[S_m]}} + N_{[S_m]} x_{ijk[S_m]} \le N - 1, \quad \forall l, j \in J_{[S_m]}, k \in K_{[S_m]} \quad (15)$$

$$\sum_{i \in I_{[S_m]} \cup J_{[S_m]}} x_{ijk[S_m]} - \sum_{i \in I_{[S_m]} \cup J_{[S_m]}} x_{jik[S_m]} = 0, \quad \forall k \in K_{[S_m]}, i \in I_{[S_m]} \cup J_{[S_m]} \quad (16)$$

$$\sum_{i \in I_{[S_m]}} \sum_{j \in J_{[S_m]}} x_{ijk[S_m]} \le 1, \quad \forall k \in K_{[S_m]} \quad (17)$$

$$\sum_{j \in J_{[S_m]}} d_i z_{ij[S_m]} \le V_i, \quad \forall i \in I_{[S_m]} \quad (18)$$

$$-z_{ij[S_m]} + \sum_{u \in I_{[S_m]} \cup J_{[S_m]}} (x_{iuk[S_m]} + x_{ujk[S_m]}) \le 1, \quad \forall i \in I_{[S_m]}, j \in J_{[S_m]}, k \in K_{[S_m]} \quad (19)$$

$$\sum_{k \in K} \sum_{j \in J} x_{ijk[S_m]} = 1, \quad \forall i \in I \quad (20)$$

$$x_{ijk[S_m]} + x_{j'i'k[S_m]} \le 1, \quad \forall i \in I_{[S_m]}, i \ne i', j \in J_{[S_m]}, k \in K_{[S_m]} \quad (21)$$

$$x_{ijk[S_m]} \in \{0, 1\}, \quad \forall i \in I_{[S_m]}, j \in J_{[S_m]}, k \in K_{[S_m]} \quad (22)$$

$$z_{ij[S_m]} \in \{0, 1\}, \quad \forall i \in I_{[S_m]}, j \in J_{[S_m]} \quad (23)$$

$$U_{j'k[S_m]} \ge 0, \quad \forall j' \in J_{[S_m]}, k \in K_{[S_m]} \quad (24)$$

The constraints of this model are developed from the previous model, and two new constraints are added as well. The main objective of Function (12) is to minimize sum of the total transportation cost for the vehicles of coalition S_m. Constraint set (13) guarantees that in any coalitions each customer will be visited exactly once by the existing vehicles. Constraint set (14) states the capacity for a set of vehicles of coalition S_m. Constraint set (15) is the new sub-tour elimination constraint set and the flow conservation is expressed in Constraint (16). Constraint set (17) means that each route can be used at most once in a coalition, while Constraint set (18) ensures capacity for the depots of coalition. Constraint set (19) specifies that a customer can be assigned to a depot only if there is a route from that depot going through that customer. Constraint set (20) restricts assignment of each customer to exactly one vehicle route and Constraint set (21) ensures that the route starts from a depot does not end to another depot. Constraint sets (22) and (23) denote the binary requirements on the decision variables. The auxiliary variables U_{jk} taking positive values are declared in constraint set (24).

The above model computes the total cost by determining the best route according to coalition S_m. For solving the problem, firstly, each player is considered independently that is a common MDVRP. Then, the model is solved again considering the coalitions of two players, three players and so on. When the optimal objective function for any coalitional scenario is smaller than sum of the individual optimal objective function, the players have the incentive to coordinate with each other. It means that the following equation should be established in a coalitional form:

$$\mathrm{TC}(S_m) \le \sum_{p \in S_m} \mathrm{TC}(\{p\}). \quad (25)$$

The cost savings of coalition S_m is denoted by $\mathrm{CS}(\{S_m\})$ and is obtained from the following equation:

$$\mathrm{CS}(S_m) = \sum_{p \in S_m} \mathrm{TC}(\{p\}) - \mathrm{TC}(S_m). \quad (26)$$

Cost saving $\mathrm{CS}(\{S_m\})$ represents the difference between sum of the individual objective function and that of the objective function of coalition S_m.

These cost savings can be either higher or lower, depending on the synergy between owners in different coalition. This synergy can be determined with the below function:

$$\mathrm{Synergy}(S_m) = \frac{\mathrm{CS}(S_m)}{\sum_{p \in S_m} \mathrm{TC}(\{p\})}. \quad (27)$$

Now, two numerical examples (symmetric and asymmetric examples) will be addressed to test the cooperative VRP. In both numerical examples, four companies were considered which serve some specific customers. The companies which are in fact the players are owners of depots and are illustrated by $P = \{1, 2, 3, 4\}$. Each player owns one depot with only one vehicle. The customers of each depot (i.e. players) are illustrated by $D_1 = \{c_1, c_2, c_3, c_4, c_5\}$, $D_2 = \{c_6, c_7, c_8, c_9, c_{10}\}$, $D_3 = \{c_{11}, c_{12}, c_{13}, c_{14}, c_{15}\}$,- $D_4 = \{c_{16}, c_{17}, c_{18}, c_{19}, c_{20}\}$ for symmetric example and $D_1 = \{c_1, c_2, c_3, c_4, c_5\}$, $D_2 = \{c_6, c_7, c_8, c_9\}$, D_3 - $= \{c_{10}, c_{11}, c_{12}, c_{13}\}$, $D_4 = \{c_{14}, c_{15}, c_{16}\}$ for asymmetric example, respectively. Maximum throughputs at depots are $V_1 = V_2 = V_3 = V_4 = 200$ for symmetric and for asymmetric are $V_1 = 220$, $V_2 = 200$, $V_3 = 210$, and $V_4 = 195$. Meanwhile, each company has one vehicle which in symmetric example has the same capacity $Q_k = 220$ and in asymmetric example capacity of vehicles are $Q_1 = 240$, $Q_2 = 200$, $Q_3 = 210$, and $Q_4 = 195$. Figure 2 depicts the framework of these examples. Each depot and their customers are shown in the same color. The travel costs spent to go from point i to j and the demand of each customer are shown in Tables 1 and 2, and the demand of each customer are shown in Tables 3 and 4.

The model of CoMDVRP (12)–(24) is first solved for each possible coalition of players S_m:{1},{2},{3},{4},{1, 2},{1, 3},{1, 4},{2, 3},{2, 4},{3, 4}{1, 2, 3},{1, 2, 4},{1, 3, 4},{2, 3, 4}, and {1, 2, 3, 4} by Gams software (Rosenthal 1988). Table 5 and Table 6 present the best route of the CoMDVRP for each of two examples. Moreover, Fig. 3 illustrates the optimal solution of the examples after the grand coalition (i.e. $S_{15} = \{1, 2, 3, 4\}$) and demonstrates that each customer is allocated to which depots. It also shows the best route among the depots and customers in grand coalition. The main idea is that the total transportation costs can be decreased if the players cooperate with each other. Table 7 summarizes values of the objective function for each coalition obtained from the CoMDVRP. This table also lists the cost saving for each possible coalition obtained from Eq. (26) and synergy obtained from Eq. (27).

Now, for a fair distribution of the cost saving among the players, the techniques of CGT are utilized. In the next section, some economic concepts are described for the cost saving allocation.

Cooperative techniques for cost saving allocation

The cooperative techniques are used to fairly assign the cost saving to each member of the coalition such that all the players (owners) might receive more than they could individually get on their own. The fair allocation is recognized according to the amount that is added by each member to a coalition. Thus, depending on the amount that each player adds to a coalition they receive a percent of cost saving. A number of sharing mechanisms of cost saving are suggested based on the economic models including Shapley value, Core, the τ value and ECSM.

