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ORIGINAL RESEARCH

Economic design of $\bar{x}$ control charts considering process shift distributions

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Abstract Process shift is an important input parameter in the economic design of control charts. Earlier $\bar{x}$ control chart designs considered constant shifts to occur in the mean of the process for a given assignable cause. This assumption has been criticized by many researchers since it may not be realistic to produce a constant shift whenever an assignable cause occurs. To overcome this difficulty, in the present work, a distribution for the shift parameter has been considered instead of a single value for a given assignable cause. Duncan's economic design model for $\bar{x}$ chart has been extended to incorporate the distribution for the process shift parameter. It is proposed to minimize total expected loss-cost to obtain the control chart parameters. Further, three types of process shifts namely, positively skewed, uniform and negatively skewed distributions are considered and the situations where it is appropriate to use the suggested methodology are recommended.

Keywords Genetic algorithms · Economic design · Control chart · Process shift distribution

Notation

a_1	Fixed cost of sampling ($)
a_2	Variable cost of sampling ($)
a_3	Cost of finding an assignable cause ($)
a_4	Cost of investigating a false alarm ($)
a_5	Hourly penalty cost for operating in the out-of-control state ($)
λ	Process failure rate (h^{-1})
g	Time to test and interpret the result per sample unit (h)
D	Time required to find the assignable cause (h)
δ	Process shift parameter when a single shift is considered
α	Probability of type I error
β	Probability of type II error
τ	Expected time of occurrence of the assignable cause between consecutive samples
$\phi(\cdot)$	Distribution function of standard normal random variable
n	Sample size
h	Sampling interval (h)
k	Width of the control limit
$\Gamma(\cdot)$	Gamma function
y	Beta distributed random variable of process shift
δ_1	Lower limit of the process shift parameter range
δ_2	Upper limit of the process shift parameter range
p, q	Parameters of beta distribution
$E(L)$	Expected loss-cost per hour ($ h^{-1})
TE(L)	Total expected loss-cost per hour ($ h^{-1})

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Introduction

In the economic design of control charts a set of cost and process parameters are used to obtain the control chart parameters. For an $\bar{x}$ control chart the input parameters include certain cost parameters such as cost of false alarm, the cost of finding an assignable cause etc., and certain process parameters like failure rate of the process and process shift parameter. Economic designs are observed to be very sensitive to process shift parameter (δ) in comparison with other parameters (Montgomery 1980). In order to obtain the full benefits of economic designs, one has to consider the appropriate values of shift parameter. Earlier designs relied on the use of constant process shift for a single assignable cause. This has been criticized by some of the researchers since it is not realistic to assume that a single assignable cause would always produce the same shift whenever it occurs. The following statements support the significance of the process shift in the economic designs:

> Chiu and Wetherill (1974): "δ is a critical risk parameter; of which we must strive to obtain an accurate estimate."
> Saniga (1992): "economic models are economic only if the shift upon which the model is developed is the shift that occurs".
> Ho and Case (1994): "the assumption that a certain cause will shift the process by a known shift is totally unrealistic."

Hence, the economic designs entail the use of appropriate process shift parameter for obtaining better designs. Since an assignable cause does not always produce a single shift, the process shifts can be considered in the following three ways:

1. Different known discrete values of the shifts.
2. A possible range for shift values with unknown distribution.
3. A known or assumed distribution of the shift values.

The procedures to find the optimal designs have been discussed for the first and second cases (Pignatillo and Tsai 1988; Linderman and Choo 2002; Vommi and Seetala 2007a, b). The present work deals with finding the optimal control chart design when the process shift follows a distribution within a range of values. Everlasting interest in finding the better control charts in process monitoring is evident from Niaki and Khedmati (2013) who proposed a new control chart to monitor the change time of multivariate binomial processes for step changes and drifts.

In the economic designs, various authors have studied genetic algorithms as a search tool when it is difficult to obtain closed form solutions by differentiating the cost functions. Arunkumar et al. (2007) studied the use of genetic algorithm for selection of vendors offering quantity discounts. Izadi and Kimiagari (2014) designed a distribution network under uncertain demand using genetic algorithm to minimize various costs.

