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ORIGINAL RESEARCH

Comparative analysis of profit between three dissimilar repairable redundant systems using supporting external device for operation

Ibrahim Yusuf

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Abstract The importance in promoting, sustaining industries, manufacturing systems and economy through reliability measurement has become an area of interest. The profit of a system may be enhanced using highly reliable structural design of the system or subsystem of higher reliability. On improving the reliability and availability of a system, the production and associated profit will also increase. Reliability, availability and profit are some of the most important factors in any successful industry and manufacturing settings. In this paper, we compare three different repairable redundant systems using an external supporting device for operation based on the profit. Explicit expressions for the busy period of repairmen, steady-state availability and profit function are derived using linear first-order differential equations. Furthermore, we compare the profit for the three systems and find that system I is more profitable than systems II and III.

Keywords Supporting device · Profit · Redundant system

List of symbols

S_i	Transition states, $i = 0, 1, 2, 3, 4, 5, 6$ for system I, $i = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ for system II and $i = 0, 1, 2, 3, 4, 5, 6, 7, 8$ for system III
α_1	Repair rate of unit A_k for both systems, $k = 1, 2$
α_2	Repair rate of unit B_k for both systems, $k = 1, 2$
β_1	Failure rate of unit A_k for both systems, $k = 1, 2$
β_2	Failure rate of unit B_k for both systems, $k = 1, 2$
α_3	Repair rate of the supporting unit for both systems
β_3	Failure rate of the supporting unit for both systems
$P^k(t),\ \ k = \text{I,II,II}$	Probability row vector
$P_y(t),\ y = 1, 2, 3$	Probability that the system is in state S_i
$P_{iO}/P_{iR}/P_{iG}$	Supporting external device is operation/under repair/is idle, $i = 1$ for system I and $i = 1, 2$ in systems II and III
$A_{kO}/A_{KR}/A_{kW}/A_{kG}$	Unit is in operation/under repair/waiting for repair/unit is idle in subsystem A
$B_{kO}/B_{KR}/B_{kW}/B_{kG}$	Unit is in operation/under repair/waiting for repair/unit is idle in subsystem B

Introduction

Proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order. System availability represents the percentage of time the system is available to users. Availability and profit of an industrial system are becoming an increasingly important issue. Where the availability of a system increases, the related profit will also increase. In real-life situations, we often encounter cases where the systems that cannot work without

I. Yusuf (✉)
Dutse, Nigeria
e-mail: ibrahimyusuffagge@gmail.com

the help of external supporting devices connect to such systems. These external supporting devices are systems themselves that are bound to fail. Such systems are found in power plants, manufacturing systems and industrial systems. Failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order. Availability and profit of an industrial system may be enhanced using highly reliable structural design of the system or subsystem of higher reliability. On improving the reliability and availability of a system/subsystem, the production and associated profit will also increase. Increase in production leads to the increase of profit. This can be achieved by maintaining reliability and availability at the highest order. To achieve high production and profit, the system should remain operative for a maximum possible duration. It is important to consider profit as well as the quality requirement.

The problem considered in this paper is more general than the work of Yusuf (2013). This paper is devoted to deal with profit comparison between three dissimilar redundant systems that worked with the help of an external supporting device. The contributions of this paper are twofold. The first is to develop the explicit expressions for steady-state availability, steady-state busy period due to failure units in subsystems and steady-state busy period due to failure of supporting unit and profit function for the three systems under study. The second is to perform a numerical investigation of the effect of the system parameters on profit.

Literature review

Reliability plays a role in the overall system performance. System reliability has been considered as a significant factor in most of the system performance-related studies (Farooquie et al. 2012; Faghihinia and Mollaverdi 2012; Khalili-Damghani and Amiri 2012; Khalili-Damghani et al. 2013; Kumar and Jain 2013; Lal et al. 2013; Taghizade and Hafezi 2012; Tewari et al. 2012). Various systems under different operational situations and circumstances in assessing the reliability and availability characteristics have been analyzed by different researchers. Such analyses include multiple vacation policies with an unreliable server (Jain et al. 2013), queuing model with state dependence and vacations (Singh et al. 2012), comparative analysis of availability or redundant system (Ke and Chu 2007), comparison between two units of cold and warm standby systems in changing weather (Mokaddis et al. 2010), comparative analysis of availability of three systems with general repairs, reboot delay and switching failure (Wang and Chen 2009), comparison of availability between two systems with warm standby units and different imperfect coverage (Wang et al. 2012), comparison of reliability and availability between four systems with warm standby components standby switching failures (Wang et al. 2006) and comparative analysis of some reliability characteristics between two systems requiring supporting devices for operation (Yusuf 2013). Recently, Izadi and Kimiagari (2014) developed an approach base on genetic algorithm and Monte Carlo simulation toward the design of a distribution network under demand uncertainty.

