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ORIGINAL RESEARCH

Combining data envelopment analysis and multi-objective model for the efficient facility location–allocation decision

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Abstract
This paper proposes an innovative procedure of finding efficient facility location–allocation (FLA) schemes, integrating data envelopment analysis (DEA) and a multi-objective programming (MOP) model methodology. FLA decisions provide a basic foundation for designing efficient supply chain network in many practical applications. The procedure proposed in this paper would be applied to the FLA problems where various conflicting performance measures are considered. The procedure requires that conflicting performance measures classified as inputs to be minimized, or outputs to be maximized. Solving an MOP problem generates diverse alternative FLA schemes along with multi-objective values. DEA evaluates these schemes to generate a relative efficiency score for each scheme. Then, using stratification DEA, all of these FLA schemes are stratified into several levels, from the most efficient to the most inefficient levels. A case study is presented to demonstrate the effectiveness and efficiency of the proposed integrating method. We observe that the combined approach in this paper performs well and would provide many insights to academians as well as practitioners and researchers.

Keywords Facility location–allocation · Data envelopment analysis · Multi-objective programming · Performance measures · Relative efficiency score

Abbreviations

AAS	Average attractiveness score
CDE	Covered demands in case of emergency
DEA	Data envelopment analysis
DMU	Decision-making unit
DRC	Disaster recovery center
ES	Efficiency score
ENDS	Expected number of non-disrupted supplies
FLA	Facility location–allocation
FE	Fully efficient
MCD	Maximum coverage distance
MONLP	Multi-objective nonlinear programming
MOP	Multi-objective programming
MDWCD	Maximum demand-weighted coverage distance
nCDE	Non-covered demand in case of emergency
PM	Performance measure
TLC	Total logistics cost

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Introduction

The facility location–allocation (FLA) decision is often considered the most important factor leading to the success of a private- or public-sector organization. Daskin (2013) emphasizes the importance of facility location problems by asserting in his recent book that *in short, the success or failure of both private and public-sector facilities depends in part on the locations chosen for those facilities*. Since the design of efficient supply chain networks starts from efficient FLA decisions (see Olivares-Benitez et al. 2012), the FLA models have been widely used in practical life as well as in many academic disciplines. Consequently, the topic of FLA problem has received considerable attention in the literature.

The basic issues of FLA are where to locate and how to size facilities, and how to meet demands from the facilities. FLA decisions inherently consist of two kinds of decision plans. One is a strategic decision plan on the facility location, while the other one is an operational decision plan on the facility allocation. The goal of FLA decision is to locate the facilities in a way that satisfies/covers the demand points most efficiently. In fact, various types of FLA models have been developed to answer questions such as how many facilities to locate, where to locate facilities, and how to distribute the items to the customers concerning different location criteria.

Many references cited in Farahani et al. (2010) demonstrate that the FLA problems are inherently multi-objective, where those objectives sometimes conflict with each other in nature (see Lee et al. 1981). The traditional FLA models deal with the objective of cost minimization, whereas demand-oriented objectives focus on measuring the 'proximity or closeness' of the facilities. The profit maximization objective may be achieved by either cost minimization or maximization of demand satisfied/covered, or both. They (2010) emphasize the importance of multi-objective facility location–allocation (MOFLA) problems after observing the substantial growth of the literature on MOFLA problems. Thus, the growing attention and interest in these problems are due to the recognition of the need to consider more objectives/criteria to achieve closer solutions to reality.

Multi-objective programming (MOP) technique provides an analytical framework where a variety of objectives can be focused on simultaneously so that a decision maker can use to provide optimal solutions. But most of the MOP techniques require decision-makers' judgment to provide weights to the deviational variables in the objective function to appropriately reflect the importance and desirability of deviations from the various target values. As the number of performance measures increases, solving the MOP model will yield a great number of alternative options. The reason is that each different weight factor set for performance measures may generate a different option. As Ragsdale (2015) states, no standard procedure is available to assign values to the weight factors in a way that guarantees the decision-makers find the most desirable solution. Evaluating alternatives generated by solving the MOP model can be viewed as a multiple-criteria decision-making (MCDM) problem, requiring a systematic solution evaluation system.

In this paper, we utilize data envelopment analysis (DEA) technique for such a systematic solution evaluation system. DEA yields relative efficiency of comparable units, which are called decision-making units (DMUs) in DEA parlance, that employ multiple outputs and inputs. To denote the relative efficiency for each DMU, DEA produces an efficiency score that is defined as the ratio of the sum of weighted outputs to the sum of weighted inputs.

The objective of this paper is to present and demonstrate how to combine DEA and MOP techniques for the efficient FLA decisions and patterns to help practitioners as well as decision-makers who are responsible for the strategic and operational decision plans. To combine DEA and MOP, we classify all performance measures into inputs or outputs to formulate the FLA problem as an MOP model. Then we solve the model for various values of weights given to the performance measures. Considering each generated alternative option for a given set of weight as a DMU, we evaluate all alternative options by utilizing DEA technique to find the efficiency of each alternative option and identify the most efficient FLA schemes. In this way, decision-makers evaluate and identify efficient and robust FLA decisions without any subjective judgment. Furthermore, once decision-makers identify efficient FLA patterns through the proposed procedure, they can modify their operational decisions without sacrificing the efficiency heavily under unexpected disruptions. We demonstrate our procedure through a case study.

This paper is organized as follows. After the literature review of FLA with DEA, we provide a brief introduction to general FLA models with MOP with the minimax objective approach and DEA. The proposed method of combining an MOFLA model and DEA is discussed. Next, we demonstrate our proposed method by MOFLA formulation and DEA evaluation through a case study using actual data in South Carolina, followed by conclusions.

Literature review

The FLA problem is widely used in practical life. The pioneering work of Weber (1929) on the location theory prompted the formulation of various mathematical programming models. After Cooper (1963) initially proposes an FLA problem by presenting a heuristic method, Hakimi (1964, 1965) applies the FLA problem to network design as a powerful tool. Several heuristic methods have been developed for the traditional FLA problem. Recently, Mesa et al. (2017) applied Cuckoo search via Levy flights (CS-LF) to uncapacitated FLA problem, claiming that applying CS-LF yields better facility locations compared to particle swarm optimization and other existing algorithms. Traditionally, minimizing cost/time/distance has been the primal objective. Some authors (Askin et al. 2014; Manatkar et al. 2016) consider also maintaining desired service level in addition to reducing the costs. Recently, contrary to the most single-period FLA problem, Manzini and Gebennini (2008) and Manatkar et al. (2016) apply mixed integer programming optimization models to design and manage multi-period, multi-stage and multi-commodity FLA problem. Hotelling (1929) introduces an influential competitive facility location model where each of two players selects a location in a linear segment, and a continuum of uniformly-distributed consumers along that segment select the closest facility. Hakimi (1983), Aboolian et al. (2007), Zhang et al. (2016a, b), and Bagherinejad and Niknam (2017) are a few of them who study and develop various algorithms and procedures for the competitive facility location problems. Balcik and Beamon (2008) consider facility location decisions for a humanitarian relief chain responding to quick-on-set disasters. Habib et al. (2016) conduct an up-to-date survey of mathematical models developed in humanitarian supply chain area and highlight the potential research areas which require the attention of the researchers. Recently, Fereiduni and Shahanaghi (2017) present a robust network design model for humanitarian logistics which will assist in location and allocation decision for multiple disaster periods. They (2017) also use the Monte Carlo simulation for generating related random numbers and different scenarios and utilize the *p*-robust approach to formulate the new network.

As shown in Farahani et al. (2010), various researchers have worked on multi-objective/criteria facility location–allocation (MOFLA) problems, such as Blake and Carter (2002), Drezner et al. (2006), Yang et al. (2007), Ho et al. (2008), Farahani et al. (2010), Hong et al. (2015), Farahani et al. (2015), Fang and Li (2015), and Manatkar et al. (2016). Khalili-Damghani et al. (2015) propose a bi-objective mixed integer mathematical programming for locations of warehouses and routing of vehicles to reduce the total cost of the supply chain and to balance the workload of distribution centers while the due dates of delivery of perishable product are met, concurrently. To solve their model, they use an evolutionary algorithm called Non-Dominated Sorting Generic Algorithm-II. Oddoye et al. (2009), Beheshtifar and Alimoahmmadi (2015), Zhang et al. (2016a, b) and Ahmadi-Javid et al. (2017) apply MOFLA models to solving healthcare related facility location–allocation problems.