Extension of Duncan's economic model

Duncan (1956) proposed an economic model for the optimum economic design of the $\bar{x}$ control chart. Duncan's economic model minimizes $E(L)$, the expected loss-cost per hour incurred by the process, to obtain the best parameters of the control chart. The expected loss-cost function is given below:

$$E(L) = \frac{(a_1 + a_2 n)}{h} + \frac{a_5\left[\frac{h}{1-\beta} - \tau + gn + D\right] + a_3 + a_4 \alpha e^{-\lambda h}/(1 - e^{-\lambda h})}{\left[\frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + D\right]} \quad (1)$$

The above expected loss-cost is based on the assumption that an assignable cause always produces a single known shift. In the present study, the assignable cause is assumed to produce different shift values within a known range. The distribution of the shift parameter δ can be known in the long run. Hence, the shift distribution can be assumed to follow a probability distribution. In order to extend the economic design for different process shifts, the expected loss-cost function given in Eq. (1) has to be modified such that it accommodates the process shift distribution. As the range of the process shift distribution will be finite and occurs between two possible values, a beta distribution function is considered for representing the process shift parameter (δ).

The probability density function of the process shift can be given as:

$$f_Y(y) = \frac{(y - \delta_1)^{p-1}(\delta_2 - y)^{q-1}}{B(p,q)(\delta_2 - \delta_1)^{p+q-1}}, \delta_1 \le y \le \delta_2 \quad (2)$$

where Y is the beta distributed random variable of process shift with δ_1 and δ_2 as lower and upper values, respectively. $B(p, q)$ is the beta function with p and q as its parameters and it is calculated as follows:

$$B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \quad (3)$$

Depending on the parameters p and q, the density function of the beta random variable will have different shapes as shown in the Fig. 1. For example, $p = 2$ and $q = 4$ represents a positively skewed distribution. Similarly, $p = 4$ and $q = 2$ represents a negatively skewed distribution. Whenever p and q take non-integer values, the beta function is called the incomplete beta function. The cumulative probability of the incomplete beta function is tabulated by Pearson as $\boldsymbol{B}_Y(p, q)$. Hence, Pearson's tables can be used to calculate the cumulative probability of a beta random variable, y. It can be noted that the tables are given for $p \ge q$. For $p < q$, $\boldsymbol{B}_Y(p,q) = 1 - \boldsymbol{B}_{(1-Y)}(p,q)$.

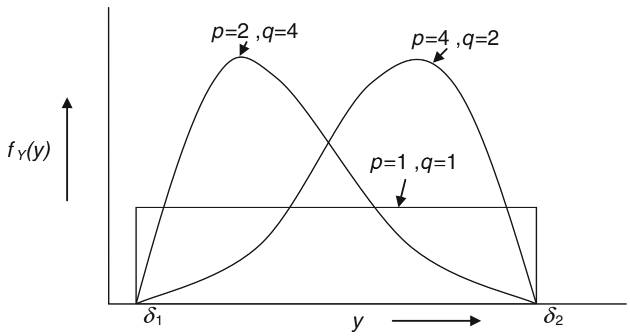

Fig. 1 Beta distribution

Considering a beta probability distribution function for the shift parameter, the probability that the shift parameter, y occurs is $f_Y(y)\mathrm{d}y$. Hence, the expected loss-cost is given by:

$$E(L|y)f_Y(y)\mathrm{d}y \tag{4}$$

In order to find out the best control chart parameters in the present case, the total expected loss-cost function as given by the following equation must be minimized.

$$\text{Total expected loss} - \text{cost} = \int_{\delta_1}^{\delta_2} [E(L|y)]f_Y(y)\mathrm{d}y \tag{5}$$

Using Duncan's loss-cost function $E(L)$, the extended total expected loss-cost per hour (TE(L)) incurred by the process can be written as:

$$\int_{\delta_1}^{\delta_2} [E(L|y)]f_Y(y)\mathrm{d}y = \int_{\delta_1}^{\delta_2} \left[\frac{(a_1 + a_2 n)}{h} + \frac{a_5\left[\frac{h}{1-\beta} - \tau + gn + D\right] + a_3 + a_4 \alpha e^{-\lambda h}/(1 - e^{-\lambda h})}{\left[\frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + D\right]} \right] [f_Y(y)]\mathrm{d}y \tag{6}$$

The probabilities of type I and type II error are computed as follows:

Probability of type I error, $\alpha = 2\varphi(-k)$.

Probability of type II error, $\beta = \int_{\delta_1}^{\delta_2} [1 - (\varphi(-k - y\sqrt{n}) + \varphi(y\sqrt{n} - k))]f_y(y)\mathrm{d}y$.