Description of the systems and states of the systems

Both systems consist of two subsystems A and B in series. System I consists of subsystem A containing two units A_1 and A_2 in cold standby; subsystem B has two units B_1 and B_2 units in cold standby and with one external supporting unit connected to subsystems A and B. System II consists of subsystem A containing two units A_1 and A_2 in cold standby; subsystem B has two units B_1 and B_2 unit in cold standby and with two external supporting units, one connected each to subsystems A and B. System III consists of subsystem A containing two units A_1 and A_2 in active parallel; subsystem B contains two units B_1 and B_2 in cold standby and with two external supporting units, one connected each to subsystems A and B. A unit in subsystem A for both systems failed with a failure rate of β_1 and repair rate of α_1. A unit in subsystem B for both systems failed with a failure rate of β_2 and repair rate of α_2. The supporting unit for both systems failed with a failure rate of β_3 and a repair rate of α_3. Systems work if one unit of subsystem A and one unit of subsystem B with corresponding supporting unit work. System failure occurs when both A_1 and A_2 or B_1 and B_2 or a supporting unit failed. Each system is attended by three repairmen: one attends to subsystem A, one to subsystem B and one to the supporting unit. The states of the systems are as follows:

System I

Up states

$S_0(A_{1O}, A_{2S}, B_{1O}, B_{2S}, P_O)$,
$S_1(A_{1R}, A_{2O}, B_{1O}, B_{2S}, P_O)$,
$S_2(A_{1O}, A_{2S}, B_{1R}, B_{2S}, P_O)$,
$S_3(A_{1R}, A_{2O}, B_{1R}, B_{2O}, P_O)$.

Down states

$S_4(A_{1R}, A_{2G}, B_{1G}, B_{2S}, P_R)$,
$S_5(A_{1W}, A_{2R}, B_{1R}, B_{2G}, P_G)$,
$S_6(A_{1R}, A_{2G}, B_{1W}, B_{2R}, P_G)$.

System II

Up states

$S_0((A_{1O}, A_{2S}), P_{1O}, (B_{1O}, B_{2S}), P_{2O})$,
$S_1((A_{1R}, A_{2O}), P_{1O}, (B_{1O}, B_{2S}), P_{2O})$,
$S_2((A_{1O}, A_{2S}), P_{1O}, (B_{1R}, B_{2S}), P_{2O})$,
$S_3((A_{1R}, A_{2O}), P_{1O}, (B_{1R}, B_{2O}), P_{2O})$.

Down states

$S_4((A_{1W}, A_{2R}), P_{1G}, (B_{1G}, B_{2S}), P_{2G})$,
$S_5((A_{1R}, A_{2S}), P_{1R}, (B_{1G}, B_{2S}), P_{2G})$,
$S_6((A_{1G}, A_{2S}), P_{1G}, (B_{1W}, B_{2R}), P_{2G})$,
$S_7((A_{1G}, A_{2S}), P_{1G}, (B_{1R}, B_{2G}), P_{2R})$,
$S_8((A_{1W}, A_{2R}), P_{1G}, (B_{1R}, B_{2G}), P_{2G})$,
$S_9((A_{1R}, A_{2G}), P_{1G}, (B_{1W}, B_{2R}), P_{2G})$.

System III

Up states

$S_0((A_{1O}, A_{2O}), P_{1O}, (B_{1O}, B_{2S}), P_{2O})$,
$S_1((A_{1R}, A_{2O}), P_{1O}, (B_{1O}, B_{2S}), P_{2O})$,
$S_2((A_{1O}, A_{2O}), P_{1O}, (B_{1R}, B_{2O}), P_{2O})$,
$S_3((A_{1R}, A_{2O}), P_{1O}, (B_{1R}, B_{2O}), P_{2O})$.

