Original Research

Developing New Methods to Monitor the Fuzzy Logistic Regression Profiles in Phase II
(A case study in health-care)

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Abstract
In real quality control applications, the performance of a process or the quality of a product is described by the relationship between a non-metric response variable and one or more control variables. Furthermore, the quality characteristic of a product or process is vague, unreliable, and linguistic and cannot be accurately expressed in most practical applications. This study was carried out aimed to provide a method for monitoring the fuzzy logistic regression profile in Phase II. In these circumstances, there is a need for special diagrams to monitor the performance of this fuzzy data. To this aim, some powerful control charts including Fuzzy exponentially weighted moving average (FEWMA), fuzzy T^2 (FT^2) control chart have been developed. In addition, to show the performance of the proposed control charts, the fuzzy hypothesis test along with average Run Length (ARL) criterion is used in Phase II. In addition, to show the efficiency of the proposed control chart in real applications, a real case study in health-care has been applied.

Keywords - Statistical quality control; Fuzzy Quality Profile; FEWMA Fuzzy Statistics; Hotelling'sF T^2; ARL

1. INTRODUCTION

Statistical quality control (SQC) has a long history. This science arose after the beginning of production and the formation of competition in the industry. Customers began to compare products and tried to choose the best product both in terms of price and quality. In the meantime, industries have put product quality control in their agenda and increasing attention has been paid to this science from then until today. quality control courses, including the era of Quality Control Worker, the era of Quality Control (QC) Supervisor, the era of Quality control inspectors and the era of Statistical quality control.

The first application of statistical methods in the form of quality control was modern control charts introduced by Shewhart in 1924. Statistical quality control consists of three main groups, including Acceptance sampling, Design of experiments (DOE) and control of statistical process (Koosha 2011) [1].

Monitoring the consequences of medical processes such as surgery can be helpful in detecting changes in performance and take corrective measures for improving them. Statistical process control (SPC) in the medical sciences as a valuable quality improvement tool. The control chart is used in health sciences because in this context, patients have different clinical presentation such as age, gender, blood pressure, renal function and others which are explanatory variables and so have different previous risks (Lovegrove et al) [2] [3]. Statistical process control is considered as a statistical technique to reduce dispersion and thus improve quality. In other words, any statistical method designed to reduce dispersion over time, It can be said that science is statistical process control (Woodall and Montgomery (1999) [4], which the control chart is the most important one (Woodall et al. 2004) [6]. They introduced a new field in the science of statistical process control and presented many applications of this science with examples in industrial and service units. They included that sometimes, the performance of a process can be described by the relationship between the response variable and one or more independent variables instead of describing a qualitative characteristic by one or more response variables.

Many researchers all over the world have investigated and presented methods for monitoring profiles and nowadays this branch has attracted much attention of scientists as one of the most attractive branches of statistical process control science. Woodall (2007) [7] has examined the areas and gaps of research monitoring profiles. Profiles can be linear, non-linear or even more complex. In most profile applications, the distribution of response variables is not normal, and if they are estimated with a normal function, the analysis error will be very high. In this case, it is necessary to examine another type of profiles, which are called generalized linear models (GLMs)-based profiles, such as logistic profiles. Furthermore, if the quality characteristic of a product or process is vague, unreliable, and linguistic so that it may not be possible to express it accurately, then input information is non-fuzzy but output information or control variable is fuzzy that fuzzy logistic regression should be used (Cheng 2005) [8].

In this paper, after investigating the researches on fuzzy profiles , it can be classified as linear and non - linear and the fuzzy logistic regression profile is discussed . In the following , the concepts and formula of fuzzy numbers are introduced . In the next section , the coefficients of the fuzzy logistic regression model are estimated using least squares method . in the fourth and fifth section , the proposed control charts FEWMA and FT2are introduced using the spread principle for fuzzy data and are compared with the using ARL method . This comparison is carried out by the simulated and actual data in the field of medical science.

2. LITERATURE REVIEW

Shang et al. (2011) investigated the binary logistics profiles monitoring in the presence of independent variables with random values in phase II. Given that independent variables are defined randomly, a parameter for profile monitoring will be added for each independent variable. They proposed a new approach for binary logistics profiles monitoring by combining Exponentially Weighted Moving Average (EWMA) and Likelihood Ratio Test (LRT) methods. They also compared their new approach with the Hoteling’s T-squared distribution (T2) and the MEWMA methods. According to the results, their proposed methods had better performance than the two existing methods under most shifts, because the Hoteling’s T-squared distribution (T2) method does not have high efficiency in discovering small to medium shifts and given that the shifts created in phase II of their research are small, therefore this method will not have high efficiency in monitoring profiles in phase II [11].

Paynaber et al. (2012) used the results of the results to monitor binary profiles in Phase I using Risk-Adjusted Control Charts. They developed the discussion of monitoring profiles for non-industrial applications of medical sciences in a cardiac surgery center on patients undergoing heart surgery. They defined a measure of patient mortality after surgery their response variable, which has measured a binary variable measured by whether does the patient survive after surgery? Furthermore, patient age and type of surgery were defined as control variables. In the present study, the mean values of response variables were linked to independent variables by a logistic communication function and the purpose has been monitoring the change the patient mortality rate by considering different values of control variables. The adjusted risk in this particular application means that the outcome of the surgery is highly dependent on the patient's condition prior to surgery. So, surgery should be adjusted to the risk of this factor [12].

Amiri et al. (2012) proposed a new approach for reducing the dimensions to overcome the large-scale problem in existing methods focusing on monitoring multiple linear profiles in phase two. They believe that increasing the scale of the parameter vector makes the performance of existing methods for monitoring multiple linear profiles worse, so there was a new approach. According to the results of (MEWMA), the estimation test, related to estimation of new parameter used in the simulation approach in this research, shows the proposed approach has a good effectiveness compared to existing methods for monitoring large-scale profiles [13]. Saghaei et al. (2012) focus on binary response and seek its application in many fields of science and engineering. They proposed some methods for monitoring such profiles in phase I. However, there is no method for monitoring them in the phase II, the tracking phase. This study has proposed two methods for monitoring logic profiles in phase II. The first method is a combination of EWMA diagrams and residual variance deviation in logistic regression models and the second method is multivariate T^2 diagram for monitoring model parameters [14].

Soleimanian et al. (2013) during a study presented four methods of Hotelling's T-squared distribution (T2), MEWMA mean, likelihood ratio test (LRT) and EWMA for monitoring binary logistics profiles in phase two and Using simulation showed better performance EWMA with respect to small and medium shifts [15].