DEA has been a widely used mathematical programming technique that evaluates the performance of a set of homogenous DMUs. Hafezalkotob et al. (2015) propose a robust DEA (RDEA) to investigate the efficiencies of DMU where there are discrete input and output data. Khalili-Damghani et al. (2016), also pointing out that the observed values of the input and output data in real-world problems are sometimes imprecise or vague, present a comprehensive fuzzy DEA framework for solving performance evaluation problems with coexisting desirable input and undesirable output data in the presence of simultaneous input–output projection. In conventional DEA, DMUs are represented as black boxes where only the initial inputs and final outputs are considered to measure their efficiency, neglecting intervening processes, i.e., different series or parallel function. A new DEA model called network DEA (NDEA) accounts for divisional efficiencies as well as the overall efficiency (see Cook et al. 2010). Saniee-Monfared and Safi (2013) propose a set of performance indicators to enable efficiency-analysis of academic activities and apply NDEA structure to account for sub-functional efficiencies such as teaching quality, research productivity, as well as overall efficiency. Tavana et al. (2016) apply NDEA to evaluate the performance of three-level supply chains and shows the applicability and efficiency of the proposed framework, insisting that the proposed method can be easily implemented in any multi-level supply chain.

Thomas et al. (2002) propose the combined obnoxious-facility location/DEA model, assuming that the number of facilities to be opened is predetermined. Klimberg and Ratick (2008) develop and test location modeling formulations by utilizing DEA to find optimal and efficient FLA patterns. Following Klimberg and Ratick (2008), Fang and Li (2015) propose a combined DEA and goal programming (GP) approach. But these two approaches assume both inputs and outputs are given to model DEA for the efficient FLA decision, whereas our paper generates all inputs and outputs from solving the MOP model. Klimberg et al. (2011) extend Klimberg and Ratick (2008) to consider the more realistic situation in which the units of products from the facilities are decision variables. Another example using the combining DEA and MOP approach is found in Ghoushchi et al. (2017) who develop the DEA model using imprecise data based on GP to evaluate and select sustainable suppliers in the supply chain.

This paper is motivated by Klimberg and Ratick (2008), Klimberg et al. (2011) and Fang and Li (2015). All of them develop and test location modeling formulations by utilizing DEA to find optimal and efficient FLA patterns. Klimberg and Ratick (2008) and Klimberg et al. (2011) postulate that locating facilities at different potential sites may affect the performance of the facility's ability to transform inputs into usable outputs. Their model formulation simultaneously considers the interaction of spatial efficiencies of different location patterns through the use of least cost objective and the facility efficiencies at those sites through the use of DEA objective. Their models (2008 and 2011) require the huge amount of the predetermined input and output data and consequently the huge number of the constraints for their combined location and simultaneous DEA model, as the numbers of facilities and their potential sites increase. In addition to those huge data and constraints required by their models, it would be not only difficult to quantify all inputs and outputs for a facility to be located to cover the allocated sites, but also very subjective for a decision maker to decide these magnitudes of such inputs and outputs. In addition, they apply the results obtained by DEA as one goal/objective for their MOP models. That is not the way that DEA has been developed and applied in many researches on the topic of DEA. For example, suppose that one of the performance measures is the total cost. The total cost should be used as an input to apply DEA method. In other words, the decision of locating a facility at a site incurs a part of the total cost and will affect the relative efficiency of FLA scheme when DEA is applied. That is the way that DEA has been invented and applied to find the efficiency of DMUs.

In this paper, we propose a more reasonable and practical approach to finding efficient FLA decisions by generating the inputs and outputs directly through formulating and solving the MOP model and by applying DEA for those inputs and outputs generated by the MOP model. No literature on our innovative approach to FLA decisions has been known.

FLA model with MOP with minimax objective approach

The following nomenclature is used:

Sets:

M Index set of potential facility sites ($j=1, 2, \ldots, M$ and $m=1, 2\ldots M$).

Parameters:

b_j Minimum number of sites that facility j can cover
B_j Maximum number of sites that facility j can cover
c_{jm} Cost of shipping one unit of demand per mile from facility j to site\demand point m
$C^{\max}$ Maximum number of facilities can be built
$CAP_j^{\max}$ Capacity of facility located at site j
d_{jm} Distance between facility located at site j and site m
D_m Demand of site m
h_j Holding cost per item per time unit at facility located at site j
F_j Fixed cost for constructing and operating facility located at site j
L_j Replenishment lead time for facility located at site j
S_j Ordering cost for facility at site j to place an order
β Desired service level
σ_m Standard deviation of demand at site m

Decision variables:

C_j Binary variable deciding whether a facility is located at site j
y_{jm} Binary variable deciding whether site m is covered by facility located at site j

In above nomenclature, we assume a_{jm} and d_{jm} equal to zero if $j=m$.

Let I and O denote the index set of performance metrics for inputs ($i=1, 2, \ldots, p$) and outputs ($r=1, 2, \ldots, s$). Let the nonnegative deviation variables, ($\omega_1^+, \omega_2^+, \ldots \omega_p^+$) and ($\omega_1^-, \omega_2^-, \ldots \omega_s^-$), denote the amounts by which each value of performance metrics deviates from the minimum and maximum values, respectively, which are called overachievement and underachievement deviation n variables. Then, the deviation variables are expressed as

$$\omega_i^+ = X_i^+ - TGV_i^+$$

and

$$\omega_r^- = TGV_r^- - X_r^-,$$

where TGV_i^+ and TGV_r^- represent the target value of performance metric, I^+ and O^-, respectively. Now, the minimax objective can be expressed as

Minimize the maximum of

$$\left\{ \alpha_1^+ \frac{\omega_1^+}{TGV_1^+}, \ldots, \alpha_p^+ \frac{\omega_p^+}{TGV_p^+}, \alpha_1^- \frac{\omega_1^-}{TGV_1^-}, \ldots, \alpha_s^- \frac{\omega_s^-}{TGV_s^-} \right\},$$

where α_i^+ and α_r^- are relative importance weights attached to the overachievement and underachievement deviation variables and the sum of all weights equals one for the purpose of analysis. Now, set Q equal to the maximum variable, such as

$$Q = \max \left\{ \alpha_1^+ \frac{\omega_1^+}{TGV_1^+}, \ldots, \alpha_p^+ \frac{\omega_p^+}{TGV_p^+}, \alpha_1^- \frac{\omega_1^-}{TGV_1^-}, \ldots, \alpha_s^- \frac{\omega_s^-}{TGV_s^-} \right\}. \tag{1}$$

The formulation of MOFLA model with MOP with the minimax objective is given as follows:

$$\text{Minimize } Q \text{ in (1)} \tag{2}$$

subject to

$$\alpha_1^+ \frac{\omega_1^+}{TGV_1^+} \le Q, \ldots, \alpha_p^+ \frac{\omega_p^+}{TGV_p^+} \le Q, \alpha_1^- \frac{\omega_1^-}{TGV_1^-} \le Q, \ldots, \alpha_s^- \frac{\omega_s^-}{TGV_s^-} \le Q, \tag{3}$$

$$X_1^+ - \omega_1^+ = TGV_1^+, \ldots, X_p^+ - \omega_p^+ = TGV_p^+, X_1^- + \omega_1^- = TGV_1^-, \ldots X_s^- + \omega_s^- = TGV_s^-, \tag{4}$$

$$\sum_{j \in M} y_{jm} = 1, \quad \forall m \in M \tag{5}$$

$$\sum_{j \in M} C_j \le C^{\max}, \tag{6}$$

$$y_{jm} \le C_j, \quad \forall j \text{ and } \forall m \in M \tag{7}$$

$$C_j \cdot b_j \le \sum_{m \in M} y_{jm} \le C_j \cdot B_j, \quad \forall j \in M \tag{8}$$

$$\sum_{m \in M} D_m y_{jm} \le CAP_j^{\max}, \quad \forall j \in M. \tag{9}$$

Constraints (5) make certain that each site is covered by a facility. Constraints (6) define the maximum number of facilities to be built. Constraints (7) ensure that each site can only be covered by a selected facility. Constraints (8) make sure that the facility j must cover at most B_j and at least b_j sites. Constraints (9) show the capacity of the facility j.

Data envelopment analysis (DEA) models

Among many performance evaluation methods, data envelopment analysis (DEA) has been widely used to evaluate the relative efficiency of a set of peer organizations called decision-making units (DMUs) that have multiple inputs and outputs. The main reason might be that DEA models need not recourse to the exact behavior function of those organizations regarding the transformation of multiple inputs to outputs. DEA defines relative efficiency as the ratio of the sum of weighted outputs to the sum of weighted inputs. The mathematical model of DEA (see Cooper et al. 2011) may be stated as:

$$\max \varphi = \sum_{r=1}^{s} u_r O_{ro}, \tag{10}$$

subject to

$$\sum_{i=1}^{p} v_i I_{io} = 1, \tag{11}$$

$$\begin{aligned} &\sum_{r=1}^{s} u_r O_{r\delta} - \sum_{i=1}^{p} v_i I_{i\delta} \le 0, \quad \delta = 1, \dots, n, \\ &u_r, v_i \ge \varepsilon, r = 1 \dots, s; i = 1 \dots, p \end{aligned} \tag{12}$$

where n number of DMUs being compared in the DEA analysis, φ efficiency rating of the DMU_o being evaluated by DEA, O_r amount of output r generated by DMU, I_i amount of input i used by DMU, p number of inputs used by the DMUs, s number of outputs generated by the DMUs, u_r coefficient or weight assigned by DEA to output r, v_i coefficient or weight assigned by DEA to input i, ε non-Archimedean.