Optimization of the above Eq. (6) for finding the optimal control chart parameters is considered in the present economic design of control chart problem with a known process shift distribution. Since the optimization of the Eq. (6) involves finding the closed form solutions for the following Eqs. (7),(8) and (9), which poses difficulties in obtaining the control chart parameters, an evolutionary optimization technique, namely a genetic algorithm (GA) has been used to obtain near global optimum results.

$$\frac{\partial}{\partial n} \int_{\delta_1}^{\delta_2} [E(L|y)]f_Y(y)\mathrm{d}y = 0 \tag{7}$$

$$\frac{\partial}{\partial h} \int_{\delta_1}^{\delta_2} [E(L|y)]f_Y(y)\mathrm{d}y = 0 \tag{8}$$

$$\frac{\partial}{\partial k} \int_{\delta_1}^{\delta_2} [E(L|y)]f_Y(y)\mathrm{d}y = 0 \tag{9}$$

The total expected loss as given in Eq. (6) considers only the shift parameter ranges and neglects the variation in the hourly cost of not detecting the shifts. There is definitely a variation in the cost parameter a_5 since larger shifts tend to produce higher values of a_5 and smaller ones relatively a lower value of a_5. But, the variation in this cost parameter is omitted in the present analysis owing to the simplicity of the model on the following grounds: (1) the effect of cost parameters is not considerable in comparison with the shift parameter, (2) the larger shifts have the tendency to be detected early and the smaller shifts may take more time to be detected. Hence, on average the penalty of not detecting the shifts per hour can be taken as a constant.

Optimal designs considering shift distributions using GA

The best design parameters of the control chart are to be obtained by minimizing the total expected loss-cost function for a given set of cost and process parameters. Employing an appropriate optimization technique may not pose difficulties in optimizing the total loss-cost Eq. (6) for a given set of cost and process parameters. Use of GA in the economic designs is very common, since it helps to find the near global solutions for very complicated objective functions (Chen and Yeh 2009). Present study employs a binary coded genetic algorithm in optimization. The ranges of control chart parameters considered are given in the Table 1. The cost and process parameters for the present designs are chosen from the earlier works of Duncan (1956) and Panagos et al. (1985) corresponding to the shift parameter of value 2. It can be noted that Duncan's work utilizes a wide range of cost parameters as compared to the most recent examples of the control chart designs. Since the present methodology utilizes the process shift distribution, in place of a constant process shift parameter $\delta = 2$, process shift parameter ranges are chosen as [0.5 1.5], [0.5 2.5] and [0.5 3.5]. The lower value of the shift parameter is taken as 0.5 because the Shewhart's control

Table 1 Control chart parameters range

S. no	Control chart parameters	Range of each parameter
1	Sample size (n)	$2 \leq n \leq 33$
2	Sampling interval (h)	$0.08 \leq h \leq 8$
3	Width of control limit (k)	$1 \leq k \leq 4.5$

Table 2 GA parameters used in the study

S. no	GA parameters	Magnitude/method
1	Population size	100
2	Selection method	Tournament selection
3	Type of crossover	(3)-point
4	Probability of crossover	0.95
5	Probability of mutation	0.05
6	Strategy	Elitist
7	Maximum generations	300

charts are not preferred for smaller shifts. Different upper values, namely 1.5, 2.5 and 3.5 are considered for the shift parameters. Control chart designs are obtained using different cost and process parameters and assuming positively skewed ($p = 2$, $q = 4$), negatively skewed ($p = 4$, $q = 2$) and uniformly distributed ($p = 1$, $q = 1$) beta random variable as process shifts.

Genetic algorithms are optimization algorithms based on the natural evolution of the species (Goldberg 1984). The search for the global optimum value in optimization problems is carried out by randomly choosing an initial population from the feasible solution space and creating a new population containing possibly better solutions through the application of genetic operators. Since, the parameters of GA are specific, parametric tuning has been carried out and the GA parameters used in the study have been shown in the Table 2.

The optimization process can either be terminated when there is no improvement in the objective function value for a specified number of generations or when the specified number of generations is completed. Present problem is observed to converge before 300 generations in all cases; hence the maximum number of generations is taken as 300. A linear ranking method with selective pressure of two is used for fitness values. The optimal control chart design parameters obtained by the present methodology are presented in the Tables 3, 4 and 5. The example problems extracted from Duncan (1956) are numbered as D1, D2 etc., and those from Panagos et al. (1985) are numbered as P1, P2 etc.