Koosha and Amiri [16] (2013) have evaluated the autocorrelation effect of observations within binary logistics profiles in phase I and have developed relevant methods. First, the effect of autocorrelation with a symmetrical structure in which the correlation coefficients between all observations are assumed to be the same on the methods proposed by Yeh et al. [17] (2009) has been investigated by them. Thus, the negative effect of autocorrelation showed [17] (2009) based on two types of step-like gradual shifts.
According to their results, the presence of autocorrelation increases the probability of the type I error and as the autocorrelation coefficient increases, the amount of this increase will also increase. Then, they proposed two methods to solve the autocorrelation problem within logistics profiles. The first method is the threshold adjustment method in which the value of the upper limit of control for each value of the correlation coefficient is determined by simulation in such a way that the probability of type I error has a specific and predetermined value. Also, the second method is based on generalized linear mixed models (GLMMs) in which first the profile parameters are estimated using it and then the profile is monitored using one of the Hotelling's T-squared distribution (T^2) statistics used by Yeh et al. [17] (2009). Finally, they evaluated these two solutions using simulations and showed that the GLMM-based method has a better performance than the threshold adjustment method in all conditions and for all statistics of Hotelling's T-squared distribution (T^2) and under step-like gradual shifts.
Amiri and Imani (2013) presented a control chart based on extended exponentially weighted moving average (EEWMA) statistic based on the deviation statistics of Skinner's residual values for monitoring logistics profiles in phase two. [18].
Moghadam et al. (2015) investigated the quality of a product if the quality characteristics of a product or service are vague, unreliable and linguistic and it is impossible to express it. This study presents a method for monitoring fuzzy linear profiles in phase two. For this purpose, FEWMA and Hotelling's T-squared distribution (T^2) charts are used using the extended extension principle. A case study is presented based on performance evaluation simulation of the proposed methods in terms of the ARL in the ceramic tile industry; finally these researchers concluded that the FEWMA method has the best performance [19].
Kamran Rad (2016) during his doctoral dissertation examined the monitoring multivariate process with classified characteristics, of which the contingency table is one of the most important tools for its analysis. This table can examine the relationship of two or more correlated classified qualitative characteristics simultaneously and determine the effect of each change on other characteristics. However, it should be noted that the position of generalized linear models is very obvious in this type of problems and also models called log-linear models have been developed to analyze such tables that this model is able to determine the relationship between each cell of the contingency table with different levels of each quality characteristic.
They will be monitored in these contingency tables. For this purpose, different monitoring approaches have been developed for this type of tables for Phase I and Phase II. Furthermore, approaches will be provided to estimate the change point for this type of tables. Furthermore, this study has presented and developed various approaches to monitor processes based on the contingency tables. In Phase I of monitoring this type of process, test approaches are presented for the likelihood ratio test, T^2, F.
In this phase, the problem is classified into two parts: process monitoring based on 1) log-linear models and (2 contingency tables. In the first case, the generalized linear test (GLT) approach and its combined statistics with EWMA are developed and in the second case, the statistics WALDs, the Stuart Scoring Test (SST) as well as their combined statistics with EWMA have been developed, and the results of all of these approaches have been compared with approaches in the research literature. He has presented various approaches for detecting alarm factor parameters in phase II.
The generalized linear test (GLT) approach and its combined statistics with EWMA have been developed, and in the second case, WALD statistics, SST as well as their combined statistics with EWMA have been developed, the results of all these approaches have been compared with approaches in the research literature. Furthermore, this study has proposed various approaches to detect the warning factor parameters in phase II. Also, this study has investigated the effect of parameter estimation in phase I on the performance of control charts in phase II. Finally, one of the results of this study is the investigation of one of the most important issues that has attracted much attention of many researchers, the effect of estimating model parameters in phase I on the performance of control charts in process monitoring in phase II has been investigated in this study. According to the results of this study, process parameters are estimated more accurately by increasing the number of historical observations in phase I. The result of this correct estimate is that the average run length, ARL, was able to approach their desired value in the controlled state. While, ARL in the controlled state was significantly different from its desired value using less historical observations [20].
Rezaeifar et al. (2020) in this article, quality control in health sciences and determining the performance of surgeons has been used because the performance of surgeons depends on the underlying diseases of patients. Fuzzy numbers are used to determine the risk of the underlying disease. They test their control with real data from specific charts. [21]

3. PROBLEM STATMENT

Recently, the concept of quality has been developed into the concept of appropriate quality by considering the opinions of users on products. Fuzzy sets are used as a tool for expressing the quality and characteristics of the product, which are expressed using some appropriate linguistic values or using some distance points. It is a method for generating a fuzzy logistic regression number. As mentioned earlier, many benefits have been reported for monitoring model-based process in order to control qualitative characteristics and using drawing control charts for each separately. So, this study was carried out aimed to develop and improve the generalized linear model-based monitoring techniques for fuzzy data to increase the sensitivity of control charts to changes in process parameters. This will improve the detection capability, which is especially of utmost importance for finding the roots of the out-process problems. In the following, it is explained how to fit the generalized linear model on the available data X and Y in order to estimate the regression relationship between these two qualitative characteristics.

4. BASIC CONCEPTS

I. *Logistic Regression Model*

The logistic regression analysis is to examine and model the relationship between a response variable and one or more predictor variables. [22].

The variable binary logistic regression is not continuous and has zero and one expression is expressed.

The value of 1 can be obtained with the probability of success π, or a value of 0 with the probability of failure of 1-π.

When the response is binary, the response variable is nonlinear after logical regression and / or logistic response function is used:

$$g(x) = a_0 + a_1 x_1 + \cdots + a_n m$$
$$\pi = \frac{e^{g(x)}}{1+e^{g(x)}} = \frac{1}{1+e^{-g(x)}}$$
So,
$$g(x) = ln\frac{\pi}{1-\pi} \quad (1)$$

Where, ($\ln\frac{\pi}{1-\pi}$)is called conditional probabilities and a_j, j = 0, 1, ... , m are the parameters of the model. In this model, the parameters are estimated by the maximum likelihood estimation (MLE) methods. The importance of each Y is evaluated by performing statistically significant coefficient tests. To determine the goodness-of- fit of the model is tested with X^2.

II. *The Basic Concepts Of Fuzzy Set*

This section provides a brief overview of the rules and definitions of the fuzzy set that will be referred to throughout this study.

Definition 1

Fuzzy numbers $\tilde{B}$ = (a, λ, β) is an asymmetric triangular fuzzy number, where (a) has extended in the center, λ has extended from the left and β has extended from the right, and its membership function is as follows:

$$\mu_{\tilde{\mu}(X)} = \begin{cases} \frac{x-(a-\lambda)}{\lambda} & a-\lambda < x \le a \\ \frac{(a+b)-x}{\beta} & a \le x \le a+\beta \\ 0 & otherwise \end{cases} \quad (2)$$

If Left-and-right areas of triangular fuzzy numbers are equal (λ = β = s), the fuzzy number is called a symmetric triangle fuzzy number and is represented by (a, s).