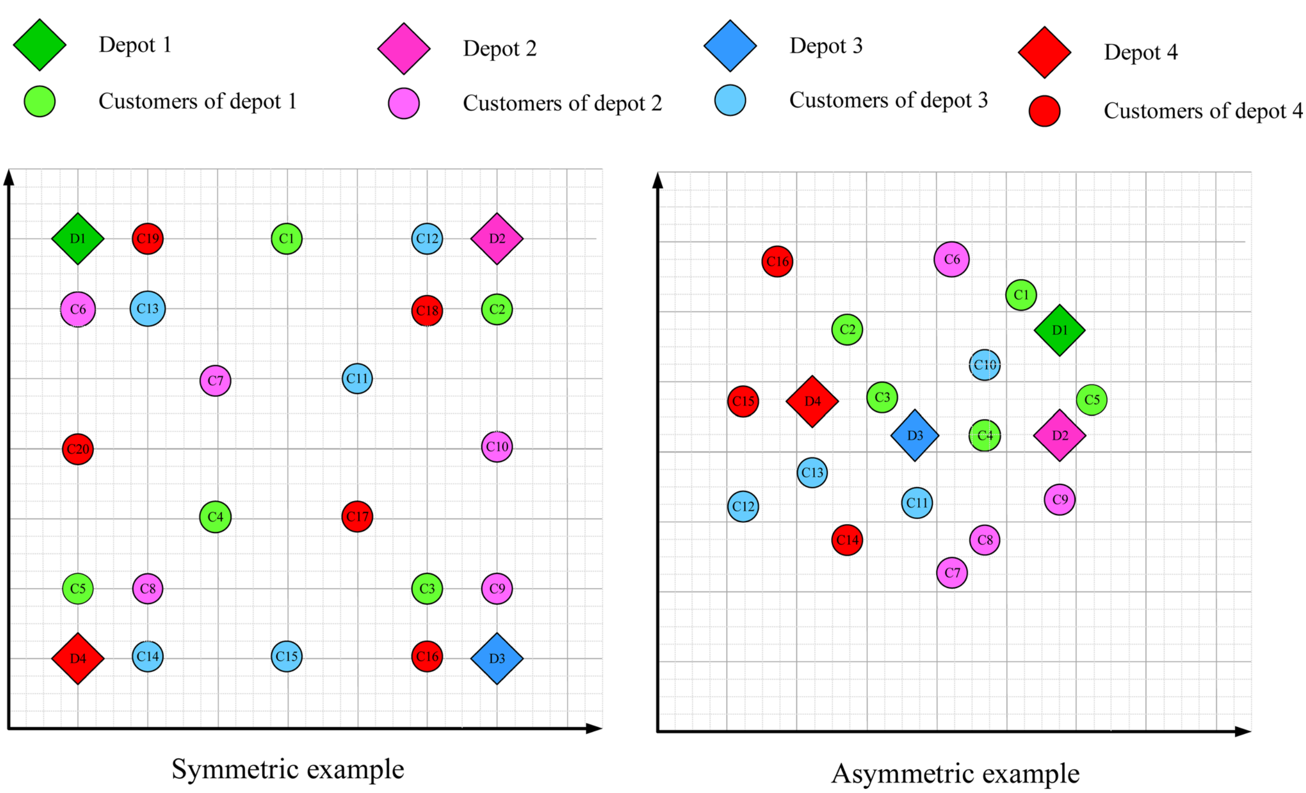

Fig. 2 Examples of coalition between 4 companies

Table 1 Travel costs (C_{ij}) spent to go from point i to j for symmetric

	D1	D2	D3	D4	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20
D1	0.00	0.00	0.00	0.00	3.00	6.08	7.07	4.47	5.00	1.00	2.83	5.10	7.81	6.71	4.47	5.00	1.41	6.08	6.71	7.81	5.66	5.10	1.00	3.00
D2	0.00	0.00	0.00	0.00	3.00	1.00	5.10	5.66	7.81	6.08	4.47	7.07	5.00	3.00	2.83	1.00	5.10	7.81	6.71	6.08	4.47	1.41	5.00	6.71
D3	0.00	0.00	0.00	0.00	6.71	5.00	1.41	4.47	6.08	7.81	5.66	5.10	1.00	3.00	4.47	6.08	7.07	5.00	3.00	1.00	2.83	5.10	7.81	6.71
D4	0.00	0.00	0.00	0.00	6.71	7.81	5.10	2.83	1.00	5.00	4.47	1.41	6.08	6.71	5.66	7.81	5.10	1.00	3.00	5.00	4.47	7.07	6.08	3.00
C1	3.00	3.00	6.71	6.71	0.00	3.16	5.39	4.12	5.83	3.16	2.24	5.39	5.83	4.24	2.24	2.00	2.24	6.32	6.00	6.32	4.12	2.24	2.00	4.24
C2	6.08	1.00	5.00	7.81	3.16	0.00	4.12	5.00	7.21	6.00	4.12	6.40	4.00	2.00	2.24	1.41	5.00	7.07	5.83	5.10	3.61	1.00	5.10	6.32
C3	7.07	5.10	1.41	5.10	5.39	4.12	0.00	3.16	5.00	6.40	4.24	4.00	1.00	2.24	3.16	5.00	5.66	4.12	2.24	1.00	1.41	4.00	6.40	5.39
C4	4.47	5.66	4.47	2.83	4.12	5.00	3.16	0.00	2.24	3.61	2.00	1.41	4.12	4.12	2.83	5.00	3.16	2.24	2.24	3.61	2.00	4.24	4.12	2.24
C5	5.00	7.81	6.08	1.00	5.83	7.21	5.00	2.24	0.00	4.00	3.61	1.00	6.00	6.32	5.00	7.07	4.12	1.41	3.16	5.10	4.12	6.40	5.10	2.00
C6	1.00	6.08	7.81	5.00	3.16	6.00	6.40	3.61	4.00	0.00	2.24	4.12	7.21	6.32	4.12	5.10	1.00	5.10	5.83	7.07	5.00	5.00	1.41	2.00
C7	2.83	4.47	5.66	4.47	2.24	4.12	4.24	2.00	3.61	2.24	0.00	3.16	5.00	4.12	2.00	3.61	1.41	4.12	4.12	5.00	2.83	3.16	2.24	2.24
C8	5.10	7.07	5.10	1.41	5.39	6.40	4.00	1.41	1.00	4.12	3.16	0.00	5.00	5.39	4.24	6.40	4.00	1.00	2.24	4.12	3.16	5.66	5.00	2.24
C9	7.81	5.00	1.00	6.08	5.83	4.00	1.00	4.12	6.00	7.21	5.00	5.00	0.00	2.00	3.61	5.10	6.40	5.10	3.16	1.41	2.24	4.12	7.07	6.32
C10	6.71	3.00	3.00	6.71	4.24	2.00	2.24	4.12	6.32	6.32	4.12	5.39	2.00	0.00	2.24	3.16	5.39	5.83	4.24	3.16	2.24	2.24	5.83	6.00
C11	4.47	2.83	4.47	5.66	2.24	2.24	3.16	2.83	5.00	4.12	2.00	4.24	3.61	2.24	0.00	2.24	3.16	5.00	4.12	4.12	2.00	1.41	3.61	4.12
C12	5.00	1.00	6.08	7.81	2.00	1.41	5.00	5.00	7.07	5.10	3.61	6.40	5.10	3.16	2.24	0.00	4.12	7.21	6.32	6.00	4.12	1.00	4.00	5.83
C13	1.41	5.10	7.07	5.10	2.24	5.00	5.66	3.16	4.12	1.00	1.41	4.00	6.40	5.39	3.16	4.12	0.00	5.00	5.39	6.40	4.24	4.00	1.00	2.24
C14	6.08	7.81	5.00	1.00	6.32	7.07	4.12	2.24	1.41	5.10	4.12	1.00	5.10	5.83	5.00	7.21	5.00	0.00	2.00	4.00	3.61	6.40	6.00	3.16
C15	6.71	6.71	3.00	3.00	6.00	5.83	2.24	2.24	3.16	5.83	4.12	2.24	3.16	4.24	4.12	6.32	5.39	2.00	0.00	2.00	2.24	5.39	6.32	4.24
C16	7.81	6.08	1.00	5.00	6.32	5.10	1.00	3.61	5.10	7.07	5.00	4.12	1.41	3.16	4.12	6.00	6.40	4.00	2.00	0.00	2.24	5.00	7.21	5.83
C17	5.66	4.47	2.83	4.47	4.12	3.61	1.41	2.00	4.12	5.00	2.83	3.16	2.24	2.24	2.00	4.12	4.24	3.61	2.24	2.24	0.00	3.16	5.00	4.12
C18	5.10	1.41	5.10	7.07	2.24	1.00	4.00	4.24	6.40	5.00	3.16	5.66	4.12	2.24	1.41	1.00	4.00	6.40	5.39	5.00	3.16	0.00	4.12	5.39
C19	1.00	5.00	7.81	6.08	2.00	5.10	6.40	4.12	5.10	1.41	2.24	5.00	7.07	5.83	3.61	4.00	1.00	6.00	6.32	7.21	5.00	4.12	0.00	3.16
C20	3.00	6.71	6.71	3.00	4.24	6.32	5.39	2.24	2.00	2.00	2.24	2.24	6.32	6.00	4.12	5.83	2.24	3.16	4.24	5.83	4.12	5.39	3.16	0.00