Advantage of using process shift distributions in control chart designs

Obviously, the control chart designs considering a single shift parameter are simple and can be solved without using an evolutionary optimization technique. Considering shift distributions in the economic designs complicates the optimization process. Hence, the use of shift distribution has to be justified over using a single parameter for the process shift. It is assumed that the designer would consider the average of the extreme shift values in finding the single shift parameter for the design. The advantage of using shift distribution, which results in cost reduction, can be realized by using the following procedure:

1. Minimize the expected loss-cost function, $E(L)$ and obtain the optimal control chart parameters $(n_0 h_0 k_0)|_{\delta=(\delta_1+\delta_2)/2}$ at single shift parameter which is the average of the minimum and maximum values of the shift parameter.
2. Minimize the total expected loss-cost function TE(L) and obtain the optimal control chart parameters ($n^*h^*k^*$) by considering the shift distribution in the range $[\delta_1\ \delta_2]$. The minimum total expected cost pertaining to these control chart parameters is TE(L^*).

(3) Substitute the control chart parameters $(n_0 h_0 k_0)|_{\delta=(\delta_1+\delta_2)/2}$ obtained by minimizing the $E(L)$ in the total expected loss-cost function [Eq. (6)] in order to find out TE(L_0).

(4) Finally, the percentage of cost reduction due to the consideration of shift distribution over a single parameter for process shift can be calculated as:

$$\frac{\mathrm{TE}(L_0) - \mathrm{TE}(L^*)}{\mathrm{TE}(L_0)} \times 100.$$

The cost values obtained for the designs under consideration are shown in the Table 6.

Results

It can be observed from the results that the cost values are influenced by the type of shift distribution and the range of the shift parameter. The advantage of using the present methodology is striking for positively skewed distribution and also for uniformly distributed shift parameters. The benefits are not remarkable for negatively skewed shift distributions. Also, when the shift range is wide, the advantage of using the present methodology is high. Figure 2 shows the variation of the maximum reductions in loss-cost obtained under consideration of different beta parameters and shift ranges.

Table 3 Control chart designs with a positively skewed shift distribution ($p = 2$, $q = 4$) with shift parameter range [0.5 3.5]

S. no	Reference	a_1	a_2	a_3	a_4	a_5	λ	g	D	n^*	h^*	k^*	TE(L^*)
1	D1	0.5	0.1	25	50	100	0.01	0.05	2	9	1.5000	2.616	5.054
2	D2	0.5	0.1	25	50	100	0.02	0.05	2	8	1.0455	2.575	8.449
3	D3	0.5	0.1	25	50	100	0.03	0.05	2	8	0.8864	2.568	11.441
4	D4	0.5	0.1	25	50	50	0.02	0.05	2	9	1.5909	2.6096	5.130
5	D5	0.5	0.1	25	50	1,000	0.01	0.05	2	6	0.3789	2.5205	31.341
6	D6	0.5	0.1	25	50	10,000	0.01	0.05	2	3	0.0877	2.3888	247.660
7	D7	0.5	0.1	25	50	100	0.01	0.5	2	3	0.9318	2.3836	7.1704
8	D8	0.5	0.1	25	50	100	0.01	0.05	20	9	1.8409	2.5616	19.177
9	D9	0.5	0.1	2.5	50	100	0.01	0.05	2	9	1.5000	2.6233	4.837
10	D10	0.5	0.1	250	500	100	0.01	0.05	2	14	1.7500	3.274	7.844
11	D11	0.5	0.1	2500	5,000	100	0.01	0.05	2	21	2.2955	3.8836	29.866
12	D12	5.0	0.1	25	50	100	0.01	0.05	2	12	3.538	2.3698	6.816
13	D13	0.5	1	25	50	100	0.01	0.05	2	5	3.197	1.959	7.457
14	D14	0.5	10	25	50	100	0.01	0.05	2	2	7.349	1.000	12.147
15	D15	0.5	1	25	50	1,000	0.01	0.05	2	4	0.833	1.9932	38.161
16	P3	5	1	250	50	50	0.01	0.05	3	6	7.0303	1.829	8.125
17	P4	0.5	0.1	35	500	100	0.01	0.05	3	14	1.7500	3.2739	6.689
18	P7	0.5	1	250	500	50	0.05	0.05	3	8	3.4015	2.6849	25.011
19	P8	5	0.1	35	50	100	0.05	0.05	3	9	1.826	2.178	24.198
20	P11	5	0.1	250	500	50	0.01	0.5	3	9	4.7272	2.8287	9.3154
21	P12	0.5	1	35	50	100	0.01	0.5	3	3	2.4394	1.9041	9.792
22	P15	0.5	0.1	250	50	50	0.05	0.5	3	3	0.9167	2.3151	23.083
23	P16	5	1	35	500	100	0.05	0.5	3	4	2.1894	2.3699	36.948
24	P19	5	1	35	50	50	0.01	0.05	20	6	7.9924	1.7808	12.272
25	P20	0.5	0.1	250	500	100	0.01	0.05	20	15	2.1894	3.2534	21.522
26	P23	0.5	1	35	500	50	0.05	0.05	20	8	5.3182	2.5068	30.5288
27	P24	5	0.1	250	50	100	0.05	0.05	20	10	3.3788	2.0548	60.844
28	P27	5	0.1	35	500	50	0.01	0.5	20	9	5.6742	2.7603	13.153
29	P28	0.5	1	250	50	100	0.01	0.5	20	3	2.9167	1.8493	23.969
30	P31	0.5	0.1	35	50	50	0.05	0.5	20	3	1.3864	2.1712	28.9380
31	P32	5	1	250	500	100	0.05	0.5	20	5	4.3788	2.2534	66.080