Down states

$S_4((A_{1G}, A_{2G}), P_{1R}, (B_{1G}, B_{2S}), P_{2G})$,
$S_5((A_{1W}, A_{2R}), P_{1G}, (B_{1R}, B_{2G}), P_{2G})$,
$S_6((A_{1R}, A_{2G}), P_{1G}, (B_{1W}, B_{2R}), P_{2G})$,
$S_7((A_{1W}, A_{2R}), P_{1G}, (B_{1G}, B_{2G}), P_{2G})$,
$S_8((A_{1G}, A_{2G}), P_{1G}, (B_{1W}, B_{2R}), P_{2G})$.

Model formulation

Analysis of system I

Let $P(t) = [P_0(t), P_1(t), P_2(t), P_3(t), P_4(t), P_5(t), P_6(t)]$ be the probability vector for system I at time $t \geq 0$. Relating the state of the system at time t and $t + \mathrm{d}t$, the steady state for system I can be expressed in the form:

$$\frac{\mathrm{d}}{\mathrm{d}t}(P(t)) = T_1 P(t) \tag{1}$$

where

$$T_1 = \begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\beta_2) & 0 & \alpha_2 & 0 & 0 & 0 \\ \beta_2 & 0 & -(\alpha_2+\beta_1) & \alpha_1 & 0 & 0 & 0 \\ 0 & \beta_2 & \beta_1 & -(\sum_{m=1}^{2}\alpha_m + \sum_{n=1}^{3}\beta_n) & \alpha_3 & \alpha_1 & \alpha_2 \\ 0 & 0 & 0 & \beta_3 & -\alpha_3 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & \alpha_1 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_2 \end{bmatrix}.$$

Availability, busy period and profit analysis

For the analysis of availability case of system I, we use the following procedure to obtain the steady-state availability, busy period and profit function. In steady state, the derivatives of the state probabilities become zero and we obtain

$$\begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\beta_2) & 0 & \alpha_2 & 0 & 0 & 0 \\ \beta_2 & 0 & -(\alpha_2+\beta_1) & \alpha_1 & 0 & 0 & 0 \\ 0 & \beta_2 & \beta_1 & -(\sum_{m=1}^{2}\alpha_m + \sum_{n=1}^{3}\beta_n) & \alpha_3 & \alpha_1 & \alpha_2 \\ 0 & 0 & 0 & \beta_3 & -\alpha_3 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & \alpha_1 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_2 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{2}$$

Solving (2) and using the following normalizing condition

$$\sum_{l=0}^{6} P_l(\infty) = 1 \tag{3}$$

we obtain $P_0(\infty),\ P_1(\infty),\ \ldots,P_6(\infty)$.

Let V_1 be the time to failure of the system for system *I*. The explicit expressions for the steady-state availability, state busy period of repairman due to failure of units A_k and B_k, busy period of repairman due to failure of supporting are as follows:

$$\begin{aligned}A_{V_1^!}(\infty) &= (P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty)) \\ &= \frac{b_1}{b_2}\end{aligned} \tag{4}$$

$$B_{V_1^!}(\infty) = \begin{pmatrix} P_1(\infty) + P_2(\infty) + P_3(\infty) \\ +P_5(\infty) + P_6(\infty) \end{pmatrix} = \frac{b_3}{b_2} \tag{5}$$

$$B^*_{V_1^!}(\infty) = P_4(\infty) = \frac{b_4}{b_2}. \tag{6}$$

From the states of system I as given above, the units and supporting unit are subjected to corrective maintenance in states 1, 2, 3, 5 and 6 and 4, respectively. Let C_0, C_1 and C_2 be the revenue generated when the system is in working state and has no income when in failed state, the cost of each repair for failed units (corrective maintenance) and repair of supporting unit, respectively. The expected total profit per unit time incurred to the system in the steady state is

Profit = total revenue generated − cost incurred by the repairman due to failure of units − cost incurred when repairing the failed supporting units.

