

# Probabilistic Multi-Period Berth Allocation Problem: Continuous and Discrete Approaches

S. Farid Mousavi<sup>1</sup>, Zahra Mahdavi<sup>2</sup>, Kaveh Khalili-Damghani\*<sup>2</sup>, Arezoo Gazori-Nishabori<sup>2</sup>

Received: 2021-04-27 / Accepted: 2021-12-30 / Published online: 2021-12-30

\* Corresponding author: kaveh.khalili@gmail.com

1- Department of Operations Management and Information Technology, Kharazmi University, Tehran, Iran

2- Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran

## Abstract

Terminal administration tasks is very important due to high proportion of the transportation cost in the businesses. Berth Allocation Problem (BAP) is one of the crucial points arise in terminal administration. One of the main issues arises in real BAP is uncertainty of planning parameters. Although several researches have been accomplished in the field of BAP, uncertain parameters have not effectively been considered. In this paper, a stochastic BAP is developed. Operation time and cost of a ship are assumed to be probabilistic with both discrete and continuous distribution function in the proposed SBAP. To solve the SBAP, two solution approaches, i.e., Stochastic Chance Constraint Programming (SCCP) and Two-Stage Stochastic Linear Programming with Recourse (TSSLPR) are proposed. A numerical example is solved to show the applicability of the suggested model and the solution approaches.

**Keywords:** Berth Allocation Problem; Stochastic Programming; Multi-Period; Stochastic Optimization

## 1. INTRODUCTION

Berth allocation problem (BAP) is a well-known optimization issue. BAP is about setting a allocate a certain berth to each of the entering ship to optimize a measurement function considering a set of constraints. The characteristics of the real BAP limit the eligible set of solutions. Finding an optimal allocation for the ships and assigning ship arrivals to the ports, when assigned by the terminal operators is the main issues in BAPs.

Generally, many ports use conventional berthing programs and standards, meaning that Ships begin to discharge or download based on their arrival time. (The ship that comes earlier will be the first to serve and

handle). However, container shipping, the arrival of ships should be necessarily made in advance. The program is used by the terminal, for berth and cargo capacity, to improve terminal performance. Secondly, by transport and ships, the timetable and service must be accurate to reach the maximum allocation and scheduling of travel with the ability to access port services. Therefore, it is necessary to optimize schedules and arrivals of ships to adapt to the required time windows.

In the real world, the servicing and handling times of ships (unloading and loading) depending on the conditions of the port, personnel, and equipment of the ship and the berth may not be real or certain, or there is

fluctuation in the cost of waiting for berthing or delay in the departure of ships. In this paper, we plan to design a model that can solve this problem under uncertainty conditions. Despite the problem assumptions, we intend to add cases that provide the uncertain conditions for this multi-period allocation. In this case, we consider uncertainty for ship handling times, mooring costs and delay costs of ships leaving the terminal.

Therefore, we plan to model this issue with possible mathematical programming.

Research innovations are summarized as follows:

- Considering irregular layouts in the ports that fit real-world conditions.
- Considering uncertain waiting costs of mooring and demurrage of vessels.
- Considering uncertain service and handling time of vessels.
- Application and comparison of different approach of probabilistic programming for modeling the uncertainty.
- Extension of the problem in a multi-period planning horizon for the allocation of vessels to ports and berth.

The multi-period allocation property leads to a kind of scheduling.

The next parts of the manuscript are arranged as presented here. The literature is reviewed in Section 2. The proposed model is developed in the Section 3. In Section 4, the numerical example and results are presented. Conclusions and future researches are presented in Section 5.

## 2. LITERATURE REVIEW

In this section a brief literature about probabilistic mathematical planning and ship scheduling are presented.