Table 2 Travel costs (C_{ij}) spent to go from point i to j for asymmetric

	*D*1	*D*2	*D*3	*D*4	*C*1	*C*2	*C*3	*C*4	*C*5	*C*6	*C*7	*C*8	*C*9	*C*10	*C*11	*C*12	*C*13	*C*14	*C*15	*C*16
*D*1	0.00	0.00	0.00	0.00	0.71	3.00	2.69	1.80	1.12	1.80	3.81	3.16	2.50	1.12	3.20	5.15	4.03	4.24	4.61	4.12
*D*2	0.00	0.00	0.00	0.00	2.06	3.35	2.55	1.00	0.71	2.92	2.50	1.80	1.00	1.41	1.41	4.61	3.54	3.35	4.53	4.72
*D*3	0.00	0.00	0.00	0.00	2.50	1.80	0.71	1.00	2.55	2.55	2.06	1.80	2.24	1.41	1.00	2.69	1.58	1.80	2.55	3.20
*D*4	0.00	0.00	0.00	0.00	3.35	1.12	1.00	2.55	4.00	2.83	3.20	3.20	3.81	2.55	2.12	1.80	1.00	2.06	1.00	2.06
*C*1	0.71	2.06	2.50	3.35	0.00	2.55	2.50	2.06	1.80	1.12	4.12	3.54	3.04	1.12	3.35	3.35	3.91	4.30	4.27	3.54
*C*2	3.00	3.35	1.80	1.12	2.55	0.00	1.12	2.50	3.64	1.80	4.27	4.03	3.50	2.06	2.69	2.92	2.06	3.00	1.80	1.41
*C*3	2.69	2.55	0.71	1.00	2.50	1.12	0.00	1.58	3.00	2.24	2.69	3.61	2.92	1.58	1.58	2.92	1.41	2.06	2.00	2.50
*C*4	1.80	1.00	1.00	2.55	2.06	2.06	1.58	0.00	1.58	2.55	2.06	1.50	1.41	1.00	1.41	3.64	2.55	2.50	3.54	3.91
*C*5	1.12	0.71	2.55	4.00	1.80	1.80	3.00	1.58	0.00	2.83	3.20	2.50	1.58	1.58	2.92	5.22	4.12	4.03	5.00	4.92
*C*6	1.80	2.92	2.55	2.83	1.12	1.12	2.24	2.55	2.83	0.00	4.50	4.03	3.81	1.58	3.54	4.30	3.61	4.27	3.61	2.50
*C*7	3.81	2.50	2.06	3.20	4.12	4.27	2.69	2.06	3.20	4.50	0.00	0.71	1.80	3.04	1.12	3.16	2.50	1.58	3.91	5.15
*C*8	3.16	1.80	1.80	3.20	3.54	3.54	3.61	1.50	2.50	4.03	0.71	0.00	1.12	2.50	1.12	3.54	2.69	2.00	4.03	5.00
*C*9	2.50	1.00	2.24	3.81	3.04	3.04	2.92	1.41	1.58	3.81	1.80	1.12	0.00	2.24	2.00	4.50	3.54	3.04	4.74	5.32
*C*10	1.12	1.41	1.41	2.55	1.12	1.12	1.58	1.00	1.58	1.58	3.04	2.50	2.24	0.00	2.24	4.03	2.92	3.20	3.54	3.35
*C*11	3.20	1.41	1.00	2.12	3.35	3.35	1.58	1.41	2.92	3.54	1.12	1.12	2.00	2.24	0.00	2.50	1.58	1.12	2.92	4.03
*C*12	5.15	4.61	2.69	1.80	3.35	5.00	2.92	3.64	5.22	4.30	3.16	3.54	4.50	4.03	2.50	0.00	1.12	1.58	1.50	3.54
*C*13	4.03	3.54	1.58	1.00	3.91	3.91	1.41	2.55	4.12	3.61	2.50	2.69	3.54	2.92	1.58	1.12	0.00	1.12	1.41	3.04
*C*14	4.24	3.35	1.80	2.06	4.30	4.30	2.06	2.50	4.03	4.27	1.58	2.00	3.04	3.20	1.12	1.58	1.12	0.00	2.50	4.12
*C*15	4.61	4.53	2.55	1.00	4.27	4.27	2.00	3.54	5.00	3.61	3.91	4.03	4.74	3.54	2.92	1.50	1.41	2.50	0.00	2.06
*C*16	4.12	4.72	3.20	2.06	3.54	3.54	2.50	3.91	4.92	2.50	5.15	5.00	5.32	3.35	4.03	3.54	3.04	4.12	2.06	0.00

Table 3 Demand of each customer for symmetric

d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
45	35	30	50	40	35	50	30	40	45
d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}
50	40	30	35	45	35	50	30	40	45

Table 4 Demand of each customer for asymmetric

d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
40	35	40	60	45	50	45	65
d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}
40	55	70	35	50	85	50	60