$$\mathrm{PF}_1 = C_0 A_{V_1^!}(\infty) - C_1 B_{V_1^!}(\infty) - C_2 B_{V_2^!}(\infty) \tag{5}$$

where

PF_1 is the profit incurred to system I,

$$b_1 = \alpha_1^2\alpha_2^2\alpha_3 + \alpha_1\alpha_2^2\alpha_3\beta_1 + \alpha_1^2\alpha_2\alpha_3\beta_2 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2$$

$$\begin{aligned}b_2 = {}& \alpha_1^2\alpha_2^2\alpha_3 + \alpha_1^2\alpha_2\alpha_3\beta_2 + \alpha_1\alpha_2^2\alpha_3\beta_1 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2 + \\ & \alpha_1\alpha_2\beta_1\beta_2\beta_3 + \alpha_1\alpha_3\beta_1\beta_2^2 + \alpha_2\alpha_3\beta_1^2\beta_2\end{aligned}$$

$$\begin{aligned}b_3 = {}& \alpha_1\alpha_2^2\alpha_3\beta_1 + \alpha_1^2\alpha_2\alpha_3\beta_2 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2 \\ & + \alpha_2\alpha_3\beta_1^2\beta_2 + \alpha_1\alpha_3\beta_1\beta_2^2\end{aligned}$$

$$b_4 = \alpha_1\alpha_2\beta_1\beta_2\beta_3.$$

Analysis of system II

Let $P(t) = [P_0(t), P_1(t), P_2(t), P_3(t), \ldots, P_9(t)]$ be the probability vector for system II at time $t \geq 0$. Relating the state of the system at time t and $t + \mathrm{d}t$, the steady state for system I can be expressed in the form:

$$\frac{\mathrm{d}}{\mathrm{d}t}(P(t)) = T_2 P(t) \tag{6}$$

where

$$T_2 = \begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1 + \sum_{n=1}^{3}\beta_n) & 0 & \alpha_2 & \alpha_1 & \alpha_3 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & -(\alpha_2 + \sum_{n=1}^{3}\beta_n) & \alpha_1 & 0 & 0 & \alpha_2 & \alpha_3 & 0 & 0 \\ 0 & \beta_2 & \beta_1 & -(\sum_{m=1}^{2}\alpha_m + \sum_{n=1}^{2}\beta_n) & 0 & 0 & 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_3 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 \end{bmatrix}$$

Availability, busy period and profit analysis

For the analysis of availability case of system II, we use the following procedure to obtain the steady-state availability, busy period and profit function. In steady state, the derivatives of the state probabilities become zero and we obtain

$$\begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\sum_{n=1}^{3}\beta_n) & 0 & \alpha_2 & \alpha_1 & \alpha_3 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & -(\alpha_2+\sum_{n=1}^{3}\beta_n) & \alpha_1 & 0 & 0 & \alpha_2 & \alpha_3 & 0 & 0 \\ 0 & \beta_2 & \beta_1 & -(\sum_{m=1}^{2}\alpha_m+\sum_{n=1}^{2}\beta_n) & 0 & 0 & 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_3 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \\ P_8(t) \\ P_9(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{7}$$

Solving (7) and using the following normalizing condition

$$\sum_{l=0}^{9} P_l(\infty) = 1, \tag{8}$$

we obtain $P_0(\infty),\ P_1(\infty),\ \ldots, P_9(\infty)$.

Let V_2 be the time to failure of the system for system II. The explicit expressions for the steady-state availability, busy period of repairman due to failure of units A_k and B_k and busy period of repairman due to failure of supporting unit are as follows:

$$A_{V_2^2}(\infty) = (P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)) = \frac{a_1}{a_2} \tag{9}$$

$$B_{V_2^2}(\infty) = \begin{pmatrix} P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+ \\ P_6(\infty)+P_8(\infty)+P_9(\infty) \end{pmatrix} = \frac{a_3}{a_2} \tag{10}$$

$$B^*_{V_2^2}(\infty) = P_5(\infty)+P_7(\infty) = \frac{a_4}{a_2}. \tag{11}$$

From the states of system II as given above, the units and supporting unit are subjected to corrective maintenance in states 1, 2, 3, 4, 6, 8 and 9 and 5 and 7. Let C_0, C_1 and C_2 be the revenue generated when the system is in working state and has no income when in failed state, the cost of each repair for failed units (corrective maintenance) and repair of supporting unit, respectively. The expected total profit per unit time incurred to the system in the steady state is

Profit = total revenue generated − cost incurred by the repairman due to preventive maintenance − cost incurred when repairing the failed units.