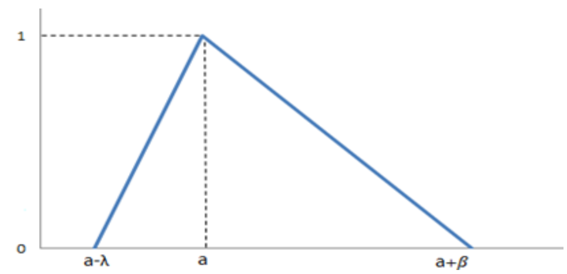

FIGURE 1
TRIANGULAR FUZZY NUMBER

Definition 2

The α-cut of the fuzzy set $\tilde{B}$ is a set of crisp elements from the reference set X with a degree of membership of the fuzzy set $\tilde{B}$ at least as large as α (0 <α≤1). The α-cut of the fuzzy set $\tilde{B}$ is shown by$C_\alpha(\tilde{B})$.

$$C_\alpha(\tilde{B}) = \{x \in X \,|\, \mu_{\tilde{\mu}(X)} \ge \alpha\} \quad (3)$$

If A˜ is an asymmetric triangular fuzzy number$C_\alpha(\tilde{B})$:

$$C_\alpha(\tilde{B}) = [(\alpha-1)\lambda + a, (1-\alpha)\beta + a] = [a^L, a^R] \quad (4)$$

Definition 3

The distance between two asymmetric triangular fuzzy numbers $\tilde{A}, \tilde{B} (d(\tilde{A}, \tilde{B})$ is defined as follows:

$$d(\tilde{A}, \tilde{B}) = \int_0^1 d^2(C_\alpha(\tilde{A}), C_\alpha(\tilde{B}))d\alpha)^{1/2}$$

Where (5)

$$d^2\left(C_\alpha(\tilde{A}), C_\alpha(\tilde{B})\right) = [a^R(\alpha) - b^R(\alpha)]^2 + [a^L(\alpha) - b^L(\alpha)]^2$$

If we write $C_\alpha(\tilde{A})$= [$a^R(\alpha) - b^R(\alpha)$] and $C_\alpha(\tilde{B})$= [$a^L(\alpha) - b^L(\alpha)$], the square of the distance between the two asymmetric triangular fuzzy numbers will be, $\tilde{A} = (a, \lambda_a, \beta_a)$ and $\tilde{B} = (b, \lambda_b, \beta_b)$:

$$d^2(\tilde{A},\tilde{B}) = 2(a-b)^2 + \frac{1}{3}\,[(\lambda_a-\lambda_b)^2 + (\beta_a - \beta_b)^2] + (a-b)\big((\beta_a-\lambda_a)-(\beta_b-\lambda_b)\big) \quad (6)$$

Note

Definition of a special case of fuzzy numbers called (L-R) fuzzy numbers is shown:

$$u(t) = \begin{cases} l\left(\dfrac{a-t}{\lambda}\right) & t \le a\,, \lambda > \beta \\ R\left(\dfrac{t-a}{\beta}\right) & t > a\,, \beta > 0 \end{cases} \quad (7)$$

Where (L, R) are defined functions with real value [0, ∞] and [l (0) =1 , R (0) = 1] and have compact support if L (h) = R (h) = H (h).

$$H(h) = \begin{cases} 1-h\,, 0 \le h \le 1 \\ 0\,, \quad h > 1 \end{cases} \quad (8)$$

Definition 6

Suppose F indicates the space of fuzzy sets and also (E⊑F), therefore for each

$m \in F\,, m: R \to [0,1]$

Consider $\mathcal{X}$ to be the product of the global Cartesian, $X_1, \ldots, X_n$.

$x_1 \times, \ldots, \times x_n$and $m_1, \ldots, m_n$, n Fuzzy set of $X_1, \ldots, X_n$ Also assume that f is the mapping from x to a whole y and $y = f(X_1, \ldots, X_n)$, It is defined according to the principle of extension(y) as follows : [23]

$$l = \{(y, l(y) | y = f(X_1, \ldots, X_n), (X_1, \ldots, X_n) \in X\}$$
$$l(y) = \begin{cases} sup_{(x_1,x_2,\ldots,x_n)\in f^{-1}} \ \min\big(m_1(x_1), \ldots, m_n(x_n)\big) & f^{-1}(y) \neq 0 \\ 0 & other\ wise \end{cases} \quad (9)$$

Where, f^{-1} is the inverse of.

And another case:

$$l(y) = \begin{cases} sup\ m(x)_{x\in f^{-1}} & f^{-1}(y) \neq 0 \\ 0 & other\ wise \end{cases} \quad (10)$$

Theorem 1

Let $u = (a, \lambda, \beta)_{LR}$ are fuzzy numbers and $\theta \in$ R.

$$\lambda u = \begin{cases} (\theta a, \theta\lambda, \theta\beta) & \theta \ge 0 \\ (\theta a, -\theta\lambda, -\theta\beta) & \theta < 0 \end{cases} \quad (11)$$

Theorem 2

Let $k = (a, \lambda, \beta)_{LR}$ and $h = (b, \gamma, \alpha)_{LR}$ be two fuzzy numbers.

$$k + h = (a, \lambda, \beta)_{LR} + (b, \gamma, \alpha)_{LR} = (a + b\,, \lambda + \gamma, \beta + \alpha\ )_{LR} \quad (12)$$

Theorem 3

Assume that $f: R^n \to R$ indicates a continuous function and $\underline{x}^*$ indicates a fuzzy n-dimensional distance [24].

a. $f(\underline{x}^*)$ defined by the extension principle is a fuzzy interval.

b. $C_\alpha[f(\underline{x}^*)] = \left[\min_{\underline{x}\epsilon C_\alpha(\underline{x}^*)} f(x) \quad \max_{\underline{x}\epsilon C_\alpha(\underline{x}^*)} f(x)\right]$ (13)

Let $X_1, X_2, \ldots, X_n$be a sample of sample size n of the fuzzy logistic regression random variable where $X = (m, l, r)_{LR}$can be defined as the mean of the sample $\bar{X}$ and the variance of the sample s_d^2 as follows:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i = (\bar{m}, \bar{l}, \bar{r})_{LR} \quad (14)$$

$$s_d^2 = \frac{1}{n}\sum_{i=1}^{n} d_*^2(x_i, \bar{x})$$

Where: (15)

$$s_d^2 = \frac{1}{n}\sum_{i=1}^{n}\int_0^1 [((m_i-\bar{m}) + (\bar{l} - l_i)L_\alpha^{(-1)})^2 + ((m_i-\bar{m}) + (r_i-\bar{r})R_\alpha^{(-1)})^2]d\alpha$$

Note

According to [25], [24] for a random sample of fuzzy logistic regression $\tilde{x}_1^*\tilde{x}_2^*, \ldots, \tilde{x}_n^*$where defined $\tilde{x}_i^* = (x_i, \lambda_i, \beta_i)$ as follows:

$$s_{\bar{x}^*}^2 = s_{x_i}^2 + \frac{1}{6}\left(s_{\lambda_i}^2 + s_{\beta_i}^2\right) + \frac{1}{2}\left(s_{x_i,\beta_i} - s_{x_i,\lambda_i}\right) \quad (16)$$

III. *FUZZY LOGISTIC REGRESSION*

Few articles have been published in the field of fuzzy logistic regression. [24]. An algorithm is proposed for fitting fuzzy nonlinear models. In fact, we used logistic regression when there is no normality condition. The researchers considered the data as definite input-fuzzy outputs in fuzzy nonlinear regression [26,27]. Fuzzy logistic regression and control limits on fuzzy data base require development in order to implement real data. The relationship can be linearly related to the variation in the logistic regression. Logit conversion is used to be linear. It is used as model output. So, the Tanaka's fuzzy approach [28] and the(least squares) method [29] are used to estimate the parameter in this adaptive model. Fuzzy qualitative variables can be equipped with a suitable probability distribution difficulty, given that such data do not have a known distribution model.Assume that in the process is under control phase I and we calculate the mean of the fuzzy process $\mu_0$and the variance s_{d0} in a fuzzy process in which the fuzzy logistic regression variables are symmetric in a group of k with sample size m.