### 2.1. BERTH ALLOCATION PROBLEM

Imai et al., (1997) proposed a BAP model while the traditional assumption about first in first out (FIFO) was not considered. The model was formulated through mathematical programming. Imai et al., (2001) developed the model proposed Imai et al., (1997) for a dynamic situation. The handling time was assumed to be deterministic. The objective was to minimize sum of the operation times of the vessels. Nishimura et al., (2001) a BAP considering a multi-water depth. Nishimura et al., (2001) assumed that each berth served

multiple vessels till the sum of the vessels are less than the length of the berth. Guan and Cheung (2004) developed a couple of BAP models for continues cases. In the first model, Guan and Cheung (2004) considered the Relative Position formulation. The objective function of the first model was to minimize the total weighted flow time. The second model proposed by Guan and Cheung (2004) was the Position Assignment formulation. Monaco and Sammarra (2007) developed the model proposed by Imai et al. (2001) using fewer variables and constraints. Imai, et al., (2007) investigated a BAP in which a mega-ship or some small vessel can be served.

Bierwirth and Meisel (2010, 2015) and Carlo et al., (2015) effectively reviewed the BAP literature. Imai et al., (2013) proposed a BAP in which two parallel quays form a channel were considered. A genetic algorithm was proposed for solving the proposed BAP. Zhen et al., (2017) presented another research in which channel berths were considered. Zhen et al. (2017) proposed mathematical programming approach to solve the BAP. Correcher et al., (2019) proposed a BAP in terminals with irregular layouts. Correcher et al., (2019) considered the specific features of the berths (adjacent berths, oppositional berths, Structural features such as irregular berths due to irregular quay). The goal was to minimize the total cost of the allocation, which was the total waiting cost for the mooring and the cost of delaying each ship on departure.

### 2.2. PROBABILISTIC MATHEMATICAL PLANNING

Khalili et al., (2017) proposed a multi-objective, multi-product, multi-period production planning model for supply chain. Khalili et al., (2017) presented a new model for the project selection problem in which some parameters were assumed to be probabilistic. The issue of project selection was one of the most essential decisions for investment organizations. Reza-Pour and Khalili (2017) proposed a stochastic time-cost project scheduling problem. Ghasemi et al., (2019) proposed a mathematical programming for the various stages of the crisis management cycle. Ghasemi et al., (2019) proposed a mathematical programming model for earthquake response in presence of uncertainty. Lozkins et al., (2019) presented a mathematical programming to minimize transportation costs under demand uncertainty. Hosseini and Sahlin (2019) proposed a multi-period optimization model for tactical decision

making under uncertainty to solve the problem of relocating empty containers. Lashgari et al., (2019) proposed a supplier selection model considering uncertainty and backorder. Javid et al. (2020) proposed a mathematical programming to model the complexity and flexibility of a production system. Adarang et al. (2020) discussed and modeled the problem of location-routing in presence of uncertainty for the provision of emergency medical services during disasters. The goal was to minimize relief time and costs (location-vehicles). Ghasemi et al., (2020) proposed a stochastic mathematical model for earthquake planning. Random demands were developed as input to the model and a simulated model was set up to generate a possible demand distribution through several scenarios. Nourzadeh et al., (2020) formulated an integer programming model for uncertain airline hub location problem. Nourzadeh et al., (2020) proposed a competitive hub allocation problem in presence of uncertainty for the transportation problem in which the travel costs and travel time in the airlines showed considerable reductions. Sajedi et al., (2020) presented a multi-period mathematical model for supply chain network desing considering demand uncertainty which was modeled through fuzzy sets. Ghasemi and Khalili (2021) proposed a simulation approach to plan the pre-earthquake stage. The demand behavior was assumed to be uncertain. The Khalili et al., (2021) proposed a mathematical model to minimize pre-disaster and post-disaster operations. The relief resources were assumed to be uncertain.