Table 4 Control chart designs with a uniformly distributed shift values ($p = 1$, $q = 1$) with shift parameter range [0.5 3.5]

S. no	Reference	a_1	a_2	a_3	a_4	a_5	λ	g	D	n^*	h^*	k^*	TE(L^*)
1	D1	0.5	0.1	25	50	100	0.01	0.05	2	9	1.5379	2.5685	5.138
2	D2	0.5	0.1	25	50	100	0.02	0.05	2	8	1.0530	2.5616	8.542
3	D3	0.5	0.1	25	50	100	0.03	0.05	2	7	0.8409	2.5068	11.528
4	D4	0.5	0.1	25	50	50	0.02	0.05	2	8	1.5151	2.5616	5.1993
5	D5	0.5	0.1	25	50	1,000	0.01	0.05	2	6	0.4015	2.4795	31.483
6	D6	0.5	0.1	25	50	10,000	0.01	0.05	2	3	0.0932	2.3814	245.837
7	D7	0.5	0.1	25	50	100	0.01	0.5	2	3	0.9924	2.3836	6.999
8	D8	0.5	0.1	25	50	100	0.01	0.05	20	9	1.8410	2.5342	19.273
9	D9	0.5	0.1	2.5	50	100	0.01	0.05	2	9	1.5379	2.5685	4.923
10	D10	0.5	0.1	250	500	100	0.01	0.05	2	15	1.7879	3.2466	8.072
11	D11	0.5	0.1	2500	5000	100	0.01	0.05	2	22	2.3182	3.8082	30.232
12	D12	5.0	0.1	25	50	100	0.01	0.05	2	12	3.5151	2.3082	6.9352
13	D13	0.5	1	25	50	100	0.01	0.05	2	4	2.7879	1.9726	7.361
14	D14	0.5	10	25	50	100	0.01	0.05	2	2	7.0682	1.1370	11.734
15	D15	0.5	1	25	50	1000	0.01	0.05	2	3	0.7424	1.9383	37.674

Table 4 continued

S. no	Reference	a_1	a_2	a_3	a_4	a_5	λ	g	D	n^*	h^*	k^*	TE(L^*)
16	P3	5	1	250	50	50	0.01	0.05	3	5	6.6591	1.8014	8.130
17	P4	0.5	0.1	35	500	100	0.01	0.05	3	14	1.7045	3.2329	6.916
18	P7	0.5	1	250	500	50	0.05	0.05	3	7	3.2803	2.6986	24.949
19	P8	5	0.1	35	50	100	0.05	0.05	3	8	1.7803	2.1233	24.356
20	P11	5	0.1	250	500	50	0.01	0.5	3	8	4.6288	2.7671	9.397
21	P12	0.5	1	35	50	100	0.01	0.5	3	2	1.9470	1.9384	9.395
22	P15	0.5	0.1	250	50	50	0.05	0.5	3	2	0.8258	2.3082	22.864
23	P16	5	1	35	500	100	0.05	0.5	3	3	2.0758	2.3836	36.086
24	P19	5	1	35	50	50	0.01	0.05	20	5	7.6136	1.7466	12.281
25	P20	0.5	0.1	250	500	100	0.01	0.05	20	15	2.1364	3.1849	21.732
26	P23	0.5	1	35	500	50	0.05	0.05	20	6	4.5682	2.5137	30.527
27	P24	5	0.1	250	50	100	0.05	0.05	20	9	3.3258	2.0000	61.019
28	P27	5	0.1	35	500	50	0.01	0.5	20	8	5.2803	2.7397	13.241
29	P28	0.5	1	250	50	100	0.01	0.5	20	2	2.4242	1.8630	23.691
30	P31	0.5	0.1	35	50	50	0.05	0.5	20	3	1.4697	21918	28.856
31	P32	5	1	250	500	100	0.05	0.5	20	4	4.1434	2.2603	65.883