5. THE PROPOSED MODEL

The relationship between the X variable and the Y variables, which include observations, are:

$$(\boldsymbol{x_{i0}}, \boldsymbol{x_{i1}}, \dots, \boldsymbol{x_{in}}, \widetilde{\boldsymbol{y}}_{\boldsymbol{i}}) \quad \boldsymbol{1 \leq i \leq n} \tag{17}$$

Where x_{ij} and $j = 0,1, \dots, m$ ($x_{ij} \in$ R) and $\tilde{y}_i$ is a fuzzy data that determines the position of each relative to the binary response sets. That is the binary response variable and due to the fuzzy logic of the variables; the answers are not very accurate. So, it is not possible to calculate the probability of success (P (yi = 1) = πi) and to model based on Forecast variables.

Then, the probability of expressing odds $\left(\frac{\pi_i}{1-\pi_i}\right)$is not significant. Here, instead of possibilities, we use the proportion of odds. [30]

The probability of success μi = poss (yi≈1) can be defined (μi , i = 1,…, n) :

a) A real value $\mu_i \in$ R; $\mu_i \in [1.0]$
b) A Fuzzy data, $\mu_i \in \{\dots, low, medium, high, \dots\}$

These items should be defined in such a way as to cover the required area. Then, $\frac{\mu_i}{1-\mu_i}$ is considered as the probability of success, which reveals the possibility of relative success in the possibility of failure[30]. Here is the second case, which is a binary logistic regression model in fuzzy mode.

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{i}} = \boldsymbol{ln}\frac{\widetilde{\boldsymbol{\mu}}_{\boldsymbol{i}}}{\boldsymbol{1} - \widetilde{\boldsymbol{\mu}}_{\boldsymbol{i}}} = \boldsymbol{A_0} + \boldsymbol{A_1 x_{i1}} + \dots + \boldsymbol{A_n x_{in}} \quad \boldsymbol{i = 1}, \dots, \boldsymbol{n} \tag{18}$$

Where $A_{.}, A_1, \dots, A_n$are triangular fuzzy numbers. The estimated output can be changed to$(\widetilde{w}_i)$. The probability of success $\left(\tilde{\mu}_i(x)\right)$can be considered using the extension principle [31].

Linguistic definition of the probability of success $\tilde{\mu}_i \in \{very\ low, low, medium, high, very\ high\}$and then the probability of success of the probabilities $\widetilde{w}_i = ln\frac{\tilde{\mu}_i}{1-\tilde{\mu}_i}$ and $i = 1,2, \dots, n$ are considered as observed outputs.

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{i}}(\boldsymbol{y}) = \boldsymbol{sup}_{\forall x_i : \boldsymbol{ln}\frac{x}{1-x}=y}\, \widetilde{\boldsymbol{\mu}}_{\boldsymbol{i}}(\boldsymbol{x}) \tag{19}$$

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{i}}\left(\boldsymbol{y} = \boldsymbol{ln}\frac{\boldsymbol{x}}{\boldsymbol{1 - x}}\right) = \widetilde{\boldsymbol{\mu}}_{\boldsymbol{i}}\left(\frac{\mathbf{exp}(\boldsymbol{x})}{\mathbf{1} + \mathbf{exp}(\boldsymbol{x})}\right) \tag{20}$$

I. *LEAST SQUARES(LS) METHOD FOR F-DATA*

The (LS) method of distance between predicted values and data is observed in reality but since data are determined fuzzy, the definition of the function for distance between two fuzzy numbers is a difficult task [32]. The advantage of using this method is its high accuracy [33]. The (LS) method for m real numbers $(\boldsymbol{x_i}, \boldsymbol{y_i})$, $\boldsymbol{1 \leq i \leq n}$, includes finding (a,b∈R) of this value.

$$\boldsymbol{r}(\boldsymbol{a}, \boldsymbol{b}) = \sum (\boldsymbol{a} + \boldsymbol{b x_i} - \boldsymbol{y_i})^{\boldsymbol{2}} \tag{21}$$

For fuzzy data $= \boldsymbol{A_1} + \boldsymbol{A_2 v}\ \boldsymbol{u}, \boldsymbol{v} \in \boldsymbol{E}$, we look for the numbers $\boldsymbol{A_1}$and $\boldsymbol{A_2}$to minimize the distance between observations and estimates. So, for the appropriate definition of the distance function between the fuzzy objects, the equations (5) and (6) are used. [34].

II. *Estimation of parameters*

The sum of the square error (SSE) between $\widetilde{w}_i$,$\widetilde{W}_i$ $i = 1, \dots, m$, that is, m must be minimized using the (LS) method. The use of distance d:

$$\boldsymbol{SSE} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{M}} (\boldsymbol{d}(\widetilde{\boldsymbol{w}}_{\boldsymbol{i}}, \widetilde{\boldsymbol{W}}_{\boldsymbol{i}}))^{\boldsymbol{2}}$$

Where: (22)

$$\boldsymbol{d}(\widetilde{\boldsymbol{w}}_{\boldsymbol{i}}, \widetilde{\boldsymbol{W}}_{\boldsymbol{i}}) = \left[\int_{\boldsymbol{0}}^{\boldsymbol{1}} \boldsymbol{f}(\boldsymbol{\alpha}) \boldsymbol{d}^{\boldsymbol{2}}((\boldsymbol{d}(\widetilde{\boldsymbol{w}}_{\boldsymbol{i}})_{\boldsymbol{\alpha}}, (\widetilde{\boldsymbol{W}}_{\boldsymbol{i}})_{\boldsymbol{\alpha}}) \boldsymbol{d\alpha}\right]^{\frac{\boldsymbol{1}}{\boldsymbol{2}}}$$

Without loss of generality, we assume that $A_j = (a_j, s_j)_T, j = 1, \dots, m$ then the estimated outputs are symmetric triangular fuzzy numbers.

$$(\widetilde{W}_i) = (f(a), f(s))_T \quad i = 1, \dots, n$$
$$f_i(a) = a_0 + a_1 x_{i1} + \dots + a_n x_{in}$$
$$f_i(s) = s_0 + s_1 x_{i1} + \dots + s_n x_{in}$$

The variables $a_0, a_1, \dots, a_n$and $s_0, s_1, \dots, s_n$are estimated using the least squares method, and then use the extension principle. $\tilde{p}_n = (m_n, a_n, b_n)$

$$\boldsymbol{m_n} = \frac{\boldsymbol{exp}\ (\boldsymbol{f_n}(\boldsymbol{a}))}{\boldsymbol{1} + \boldsymbol{exp}\ (\boldsymbol{f_n}(\boldsymbol{a}))}$$

$$a_n = \frac{\exp\ (f_n(a))}{1 + \exp\ (f_n(a))} - \frac{\exp\ (f_n(a) - f_n(s))}{1 + \exp\ (f_n(a) - f_n(s))} \tag{23}$$

$$b_n = \frac{\exp\,(f_n(a)+f_n(s))}{1+\exp\,(f_n(a)+f_n(s))} - \frac{\exp\,(f_n(a))}{1+\exp\,(f_n(a))}$$