### 2.3. SCHEDULING PROBLEM

Li et al., (1998) presented a BAP considering constant operation time. Guan et al., (2002) proposed a BAP in which a multiprocessor task scheduling was considered. Guan et al., (2002) tried to minimize the total operation time of ships. Zhen et al., (2011) developed a BAP in presence of uncertainty of ship operations. Taviana et al., (2017) proposed mathematical programming for truck-scheduling with direct drone. Khalili et al., (2017) proposed a genetic algorithm for solving cross-dock truck scheduling problems. Xi et al., (2017) examined a BAP problem considering uncertain passenger flow. Shahabi et al., (2019) proposed a simulation-optimization method to improve the flexibility of the train schedule. Schepler et al., (2019) proposed a

stochastic version of BAP to minimize the waiting total time for turnaround vessels. Barbosa et al., (2019) proposed a meta-heuristic approach to solve a dynamic BAP. Wawrzyniak et al., (2020) considered an algorithm selection for the BAP under uncertainly. Guo et al., (2021) proposed a mathematical planning model for a BAP in presence of uncertain ships' operation time. Chargui et al., (2021) developed a BAP considering worker performance variability and yard truck deployment. Rodrigues and Agra (2021) studied a robust BAP and crane assignment problem under uncertainty. Rodrigues and Agra (2021) modeled a decomposition algorithm under uncertain arrival times. Xiang and Liu (2021) considered tactical BAP with uncertain operation time. Liu et al., (2021) proposed a mixed-integer linear programming model for BAP in seaport. Bacalhau et al., (2021) presented hybrid metaheuristic approach to solve a BAP.

### 3. PROPOSED MODEL

In this section, the proposed stochastic BAP in Different Terminals with Irregular Layouts (SBAP) considering multiple-period of planning is proposed. The basic assumptions of BAP are those by Correcher et al., (2019). We do not revisit the model proposed by Correcher et al., (2019) for sake of bravery. Interested readers are referred to the model proposed by Correcher et al., (2019) for more details. In this paper, we are going to extend the proposed model by Correcher et al. (2019) in presence of uncertainty.

#### 3.1. THE MAIN ASSUMPTIONS OF THE PROPOSED MODEL

As mentioned, the properties of the proposed model by Correcher et al. (2019) are held here. Moreover, the following assumptions are provided to handle the uncertainty in the BAP:

- Waiting costs from mooring and costs of delaying the departure of the ship are considered in stochastic form.
- Ship handling time is considered in stochastic form.
- For continuous contingency modeling, we use the standard normal distribution by considering 5 scenarios with the same probability of occurrence.
- we used from Discrete stochastic programming approaches (i.e., wait and see, Here and Now, and Expected value)

Table 1 presents the notations used in the mathematical modeling procedure.

TABLE 1

SETS, INDICES, PARAMETERS, AND DECISION VARIABLES (ADAPTED FROM JUAN FRANCISCO CORRECHER ET AL., 2019)

Indices	
$i, j$	Indicator of vessels
$f, k$	Indicator of berth
Sets	
Vessels	
$l_i$	Length
$w_i$	Ship's Width
$a_i$	Time of arriving ship $i$
$s_i$	Time of departure ship $i$
$h_i^k$	handling time in berth $k$
$B_i \subseteq B$	Set of compatible berths
$C^w_i$	Delay cost
$C^d_i$	Waiting cost
Berths	
	$B^o$ : Set of opposite berths
	$B^a$ : Set of adjacent berths
$rel_k$	the release time for each berth $k \in B$
Precalculated sets	
	$A = \{(f, i, k, j) \mid i, j \in V; f \in B; k \in B; i < j; f \neq k; \frac{l_i}{2} + \frac{l_j}{2} + c_{fk}^n > d_{fk}^n\}$
	$O = \{(f, i, k, j) \mid i, j \in V; f \in B; k \in B; i < j; f \neq k; w_i + w_j + c_{fk}^o > d_{fk}^o\}$
	$C = \{(f, i, k, j) \mid i, j \in V; k \in B; k \in B; i \neq j\}$
Decision variables	
$t_i$	berthing time of ship $i$
$r_i$	departure time of ship $i$
$u_i$	delay of ship $i$
	$M_i^k = \begin{cases} 1 & \text{if ship } i \text{ is moored on berth } k \\ 0 & \text{otherwise} \end{cases}$
	$\sigma_{ij} = \begin{cases} 1 & \text{if } r_i \leq t_j, i, j \in V \\ 0 & \text{otherwise} \end{cases}$
	$y_{ij} = \begin{cases} 1 & \text{if } r_i \leq t_j, i, j \in V \\ 0 & \text{otherwise} \end{cases}$
	$\varphi_{ij} = \begin{cases} 1 & \text{if } r_i \leq t_j, i, j \in V \\ 0 & \text{otherwise} \end{cases}$