$$\mathrm{PF}_2 = C_0 A_{V_1^2}(\infty) - C_1 B_{V_1^2}(\infty) - C_2 B_{V_2^2}(\infty). \tag{12}$$

where

PF_2 is the profit incurred to system II

$$a_1 = \alpha_1^2\alpha_2^2\alpha_3 + \alpha_1\alpha_2^2\alpha_3\beta_1 + \alpha_1^2\alpha_2\alpha_3\beta_2 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2$$

$$a_2 = \alpha_1^2\alpha_2^2\alpha_3 + \alpha_1^2\alpha_2\alpha_3\beta_2 + \alpha_1\alpha_2^2\alpha_3\beta_1 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2 + \alpha_1^2\alpha_3\beta_2^2 + \alpha_1^2\alpha_2\beta_2\beta_3 + \alpha_2\alpha_3\beta_1^2\beta_2 + \alpha_1\alpha_2^2\beta_1\beta_3 + \alpha_1\alpha_3\beta_1\beta_2^2 + \alpha_2^2\alpha_3\beta_1^2$$

$$a_3 = \alpha_1\alpha_2^2\alpha_3\beta_1 + \alpha_1^2\alpha_2\alpha_3\beta_2 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2 + \alpha_2\alpha_3\beta_1^2\beta_2 + \alpha_1\alpha_3\beta_1\beta_2^2 + \alpha_2^2\alpha_3\beta_1^2 + \alpha_1^2\alpha_3\beta_2^2$$

$$a_4 = \alpha_1\alpha_2^2\beta_1\beta_3 + \alpha_1^2\alpha_2\beta_2\beta_3.$$

Analysis of system III

Let $P(t) = [P_0(t), P_1(t), P_2(t), P_3(t), \ldots, P_8(t)]$ be the probability vector for system III at time $t \geq 0$. Relating the state of the system at time t and $t + \mathrm{d}t$, the steady state for system III can be expressed in the form:

$$\frac{\mathrm{d}}{\mathrm{d}t}(P(t)) = T_3 P(t) \tag{13}$$

where

$$T_3 = \begin{bmatrix} -\sum_{n=1}^{3}\beta_n & \alpha_1 & \alpha_2 & 0 & \alpha_3 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\sum_{n=1}^{2}\beta_n) & 0 & \alpha_2 & 0 & 0 & 0 & \alpha_1 & 0 \\ \beta_2 & 0 & -(\alpha_2+\sum_{n=1}^{2}\beta_n) & \alpha_1 & 0 & 0 & 0 & 0 & \alpha_2 \\ 0 & \beta_2 & \beta_1 & -(\sum_{m=1}^{2}\alpha_m+\sum_{n=1}^{2}\beta_n) & 0 & \alpha_1 & \alpha_2 & 0 & 0 \\ \beta_3 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_2 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 \end{bmatrix}.$$

Availability and busy period analysis

For the analysis of availability case of system III, we use the following procedure to obtain the steady-state availability, busy period and profit function. In the steady state, the derivatives of the state probabilities become zero and we obtain

$$\begin{bmatrix} -\sum_{n=1}^{3}\beta_n & \alpha_1 & \alpha_2 & 0 & \alpha_3 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\sum_{n=1}^{2}\beta_n) & 0 & \alpha_2 & 0 & 0 & 0 & \alpha_1 & 0 \\ \beta_2 & 0 & -(\alpha_2+\sum_{n=1}^{2}\beta_n) & \alpha_1 & 0 & 0 & 0 & 0 & \alpha_2 \\ 0 & \beta_2 & \beta_1 & -(\sum_{m=1}^{2}\alpha_m+\sum_{n=1}^{2}\beta_n) & 0 & \alpha_1 & \alpha_2 & 0 & 0 \\ \beta_3 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_2 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \\ P_8(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{14}$$

Solving (14) and using the following normalizing condition

$$\sum_{l=0}^{8} P_l(\infty) = 1, \tag{15}$$

we obtain $P_0(\infty), P_1(\infty), \ldots, P_8(\infty)$.

Let V_3 be the time to failure of the system for system III. The explicit expressions for the steady-state availability, busy period of repairman due to failure of units A_k and B_k and busy period of repairman due to failure of supporting unit are as follows:

$$A_{V_3^3}(\infty) = (P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty)) = \frac{d_1}{d_2} \quad (16)$$

$$B_{V_3^3}(\infty) = \begin{pmatrix} P_1(\infty) + P_2(\infty) + P_3(\infty) + P_5(\infty) \\ +P_6(\infty) + P_7(\infty) + P_8(\infty) \end{pmatrix} = \frac{d_3}{d_2} \quad (17)$$

$$B^*_{V_3^3}(\infty) = P_4(\infty) = \frac{d_4}{d_2}. \quad (18)$$

From the states of system III as given above, the units and supporting unit are subjected to corrective maintenance in states 1, 2, 3, 4, 5, 6, 7, 8 and 4. Let C_0, C_1 and C_2 be the revenue generated when the system is in working state and has no income when in failed state, the cost of each repair for failed units (corrective maintenance) and repair of supporting unit, respectively. The expected total profit per unit time incurred to the system in the steady state is