6. The Exponentially Weighted Moving Average (EWMA) control chart

Exponentially Weighted Moving Average (EWMA) control chart are commonly used to identify continuous changes in a process. The parameter under monitoring is usually the process average for a given sequence of definite observations $\{x_n, n = 1,2,3,..\}$. In normal mode, $\mu_n = E(x_n)$when the goal is to detect a small change in the process average, an individual may specify levels. If $\mu_1 > \mu_0$or $\mu_1 < \mu_0$changes under conditions ofμ_1, undesirable conditions should be identified. X_n is the sample mean.

$$\boldsymbol{EWMA_i = \theta(\bar{x}_n) + (1-\theta)EWMA_{i-1}} \qquad (24)$$

θ is a smoothing constant which varies between zero and one (0 <θ≤1). A small value of θ increases the sensitivity of the graph to small changes and a large value of θ increases the sensitivity of the graph to large changes. When i → ∞, $[1-(1-\ \theta)^{2i}]$ is equal to 1. It's difficult to equip fuzzy qualitative variables with a suitable possible distribution. If the process is “under control” and the mean and variance of fuzzy numbers can be calculated based on a group (m) of the number (n) using the cheng method: [35]

$$\tilde{x}_{ij} = (m_{ij}, R_{ij}, R_{ij})_{RR} \quad i = 1,\dots,n \ \ , j = 1,\dots,m$$
$$\tilde{\mu}_0 = (\mu_0, R_0, R_0)_{RR}$$
$$\bar{\bar{x}} = (\frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m} m_{ij}\,, \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m} r_{ij}\,, \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m} r_{ij})_{RR}$$
$$S_{d_0} = \frac{1}{n-1}\sum_{i=1}^{n}(\frac{1}{m-1}\sum_{j=1}^{m} d_*^2(\tilde{x}_{ij}\,,\bar{\bar{x}}_i))^{\frac{1}{2}}$$

We assume that all samples include observations described by fuzzy logistic regression numbers that can be "standard".

$$\tilde{Y}_i = (y_i, r_{y_i}, r_{y_i})_{RR} = \frac{1}{s_{d0}} \odot (\bar{\bar{x}} \ominus \mu_0) = (\frac{\bar{\mu}_i-\mu_0}{s_{d_0}}, \frac{\bar{r}_i-r_0}{s_{d_0}}, \frac{\bar{r}_i-r_0}{s_{d_0}})_{RR}$$

Note that this "standard" method is considered as only an of the standardization of a random variable, and the result of the "standard" $(y_i, r_{y_i}, r_{y_i})_{RR}$ central variable y_i and r_{y_i} is an approximate of the original variable. However, may not be an approximation of the left or right emission variable:

$$EWMA_{Ei}^{a}(\alpha)^{+} = \theta((y_i + r_\alpha^{(-1)} r_{y_i}) - F) + (1-\theta)EWMA_{E_{i-1}}^{a}(\alpha)$$
$$EWMA_{Ei}^{a}(\alpha)^{-} = \theta((y_i - r_\alpha^{(-1)} r_{y_i}) - F) + (1-\theta)EWMA_{E_{i-1}}^{a}(\alpha) \qquad (25)$$

(F) is the reference value which is defined as follows: $F = \frac{\delta}{2}\, s_{d_0}$, with$(\delta)$, the change in shift can be determined based on the standard deviation.

$$\tilde{Y}_i \ominus F = (y_i - F, r_{y_i}, r_{y_i})_{RR}$$

The fuzzy random data can hardly be approximated by a function. However, it can be assumed that the standardized fuzzy numbers of the sample $\tilde{Y}_i \ \ i = 1,\dots,n$ obtained during phase I. can form our Bootstrap population using SPSS software.

So, the Bootstrap(can form our Bootstrap population using SPSS software) sampling method from this population can be used, a repeated sampling is performed B times by substituting and the sample is obtained $\tilde{Y}^{*^B} = (y^{*^B}, r_y^{*^B}, r_y^{*^B})_{RR}$, where $B = 1,2,\dots,B_{max}$ and B_{max} indicates many number and is greater than the number of original samples. if the histogram of the variables l_y^* and y^* indicate that the y^* parameter follows approximately the standard normal distribution and the random variable y^* is approximately equal to the distribution x^2 with $e = [\overline{r_y^{*^B}}] + 1$ degree of freedom that $[\overline{r_y^{*^B}}]$ represents the largest integer is smaller than the mean sample $\bar{r}$ of r Bootstrap. The bootstrap distribution is used to standardize fuzzy variables. $\tilde{\mu}_0$ and s_{d_0} are the mean and variance of fuzzy data, respectively .

$$\tilde{Y} = (y, r_y, r_y)_{RR}$$
$$var(y_i + r_\alpha^{(-1)} l_{y_i}) = 1 + 2er_\alpha^{(-1)^2} + 2r_\alpha^{(-1)} E(y_i r y_i) \qquad (26)$$

Where $EWMA_{E\,0}^{a}$ = 0 and θ is smoothing coefficient 0 <θ≤1:

$$h_1^+ = k\sqrt{\frac{\theta}{(2-\theta)} 1 + 2er_\alpha^{(-1)^2} + 2r_\alpha^{(-1)} E(y_i r y_i)}$$
$$h_1^- = -k\sqrt{\frac{\theta}{(2-\theta)} 1 + 2er_\alpha^{(-1)^2} - 2r_\alpha^{(-1)} E(y_i r y_i)}$$

The distance $D_{p,q}$ proposed by Gildeh and Jane [36]. $D_{p,q}$ depends on the parameters p and q which q is the weight given to the sides of fuzzy numbers. It is necessary to merge all values of α∈ [0,1] to define the distance $D_{p,q}$. Therefore, the distance between EWMA and h with a fixed distance, for example $c_1 = [c_1^-, c_1^+]$:

$$d_{E_n} = D_{p,q}(E_{n,t}, c_1) = [(1-q)|E_n^-(\alpha) - c_2^-|^p + q|E_n^+(\alpha) - c_2^+|^p]^2$$
$$d_{H_1} = D_{p,q}(H_2, c_1) = [(1-q)|h_1^-(\alpha) - c_2^-|^p + q|h_1^+(\alpha) - c_2^+|^p]^2$$

7. FUZZY $T^2(\widetilde{FT}^2)$ CONTROL CHART

A method is provided for monitoring logistic regression techniques in phase II based on T^2 statistics.

$$T_i^2 = (z_i - \mu)^T \sum^{-1} (z_i - \mu) \quad (27)$$

In the above equation, $z_i = (a_{0i}, a_{1i})$ is a vector of the estimated parameters in the profile that is calculated using the sample data.

$$\mu = (A_0, A_1)^T$$
$$\sum = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} \\ \sigma_{1,2} & \sigma_2^2 \end{bmatrix}$$

They are the mean vector and the covariance matrix Z, respectively.

Therefore, when the data set is definite, T^2 statistics are calculated as follows:

$$T_i^2 = \frac{1}{\sigma_1^2\sigma_2^2 - \sigma_{12}^2}(\sigma_2^2(a_{0i} - A_0)^2 + \sigma_1^2(a_{1i} - A_1)^2 - 2\sigma_{12}(a_{0i} - A_0)(a_{1i} - A_1)) \quad (28)$$

In phase II, if μ and $\sum$ are known, the statistic T_i^2 follows the distribution x^2 with two degrees of freedom. Otherwise, μ and $\sum$, are estimated based on the controlled data set from Phase one, the statistic T_i^2 is calculated using the distribution$\frac{2m}{m+1}F_{2,m(n-2)}$. However, if μ and $\sum$ are estimated from a large number of sample in phase I with the function x_2^2, the T_i^2 statistic is estimated in phases I and II.