Let φ^* denote the optimal value of the objective function corresponding to the optimal solution (u^*, v^*). DMU_o is said to be efficient if $\varphi^* = 1$. DEA models can be either input-oriented or output-oriented, depending upon the rationale for conducting DEA. The model given by (10)–(12) is called an input-oriented CCR model, which was initially proposed by Charnes et al. (1978), and φ^* is called constant returns to scale (CRS) efficient score (ES).

Now, the dual program of (10)–(12), which is called input-oriented envelopment DEA model (e-DEA), is formulated:

$$\min \theta - \varepsilon \left\{ \sum_{i=1}^{p} \xi_i^- + \sum_{r=1}^{s} \xi_r^+ \right\} \tag{13}$$

subject to

$$\sum_{\delta=1}^{n} \lambda_\delta I_{i\delta} - \theta I_{io} + \xi_i^- = 0, \quad i = 1, \dots, p, \tag{14}$$

$$\begin{aligned} &\sum_{\delta=1}^{n} \lambda_\delta O_{r\delta} - O_{ro} - \xi_r^+ = 0, \quad r = 1, \dots, s, \\ &\xi_i^-, \xi_r^+, \lambda_\delta \ge 0, \quad \delta = 1, \dots n. \end{aligned} \tag{15}$$

In the above dual model given by (13)–(15), θ is the efficient score (ES) and DMU_o is said to be efficient if $\theta^* = 1$ and λ_δ is the dual variable, used to indicate benchmark information. ξ_r^+ and ξ_i^- are slack variables used to calculate the target input and output variables for an inefficient DMU. For $\theta^* = 1$, the performance of DMU_o is *fully efficient* if and only if all slacks $\xi_r^{+*} = \xi_i^{-*} = 0$, otherwise is *weakly efficient*. See Cooper et al. (2011) for details. If the performance of a DMU is *weakly efficient*, it implies that the DMU is on the best-practice frontier, but the performance can be still improved by reducing input(s) or increasing output(s). Thus, weakly efficient DMUS are not classified as really efficient. By solving the above model, we can obtain the efficiency score of each DMU and check if the performance of the target DMU is fully efficient or weakly efficient.

Performance evaluation or measurement often depends upon by the context. One could ask "what is the relative attractiveness of a particular DMU when compared to others?" Following this vein, Seiford and Zhu (2003) propose the stratification/context-dependent DEA method to measure the attractiveness score and progress of DMUs with respect to a given evaluation context. For this, they stratify DMUs into different efficiency levels. Let $J^1 = \{DMU_\delta, \delta = 1, 2, \dots, n\}$ be the whole set of n number of DMUs and iteratively define $J^{\ell+1} = J^\ell - E^\ell$, until $J^{\ell+1}$ becomes null. E^ℓ consists of all the efficient DMUs on the ℓth level, that is, $E^\ell = \{DMU_k \in J^\ell | \theta^*(\ell, k) = 1\}$, and $\theta^*(\ell, k)$ is the optimal value to the following CRS model when DMU_k is under evaluation.

$$\theta^*(\ell, k) = \min_{\lambda_\delta, \theta(\ell, k)} \theta(\ell, k) \tag{16}$$

Subject to

$$\sum_{\delta \in F(J^{\ell})}^{K} \lambda_\delta I_{i\delta} - \theta(\ell, k) I_{ik} \le 0, \quad i = 1, \dots, p, \tag{17}$$

$$\sum_{\delta \in F(J^{\ell})}^{K} \lambda_\delta O_{r\delta} - O_{rk} \ge 0, \quad r = 1, \dots, s, \tag{18}$$

$$\lambda_\delta \ge 0, \delta = 1, \dots n,$$

where $\delta \in F\left(J^{\ell}\right)$ means $DMU_\delta \epsilon J^{\ell}$, i.e., $F(\cdot)$ represents the correspondence from a DMU set to the corresponding subscript index set. In fact, all DMUs in E^{ℓ} are equivalent from the traditional DEA perspective. The DEA stratification model given by (16)–(18) partitions the set of DMUs into different frontier levels characterized by E^{ℓ}. The attractiveness score for each DMU in the ℓth level (E^{ℓ}) is computed against DMUs in the $(\ell + 1)$th and lower levels as the evaluation context (see Zhu 2014). For an example, to find an attractiveness score for $DMU_q = \left(I_q, O_q\right)$ from a specific level, $E^{\ell_o}, \ell_o \in \{1, 2, \dots L-1\}$, we solve the following model:

$$H_q^*(d) = \min H_q(d), \quad d = 1, \dots L - \ell_o \tag{19}$$

Subject to

$$\sum_{\delta \in F(E^{\ell_o+d})}^{K} \lambda_\delta I_\delta - H_q(d) I_q \le 0 \tag{20}$$

$$\sum_{\delta \in F(E^{\ell_o+d})}^{K} \lambda_\delta O_\delta - O_q \ge 0, \tag{21}$$

$$\lambda_\delta \ge 0, \quad \delta \in F\left(E^{\ell_o+d}\right).$$

$H_q^*(d)$ is called *d-degree* attractiveness of DMU_q from a specific level E^{ℓ_o}. In this way, the stratification/context-dependent DEA can have more discriminating power on each stratification level. In this paper, we adopt the DEA-based stratification concept for the FLA problem. First, we stratify DMUs into efficiency level, such as $E^1, E^2, E^3, \dots E^L$, using (16)–(18). Then we compute the attractiveness score (AS) for each DMU in the first level E^1 against the DMUs in each lower level, such as $E^2, E^3, \dots E^L$, using (19)–(21) after setting $\ell_o = 1$. Then, we compute the average attractiveness score (AAS) for DMU_q in E^1, AAS_q, which is defined as

$$AAS_q = \frac{\sum_{d=1}^{L-1} H_q^*(d)}{L-1}. \tag{22}$$

Note that all DMUs in the first level E^1 are same in terms of efficiency score $\varphi^* = 1$ in (10) or $\theta^* = 1$ in (13). We can identify the most efficient DMUs which have the highest value of AAS.

Now, we propose the following formal procedure of combining DEA and MOP model for the efficient FLA decision:

Step 1 (MOFLA formulation and pre-stratification)

(1) Define objectives/goals for performance measures (PMs) to be considered. Then, classify PMs into p inputs and r outputs.
(2) Formulate MOFLA problem as MOP model shown in Eqs. (1)–(9).
(3) Set the value of weight for each PM, where each weight changes between 0 and 1 with an increment of Δ, where $0 \le \Delta \le 1$.
(4) For each set of weights, solve the MOP model and call each solution as $DMU_\delta, \quad \delta = 1, 2, \dots, n.$

Step 2 (Stratification DEA)

(1) Set $\ell = 1$ and construct E^{ℓ} by evaluating each DMU in J^{ℓ}.
(2) Set $J^{\ell+1} = J^{\ell} - E^{\ell}$. If $J^{\ell+1} = \emptyset$, go to (3) after setting $\ell = L$. Otherwise, repeat (2) after setting $\ell = \ell + 1$.
(3) Setting $\ell_o = 1$, compute $H_q^*(d), d = 1, 2, \dots, L-1$, and compute AAS_q, given in (22), for DMU_q in J^{ℓ_o}.

Step 3 Rank the DMUs in J^{ℓ_o} based on the value of AAS, from highest to lowest.

Case study in South Carolina

Historic flooding tore through South Carolina (SC) in October 2015 when numerous rivers burst their banks, washing away roads, bridges, vehicles, and homes. Hundreds of people required rescue and the state's emergency management department urged everyone in the state not to travel. The Federal Emergency Management Agency (FEMA) opened disaster recovery centers (DRCs) in several SC counties to help SC flood survivors. We use the problem of locating DRCs in SC as our case study. We assume that each DRC follows an (s, Q) policy to maintain its inventory and carries a safety stock to maintain the desired service level of β. To follow Step 1 in the formal procedure, we define objectives for PMs for our case study.