The number of players (owners in CoMDVRP) is denoted by n and the set of all the players is denoted by P such that $P = \{1, 2,\ldots, n\}$. A cooperative game is considered in which the players choose to cooperate in the coalitions if they are profitable. A coalition is any subset $S \subseteq P$, and hence there are $2^N - 1$ possible coalitions. For an N-person cooperative game among owners of CoMDVRP, a characteristic function CS(S)is considered. For owners set p, CS(P) represents the possible cost saving when all owners cooperate (i.e., the characteristic function of grand coalition). The game has a super-additive property if:

$$\mathrm{CS}(S) \geq \sum_{p \in S} \mathrm{CS}(\{p\}) \tag{28}$$

A cost allocation vector $\vec{y} = (y_1, y_2, \ldots, y_n)$ assigns a quantity y_p to each player p in P and we know that $\sum_{p \in P} y_p \leq \mathrm{CS}(P)$. A vector $\vec{y} = (y_1, y_2, \ldots, y_n)$ is an imputation for cost saving assignment if it satisfies the *individual rationality condition* $y_p \geq \mathrm{CS}(\{p\})$ for all $p \in P$, and *efficient condition* $\sum_{p \in P} y_p = \mathrm{CS}(P)$, respectively (Barron 2013). Actually, an imputation shows that how CS(P)should be distributed among the owners of VRP system such that no one will reject the allocated assignment. The set of all feasible imputations for the cooperative game is defined as

$$Y = \left\{ \vec{y} = (y_1, y_2, \ldots, y_n) \middle| y_p \geq \mathrm{CS}(\{p\}), \sum_{p \in P} y_k = \mathrm{CS}(P) \right\}. \tag{29}$$

The main challenge of CGT is to fairly assign the payoff CS(P) among the players (Barron 2013). According to different interpretations of the fairness, previous researchers have suggested various methods. In the following sections, we propose some of them for cost saving allocation problem in the cooperative VRP, however, the readers may

Table 5 Best route for each vehicle in different coalitions for symmetric example

Coalition	Route 1	Route 2	Route 3	Route 4
$S_1 = \{1\}$	D_1–c_1–c_2–c_3–c_4–c_5–D_1	–	–	–
$S_2 = \{2\}$	–	D_2–c_{10}–c_9–c_8–c_7–c_6–D_2	–	–
$S_3 = \{3\}$	–	–	D_3–c_{15}–c_{14}–c_{13}–c_{11}–c_{12}–D_3	–
$S_4 = \{4\}$	–	–	–	D_4–c_{20}–c_{19}–c_{18}–c_{17}–c_{16}–D_4
$S_5 = \{1, 2\}$	D_1–c_6–c_1–c_7–c_5–c_8–D_1	D_2–c_2–c_{10}–c_9–c_3–c_4–D_2	–	–
$S_6 = \{1, 3\}$	D_1–c_{13}–c_1–c_{12}–c_2–c_{11}–D_1	–	D_3–c_3–c_{15}–c_{14}–c_5–c_4–D_3	–
$S_7 = \{1, 4\}$	D_1–c_{19}–c_1–c_{18}–c_2–c_{17}–D_1	–	–	D_4–c_5–c_{20}–c_4–c_3–c_{16}–D_4
$S_8 = \{2, 3\}$	–	D_2–c_{12}–c_{11}–c_{15}–c_{14}–c_8–D_2	D_3–c_9–c_{10}–c_7–c_{13}–c_6–D_3	–
$S_9 = \{2, 4\}$	–	D_2–c_{18}–c_{10}–c_9–c_{16}–c_{17}–D_2	–	D_4–c_8–c_{20}–c_6–c_{19}–c_7–D_4
$S_{10} = \{3, 4\}$	–	–	D_3–c_{16}–c_{15}–c_{17}–c_{18}–c_{12}–D_3	D_4–c_{14}–c_{20}–c_{13}–c_{19}–c_{11}–D_4
$S_{11} = \{1, 2, 3\}$	D_1–c_6–c_{13}–c_7–c_1–c_{12}–D_1	D_2–c_2–c_{10}–c_9–c_3–c_{11}–D_2	D_3–c_{15}–c_4–c_8–c_5–c_{14}–D_3	–
$S_{12} = \{1, 2, 4\}$	D_1–c_{19}–c_7–c_1–c_{18}–c_{11}–D_1	D_2–c_{10}–c_9–c_{17}–c_3–c_{16}–D_2	–	D_4–c_5–c_8–c_4–c_{20}–c_6–D_4
$S_{13} = \{1, 3, 4\}$	D_1–c_1–c_{12}–c_2–c_{18}–c_9–D_1	–	D_3–c_{16}–c_3–c_{17}–c_{15}–c_5–D_3	D_4–c_{14}–c_4–c_{20}–c_{13}–c_{19}–D_4
$S_{14} = \{2, 3, 4\}$	–	D_2–c_{12}–c_{18}–c_{11}–c_{10}–c_{16}–D_2	D_3–c_9–c_{17}–c_{15}–c_{14}–c_8–D_3	D_4–c_{20}–c_7–c_{13}–c_6–c_{19}–D_4
$S_{15} = \{1, 2, 3, 4\}$	D_1–c_{19}–c_{13}–c_6–c_{20}–c_{17}–D_1	D_2–c_2–c_{18}–c_{12}–c_1–c_7–D_2	D_3–c_{16}–c_3–c_9–c_{10}–c_{11}–D_3	D_4–c_5–c_8–c_{14}–c_{15}–c_4–D_4

Table 6 Best route for each vehicle in different coalitions for asymmetric example

Coalition	Route 1	Route 2	Route 3	Route 4
$S_1 = \{1\}$	D_1–c_1–c_5–c_4–c_3–c_2–D_1	–	–	–
$S_2 = \{2\}$	–	D_2–c_9–c_8–c_7–c_6–D_2	–	–
$S_3 = \{3\}$	–	–	D_3–c_{10}–c_{11}–c_{13}–c_{12}–D_3	–
$S_4 = \{4\}$	–	–	–	D_4–c_{14}–c_{15}–c_{16}–D_4
$S_5 = \{1, 2\}$	D_1–c_1–c_6–c_5–c_9–c_7–D_1	D_2–c_8–c_4–c_3–c_2–D_2	–	–
$S_6 = \{1, 3\}$	D_1–c_1–c_{10}–c_3–c_{13}–c_{12}–D_1	–	D_3–c_{11}–c_4–c_5–c_2–D_3	–
$S_7 = \{1, 4\}$	D_1–c_1–c_5–c_4–c_3–c_2–D_1	–	–	D_4–c_{16}–c_{15}–c_{14}–D_4
$S_8 = \{2, 3\}$	–	D_2–c_{11}–c_7–c_{13}–c_{12}–D_2	D_3–c_8–c_9–c_{10}–c_6–D_3	–
$S_9 = \{2, 4\}$	–	D_2–c_9–c_8–c_7–c_6–D_2	–	D_4–c_{14}–c_{15}–c_{16}–D_4
$S_{10} = \{3, 4\}$	–	–	D_3–c_{10}–c_{11}–c_{14}–D_3	D_4–c_{13}–c_{12}–c_{15}–c_{16}–D_4
$S_{11} = \{1, 2, 3\}$	D_1–c_1–c_{10}–c_6–c_2–c_3–D_1	D_2–c_5–c_{11}–c_{13}–c_{12}–D_2	D_3–c_4–c_9–c_8–c_7–D_3	–
$S_{12} = \{1, 2, 4\}$	D_1–c_5–c_1–c_6–c_2–c_{15}–D_1	D_2–c_9–c_4–c_3–c_{16}–D_2	–	D_4–c_{14}–c_7–c_8–D_4
$S_{13} = \{1, 3, 4\}$	D_1–c_1–c_{10}–c_3–c_{14}–D_1	–	D_3–c_{11}–c_4–c_5–c_2–D_3	D_4–c_{13}–c_{12}–c_{15}–c_{16}–D_4
$S_{14} = \{2, 3, 4\}$	–	D_2–c_{11}–c_7–c_{14}–D_2	D_3–c_8–c_9–c_{10}–c_6–D_3	D_4–c_{13}–c_{12}–c_{15}–c_{16}–D_4
$S_{15} = \{1, 2, 3, 4\}$	D_1–c_1–c_{11}–c_8–c_7–D_1	D_2–c_5–c_9–c_4–c_{10}–D_2	D_3–c_3–c_{13}–c_{12}–c_{14}–D_3	D_4–c_{15}–c_{16}–c_6–c_2–D_4

refer to Barron (2013), Branzei et al. (2008), and Gilles (2010) for more information.