Suppose that the vectors $\tilde{\mu} = (\tilde{A}_0, \tilde{A}_1)^T$ and $\tilde{Z}_i = (\tilde{a}_{0i}, \tilde{a}_{1i})^T$ show the fuzzy values of the known and estimated parameters and $\sum = \begin{bmatrix} \sigma_{\tilde{a}_{0i}}^2 & \sigma_{\tilde{a}_{0i}\tilde{a}_{1i}} \\ \sigma_{\tilde{a}_{0i}\tilde{a}_{1i}} & \sigma_{\tilde{a}_{1i}}^2 \end{bmatrix}$ is the covariance matrix z. The elements $\sum$ are the variance and covariance $\tilde{a}_{0i}$ and $\tilde{a}_{1i}$.

It is assumed that these elements from the controlled data set are known and estimated in phase I. According to [37, 38], for a random sample of fuzzy numbers logistic regression $\tilde{x}_1^*\tilde{x}_2^*, \dots, \tilde{x}_n^*$ that $\tilde{x}_i^* = (x_i, \lambda_i, \beta_i)$, $\sigma_{\tilde{x}^*}^2$.

$$s_{\tilde{x}^*}^2 = s_{x_i}^2 + \frac{1}{6}(s_{\lambda_i}^2 + s_{\beta_i}^2) + \frac{1}{2}(s_{x_i,\beta_i} - s_{x_i,\lambda_i}) \quad (29)$$

To facilitate the calculations and avoid the computational complexity, it is assumed that the control value of these parameters used in the equation is calculated from the data of the controlled state in phase I.

If $\tilde{a}_{0i} = (a_{0i}, \lambda_{0i}, \beta_{0i})$ $\quad \tilde{A}_0 = (a_0, \lambda_0, \beta_0)$
$\tilde{a}_{1i} = (a_{1i}, \lambda_{1i}, \beta_{01i})$ $\quad \tilde{A}_1 = (a_1, \lambda_1, \beta_1)$

$$\widetilde{D}_{a_i} = \tilde{a}_{0i} - \tilde{A}_0 = (a_{0i} - a_0, \lambda_{0i} - \beta_0, \beta_{0i} - \beta_0) \quad (30)$$
$$\widetilde{D}_{a_i} = \tilde{a}_{1i} - \tilde{A}_1 = (a_{1i} - a_1, \lambda_{1i} - \beta_1, \beta_{1i} - \beta_1) \quad (31)$$

Therefore, according to the extension principle, we have:

$$C_\alpha(\widetilde{FT}^2) = \left[FT_i^{2^L}(\alpha), FT_i^{2^R}(\alpha)\right] = \begin{bmatrix} \min T_i^2(x^{"}) & \max T_i^2(x^{"}) \\ s.t & s.t \\ x^{"} \epsilon C_\alpha(x^{"*}) & x^{"} \epsilon C_\alpha(x^{"*}) \end{bmatrix} \quad (32)$$

Where, $x^{"} = (D_{a_{0i}}, D_{a_{1i}})$.
$C_\alpha(\widetilde{D}_{a_{0i}}) = [D_{a_{0i}}^l(\alpha), D_{a_{0i}}^R(\alpha)]$ and $C_\alpha(\widetilde{D}_{a_{1i}}) = [D_{a_{1i}}^l(\alpha), D_{a_{1i}}^R(\alpha)]$, the solution for optimizing the problem can be written as follows [19].

$$minFT_i^2(x^{"}) = FT_i^{2^L}(\alpha)$$
$$s.t:$$
$$x^{"} \in C_\alpha(x^{"*}) = \begin{cases} 0 & \text{If } 0 \in C_\alpha(\widetilde{D}_{a_{0i}}) \text{ and } 0 \in C_\alpha(\widetilde{D}_{a_{1i}}) \\ \frac{1}{\sigma_{\tilde{a}_{0i}}^2} \times \min\{(D_{a_{0i}}^l(\alpha))^2, (D_{a_{0i}}^R(\alpha))^2\} & \text{If } 0 \notin C_\alpha(\widetilde{D}_{a_{0i}}) \text{ and } 0 \in C_\alpha(\widetilde{D}_{a_{1i}}) \\ \frac{1}{\sigma_{\tilde{a}_{1i}}^2} \times \min\{(D_{a_{1i}}^l(\alpha))^2, (D_{a_{1i}}^R(\alpha))^2\} & \text{If } 0 \in C_\alpha(\widetilde{D}_{a_{0i}}) \text{ and } 0 \notin C_\alpha(\widetilde{D}_{a_{1i}}) \\ \frac{1}{\sigma_{\tilde{a}_{0i}}^2} \times \min\{(D_{a_{0i}}^l(\alpha))^2, (D_{a_{0i}}^R(\alpha))^2\} + \frac{1}{\sigma_{\tilde{a}_{1i}}^2} \times \min\{(D_{a_{1i}}^l(\alpha))^2, (D_{a_{1i}}^R(\alpha))^2\} & \text{else} \end{cases} \quad (33)$$

$$s.t:$$
$$\boldsymbol{x^{"} \in C_\alpha(x^{"*})} =$$
$$\boldsymbol{maxFT_i^2(x^{"}) = FT_i^{2^R}(\alpha) = \frac{1}{\sigma_{\tilde{a}_{0i}}^2} \times \min\{(D_{a_{0i}}^l(\alpha))^2, (D_{a_{0i}}^R(\alpha))^2\} + \frac{1}{\sigma_{\tilde{a}_{1i}}^2} \times \min\{(D_{a_{1i}}^l(\alpha))^2, (D_{a_{1i}}^R(\alpha))^2\}} \quad (34)$$

8. AVERAGE RUN LENGTH (ARL)

Average Run Length (ARL) is used for control chart evaluation. The number of points that occur until a signal is used to determine the control limits when the process remains in control and is shown by ARL_0. In this study, the performance of control diagrams based on (ARL) criterion is calculated based on different variations in parameters. The simulation method is used in matlab software to calculate its value. In most fuzzy literature, the mean of the mean fuzzy number and variance is deterministic.

9. SIMULATION STUDIES AND PERFORMANCE EVALUATION

The performance of control charts is simulated using MATLAB software to evaluate and compare the performance of the proposed control charts. The model parameters $A_0 = (a_0, s_0)$ and $A_1 = (a_1, s_1)$ are considered (-3.53, 0.18) and (0.0559, 0.00001), respectively.

$$f_n(a) = -3.53 + 0.18u_n \text{ , } f_n(s) = 0.0559 + 0.00001u_n$$

First, three values of the variable X are randomly generated 0 <X <1 for each simulation. Then, the model parameters are calculated in the "controlled mode" using the least squares method and then The FEWMA and FT^2 statistics, and then the values of the "control charts" are set to 200 (here) to achieve optimal ARL under optimal control. ARL values are calculated in Tables 2, 3, 4 and 5 and according to the results of analysis, the performance of FEWMA chart is better than the performance of FT^2in small and medium shifts but the performance of FT^2 chart is better in large shifts.