Objective 1: Minimize the total logistics cost (TLC)

Minimizing *TLC* has been the traditional objective of most FLA models. Given this problem setting, the *TLC* consists of the fixed cost of locating DRCs, the transportation/shipping cost from DRCs to the demand points, cycle stock cost, and safe stock cost to maintain the desired service level of β, and is given in Eq. (23):

$$\begin{aligned} \text{TLC} &= \text{fixed cost for facilities} + \text{shipping cost} + \text{cycle stock cost} + \text{safe stock cost} \\ &= \sum_{j\in M} f_j C_j + \sum_{j\in M}\sum_{m\in M} D_m d_{jm} y_{jm} c_{jm} \\ &\quad + \sum_{j\in M} C_j \left[\sqrt{2S_j h_j \sum_{m\in M} D_m y_{jm}} + h_j z_\beta \sqrt{\sum_{m\in M} L_j \sigma_m^2 y_{jm}} \right]. \end{aligned} \tag{23}$$

Objective 2: Minimize the maximum coverage distance (MCD)

Ideally, each DRC should be located near the affected sites or demand points. Our assumption is that the shorter distance is, the shorter delivery time is. The second PM is the maximum coverage distance (*MCD*) to be minimized so that each demand point is covered by one of the DRCs within the endogenously determined distance. In other words, the objective for the second PM is equivalent to attempting to minimize the longest delivery distance between DRCs and the affected sites. If *MCD* is too large, it will cause inefficiency to the resulting FLA scheme, since it will take an excessive time for a DRC to deliver to a remotely located site. Now, *MCD* is given by

$$MCD = \max\left\{d_{jm} y_{jm}\right\}, \quad \forall j \text{ and } m. \tag{24}$$

Objective 3: Minimize the maximum demand-weighted coverage distance (MDWCD)

MCD does not consider demand associated with each site but the longest distance only. Thus, it would be important to consider not only *MCD* but also the demand-weighted distance. The next PM is the maximum demand-weighted coverage distance (*MDWCD*) to be minimized, which is given by

$$MDWCD = \max\left\{D_m d_{jm} y_{jm}\right\}, \quad \forall i, j, \text{ and } m. \tag{25}$$

Objective 4: Maximize the covered demands in case of emergency (CDE)

Deckle et al. (2005) studied the problem of minimizing the total number of DRCs in Alachua County, Florida, subject to each county resident being close to a DRC must be less than a given threshold. In fact, each location should be within a certain distance of the nearest DRCs to be served in case of emergency. In addition, there may be some environmental constraints or difficulties such as road damage and weather issues, which may limit the maximum coverage distance given by *MCD* in (24). Thus, the maximum effective coverage distance (MECD), denoted by D^c, may be shorter than *MCD*. However, while minimizing *MCD*, it is desirable to maximize the covered demands within D^c. The fourth PM is the covered demands in case of emergency (*CDE*) to be maximized, which is expressed as

$$CDE = \sum_{m\in M}\sum_{j\in J} D_m \gamma_{jm} y_{jm}, \tag{26}$$

where binary parameters, γ_{jm}, are

$$\gamma_{jm} = \begin{cases} 1, & \text{if } d_{jm} \le D^c \\ 0, & \text{otherwise.} \end{cases} \tag{27}$$

Objective 5: Maximize the covered demands in case of emergency (ENDS)

To enhance DRC's resilience, it would be important to locate DRCs at the safest areas if possible, so that the chances of DRCs' being disrupted are minimized. We assume that if a DRC is disrupted, it can't handle the supplies being delivered to the affected site. Now, our fifth PM is the expected number of non-disrupted supplies (*ENDS*) to be maximized, which is given by

$$ENDS = \sum_{j\in M} C_j\left(1 - p_j\right) \sum_{m\in M} \left(y_{jm} D_m\right), \tag{28}$$

where p_j denotes the risk probability of DRC$_j$'s being disrupted. Note that both C_j and y_{jm} in (28) are decision variable, thus Eq. (28) is no more a linear combination. To linearize it, we define $Z_{jm} = C_j * y_{jm}$ and rewrite (28) as

$$ENDS = \sum_{j\in M} \left(1 - p_j\right) \sum_{m\in M} D_m Z_{jm}, \tag{29}$$

where

$$\max\left\{0, C_j + y_{jm} - 1\right\} \le Z_{jm} \le \frac{C_j + y_{jm}}{2}.$$

Now, among the above five PMs, the first three PMs, *TLC*, *MCD*, and *MDWCD*, to be minimized would be classified as inputs, $I = \{TLC, MCD, MDWCD\}$, whereas the other two, *CDE* and *ENDS*, to be maximized as outputs, $O = \{CDE, ENDS\}$. Let the nonnegative deviation variables, $\left\{\omega^+_{TLC}, \omega^+_{MCD}, \omega^+_{MDWCD}, \omega^-_{CDE}, \omega^-_{ENDS}\right\}$ denote the amounts by which each value of *TLC*, *MCD*, *MDWCD*, *CDE*, and *ENDS* deviates from each target value, the minimax variable from Eq. (1), Q, is expressed as

$$Q = \max \left\{ \alpha_1^+ \frac{\omega_{TLC}^+}{TLC_{\min}}, \alpha_2^+ \frac{\omega_{MCD}^+}{MCD_{\min}}, \alpha_3^+ \frac{\omega_{MDWCD}^+}{MDWCD_{\min}}, \alpha_1^- \frac{\omega_{CDE}^-}{CDE_{\max}}, \alpha_2^- \frac{\omega_{ENDS}^-}{ENDS_{\max}} \right\}. \quad (30)$$

Then, we formulate a multi-objective nonlinear programming (MONLP) model with the minimax objective as follows:

$$\min Q \text{ in } (30) \quad (31)$$

subject to

$$\alpha_1^+ \frac{\omega_{TLC}^+}{TLC_{\min}} \le Q, \quad (32)$$

$$\alpha_2^+ \frac{\omega_{MCD}^+}{MCD_{\min}} \le Q, \quad (33)$$

$$\alpha_3^+ \frac{\omega_{MDWCD}^+}{MDWCD_{\min}} \le Q, \quad (34)$$

$$\alpha_1^- \frac{\omega_{CDE}^-}{CDE_{\max}} \le Q, \quad (35)$$

$$\alpha_2^- \frac{\omega_{ENDS}^-}{ENDS_{\max}} \le Q, \quad (36)$$

$$TLC \text{ in } (23) - \omega_{TLC}^+ = TLC_{\min}, \quad (37)$$

$$MCD \text{ in } (24) - \omega_{MCD}^+ = MCD_{\min}, \quad (38)$$

$$MDWCD \text{ in } (25) - \omega_{MDWCD}^+ = MDWCD_{\min}, \quad (39)$$

$$CDE \text{ in } (26) + \omega_{CDE}^- = CDE_{\max}, \quad (40)$$

$$ENDS \text{ in } (29) + \omega_{ENDS}^- = ENDS_{\max}, \quad (41)$$

$$\sum_{g=1}^{3} \alpha_g^+ + \sum_{g=1}^{2} \alpha_g^- = 1, \text{ and} \quad (42)$$

Constraints (5) – (9).

This competes (1) and (2) in Step 1.

Solving MONLP and applying DEA

As Hong et al. (2015) use major disaster declaration records in South Carolina (SC) from FEMA database, we use the same data. Forty-six counties in SC are clustered based on proximity and populations into twenty counties. Then, one city from each clustered county based on a centroid approach is chosen, assuming that all population within the clustered county exists in that city. The distance between these cities is considered to be the distance between counties. We assume that when a major disaster is declared, the DRC in

Table 1 Data for locations of DRCs

No.	City	County	POP, D_m (K)	p_i	F_i (K)
1	Aiken	Aiken/Barnwell	184	0.313	700
2	Anderson	Anderson/Oconee/Pickens	373	0.125	500
3	Beaufort	Beaufort/Jasper	187	0.063	400
4	Bennettsville	Marlboro/Darlington/Chesterfield	96	0.375	700
5	Charleston	Charleston	350	0.25	600
6	Columbia	Richland/Fairfield/Kershaw	461	0.375	700
7	Conway	Horry	269	0.375	700
8	Florence	Florence/Dillon/Marion	203	0.438	800
9	Georgetown	Georgetown/Williamsburg	93	0.438	800
10	Greenville	Greenville/Laurens	521	0.125	500
11	Greenwood	Greenwood/Abbeville	92	0.125	550
12	Hampton	Hampton/Allendale	33	0.188	500
13	Lexington	Lexington/Newberry/Saluda	318	0.313	650
14	McCormick	McCormick/Edgefield	35	0.25	600
15	Moncks Corner	Berkeley	178	0.313	650
16	Orangeburg	Orangeburg/Bamberg/Calhoun	123	0.375	700
17	Rock Hill	York/Chester/Lancaster	321	0.313	700
18	Spartanburg	Spartanburg/Cherokee/Union	367	0.313	650
19	Sumter	Sumter/Clarendon/Lee	157	0.375	700
20	Walterboro	Colleton/Dorchester	135	0.25	600

Table 2 Input data used for the case study

Symbol	Meaning	Value
b_j	A minimum number of sites that DRC j can cover	2, $\forall j$
B_j	A maximum number of sites that DRC j can cover	7, $\forall j$
c_{jm}	Cost of shipping one unit of demand per mile from DRC j to site m	\$0.10, $\forall j$ and m
$C^{\max}$	Maximum number of DRCs to be built	5
$CAP_j^{\max}$	Capacity of DRC j	1500, $\forall j$
h_j	Holding cost per item per unit time at DRC j	\$5.00, $\forall j$
L_j	Replenishment lead time at DRC j	0.01, $\forall j$
S_j	Ordering cost for DRC j to place an order	\$500.00, $\forall j$
β	Desired service level for all DRCs	0.95
γ^c	Maximum effective coverage distance in case of emergency	35 miles
σ_m	Standard deviation of demand per unit time at site m	5 K, $\forall m$

that county can't function due to the damaged facility and supply items and closed or unsafe roads and highways. The FEMA database provides a list of counties where a major disaster was declared. Based on the historical record available in the FEMA database and the assumption, the risk probability for each site (a county or a clustered county) is calculated in Table 1. For example, the probability that a facility in Walterboro will suffer from a major disaster and will be shut down is 25%. For the case study, we hypothetically pre-determine and list the input parameters in Table 2.