Shapley value

The Shapley value is one of the solution methods for distribution of cost saving among the players (owners) that was put forward by Shapley in 1952 (Shapley 1952). Shapley value is a weighted average which considers contributions of the marginal cost saving given by the members in each possible coalition. An allocation $\vec{y} = (y_1, y_2, \ldots, y_n)$ is called Shapley value if:

$$y_i = \sum_{S \subset \prod^i} \frac{(|S|-1)!(|P|-|S|)!}{|P|!}[\mathrm{CS}(S) - \mathrm{CS}(S - \{p\})], \quad (30)$$

where Π^i is the set of all coalitions $S \subset P$that contain player p, and $|S|$ represents the number of members in coalition S, and $|N| = n$. $\mathrm{CS}(S) - \mathrm{CS}(S - \{p\})$ gives the amount by which the cost saving of coalition $S - \{p\}$

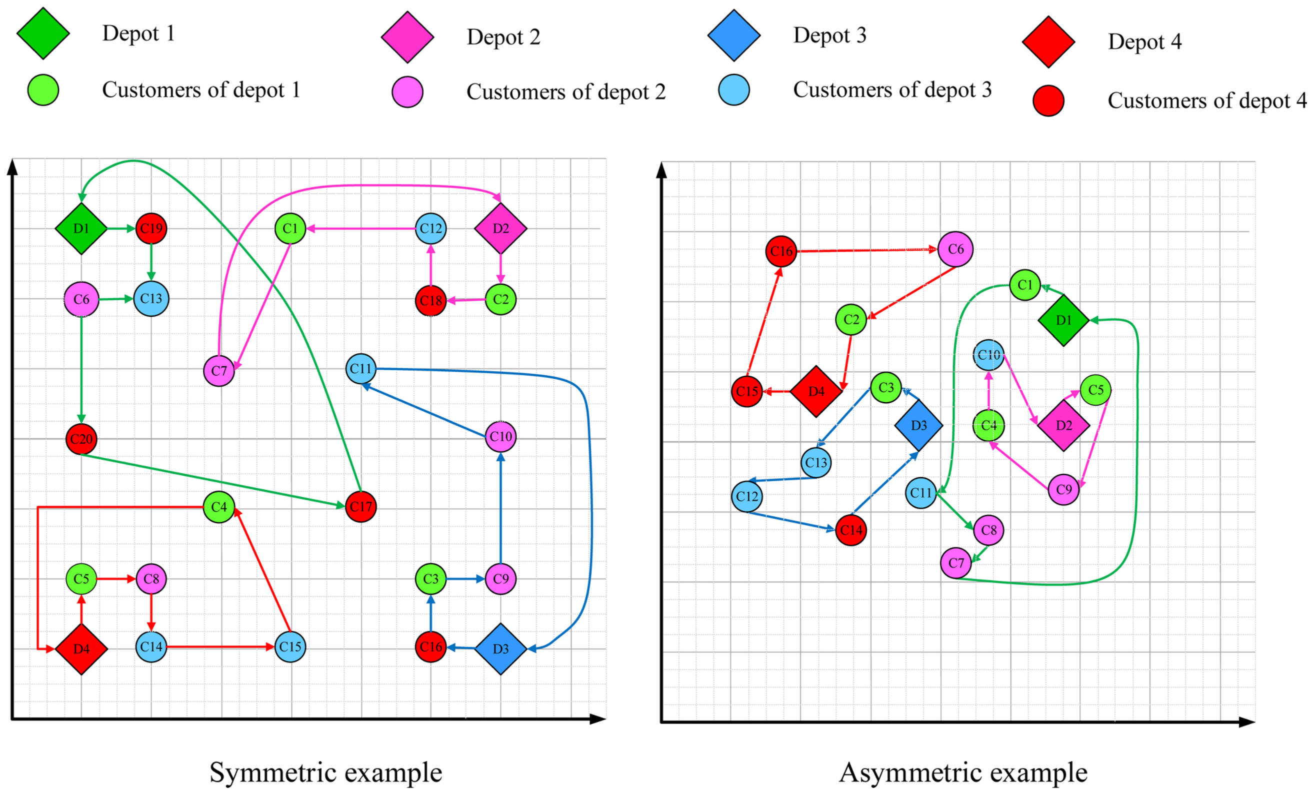

Fig. 3 Results of grand coalition in numerical examples

Table 7 Transportation cost and cost saving for each possible coalition

Coalition	Symmetric example			Asymmetric example		
	TC(S_m)	CS(S_m)	Synergy(S_m)	TC(S_m)	CS(S_m)	Synergy(S_m)
$S_1 = \{1\}$	15.684	0	0	6.79	0	0
$S_2 = \{2\}$	15.398	0	0	7.33	0	0
$S_3 = \{3\}$	14.472	0	0	6.35	0	0
$S_4 = \{4\}$	15.684	0	0	6.62	0	0
$S_5 = \{1, 2\}$	20.166	10.916	0.351	14.04	0.08	0.01
$S_6 = \{1, 3\}$	18.601	11.555	0.383	11.73	1.41	0.11
$S_7 = \{1, 4\}$	19.240	12.128	0.387	13.41	0	0
$S_8 = \{2, 3\}$	19.896	9.974	0.334	12.89	0.79	0.06
$S_9 = \{2, 4\}$	18.601	12.481	0.401	13.95	0	0
$S_{10} = \{3, 4\}$	20.402	9.754	0.258	10.45	2.52	0.10
$S_{11} = \{1, 2, 3\}$	25.877	19.677	0.432	16.22	4.25	0.21
$S_{12} = \{1, 2, 4\}$	26.009	20.757	0.444	17.8	2.94	0.14
$S_{13} = \{1, 3, 4\}$	26.349	19.491	0.425	16.94	2.82	0.14
$S_{14} = \{2, 3, 4\}$	26.349	19.205	0.421	16.53	3.77	0.19
$S_{15} = \{1, 2, 3, 4\}$	30.831	30.407	0.496	22.09	5	0.18

increases when player p joins it. Therefore, $\mathrm{CS}(S) - \mathrm{CS}(S - \{p\})$ represents the marginal cost saving of participant p with respect to coalition S. The formula of Shapley value denotes summation over all coalitions that contain player p.