**3.2. STOCHASTIC CHANCE CONSTRAINT PROGRAMMING**

Stochastic Chance Constraint Programming (SCCP) is the most common method in the field of uncertain

mathematical programming. Charnes and Cooper (1963) presented SCCP. In this approach, the goal is to optimize the expected value of objectives, while some

constraints are taken into consideration (Kim et al., 2012). The deterministic equivalent of a probabilistic mathematical programming can be achieved using the well-known SCCP approach (Kim et al., 2012).  
 3.3 Stochastic parameters of BAP

In this section the proposed stochastic BAP is addressed. Then its deterministic equivalent is developed using the SCCP approach (Charnes and Cooper, 1963). The random parameters of the proposed probabilistic BAP are presented in Table 2.

TABLE 2  
 RANDOM PARAMETERS

Deterministic parameter	Random Parameter	
$Cw_i$	$\widetilde{c}w_i$	Probabilistic cost of waiting, vessel $i$
$Cd_i$	$\widetilde{c}d_i$	Probabilistic cost of Delay, vessel $i$
$h(i,k)$	$\widetilde{h}_{(i,k)}$	Probabilistic handling time at berth $k$ , vessel $i$

Table 3 shows the discrete probability function of the stochastic parameters.

TABLE 3  
 INTRODUCE  $h(i, k)$ ,  $Cd$ ,  $Cw$

Handling time $h(i,k)$					
$h$	$A_5$	$A_4$	$A_3$	$A_2$	$A_1$
probability	0.2	0.2	0.2	0.2	0.2
The cost of delayed departure from ships $Cd$					
$Cd$	$Y_5$	$Y_4$	$Y_3$	$Y_2$	$Y_1$
probability	0.2	0.2	0.2	0.2	0.2
The cost of waiting for the berthing $Cw$					
$Cw$	$X_5$	$X_4$	$X_3$	$X_2$	$X_1$
probability	0.2	0.2	0.2	0.2	0.2

Based on the SCCP approach (Charnes and Cooper, 1963), the deterministic equivalent of the stochastic BAP is proposed as follows:

$$\begin{aligned}
 C_i^{w*} &= E(\widetilde{c}w_i) - Z(1-Bi) \sqrt{\text{var}(\widetilde{c}w_i)} \\
 C_i^{d*} &= E(\widetilde{c}d_i) - Z(1-Bi) \sqrt{\text{var}(\widetilde{c}d_i)} \\
 \text{Min } \bar{f} &= \sum_{i \in V} (C_i^{w*}(t_i - a_i) + C_i^{d*}u_i) \\
 \sum_{i \in V} (\widetilde{c}w_i(t_i - a_i) + \widetilde{c}d_i u_i) &\geq \bar{f} \\
 p \left( \sum_{i \in V} (\widetilde{c}w_i(t_i - a_i) + \widetilde{c}d_i u_i) \geq \bar{f} \right) &(1 - Bi)\% \\
 \bar{N}_i &= \sum_{i \in V} (\widetilde{c}w_i(t_i - a_i) + \widetilde{c}d_i u_i) - \bar{f}
 \end{aligned}
 \tag{1}$$

$$p(\tilde{N}_i \geq 0) \geq (1 - B_i)\%$$

$$E(\tilde{N}_i) + \varphi^{-1}(B_i) * \sqrt{\text{var}(\tilde{N}_i)} \leq 0$$

Finally:

$$E \frac{(\sum_{i \in V} (\tilde{c}w_i(t_i - a_i) + \tilde{c}d_i u_i))}{\sqrt{(\sum_{i \in V} (\text{var}(\tilde{c}w_i) * (t_i - a_i) + \text{var}(\tilde{c}d_i) * u_i))}} + \varphi^{-1}(B_i) * \tag{2}$$

s. t.