TABLE I
VALUES OF UPPER CONTROL LIMITS

N=100	Control limit	
	ucl_{T^2}	ucl_E
	11.57	0.97

TABLE II
COMPARISONS OF ARL_1 ON THE A_0 TO A_0 - K (ARL_0= 200)

		k									
	Methods	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
Proposed method	FT^2	138.24	63.89	27.86	14.5	5.78	4.2	2.63	1.87	1.56	1.21
	FEWMA	57.87	16.44	7.86	5.4	3.98	3.13	2.64	2.34	2.12	1.91
Existing method	**FCUSUM**	72.84	20.33	8.20	4.65	3.18	2.49	1.95	1.65	1.44	1.32

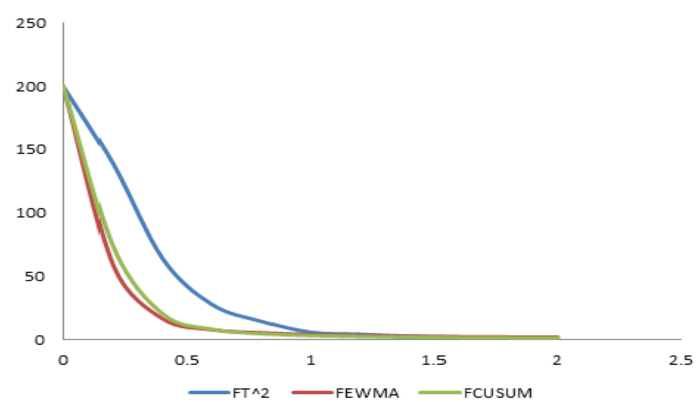

FIGURE 2
COMPARISON OF CONTROL CHARTS PERFORMANCE

Based on Table 2 and Figure 2, results show that the FEWMA chart has better performance than the FCUSUM control chart under small shifts. In addition, FT^2chart outperform other control charts under large shifts in process parameter (A_0).

TABLE III
COMPARISONS OF ARL_1 ON THE A_0 TO A_0 + K (ARL_0= 200)

		k									
	Methods	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
Proposed method	FT^2	170.22	72.75	28.78	13.62	6.98	4.98	2.62	1.79	1.51	1.21
	FEWMA	51.47	15.48	7.37	4.98	3.81	2.93	2.57	2.21	1.91	1.78
Existing method	**FCUSUM**	68.25	22.31	7.80	3.78	3.14	2.12	1.92	1.54	1.42	1.29

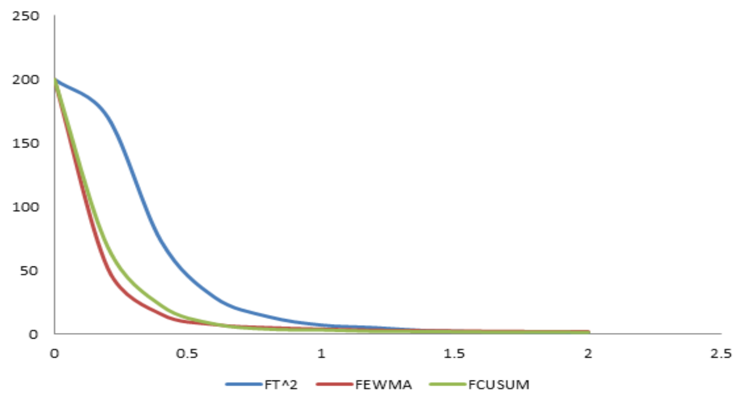

FIGURE 3
COMPARISON OF CONTROL CHARTS PERFORMANCE

Results from Table and Figure 3, proposed control charts have better performance than existing chart under both and positive small and large shifts in process parameter (A_0).

TABLE IV
COMPARISONS OF ARL_1 ON THE A1 TO A1 - K (ARL_0= 200)

	Methods	k 0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
Proposed method	FT^2	136.07	63.23	28.34	12.22	6.31	4.22	2.71	1.88	1.53	1.20
	FEWMA	88.40	25.20	11.32	5.64	4.37	3.64	3.01	2.45	2.34	2.29
Existing method	FCUSUM	85.70	19.13	7.24	3.9.	2.60	2.04	1.81	1.38	1.41	1.21

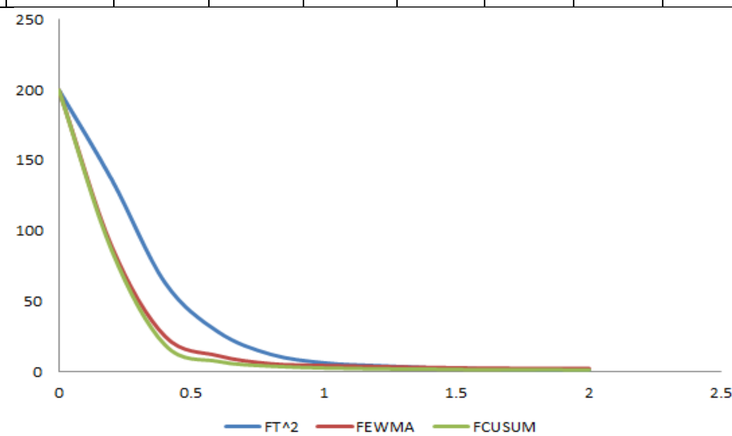

FIGURE 4
COMPARISON OF CONTROL CHARTS PERFORMANCE

According to results from Table 4 and Figure 4, FT^2 control chart has better performance than other charts under negative large shift in (A_1).

TABLE V
COMPARISONS OF ARL_1 ON THE A1 TO A1 - K (ARL_0= 200)

	Methods	k 0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
Proposed method	FT^2	140.53	71.02	38.45	13.11	6.93	4.96	3.4	2.2	1.52	1.18
	FEWMA	90.17	20.41	11.08	4.53	3.74	3.62	3.02	2.54	2.43	2.28
Existing method	FCUSUM	86.54	20.45	9.96	4.37	3.01	2.63	1.84	1.39	1.41	1.17

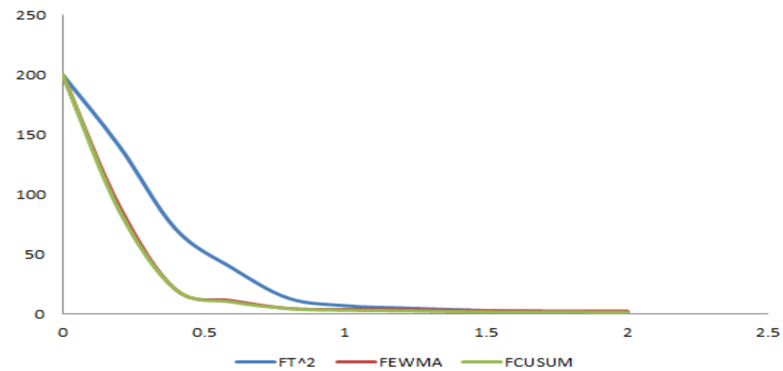

FIGURE 5
COMPARISON OF CONTROL CHARTS PERFORMANCE

As shown in Table 5 and Figure 5, FEWMA control chart outperform the FCUSUM chart in positive small shift in A_1. In addition, results show that, FT^2 control chart has similar performance with FCUSUM control chart under large shifts in mentioned parameter.