Core

Another very important concept in CGT is the core (Gillies 1953). For the cost saving allocation problem, the core of a game presents a set of imputations as follows:

$$\text{core}(0) = \{\vec{y} \in Y \mid e(S, \vec{y}) \leq 0,\ \forall S \subset P\} = \left\{ \vec{y} \in Y \,\middle|\, v(S) - \sum_{p \in P} y_p \leq 0,\ \ \forall S \subset P \right\} \quad (31)$$

The game is called stable if the core is non-empty. The core is the set of allocations so that each coalition receives at least the cost saving associated with that coalition. The ε-core, for $-\infty < \varepsilon < +\infty$ is defined as follows:

$$\text{core}(\varepsilon) = \{\vec{y} \in Y \mid e(S, \vec{y}) \leq \varepsilon,\ \forall S \subset P,\ S \neq P,\ S \neq \emptyset\} \quad (32)$$

However, the core has two drawbacks: it can be not unique and it can be empty. A relatively simple way to handle both limitations is through least core (or minimax) core methods. Actually, the least core method shrinks the core space simultaneously from each side of boundary until a single point (imputation) is achieved. A linear model of the minimax core is introduced as follows:

$$\min \varepsilon \quad (33)$$

Subject to:

$$e(S, \bar{y}) \leq \varepsilon, \quad \forall S \subset P \quad (34)$$

$$\sum_{p \in P} y_p = v(P) \quad (35)$$

$$y_p \geq 0, \quad \forall p \quad (36)$$

τ value

An extension of the Shapley value is based on the idea that if there are a priori unions. The τ value is defined as the efficient imputation τ, (i.e. $(\vec{\tau} \in Y)$) such that $\vec{\tau} = m + \alpha(M - m)$ for some α, where M and m denote the utopia payoffs and the minimum rights vectors, respectively. m and M are defined as follows:

$$m_p = \max_{S_m : p' \in S_m} \left\{ \text{CS}(S_m) - \sum_{p' \in S_m \setminus \{p\}} M_{p'} \right\} \quad (37)$$

$$M_p = \text{CS}(P) - \text{CS}(P \setminus \{p\}) \quad (38)$$

The τ value is only defined for quasi balanced games. A class of the quasi balanced games contains all games that have a non-empty core (Casas-Méndez et al. 2003). τ value method defines imputation $\vec{\tau} = (\tau_1, \tau_2, \ldots, \tau_n)$ such that:

$$\tau_k = m_k + \alpha(M_k - m_k), \quad (39)$$

in which $\alpha \in [0, 1]$ is uniquely determined by $\sum_{p \in P} \tau_p = \text{CS}(P)$.

Equal cost saving method

Equal cost saving method (ECSM) is motivated to provide a stable and uniform allocation for the players. Actually, this method minimizes the maximum differences in the pairwise relative cost saving of the owners in a coalition. The formulation of ECSM is as follows

$$\text{Min}\, z \quad (40)$$

Subject to:

$$z \geq |y_p - y_{p'}|, \quad \forall (p, p') \in P, \quad (41)$$

$$\sum_{p \in S} y_p \geq \text{CS}(S), \quad \text{for all } S \subset P,\ S \neq P, \quad (42)$$

$$\sum_{p \in P} y_p = \text{CS}(P). \quad (43)$$

Constraint set (41) measures the maximum difference between the relatives of each two players. Thus, variable z represents the largest difference that should be minimized in the objective function. Constraint sets (42) and (43) ensure stability of the imputation.

Numerical example for cost saving methods in cooperative VRP

As mentioned earlier, it is important how the total cost or cost saving should be fairly distributed among the players. CGT provides an appropriate framework to study the problems of joint cost saving allocation. For the previous numerical example, allocation of the cost saving according to different CGT methods are shown in Table 8. The Shapley value, and τ value have been computed using TUGlab (Mirás Calvo and Sánchez Rodrı guez 2006). Moreover the minimax core problem (33)–(36) and ECSM problem (40)–(43) have been computed by Lingo 11.

Table 9 lists the corresponding satisfaction values for each coalition. Consider imputation $\vec{y} = (y_1, y_2, \ldots, y_n)$, satisfaction of a coalition S_m from the imputation $\vec{y}$ is computed by $F_s(\text{CS}, \vec{y}) = \sum_{p \in S_m} y_p - \text{CS}(S_m)$.$F_s(\text{CS}, \vec{y})$ represents the extra shares of allocated cost savings that members of a coalition can obtain with respect to the cost saving of the coalition.

Table 9 shows the satisfaction value for each coalition in absolute terms and also the satisfaction value in relative terms as a percentage of the satisfaction that are obtained from the imputations, i.e. $F_s(\text{CS}, \vec{y}) / \text{TC}(S_m)$. Furthermore,

Table 8 Allocation of cost saving to VRP owners based on different CGT methods

Coalition	Symmetric example				Asymmetric example			
	Shapley	τ value	Maxmin core	ECSM	Shapley	τ value	Maxmin core	ECSM
$S_1 = \{1\}$	7.9934	8.1576	7.8135	7.6017	0.9900	0.7372	1.0300	0.7500
$S_2 = \{2\}$	7.6934	7.9149	8.0685	7.6017	1.2033	1.6872	1.1500	1.7300
$S_3 = \{3\}$	6.9234	6.6056	7.1105	7.6017	1.8050	1.9428	2.0900	1.7700
$S_4 = \{4\}$	7.7967	7.7289	7.4145	7.6017	1.0017	0.6328	0.7300	0.7500

Table 9 Coalition satisfactions from different CGT methods

Coalition	Symmetric example				Asymmetric example			
	Shapley	τ value	Maximin core	ECSM	Shapley	τ value	Maximin core	ECSM
$S_1 = \{1\}$	7.9934	8.1576	7.8135	7.6017	0.9900	0.7372	1.0300	0.7500
	(0.51 %)	(0.52 %)	(0.49 %)	(0.48 %)	(0.15 %)	(0.11 %)	(0.15 %)	(0.11 %)
$S_2 = \{2\}$	7.6934	7.9149	8.0685	7.6017	1.2033	1.6872	1.1500	1.7300
	(0.5 %)	(0.51 %)	(0.52 %)	(0.49 %)	(0.16 %)	(0.23 %)	(0.15 %)	(0.23 %)
$S_3 = \{3\}$	6.9234	6.6056	7.1105	7.6017	1.8050	1.9428	2.0900	1.7700
	(0.48 %)	(0.46 %)	(0.49 %)	(0.52 %)	(0.28 %)	(0.31 %)	(0.33 %)	(0.28 %)
$S_4 = \{4\}$	7.7967	7.7289	7.4145	7.6017	1.0017	0.6328	0.73	0.7500
	(0.5 %)	(0.49 %)	(0.47 %)	(0.48 %)	(0.15 %)	(0.10 %)	(0.11 %)	(0.11 %)
$S_5 = \{1, 2\}$	4.7708	5.1565	4.966	4.2874	2.1133	2.3444	2.1000	2.4000
	(0.24 %)	(0.26 %)	(0.24 %)	(0.21 %)	(0.15 %)	(0.16 %)	(0.19 %)	(0.17 %)
$S_6 = \{1, 3\}$	3.3618	3.2082	3.369	3.6484	1.385	1.2700	1.7100	1.1100
	(0.18 %)	(0.17 %)	(0.18 %)	(0.2 %)	(0.12 %)	(0.11 %)	(0.15 %)	(0.09 %)
$S_7 = \{1, 4\}$	3.6621	3.7585	3.100	3.6484	1.9917	1.3700	1.7600	0.0900
	(0.19 %)	(0.19 %)	(0.16 %)	(0.19 %)	(0.15 %)	(0.10 %)	(0.14 %)	(0.01 %)
$S_8 = \{2, 3\}$	4.6428	4.5465	5.205	5.2294	2.2183	2.8400	2.4500	2.7100
	(0.23 %)	(0.23 %)	(0.26 %)	(0.26 %)	(0.17 %)	(0.22 %)	(0.19 %)	(0.21 %)
$S_9 = \{2, 4\}$	3.0091	3.1628	3.002	2.7224	2.2050	2.3200	1.8800	2. 4800
	(0.16 %)	(0.17 %)	(0.16 %)	(0.15 %)	(0.16 %)	(0.17 %)	(0.13 %)	(0.18 %)
$S_{10} = \{3, 4\}$	4.9661	4.5805	4.771	5.4494	0.2867	0.0556	0.3	0
	(0.24 %)	(0.23 %)	(0.23 %)	(0.27 %)	(0.03 %)	(0.01 %)	(0.02 %)	(0 %)
$S_{11} = \{1, 2, 3\}$	2.9332	3.0011	3.316	3.1281	−0.2517	0.1172	0.0200	0
	(0.11 %)	(0.12 %)	(0.13 %)	(0.12 %)	(−0.01 %)	(0.01 %)	(0.001 %)	(0 %)
$S_{12} = \{1, 2, 4\}$	2.7265	3.0444	2.539	2.0481	0.255	0.1172	−0.0300	0.2900
	(0.1 %)	(0.12 %)	(0.09 %)	(0.08 %)	(0.1 %)	(0.01 %)	(−0.001 %)	(0.02 %)
$S_{13} = \{1, 3, 4\}$	3.2225	3.0011	2.847	3.3141	0.9767	0.4928	1.0300	0.4500
	(0.12 %)	(0.12 %)	(0.11 %)	(0.13 %)	(0.06 %)	(0.03 %)	(0.06 %)	(0.03 %)
$S_{14} = \{2, 3, 4\}$	3.2085	3.0444	3.3880	3.6001	0.2400	0.4928	0.2	0.4800
	(0.12 %)	(0.12 %)	(0.13 %)	(0.14 %)	(0.01 %)	(0.03 %)	(−0.001 %)	(0.03 %)
Min F_S(CS, y)	2.7265	3.0011	2.5395	2.0481	−0.2517	0.0556	−0.0300	0
(Min F_S(CS, y)/ TC(S_m))	(0.1 %)	(0.12 %)	(0.09 %)	(0.08 %)	(−0.01 %)	(0.01 %)	(0 %)	(0 %)
Max F_S(CS, y)	7.9934	8.1576	8.0685	7.6017	2.2183	2.84	2.4500	2.7100
(Max F_S(CS, y)/ TC(S_m))	(0.51 %)	(0.52 %)	(0.52 %)	(0.52 %)	(0.28 %)	(0.31 %)	(0.32 %)	(0.28 %)
Sum F_S(CS, y)	66.9103	66.911	66.911	67.4826	16.42	16.42	16.4200	15.0100
(Sum F_S(CS, y)/ TC(S_m))	(369.32 %)	(369.32 %)	(369.40 %)	(372.5 %)	(1.59 %)	(1.58 %)	(1.60 %)	(1.47 %)