As shown in the MONLP model given by Eqs. (30)–(42), it is necessary to find the target value for each PM. These values can be obtained by setting the corresponding weight equal to 1 and solving the model. For example, setting $\boldsymbol{\alpha} = (0, 1, 0, 0, 0)$ and solving the MONLP model yield the target value of MCD, $MCD_{\min}$. Now, we solve and summarize the target values of five PMs, $TLC_{\min}, MCD_{\min}, MDWCD_{\min}$, $CDE_{\max}$, and $ENDS_{\max}$ in Table 3. Using these target values in Table 3, the MONLP model is solved for various values of $\boldsymbol{\alpha}$, where each weight changes between 0 and 1 with an increment of 0.1. There are 1001 configurations arising out of the combinations of the setting of $\boldsymbol{\alpha}$ under the condition given in Eq. (42). After solving the model, we reduce 1001 configurations into 542 consolidated configurations, based upon the values of the five performance measures. To apply DEA, we consider each of 542 configurations as a DMU. In fact, each configuration or DMU denotes an FLA scheme.

There are two important issues in carrying out an efficiency study using DEA. One is the homogeneity of DMUs, and the other one is the isotonicity property. Since all DMUs are generated by the same MONLP model under different weights, the *homogeneity* of DMUs is satisfied to generate meaningful DEA results. DEA requires that there be an isotonic (order preserving) relation between inputs and outputs, i.e., an increase in any input should not result in a decrease in any output (see Golany and Roll 1989). We use the correlation analysis between all inputs and outputs to test this relationship. We find that the correlation coefficients (ρ) between TLC versus CDE and between $MDWCD$ and CDE are negative, such as $\rho(TLC \text{ vs. } CDE) = -0.6623$ and $\rho(MDWCD \text{ vs. } CDE) = -0.5467$, whereas all other correlation coefficients are positive. Negative correlation coefficients would indicate that CDE should not be treated as an output to apply DEA. As Charnes et al. (1984) invert the values of some factors before they are entered into the analysis, we define $nCDE$ as the non-covered demand in case of emergency, which is obtained by subtracting CDE from the total demand of 4,496 K as seen from Table 1. Before applying DEA, we delete CDE from the set of outputs and add $nCDE$ as a new input, which results in $I = \{TLC, MCD, MDWCD, nCDE\}$ and $O = \{ENDS\}$. Now, the correlation coefficient between $nCDE$ and $ENDS$ is positive, $\rho(\text{nCDE vs. } ENDS) = 0.1844$. In fact, all correlation coefficients between all inputs and an output become positive, which implies that the isotonicity property would not be violated to implement DEA for the case study. This completes (3) and (4) in Step 1.

Now, using the stratification DEA method explained in Step 2, we stratify 542 DMUs into 24 levels, $E^1, E^2, \ldots E^{24}$, and identify DMUs in each level. To rank DMUs in Level 1, E^1, we compute 23 attractiveness scores and then the AAS given in Eq. (22). By excluding the weakly efficient DMUs in E^1, we find that there are thirty-one (31) fully efficient DMUs (FEDMUs). Based upon the AASs, we rank all 31 FEDMUs. This completes Step 3. Each DMU represents

Table 3 The target values of five performance measures

PM	$TLC_{\min}$	$MCD_{\min}$	$MDWCD_{\min}$	$CDE_{\max}$	$ENDS_{\max}$
$\boldsymbol{\alpha} = (\alpha_1^+, \alpha_2^+, \alpha_3^+, \alpha_1^-, \alpha_2^-)$	(1, 0, 0, 0, 0)	(0, 1, 0, 0, 0)	(0, 0, 1, 0, 0)	(0, 0, 0, 1, 0)	(0, 0, 0, 0, 1)
Target value	\$24,188 (K)	65.93 miles	13,150 K	3158 K	4027 K

Table 4 DMUs in the first level and last two levels

No.	DMU #	$\boldsymbol{\alpha}=(\alpha_1^+, \alpha_2^+, \alpha_3^+, \alpha_1^-, \alpha_2^-)$	*TLC* (K$)	*MCD* (miles)	*MDWCD* (K miles)	*nCDE* (K)	*ENDS* (K)	*ES*	*AAS*	Ranking
			Input	Input	Input	Input	Output			
Level 1										
1	4	(0.0, 0.0, 0.0, 0.7, 0.3), (0.0, 0.0, 0.0, 0.6, 0.4)	$36,842	267	44,507	1420	3721	1.000	1.2587	18
2	13	(0.0, 0.0, 0.1, 0.7, 0.2), (0.1, 0.0, 0.1, 0.6, 0.2), (0.2, 0.0, 0.1, 0.5, 0.2), (0.3, 0.0, 0.1, 0.4, 0.2)	$27,673	173	17,164	1338	3412	1.000	1.2907	10
3	14	(0.0, 0.0, 0.1, 0.6, 0.3), (0.1, 0.0, 0.1, 0.5, 0.3), (0.2, 0.0, 0.1, 0.4, 0.3)	$29,858	194	19,063	1338	3423	1.000	1.2738	12
4	15	(0.0, 0.0, 0.1. 0.5, 0.4)	$27,867	194	20,999	1338	3426	1.000	1.2597	17
6	59	(0.0, 0.1, 0.1, 0.7, 0.1), (0.0, 0.3, 0.3, 0.2, 0.2), (0.0, 0.4, 0.4, 0.1, 0.1), (0.1, 0.2, 0.2, 0.3, 0.2), (0.1, 0.3, 0.3, 0.1, 0.2), (0.1, 0.4, 0.4, 0.0, 0.1), (0.2, 0.1, 0.1, 0.5, 0.1), (0.2, 0.3, 0.3, 0.0, 0.2), (0.3, 0.1, 0.1, 0.4, 0.1), (0.3, 0.2, 0.2, 0.1, 0.2), (0.4, 0.1, 0.1, 0.3, 0.1), (0.4, 0.2, 0.2, 0.1, 0.1), (0.4, 0.2, 0.2, 0.0, 0.2), (0.5, 0.1, 0.1, 0.2, 0.1), (0.5, 0.2, 0.2, 0.0, 0.1)	$24,785	87	17,416	1338	3364	1.000	1.2832	11
7	61	(0.0, 0.1, 0.1, 0.5, 0.3)	$26,027	97	19,328	1338	3396	1.000	1.2691	13
7	82	(0.0, 0.1, 0.4, 0.4, 0.1)	$24,986	87	14,216	1338	3376	1.000	1.3726	6
8	96	(0.0, 0.1, 0.7, 0.1, 0.1), (0.1, 0.1, 0.7, 0.0, 0.1), (0.2, 0.1, 0.7, 0.0, 0.0)	$24,785	87	13,759	1338	3364	1.000	1.3987	5
9	97	(0.0, 0.1, 0.7, 0. 0, 0.2)	$25,370	87	13,759	1338	3384	1.000	1.4065	4
10	104	(0.0, 0.2, 0.0, 0.5, 0.3), (0.1, 0.2, 0.0, 0.4, 0.3), (0.2, 0.2, 0.0, 0.3, 0.3)	$25,457	80	24,721	1472	3450	1.000	1.2179	23
11	130	(0.0, 0.2, 0.4, 0.3, 0.1), (0.0, 0.3, 0.6, 0.0, 0.1), (0.1, 0.2, 0.4, 0.2, 0.1), (0.2, 0.1, 0.2, 0.3, 0.2), (0.3, 0.1, 0.2, 0.3, 0.1), (0.4, 0.1, 0.2, 0.2, 0.1), (0.5, 0.1, 0.2, 0.1, 0.1)	$24,986	87	15,283	1338	3376	1.000	1.3259	7
12	158	(0.0, 0.3, 0.2, 0.1, 0.4), (0.2, 0.3, 0.2, 0.1, 0.2), (0.3, 0.3, 0.2, 0.0, 0.2), (0.4, 0.3, 0.2, 0.0, 0.1)	$24,785	87	19,549	1338	3364	1.000	1.2619	15
13	170	(0.0, 0.4, 0.0, 0.6, 0.0), (0.0, 0.4, 0.0, 0.5, 0.1), (0.0, 0.6, 0.0, 0.4, 0.0), (0.1, 0.5, 0.0, 0.3, 0.1), (0.2, 0.4, 0.0, 0.4, 0.0), (0.3, 0.3, 0.0, 0.4, 0.0), (0.3, 0.4, 0.0, 0.3, 0.0), (0.4, 0.4, 0.0, 0.2, 0.0), (0.5, 0.2, 0.0, 0.3, 0.0), (0.5, 0.3, 0.0, 0.2, 0.0), (0.6, 0.2, 0.0, 0.2, 0.0), (0.6, 0.3, 0.0, 0.1, 0.0), (0.6, 0.3, 0.0, 0.0, 0.1), (0.7, 0.3, 0.0, 0.0, 0.0)	$24,914	70	21,613	1475	3270	1.000	1.2234	22
14	176	(0.0, 0.4, 0.1, 0.5, 0.0), (0.5, 0.1, 0.0, 0.4, 0.0), (0.8, 0.1, 0.0, 0.0, 0.1)	$24,822	77	21,613	1472	3349	1.000	1.2076	27
15	188	(0.0, 0.4, 0.3, 0.1, 0.2), (0.1, 0.4, 0.3, 0.0, 0.2)	$24,785	87	18,838	1338	3364	1.000	1.2681	14
16	225	(0.0, 0.7, 0.0, 0.1, 0.2), (0.1, 0.6, 0.0, 0.1, 0.2)	$28,195	69	21,613	2276	3367	1.000	1.1989	29