Profit = total revenue generated − cost incurred by the repairman due to preventive maintenance − cost incurred when repairing the failed units.

$$PF_3 = C_0 A_{V_1^3}(\infty) - C_1 B_{V_1^3}(\infty) - C_2 B_{V_2^3}(\infty) \quad (19)$$

where

PF_3 is the profit incurred to system III.

$$d_1 = \alpha_1^2\alpha_2^2\alpha_3 + \alpha_1\alpha_2^2\alpha_3\beta_1 + \alpha_1^2\alpha_2\alpha_3\beta_2 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2$$

$$d_2 = \alpha_1^2\alpha_2^2\alpha_3 + \alpha_1^2\alpha_2\alpha_3\beta_2 + \alpha_1\alpha_2^2\alpha_3\beta_1 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2 + \alpha_1^2\alpha_3\beta_2^2 + \alpha_2\alpha_3\beta_1^2\beta_2 + \alpha_1^2\alpha_2^2\beta_3 + \alpha_1\alpha_3\beta_1\beta_2^2 + \alpha_2^2\alpha_3\beta_1^2$$

$$d_3 = \alpha_1\alpha_2^2\alpha_3\beta_1 + \alpha_1^2\alpha_2\alpha_3\beta_2 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2 + \alpha_2\alpha_3\beta_1^2\beta_2 + \alpha_1\alpha_3\beta_1\beta_2^2 + \alpha_2^2\alpha_3\beta_1^2 + \alpha_1^2\alpha_3\beta_2^2$$

$$d_4 = \alpha_1^2\alpha_2^2\beta_3.$$

Results and discussions

In this section, we numerically obtained and compared the results for mean time to system failure, system availability and profit function for all the developed models. For each model, the following set of parameter values were fixed throughout the simulations for consistency for the three cases.

$$\beta_1 = \beta_3 = 0.3, \beta_2 = 0.4, \alpha_1 = \alpha_2 = \alpha_3 = 0.5, C_0 = 2,000, C_1 = 1,000, C_2 = 500.$$

Figures 1, 2 and 3 show the profit results for the three systems being studied against the repair rate α_1, α_2 and α_3. It is clear from the figures that system I has higher profit as compared to the other two systems. The differences between the profit of system I and the other two systems widen as α_1, α_2 and α_3 increase. There is no significant difference between the availability of system II and that of system III with respect to α_1, α_2 and α_3. However, one can say that the results from Fig. 3 show slightly more distinction between the profit of system II and that of system III. These tend to suggest that system I is better than the other systems.

Figures 4, 5 and 6 show the profit for the three systems against the failure rates β_1, β_2 and β_3. In these figures, it can be seen that the profit of system I decreases more slowly than those of the other two systems. By comparing