$$p \left( t_i + \sum_{k \in B_i} m_i^k \tilde{h}_{(i,k)} - r_i \leq 0 \right) (1 - B_i)\%$$

$$A_i = t_i + \sum_{k \in B_i} m_i^k \tilde{h}_{(i,k)} - r_i$$

$$p(A_i) \geq (1 - B_i)\%$$

$$E(A_i) - \varphi^{-1}(1 - B_i) * \sqrt{\text{var}(A_i)} \leq 0$$

Finally:

$$E(t_i + \sum_{k \in B_i} m_i^k \tilde{h}_{(i,k)} - r_i) - \varphi^{-1}(1 - B_i) * \sqrt{(t_i + \sum_{k \in B_i} m_i^k * \text{var}(\tilde{h}_{(i,k)}) - r_i)} \leq 0 \quad i=1, \dots, n \tag{3}$$

### 3.4 DISCRETE PROBABILITY MODELS ( $Z_{WS}, Z_{HN}, Z_{EV}$ )<sup>1</sup>

Calculating Zws: Here, we modify our model with 5 scenarios that are occurrence probability each (0.2). Since our goal is to predict the probability of the handling time h (i, k) and the cost of waiting for the berthing Cw (i) and the cost of delayed departure from

ships Cd(i), our problem model has 18 limitations only the target function and the limitation of No. 5 varies, which is enough to calculate Zws, each of which scenarios is calculated individually, and finally Zws total obtain from the average.

$$Min \sum_{i \in V} (C_i^w(t_i - a_i) + C_i^d u_i) \tag{4}$$

s.t:

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_i^k, \quad \forall i \in V \tag{5}$$

In this model, our main model does not change. Only with each scenario, we solve the model five times.

$$Zws \text{ (total)} = Z_1 * (0.2) + Z_2 * (0.2) + Z_3 * (0.2) + Z_4 * (0.2) + Z_5 * (0.2)$$

$$Min \sum_{i \in V} 0.2 * (C_{i1}^w(t_i - a_i) + C_{i1}^d u_i) + \sum_{i \in V} 0.2 * (C_{i2}^w(t_i - a_i) + C_{i2}^d u_i) + \sum_{i \in V} 0.2 * (C_{i3}^w(t_i - a_i) + C_{i3}^d u_i) + \sum_{i \in V} 0.2 * (C_{i4}^w(t_i - a_i) + C_{i4}^d u_i) + \sum_{i \in V} 0.2 * (C_{i5}^w(t_i - a_i) + C_{i5}^d u_i) \tag{6}$$

s.t:

<sup>1</sup> WS: wait and see, HN: Here and Now, EV: expected value.

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_{i1}^k, \quad \forall i \in V \tag{7}$$

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_{i2}^k, \quad \forall i \in V \tag{8}$$

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_{i3}^k, \quad \forall i \in V \tag{9}$$

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_{i4}^k, \quad \forall i \in V \tag{10}$$

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_{i5}^k, \quad \forall i \in V \tag{11}$$

$Z_{HN}$  calculation: In this model, we put five scenarios simultaneously in the model, and then we start to solve it, and ultimately the final result is the same as the  $Z_{HN}$ .

By adding new constraints and a new target function to model,  $Z_{HN}$  is obtained.