Table 4 (continued)

No.	DMU #	$\boldsymbol{\alpha} = (\alpha_1^+, \alpha_2^+, \alpha_3^+, \alpha_1^-, \alpha_2^-)$	*TLC* (K$)	*MCD* (miles)	*MDWCD* (K miles)	*nCDE* (K)	*ENDS* (K)	*ES*	*AAS*	Ranking
			Input	Input	Input	Input	Output			
17	234	(0.0, 0.8, 0.0, 0.1, 0.1), (0.0, 0.9, 0.0, 0.1, 0.0), (0.1, 0.7, 0.0, 0.1, 0.1), (0.1, 0.8, 0.0, 0.1, 0.0), (0.2, 0.6, 0.0, 0.1, 0.1), (0.3, 0.5, 0.0, 0.1, 0.1), (0.3, 0.6, 0.0, 0.1, 0.0), (0.3, 0.6, 0.0, 0.0, 0.1), (0.3, 0.7, 0.0, 0.0, 0.0), (0.4, 0.6, 0.0, 0.0, 0.0)	$26,098	67	22,153	1905	3300	1.000	1.2078	26
18	235	(0.0, 0.8, 0.0, 0.0, 0.2), (0.1, 0.7, 0.0, 0.0, 0.2)	$30,757	69	31,040	2818	3405	1.000	1.2085	25
19	245	(0.1 0.0, 0.0, 0.6, 0.3)	$32,858	190	44,507	1420	3710	1.000	1.2566	19
20	246	(0.1, 0.0, 0.0, 0.5, 0.4)	$33,634	194	44,507	1420	3721	1.000	1.2604	16
21	325	(0.1, 0.4, 0.0, 0.5, 0.0), (0.1, 0.6, 0.0, 0.3, 0.0), (0.4, 0.3, 0.0, 0.2, 0.1)	$25,117	70	21,613	1475	3282	1.000	1.2274	20
22	375	(0.2, 0.1, 0.2, 0.4, 0.1), (0.6, 0.1, 0.2, 0.0, 0.1)	$24,785	87	15,283	1338	3364	1.000	1.3228	8
23	409	(0.2, 0.5, 0.0, 0.1, 0.2), (0.3, 0.4, 0.0, 0.1, 0.2), (0.3, 0.5, 0.0, 0.0, 0.2)	$27,767	70	21,613	2210	3408	1.000	1.1966	30
24	416	(0.2, 0.7, 0.0, 0.0, 0.1)	$27,150	68	22,153	1924	3323	1.000	1.2125	24
25	463	(0.4, 0.0, 0.0, 0.5, 0.1)	$25,387	190	13,150	1338	3394	1.000	1.4452	3
26	473	(0.4, 0.1, 0.0, 0.4, 0.1)	$25,365	77	24,721	1472	3425	1.000	1.2256	21
27	485	(0.4, 0.2, 0.3, 0.0, 0.1)	$24,785	87	15,994	1338	3364	1.000	1.3059	9
27	494	(0.5, 0.0, 0.0, 0.4, 0.1), (0.6, 0.0, 0.0, 0.3, 0.1)	$25,022	94	13,150	1338	3390	1.000	1.4545	1
29	536	(0.7, 0.1, 0.0, 0.1, 0.1)	$24,936	77	24,721	1538	3380	1.000	1.1936	31
30	539	(0.8, 0.0, 0.0, 0.1, 0.1), (0.9, 0.0, 0.0, 0.0, 0.1)	$24,628	94	13,150	1338	3382	1.000	1.4518	2
31	540	(0.8, 0.0, 0.0, 0.0, 0.2)	$25,371	84	24,721	1472	3461	1.000	1.2066	28
Level 23										
1	251	(0.1, 0.0, 0.0, 0.0, 0.9)	$35,051	151	33,566	2687	3894	0.8091	N/A	N/A
2	277	(0.1, 0.1, 0.1, 0.1, 0.6)	$32,247	116	23,172	2707	3545	0.8962	N/A	N/A
Level 24										
1	10	(0.0, 0.0, 0.0, 0.0, 1.0)	$49,394	219	61,848	3323	4027	0.5937	N/A	N/A

the optimal location of DRCs and their allocation scheme. In Table 4, we present all FEDMUs in E^1, as well as the least efficient DMUs in the last two levels, E^{23} and E^{24}. We also present the combination of weights, the value of each PM, CRS efficient score (ES), and AAS and its ranking. For example, we see that DMU_4 generated by solving MONLP

Table 5 DRC location and allocation for some efficient and inefficient DMUs

Level	AAS (Rank)	DMU #	DRC	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6
1	1.4545 (1)	494	Charleston	Beaufort	Georgetown	Hampton	Moncks Corner	Orangeburg	Walterboro
			Conway	Florence					
			Greenville	Anderson	Greenwood	McCormick	Spartanburg		
			Lexington	Aiken	Columbia	Sumter			
			Rock Hill	Bennettsville					
1	1.4518 (2)	539	Charleston	Beaufort	Georgetown	Hampton	Moncks Corner	Walterboro	
			Conway	Florence					
			Greenville	Anderson	Greenwood	McCormick	Spartanburg		
			Lexington	Aiken	Columbia	Orangeburg	Sumter		
			Rock Hill	Bennettsville					
1	1.4452 (3)	463	Charleston	Beaufort	Georgetown	Moncks Corner	Orangeburg	Walterboro	
			Conway	Florence					
			Greenville	Anderson	Greenwood	Hampton	McCormick	Spartanburg	
			Lexington	Aiken	Columbia	Sumter			
			Rock Hill	Bennettsville					
1	1.4065 (4)	97	Charleston	Beaufort	Georgetown	Hampton	Moncks Corner	Orangeburg	Walterboro
			Conway	Bennettsville	Florence				
			Greenville	Anderson	Greenwood	McCormick	Spartanburg		
			Lexington	Aiken	Columbia				
			Rock Hill	Sumter					
1	1.3987 (5)	96	Charleston	Beaufort	Hampton	Moncks Corner	Walterboro		
			Conway	Bennettsville	Florence	Georgetown			
			Greenville	Anderson	Greenwood	McCormick	Spartanburg		
			Lexington	Aiken	Columbia	Orangeburg			
			Rock Hill	Sumter					
1	1.3726 (6)	82	Charleston	Beaufort	Hampton	Georgetown	Moncks Corner	Walterboro	
			Conway	Bennettsville	Florence				
			Greenville	Anderson	Greenwood	McCormick	Spartanburg		
			Lexington	Aiken	Columbia	Orangeburg			
			Rock Hill	Sumter					
23	N/A	251	Anderson	McCormick					
			Beaufort	Charleston	Florence	Hampton	Moncks Corner	Orangeburg	Walterboro
			Conway	Bennettsville	Georgetown				
			Greenwood	Aiken	Columbia	Lexington	Sumter		
			Greenville	Rock Hill	Spartanburg				
23	N/A	277	Beaufort	Hampton	Orangeburg	Walterboro			
			Columbia	Rock Hill					
			Greenwood	Aiken	Anderson	Lexington	Spartanburg	Sumter	
			Moncks Corner	Bennettsville	Charleston	Conway	Florence	Georgetown	
			Greenville	McCormick					
24	N/A	10	Anderson	Lexington	Sumter				
			Beaufort	Bennettsville	Columbia	Charleston	Georgetown	Moncks Corner	Walterboro
			Greenwood	Aiken	Conway	Florence	Hampton	Spartanburg	
			Greenville	McCormick	Orangeburg	Rock Hill			

with either the combination of weights, (0.0, 0.0, 0.0, 0.7, 0.3) or (0.0, 0.0, 0.0, 0.6, 0.4), yields ($36,842 K, 267 miles, 44507 K miles, 3721 K, 1420 K) as the optimal values of *TLC*, *MCD*, *MDWCD*, *nCDE*, and *ENDS*. It obtains a complete and fully efficient score of 1.000, yields an AAS of 1.2587 with its ranking of 18. As shown in Table 4, all of FEDMUs in E^1 possess a perfect ES of 1.000, whereas DMU_{10} in the last level, E^{24}, is considered as the least