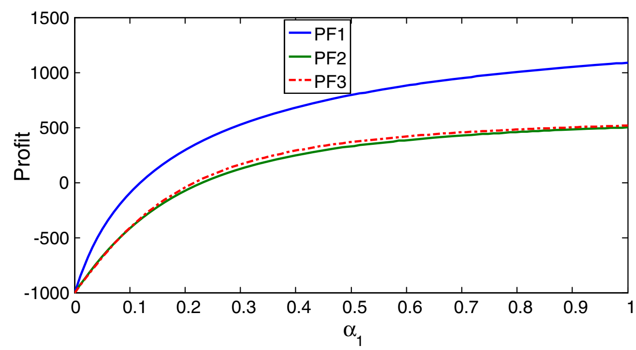

Fig. 1 Profit against α_1

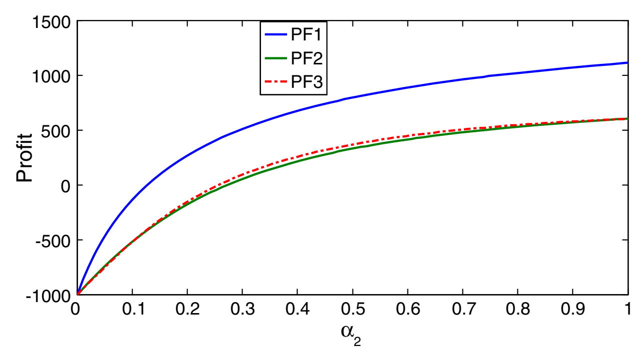

Fig. 2 Profit against α_2

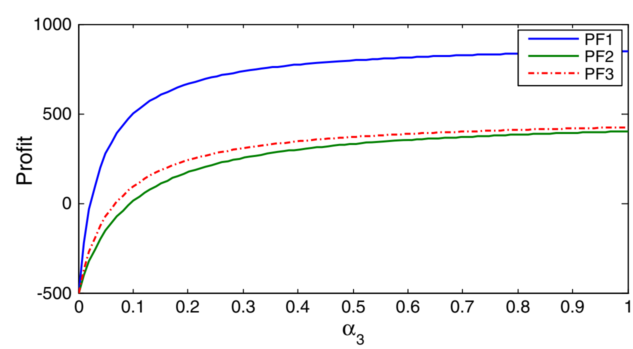

Fig. 3 Profit against α_3

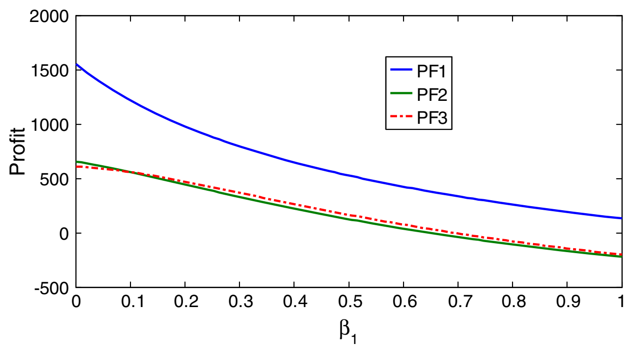

Fig. 4 Profit against β_1

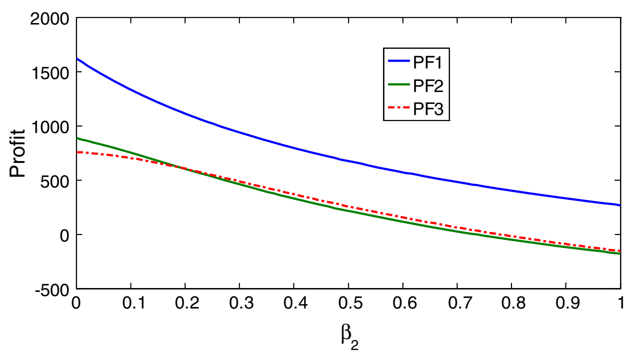

Fig. 5 Profit against β_2

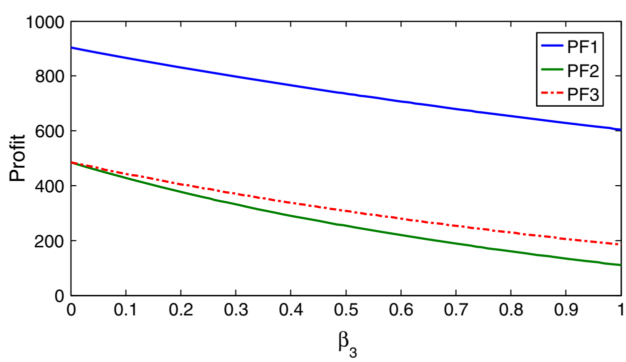

Fig. 6 Profit against β_3

systems II and III, it can be observed that there is not much difference between the two. System III decreases a little faster than system II. We can conclude as before that system I is better than the other two systems in all the three figures.

Conclusion

In this paper, we analyzed three dissimilar systems, each consisting of two subsystems A and B, each containing two units with supporting unit attached to the systems. Explicit expressions for steady-state availability, busy period and profit function for the three systems were derived and comparative analysis was also performed numerically. It is evident from Figs. 1, 2, 3, 4, 5, 6 that the optimal system is system I. Models presented in this paper are important to engineers, maintenance managers and plant management for proper maintenance analysis, decision and safety of the system as a whole. The models will also assist engineers, decision makers and plant management to avoid an incorrect reliability assessment and consequent erroneous decision making, which may lead to unnecessary expenditures, incorrect maintenance scheduling and reduction of safety standards.

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