Table 5 Control chart designs with a negatively skewed shift distribution ($p = 4$, $q = 2$) with shift parameter range [0.5 3.5]

S. no	Reference	a_1	a_2	a_3	a_4	a_5	λ	g	D	n^*	h^*	k^*	TE(L^*)
1	D1	0.5	0.1	25	50	100	0.01	0.05	2	4	1.2727	2.9726	4.034
2	D2	0.5	0.1	25	50	100	0.02	0.05	2	4	0.9090	2.9794	6.944
3	D3	0.5	0.1	25	50	100	0.03	0.05	2	4	0.7879	2.9521	9.578
4	D4	0.5	0.1	25	50	50	0.02	0.05	2	4	1.3410	2.9795	4.168
5	D5	0.5	0.1	25	50	1,000	0.01	0.05	2	3	0.3636	2.8630	26.837
6	D6	0.5	0.1	25	50	10,000	0.01	0.05	2	2	0.0985	2.7150	226.628
7	D7	0.5	0.1	25	50	100	0.01	0.5	2	2	1.0000	2.7260	5.188
8	D8	0.5	0.1	25	50	100	0.01	0.05	20	4	1.5151	2.9247	18.362
9	D9	0.5	0.1	2.5	50	100	0.01	0.05	2	4	1.2727	2.9726	3.816
10	D10	0.5	0.1	250	500	100	0.01	0.05	2	6	1.4167	3.6164	6.449
11	D11	0.5	0.1	2500	5000	100	0.01	0.05	2	5	1.5530	4.3630	27.903
12	D12	5.0	0.1	25	50	100	0.01	0.05	2	6	3.4318	2.8082	5.942
13	D13	0.5	1	25	50	100	0.01	0.05	2	2	2.0758	2.3836	5.534
14	D14	0.5	10	25	50	100	0.01	0.05	2	2	6.6391	1.7534	9.697
15	D15	0.5	1	25	50	1000	0.01	0.05	2	2	0.6515	2.3699	31.127
16	P3	5	1	250	50	50	0.01	0.05	3	3	5.9697	2.2749	7.034
17	P4	0.5	0.1	35	500	100	0.01	0.05	3	6	1.4167	3.6164	5.294
18	P7	0.5	1	250	500	50	0.05	0.05	3	4	2.5000	3.1301	22.006
19	P8	5	0.1	35	50	100	0.05	0.05	3	5	1.7651	2.6370	22.201
20	P11	5	0.1	250	500	50	0.01	0.5	3	4	4.7348	3.0890	7.490
21	P12	0.5	1	35	50	100	0.01	0.5	3	2	2.1136	2.3699	7.344
22	P15	0.5	0.1	250	50	50	0.05	0.5	3	2	0.9167	2.6849	20.989
23	P16	5	1	35	500	100	0.05	0.5	3	2	1.8712	2.6849	29.298
24	P19	5	1	35	50	50	0.01	0.05	20	3	6.7879	2.2329	11.353
25	P20	0.5	0.1	250	500	100	0.01	0.05	20	6	1.6894	3.5753	20.406
26	P23	0.5	1	35	500	50	0.05	0.05	20	3	3.2576	2.8973	28.715
27	P24	5	0.1	250	50	100	0.05	0.05	20	5	3.2197	2.5068	60.055
28	P27	5	0.1	35	500	50	0.01	0.5	20	4	5.3788	3.0548	11.672
29	P28	0.5	1	250	50	100	0.01	0.5	20	2	2.5379	2.3288	22.030
30	P31	0.5	0.1	35	50	50	0.05	0.5	20	2	1.3864	2.5342	27.824
31	P32	5	1	250	500	100	0.05	0.5	20	3	3.8636	2.6918	62.837