$Z_{EV}$  calculation: To calculate this model, the first step is to write scenarios average in the target function and write the same in limits.

$$\text{Min} \sum_{i \in V} (C_{i3}^w(t_i - a_i) + C_{i3}^d u_i) \tag{12}$$

s.t:

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k * 0.2 (h_{i1}^k + h_{i2}^k + h_{i3}^k + h_{i4}^k + h_{i5}^k) \quad i \in V \tag{13}$$

When we solve the above model, we put the obtained variables into the lower model, which is the second step. For the second step, we write a separate

model for each scenario, provided that the value of the first stage variables is equal to what was achieved in the first step.

$$\begin{aligned} &\text{Min} \sum_{i \in V} (C_{i1}^w(t_i - a_i) + C_{i1}^d u_i) \\ &\text{Min} \sum_{i \in V} (C_{i1}^w(t_i - a_i) + C_{i1}^d u_i) \end{aligned} \tag{14}$$

s.t:

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_{i1}^k, \quad \forall i \in V \tag{15}$$

$$\text{Min} \sum_{i \in V} (C_{i2}^w(t_i - a_i) + C_{i2}^d u_i) \tag{16}$$

s.t:

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_{i2}^k, \quad \forall i \in V \tag{17}$$

$$\text{Min} \sum_{i \in V} (C_{i3}^w(t_i - a_i) + C_{i3}^d u_i) \tag{18}$$

s.t:

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_{i3}^k, \quad \forall i \in V \tag{19}$$

$$\text{Min} \sum_{i \in V} (C_{i4}^w (t_i - a_i) + C_{i4}^d u_i) \tag{20}$$

s.t:

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_{i4}^k, \quad \forall i \in V \tag{21}$$

$$\text{Min} \sum_{i \in V} (C_{i5}^w (t_i - a_i) + C_{i5}^d u_i) \tag{22}$$

s.t:

$$r_i \geq t_i + \sum_{k \in B_i} m_i^k h_{i5}^k, \quad \forall i \in V \tag{23}$$

$Z_{EV}$ : (attention to uncertainty in the solving process)  
 $Z_{ws}$ : (lack of attention to uncertainty)  
 $Z_{HN}$ : (only considering the probabilities in modeling)

**4. NUMERICAL EXAMPLE AND RESULTS**

To reflect the applicability of the proposed model of this study, a mathematical example is suggested in

this part and the outcomes are discussed. First, we codify the deterministic model of our basic paper using LINGO software and then coding the probabilistic models and end compare and conclude. As shown in Figure 1 we consider 10 ships and 10 berths (with irregular layouts).

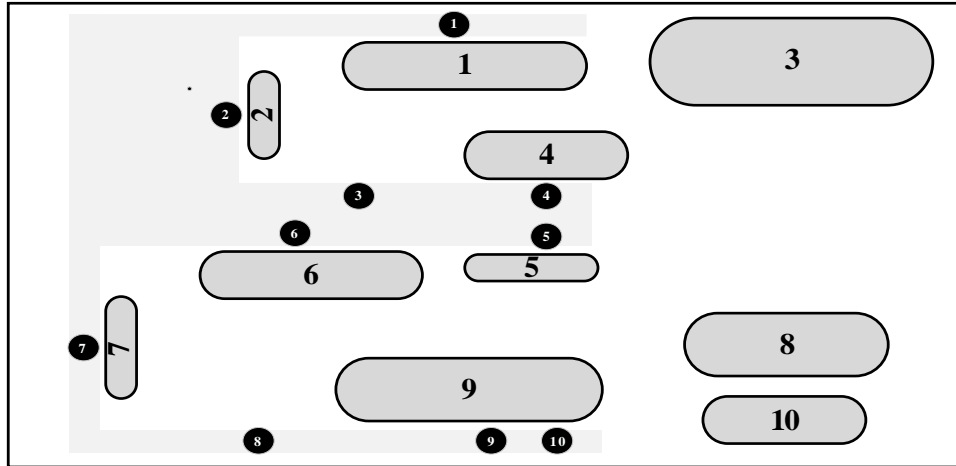


FIGURE 1  
 SCHEMATIC VIEW OF DIFFERENT TERMINALS WITH IRREGULAR LAYOUTS

According to the arrival and desired departure time and the handling time of ships at the berth and the

constraints of the deterministic model (1-18), parameters (Figure 2).