10. CASE STUDY

To illustrate the applicability of the proposed method in real world, a real data set in medical science based on the lupus patients at the Hafez Hospital in Iran has been selected from literature. (To more information about real data set, see [31]). In this case, five control variables were considered and related data are presented in Table 6.

TABLE VI
REAL DATA SET

Observation number	X_1	X_2	X_3	X_4	X_5	$\tilde{\mu}_i$
1	1	1	112	105	1	High
2	0	1	80	23	0	Medium
3	0	1	115	15	0	High
4	0	1	105	107	1	High
5	0	0	89	150	1	Medium
6	1	1	160	10	1	Very high
7	0	1	100	23	0	Medium
8	0	0	100	85	1	High
9	0	1	48	83	0	Low
10	1	0	15	19	1	Very low
11	0	0	50	91	0	Low
12	0	1	59	200	1	Medium
13	0	1	83	20	1	Low
14	0	0	15	200	0	Low
15	1	0	85	15	1	Medium

The definition of $\mu_i = 1, \dots, n$ in the form of triangular fuzzy numbers is as follows:

$$very-low(x) = \begin{cases} 1-\dfrac{0.02-x}{0.01} & 0.01 \le x \le 0.02 \\ 1-\dfrac{x-0.02}{0.18} & 0.02 \le x \le 0.18 \end{cases} \qquad low(x) = \begin{cases} 1-\dfrac{0.25-x}{0.15} & 0.01 \le x \le 0.25 \\ 1-\dfrac{x-0.25}{0.15} & 0.25 \le x \le 0.4 \end{cases}$$

$$medium(x) = \begin{cases} 1-\dfrac{0.5-x}{0.15} & 0.35 \le x \le 0.5 \\ 1-\dfrac{x-0.5}{015} & 0.5 \le x \le 0.65 \end{cases} \qquad high(x) = \begin{cases} 1-\dfrac{0.75-x}{0.15} & 0.6 \le x \le 0.75 \\ 1-\dfrac{x-0.75}{0.15} & 0.75 \le x \le 0.9 \end{cases}$$

$$very-high(x) = \begin{cases} 1-\dfrac{0.98-x}{0.18} & 0.8 \le x \le 0.98 \\ 1-\dfrac{x-0.999}{0.01} & 0.98 \le x \le 0.99 \end{cases}$$

The optimal model is defined based on the real data for:

$$\widetilde{W} = \ln\left(\frac{\tilde{\mu}}{1-\tilde{\mu}}\right) = (-3.8591,1.1617)_T + (0.4248,0.3832)_T X_1 + (-0.1309,0.0366)_T X_2 + (0.0431,-0.0043)_T X_3 + (0.0091,-0.0019)_T X_4 + (-0.6083,0.1451)_T X_5$$

We use $D_{p,q}$ distance when p=1, q=0.5 and assume $\alpha = 0.65$ and $ARL_0 = 200$

TABLE VII
VALUES OF UPPER CONTROL LIMITS

N=15	Control limit	
	ucl_{T^2}	ucl_E
	13.40	0.88

TABLE VIII
COMPARISONS OF ARL_1 ON THE A_1 TO A_1 - K (IN-CONTROL ARL = 200)

		k									
	Methods	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
Proposed method	FT^2	138.23	63.95	27.94	13.21	6.89	3.87	2.64	1.78	1.45	1.32
	FEWMA	60.11	17.01	7.15	5.21	3.87	3.15	2.68	2.31	2.12	1.99
Existing method	**FCUSUM**	73.15	20.33	8.02	4.6	3.11	2.43	1.97	1.64	1.39	1.41

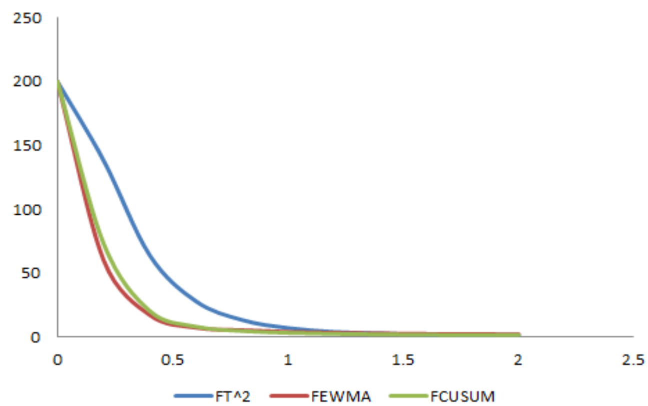

FIGURE 6
COMPARISON OF CONTROL CHARTS PERFORMANCE

TABLE IX
COMPARISONS OF ARL_1 ON THE A_0 TO A_0 + K (IN-CONTROL ARL = 200)

		k									
	Methods	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
Proposed method	FT^2	166.24	109.5	61.79	32.54	16.84	9.06	5.31	3.23	2.28	1.71
	FEWMA	63.68	17.62	8.23	5.24	3.98	3.14	2.74	2.29	2.05	1.95
Existing method	**FCUSUM**	75.4	21.12	8.2	5.12	4.01	3.4	2.71	2.45	2.1	1.69

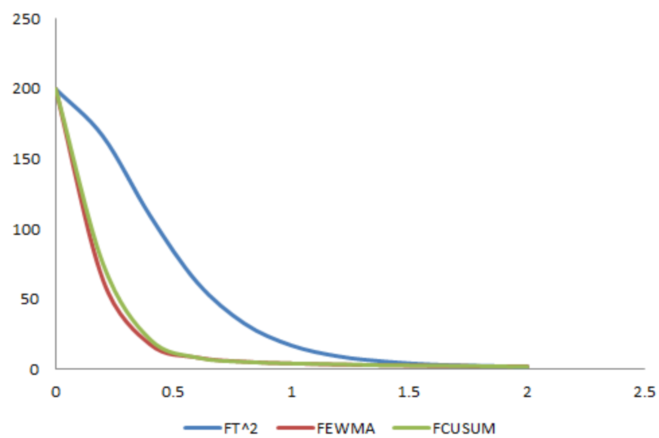

FIGURE 7
COMPARISON OF CONTROL CHARTS PERFORMANCE

According to the above results, performance of the proposed control charts is better than the existing FCUSUM chart under small and large shifts in process parameters.

11. CONCLUSION

This study was carried out aimed to monitor a logistic regression processes in the presence of ambiguous variables and a linguistic term such as {very low, low, medium,..} or other such as that is determined by experts. Two control charts are developed and presented with fuzzy numbers-based on $EWMA$,T^2. In order to evaluate the performance and monitoring approaches, simulations studies have been applied. Results of simulations showed that FEWMA has better performance than existing FCUSUM control chart under small shifts in process parameters. In addition, Results showed that, $Hotelling's\ FT^2$ control chart outperform other two control charts under large shifts. Furthermore, a real case study has been used in this paper which computation outcomes were confirmed the simulation results. It is recommended that investigating the performance of fuzzy control diagrams when the parameters are unknown and sometimes are estimated with errors, be considered as a topic for future research.

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