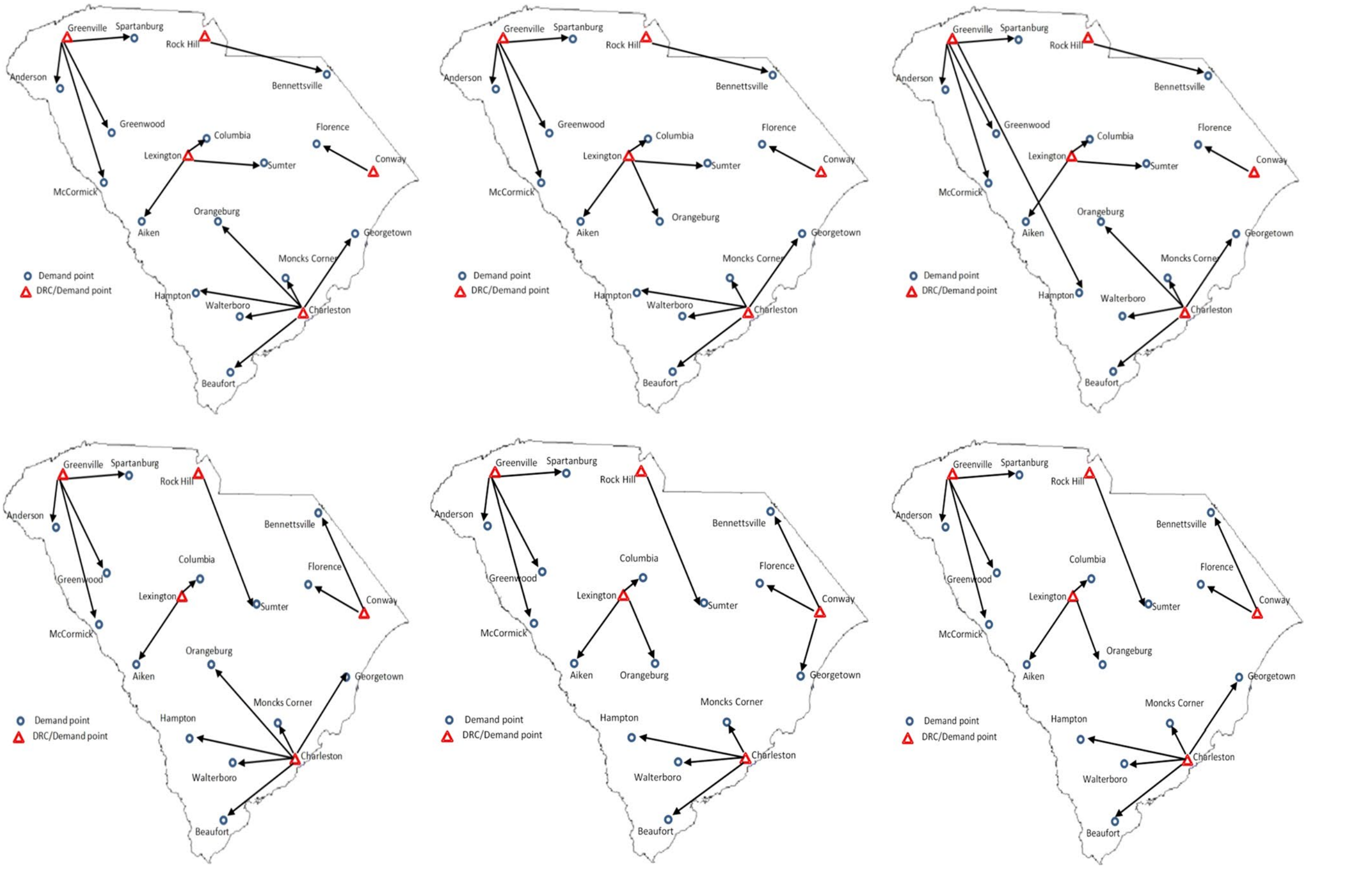

Fig. 1 Efficient facility location–allocation schemes for DMU_{494}, DMU_{539}, DMU_{463}, DMU_{97}, DMU_{96}, and DMU_{82}

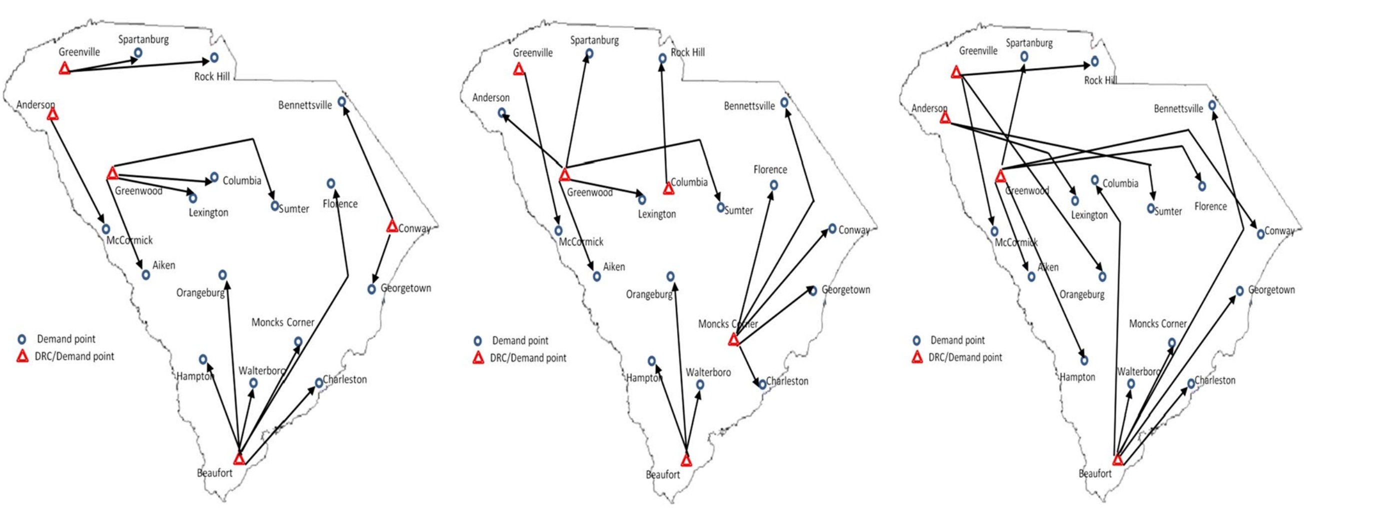

Fig. 2 Inefficient facility location–allocation schemes for DMU_{251}, DMU_{277}, and DMU_{10}

efficient with ES of 0.5970. For the purpose of comparison with inefficient DMUs, the two DMUs in E^{23}, DMU_{251} and DMU_{277}, are presented too. It appears that DMU_{494} in E^1 which has the highest AAS of 1.4545, yields the most efficient FLA scheme.

In Table 5, we present the optimal DRC location–allocation schemes for top six DMUs in E^1 and three DMUs in E^{23} and E^{24}. All of the FLA schemes in Table 5 are depicted in Figs. 1 and 2. A notable observation from Table 5 and Fig. 1 is that all six efficient DMUs in Level 1 select {Charleston, Conway, Greenville, Lexington, Rock Hill} as the optimal locations for DRCs, and their allocations show very similar patterns with just a few exceptions. For example, DRC Lexington covers {Aiken, Columbia, Sumter} in DMU_{494} and DMU_{463}, {Aiken, Columbia, Orangeburg} in DMU_{96} and DMU_{82}, {Aiken, Columbia, Orangeburg, Sumter} in DMU_{539}, and {Aiken, Columbia} in DMU_{97}. DRC Rock Hill covers {Sumter} in DMU_{90}. We can observe the only difference in allocation for the two most efficient DMUs, DMU_{494} and DMU_{539}, such that {Orangeburg} will be covered by DRC Charleston in DMU_{494} but by DRC Lexington in DMU_{539}. Also, from Table 4, we see that DMU_{494} yields higher values of input, *TLC,* as well as of output, *ENDS*, than those of DMU_{539}, while both DMUs yield the same values of other performance measures, *MCD*, *MDWCD*, and, *nCDE*. Note that the optimal location–allocation schemes of DRCs for the three DMUs in the two lowest levels, E^{23} and E^{24}, are quite different from the schemes for the four efficient DMUs in E^1. All of them commonly select {Beaufort, Greenwood, Greenville} as the optimal locations for DRCs and their allocations are quite different, as shown in Fig. 2. From Table 4, we observe that their input values, as well as outputs, are higher than the DMUs in E^1.