Table 6 Cost values of different designs considering shift distribution in the range [0.5 3.5] over a fixed single $\delta = 2$

S. no	Design	Control chart parameters using single shift parameter, $\delta = 2$				Positively skewed ($p = 2$, $q = 4$)			Uniformly distributed ($p = 1$, $q = 1$)			Negatively skewed ($p = 4$, $q = 2$)		
		n_0	h_0	k_0	$E(L)$	TE(L^*)	TE(L_0)	Cost reduction (%)	TE(L^*)	TE(L_0)	Cost reduction (%)	TE(L^*)	TE(L_0)	Cost reduction (%)
1	D1	5	1.4073	3.0822	4.0133	5.0538	7.0529	28.34	5.1384	6.9053	25.5876	4.0336	4.0540	0.5032
2	D2	5	1.0215	3.0768	6.9470	8.4493	10.9083	22.54	8.5419	10.6223	19.5852	6.9438	6.9994	0.7944
3	D3	4	0.7837	2.9423	9.5946	11.4412	14.3496	20.27	11.5283	13.7254	16.0075	9.5778	9.5908	0.1355
4	D4	5	1.4534	3.0768	4.1536	5.1304	6.7722	24.24	5.1993	6.5112	20.1484	4.1678	4.1883	0.4895
5	D5	4	0.4052	2.9538	26.9760	31.3406	37.9558	17.43	31.4834	37.3793	15.7732	26.8369	26.9906	0.5695
6	D6	2	0.0912	2.6950	228.8073	247.6600	268.6688	7.82	245.8366	261.5329	6.0000	226.6277	226.8231	0.0864
7	D7	2	0.9356	2.6903	5.4022	7.1704	8.9196	19.61	6.9992	8.1379	13.9926	5.1889	5.2063	0.3342
8	D8	5	1.6596	3.0470	18.3720	19.1769	20.8071	7.83	19.2727	20.6884	6.8430	18.3617	18.3982	0.1984
9	D9	5	1.4058	3.0819	3.7950	4.8373	6.8373	29.25	4.9217	6.6898	26.4298	3.8157	3.8359	0.5266
10	D10	6	1.4592	3.6699	6.3676	7.8441	12.3297	36.38	8.0721	12.1022	33.3006	6.4494	6.4670	0.2722
11	D11	8	1.8333	4.2688	28.2862	29.8660	36.0919	17.25	30.2323	35.5039	14.8479	27.9032	28.4523	1.9299
12	D12	6	3.4517	2.8834	5.8679	6.8160	9.4269	27.70	6.9352	9.2375	24.9234	5.9416	5.9552	0.2284
13	D13	3	2.5947	2.4293	5.6350	7.4516	9.2278	19.25	7.3614	8.5438	13.8393	5.5340	5.5551	0.3798
14	D14	2	6.7078	1.6346	9.9989	12.1470	13.6079	10.74	11.7341	12.5450	6.4639	9.6972	9.7368	0.4067
15	D15	3	0.8114	2.4310	31.7535	38.1614	44.2598	13.78	37.6741	42.2901	10.9151	31.1267	31.5167	1.2374
16	P3	4	6.4389	2.3954	7.0526	8.1250	9.4519	14.04	8.1302	9.0447	10.1109	7.0343	7.0666	0.4571
17	P4	6	1.4573	3.6679	5.2102	6.6896	11.1776	40.15	6.9158	10.9593	36.8956	5.2942	5.3102	0.3013
18	P7	5	2.8451	3.1781	22.1592	25.0115	27.2939	8.36	24.9485	26.0125	4.0903	22.0061	22.1815	0.7907
19	P8	5	1.7722	2.6806	22.1318	24.1980	27.8130	13.00	24.3557	26.9751	9.7104	22.2008	22.2318	0.1394
20	P11	5	4.9511	3.2026	7.4826	9.3154	11.8101	21.12	9.3969	11.1149	15.4567	7.4900	7.5296	0.5259
21	P12	2	2.0663	2.3014	7.6162	9.7918	11.1462	12.15	9.3954	10.2303	8.1611	7.3437	7.3690	0.3433
22	P15	2	2.8385	2.6381	21.2807	23.0825	24.2687	4.89	22.8642	23.3744	2.1827	20.9885	21.0321	0.2073
23	P16	3	2.0735	2.7672	29.9554	36.9475	41.0083	9.90	36.0861	37.7597	4.4322	29.2981	29.4076	0.3724
24	P19	4	7.3303	2.3595	11.3843	12.2719	13.3913	8.36	12.2809	13.0565	5.9403	11.3530	11.3947	0.3660
25	P20	6	1.7222	3.6376	20.3668	21.5216	25.1898	14.56	21.7319	25.0251	13.1596	20.4061	20.4404	0.1678
26	P23	4	3.7676	2.9557	28.8402	30.5288	32.1491	5.04	30.5267	31.3042	2.4837	28.7145	28.7785	0.2224
27	P24	6	3.2682	2.7504	60.1211	60.8436	62.6852	2.94	61.0196	62.4053	2.2205	60.0552	60.1623	0.1780
28	P27	5	5.6334	3.1708	11.6616	13.1529	15.3452	14.29	13.2409	14.7750	10.3831	11.6717	11.7004	0.2453
29	P28	2	2.4617	2.2545	22.2792	23.9688	25.0793	4.43	23.6910	24.3464	2.692	22.0300	22.0725	0.1925
30	P31	2	1.3243	2.4816	28.0173	28.9380	29.7029	2.58	28.8558	29.2183	1.2407	27.8241	27.8693	0.1622
31	P32	3	3.7923	2.6334	63.2168	66.0801	68.3804	3.36	65.8825	66.8962	1.5153	62.8365	62.9310	0.1502