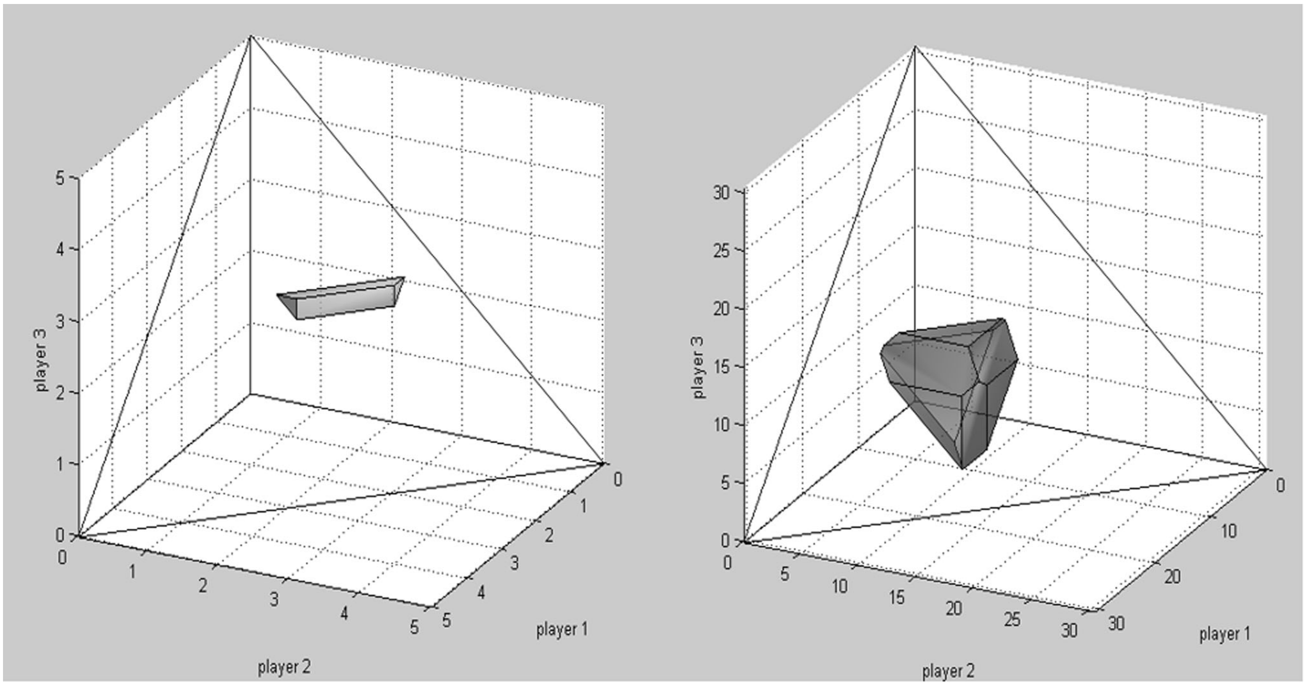

Fig. 4 Core of Co-MDVRP for examples with four players

Table 10 Similarity between solutions of CGT methods, measured by correlation

Coalition	Symmetric example				Asymmetric example			
	Shapley value	τ value	Maxmin core	ECSM	Shapley value	τ value	Maxmin core	ECSM
Shapley value	–	0.9797	0.9848	–	–	0.8559	0.7977	0.7846
τ value	–	–	0.9443	–	–	–	0.7384	0.9895
Maxmin core	–	–	–	–	–	–	–	0.6443

minimum, maximum and the overall satisfaction of the coalitions are indicated in this table.

Figure 4 illustrates the core of the four players game in barycentric coordinates which are computed by TUGlab. Note that only the cost savings allocated to the players 1, 2 and 3 are shown in this figure. The amount allocated to the fourth player is implicit and can be computed from the efficiency condition for each point.

To investigate the similarity between the four methods of CGT, we suggest correlation between the cost saving allocations. For two imputations $\vec{y} = (y_1, y_2, \ldots, y_n)$ and $y\prime = (y'_1, y'_2, \ldots, y'_n)$, correlation measure is defined as follows

$$\rho(\overrightarrow{y_p}, \overrightarrow{y'_p}) = \frac{\sum_k \left(y_p - \overline{y_p}\right)\left(y'_p - \overline{y'_p}\right)}{\sqrt{\sum_k \left(y_p - \overline{y_p}\right)^2 \sum_k \left(y'_p - \overline{y'_p}\right)^2}}. \tag{44}$$

Table 10 illustrates the correlation measures for each pair of CGT methods. From the table we know that the solutions of Shapley value, τ value, maximin core, and ECSM are close in the symmetric example. However, in asymmetric example, the solutions of different CGT methods result in different solutions.

The following observation and managerial insights are derived from the numerical examples:

1. Due to synergy effects of the owners, the cost saving of coalition in a VRP can be considerable. For instance, there are 49.6 and 18 % cost saving in the grand coalition of symmetric and asymmetric examples that could be regarded as a good motivation for the cooperation among the companies.
2. Each player has a different role in the coalition. For instance, in symmetric example, if owner 1 joins the coalition {2, 3}, the cost savings would be 19.677, and

if he joins the coalition {2, 4}, then the cost savings would be 20.757, and 19.491 if he joints the coalition {3, 4}.