In Fig. 1, we observe that Hampton is covered by DRC Greenville in the scheme of DMU_{463}, while it is covered by DRC Charleston in other efficient DMUs. Hampton is 190 miles away from Greenville. From Table 4, we see that *MCD* for DMU_{463} is 190 miles, but *MWDCD* is the target (minimum) value of 13,150. The reason is that since Hampton has a small demand of 33 K from Table 1, a relatively high value of *MCD* does not affect *MWDCD*. The two DRC locations, Lexington and Charleston, have higher risk probabilities, 0.313 and 0.25, respectively, than Greenville with 0.125. In addition, Hampton is 84.25 and 78 miles away from Lexington and Charleston, which exceed the maximum effective coverage distance, D_c, of 35 miles. Thus, without sacrificing *MWDCD* and *CDE*, DRC Greenville for DMU_{463} covers Hampton to enhance *ENDS* at the sacrifice of *MCD* and *TLC*. Note that finding the location of DRC is a strategic decision plan. In an operational level, if neither DRC Charleston nor DRC Lexington is disrupted, it would enhance efficiency to let either Charleston or Lexington, instead of DRC Greenville, cover Hampton if it has enough capacity. In this way, practitioners and decision-makers can make flexible operational decisions once they have efficient and robust FLA schemes on their hands.

Figure 2 depicts the schemes of DMU_{251}, DMU_{277}, and DMU_{10}, which generally yield high values of inputs and an output, as seen from Table 4. Note that the most inefficient scheme DMU_{10} with ES of 0.5937, which is generated with $\boldsymbol{\alpha} = (0, 0, 0, 0, 1)$, dominates DMU_{251} and DMU_{277} regarding all inputs and an output. On the contrary to other schemes, the optimal number of DRCs generated by DMU_{10} is only four and all four DRCs, {Anderson, Beaufort, Greenwood, Greenville}, are located at the sites whose risk probabilities are the lowest ones, such as 0.063, 0.125, 0.125, and 0.125. In other words, DMU_{10} will guarantee the maximum expected demand covered/satisfied under the risk of disruptions, at the sacrifice of all input performance measures. If satisfying demand under the high risk of disruptions has the highest priority, decision-makers will adopt the scheme of DMU_{10}. Decision-makers may want to consider DMU_{251} generated with $\boldsymbol{\alpha} = (0.1, 0, 0, 0, 0.9)$ rather than DMU_{10}, since they can observe that switching from DMU_{10} to DMU_{251} would reduce *ENDS* by 133 K (= 4027–3894 K) (3.3%), but all other inputs are significantly saved: *TLC*, *MCD*, *MDWCD*, and *nCDE* are reduced by $14,034 K (29%), 68 miles (31%), 28282 K miles (45%), 636 K (19%), respectively.

To identify robust locations of DRCs, we list, in Table 6, the locations for DRCs for all DMUs in E^1 and list the DRC location sets with the frequencies (≥ 2) in descending order in Table 7. In Table 8, we also find the frequency of each DRC location and list the locations, frequencies along with the corresponding percentages in descending order. As shown in Tables 6 and 7, a set of five sites for the location of DRCs, {Charleston, Conway, Greenville, Lexington, Rock Hill}, is selected 15 times out of 31 cases. As stated before and shown in Fig. 1, the six most efficient DMUs select these DRC locations. As expected, these five locations are ranked from 1st to 5th in terms of the frequency of DRC location as shown in Table 8. Identifying the robust locations of DRCs will be important for decision-makers to decide an alternative location when the risk of facility disruptions is very high. For example, from Tables 6 to 7, {Beaufort} can replace {Conway} or {Rock Hill} as a DRC location if either {Conway} or {Rock Hill} is disrupted. Similarly, {Aiken, Columbia} can replace {Lexington, Rock Hill} which is under the high risk of disruptions. In addition, decision-makers can exclude some sites from the candidate of DRC location. For example, the seven sites, {Bennettsville, Georgetown, McCormick, Moncks Corner, Orangeburg, Spartanburg, Sumter}, are never selected as a DRC location.

Table 6 Locations selected for DRCs for all DMUs in Level 1

No.	DMU	DRC 1	DRC 2	DRC 3	DRC 4	DRC 5
1	4	Charleston	Beaufort	Greenville	Lexington	Rock Hill
2	13	Charleston	Conway	Greenville	Lexington	Rock Hill
3	14	Charleston	Conway	Greenville	Lexington	Rock Hill
4	15	Charleston	Conway	Greenville	Lexington	Rock Hill
6	59	Charleston	Conway	Greenville	Lexington	Rock Hill
7	61	Charleston	Conway	Greenville	Lexington	Rock Hill
7	82	Charleston	Conway	Greenville	Lexington	Rock Hill
8	96	Charleston	Conway	Greenville	Lexington	Rock Hill
9	97	Charleston	Conway	Greenville	Lexington	Rock Hill
10	104	Charleston	Conway	Greenville	Lexington	Beaufort
11	130	Charleston	Conway	Greenville	Lexington	Rock Hill
12	158	Charleston	Conway	Greenville	Lexington	Rock Hill
13	170	Charleston	Conway	Greenville	Aiken	Columbia
14	176	Charleston	Conway	Greenville	Beaufort	Columbia
15	188	Charleston	Conway	Greenville	Lexington	Rock Hill
16	225	Florence	Greenwood	Greenville	Walterboro	Columbia
17	234	Charleston	Beaufort	Florence	Anderson	Columbia
18	235	Walterboro	Greenwood	Florence	Anderson	Rock Hill
19	245	Charleston	Beaufort	Greenville	Lexington	Rock Hill
20	246	Charleston	Beaufort	Greenville	Lexington	Rock Hill
21	325	Charleston	Conway	Greenville	Aiken	Columbia
22	375	Charleston	Conway	Greenville	Lexington	Rock Hill
23	409	Greenwood	Conway	Greenville	Walterboro	Columbia
24	416	Charleston	Anderson	Hampton	Florence	Columbia
25	463	Charleston	Conway	Greenville	Lexington	Rock Hill
26	473	Charleston	Conway	Greenville	Lexington	Beaufort
27	485	Charleston	Conway	Greenville	Lexington	Rock Hill
27	494	Charleston	Conway	Greenville	Lexington	Rock Hill
29	536	Charleston	Beaufort	Greenville	Lexington	Florence
30	539	Charleston	Conway	Greenville	Lexington	Rock Hill
31	540	Charleston	Conway	Greenville	Lexington	Beaufort

Table 7 Frequency of robust DRC locations for the DMU in Level 1

No.	DRC locations	Frequency
1	{Charleston, Conway, Greenville, Lexington, Rock Hill}	15
2	{Charleston, Beaufort, Greenville, Lexington, Rock Hill}	3
3	{Charleston, Conway, Greenville, Lexington, Beaufort}	3
4	{Charleston, Conway, Greenville, Aiken, Columbia}	2

Conclusions

In this study, we propose an innovative procedure of deciding efficient facility location–allocation (FLA) by combining data envelopment analysis (DEA) methodology with

Table 8 Frequency of locations selected for DRCs for the DMUs in Level 1

No.	Location	Frequency (max of 31)	Percentage (%)
1	Charleston	28	90.3
2	Greenville	28	90.3
3	Lexington	23	74.2
4	Conway	23	74.2
5	Rock Hill	20	64.5
6	Beaufort	9	29.0
7	Columbia	7	22.6
8	Florence	5	16.1
9	Walterboro	3	9.7
10	Anderson	3	9.7
11	Greenwood	3	9.7
12	Aiken	2	6.5
13	Hampton	1	3.2

multi-objective programming model. We accomplish by first using a multiple objective nonlinear programming (MONLP) with the minimax objective approach, which would yield more balanced options, to generate all inputs and outputs for each configuration arising out of the combinations of the weight factor $\boldsymbol{\alpha}$. Each location–allocation scheme generated by solving MONLP for a given set of $\boldsymbol{\alpha}$ is treated as a decision-making unit (DMU). For the generated DMUs, we apply DEA to identify efficient location–allocation schemes, and then we use the context-dependent/stratification DEA to rank the efficient schemes. The contribution of this paper is that our procedure generates inputs and outputs directly from the multi-objective mathematical model, so that DEA would be more realistically applied to identify the efficient schemes. As stated before, the literature available so far has assumed that all inputs and outputs for locating a facility to a site are given or known unrealistically to apply DEA for their models.

Through case study using actual major disaster declaration records in South Carolina, we demonstrate the applicability of our procedure and observe that our combined approach with MONLP model and DEA for the FLA problem performs well. The MONLP model generates various FLA schemes based upon the weights given to each performance measure which is classified as an input or output. Out of those FLA schemes, DEA identifies efficient schemes, and then context-dependent DEA ranks those efficient schemes and allows us to identify the most efficient FLA schemes. Our new approach to the MOFLA problems would provide many insights to academians as well as practitioners and researchers.

In this paper, we assume that if a facility becomes unavailable or shut down due to disruptions, all the demand for the sites assigned to the disrupted facility is lost. For future research, it would be interesting to consider the backup supply from the undisrupted facility, so that the demand for some sites would be satisfied.

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