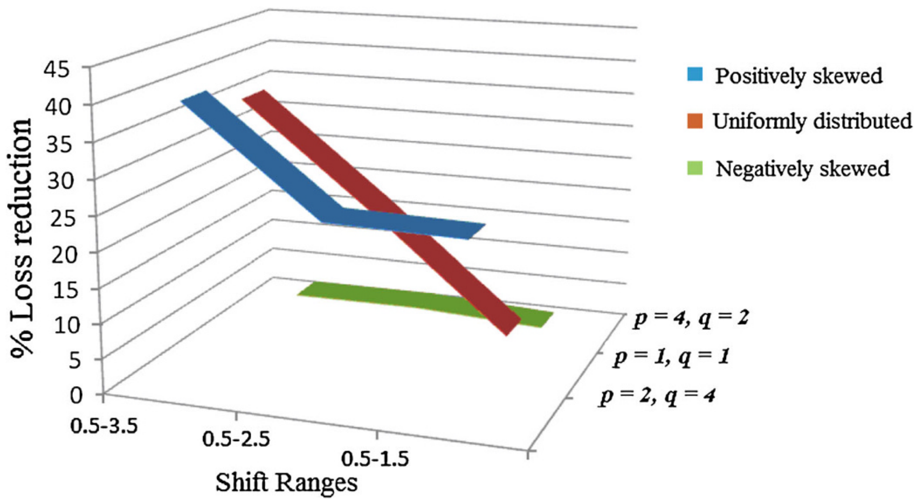

Fig. 2 Variation of maximum reductions in loss-cost for different beta parameters and shift ranges

From the present study, it can be observed that a maximum reduction in the loss-cost up to 36.90 % is achieved when the shift follows a uniform distribution. Similarly, a cost reduction of up to 40.15 % is achieved for a positively skewed shift distribution. Economic designs with the process shift following a negatively skewed distribution do not offer much monetary benefits with the present methodology.

Conclusions

Economic designs which consider a fixed shift to occur in the process for an assignable cause are criticized by earlier researchers to be totally unrealistic. Hence, there is a need to consider different shifts produced by the assignable cause. In the long run, the shifts may produce a distribution which may be positively skewed, negatively skewed or uniformly distributed. In the present work, for a given assignable cause, the process shift is considered to follow a probability distribution which can represent different situations. Beta distribution is considered for representing the process shift. Duncan's (1956) model of economic design has been extended to accommodate the process shift distribution. The extended model uses total expected loss-cost per hour incurred by the process for obtaining optimum designs. GA based search has been used for minimizing the total expected loss-cost function. The cost and process parameters have been drawn from Duncan (1956) and Panagos et al. (1985). Economic designs have been obtained when the shift follows positively skewed, negatively skewed and uniform distributions by varying the parameters of the beta distribution with different shift ranges.

The control chart designs obtained using a distribution for process shift and the designs with a single process shift based on the average of the extreme values of shift parameters have been compared. Hence, it can be concluded that the present methodology has to be adopted when the shift distributions are either positively skewed or uniform. For a negatively skewed distribution, as the benefits are not remarkable, designs can be made based on a single process shift instead of using the shift distribution.

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