3. The results from various mechanisms of the CGT are somewhat different. Therefore, these characteristics must be considered in the contracts between companies and as a result, the cost saving assigned to each player can be thoroughly specified.

Conclusion and further research

This paper proposed a new vehicle routing problem for minimizing the transportation costs when there are multiple players (i.e. owners). Considering a multi-depot VRP, the cooperation among owners of the depots was suggested by sharing their vehicles. The transportation costs of collaboration among different owners were evaluated. It was shown that the transportation costs were decreased that could lead to considerable cost savings. Hence, for a fair allocation of the cost savings among the owners, a set of methods based on the CGT theory including Shapley value, τ value, least core, and equal cost saving method were proposed. Two symmetric and asymmetric examples were provided to evaluate the proposed concept. Based on the results obtained from the cost saving allocation, the owners can decide about joining the coalitions that bring them more profit.

There are several directions and suggestions for future research works. First of all, it was assumed that in the MDVRP, the vehicles are similar. Changing this assumption in the proposed MDVRP, can offer a new CoMDVRP. Furthermore, other vehicle routing problems such as time windows and etc. can be used to present a cooperative model. Finally, this study assumes that the cost parameters of the owners are common knowledge; however, it is unlikely that the owners would be privy to real cost parameters. This situation would lead to a collaborative game model under asymmetric information that is interesting but challenging.

References

Aras N, Aksen D, Tekin MT (2011) Selective multi-depot vehicle routing problem with pricing. Transp Res Part C Emerg Technol 19(5):866–884

Barron EN (2013) Game theory: an introduction, vol 2. Wiley, NewYork

Branzei R, Dimitrov D, Tijs S (2008) Models in cooperative game theory, vol 556. Springer Science & Business Media, NewYork

Casas-Méndez B, Garcıa-Jurado I, Van den Nouweland A, Vázquez-Brage M (2003) An extension of the τ value to games with coalition structures. Eur J Oper Res 148(3):494–513

Charles SL, Hansen DR (2008) An evaluation of activity-based costing and functional-based costing: a game-theoretic approach. Int J Prod Econ 113(1):282–296

Contardo C, Martinelli R (2014) A new exact algorithm for the multi-depot vehicle routing problem under capacity and route length constraints. Discret Optim 12:129–146

Cordeau JF, Laporte G, Savelsbergh MWP, Vigo D (2007) Chapter 6 vehicle routing. Handb Oper Res Manag Sci 14:367–428

Crevier B, Cordeau JF, Laporte G (2007) The multi-depot vehicle routing problem with inter-depot routes. Eur J Oper Res 176(2):756–773

Frisk M, Göthe-Lundgren M, Jörnsten K, Rönnqvist M (2010) Cost allocation in collaborative forest transportation. Eur J Oper Res 205(2):448–458

Gilles RP (2010) The cooperative game theory of networks and Hierarchies. Springer Science & Business Media, NewYork

Gillies DB (1953) Some theorems on n-person games. Ph.D. Thesis, Princeton University Press, Princeton

Hafezalkotob A, Makui A (2015) Cooperative maximum-flow problem under uncertainty in logistic networks. Appl Math Comput 250:593–604

Hasani-Goodarzi A, Tavakkoli-Moghaddam R (2012) Capacitated vehicle routing problem for multi-product cross-docking with split deliveries and pickups. Proc Soc Behav Sci 62:1360–1365

Ho W, Ho GT, Ji P, Lau HC (2008) A hybrid genetic algorithm for the multi-depot vehicle routing problem. Eng Appl Artif Intell 21(4):548–557

Jin M, Kai L, Burak E (2008) A column generation algorithm for the vehicle routing problem with split delivery. Oper Res Lett 36(2):265–270

Kang KH, Lee YH, Lee BK. An exact algorithm for multi depot and multi period vehicle scheduling problem. Department of Industrial and Information Engineering, Yonsei University, 134 Shinchon-Dong, Seodaemoon-Gu, SEOUL 120-749 Korea

Karakati CS, Podgorelec V (2014) A survey of genetic algorithms for solving multi depot vehicle routing problem. Appl Soft Comput J

Lehoux N, Audy JF, Sophie DA, Rönnqvist M (2009) Issues and experiences in logistics collaboration. In: Leveraging knowledge for innovation in collaborative networks. Springer Berlin Heidelberg, NewYork, pp 69–76

Liu R, Jiang Z, Fung RY, Chen F, Liu X (2010) Two-phase heuristic algorithms for full truckloads multi-depot capacitated vehicle routing problem in carrier collaboration. Comput Oper Res 37(5):950–959

Lozano S, Moreno P, Adenso-Díaz B, Algaba E (2013) Cooperative game theory approach to allocating benefits of horizontal cooperation. Eur J Oper Res 229(2):444–452

McCain RA (2008) Cooperative games and cooperative organizations. J Soc Econ 37(6):2155–2167

Mirás Calvo MA, SánchezRodrıguez E (2006) TUGlab: a cooperative game theory toolbox. http://webs.uvigo.es/mmiras/TUGlab/TUGlabICM06.pdf. Accessed 08 Jan 13

Montoya-Torres JR, Franco JL, Isaza SN, Jiménez HF, Herazo-Padilla N (2014) A literature review on the vehicle routing problem with multiple depots. Comput Ind Eng

Paolo T, Daniele V (2002) Models, relaxations and exact approaches for the capacitated vehicle routing problem. Discret Appl Math 123:487–512

Prescott-Gagnon E, Desaulniers G, Rousseau LM (2014) Heuristics for an oil delivery vehicle routing problem. Flex Serv Manuf J 26(4):516–539

Ray S, Soeanu A, Berger J, Debbabi M (2014) The multi-depot split-delivery vehicle routing problem: model and solution algorithm. Knowl Based Syst 71:238–265

Rosenthal RE (1988) Chapter 2: A GAMS tutorial. GAMS: a user's guide. The Scientific Press, Redwood City

Salhi S, Imran A, Wassan NA (2014) The multi-depot vehicle routing problem with heterogeneous vehicle fleet: formulation and a variable neighborhood search implementation. Comput Oper Res 52:315–325

Shapley LS (1952) A value for n-person games. No. RAND-P-295. RAND CORP SANTA MONICA CA

Stefan E, Maud G-L, Peter V (1998) The traveling salesman game: an application of cost allocation in a gas and oil company. Ann Oper Res 82:453–471

Sterzik S, Kopfer H, Yun WY (2012) Reducing hinterland transportation costs through container sharing. Flex Serv Manuf J 1–21

Surekha P, Sumathi S (2011) Solution to multi-depot vehicle routing problem using genetic algorithms. World Appl Progr 1(3):118–131

Vanovermeire C, Sörensen K (2014) Measuring and rewarding flexibility in collaborative distribution, including two-partner coalitions. Eur J Oper Res 239(1):157–165

Vidović M, Radivojević G, Raković B (2011) Vehicle routing in containers pickup up and delivery processes. Proc Soc Behav Sci 20:335–343

Wang X, Kopfer H (2015) Rolling horizon planning for a dynamic collaborative routing problem with full-truckload pickup and delivery requests. Flex Serv Manuf J 1–25

Wasner M, Zapfel G (2004) An integrated multi-depot hub-location vehicle routing model for network planning of parcel service. Int J Prod Econ 90(3):403–419