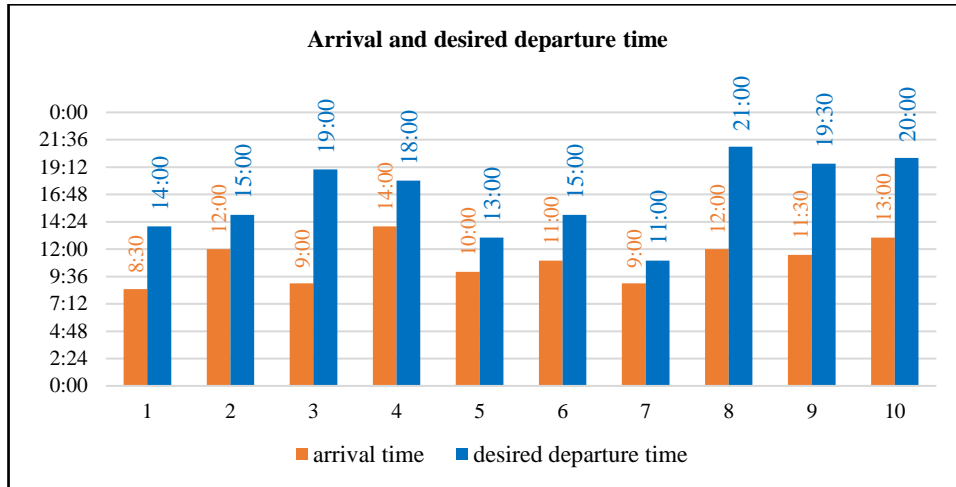


FIGURE  
THE RESULTS OF THE CODING ARE SUMMARIZED IN FIGURE 3.

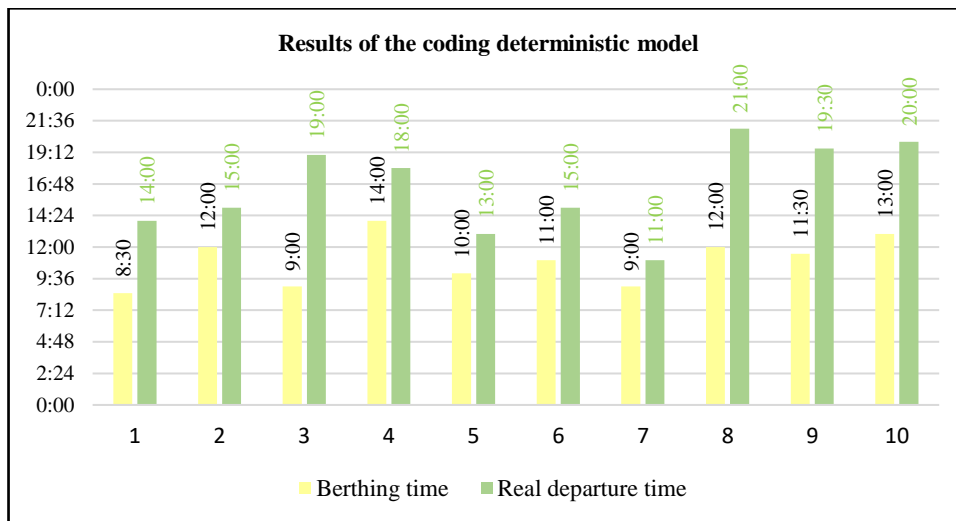


FIGURE 3  
RESULTS OF THE CODING DETERMINISTIC MODEL (Z:0)

The results of the Stochastic Scheduling Model are in Figure 4.

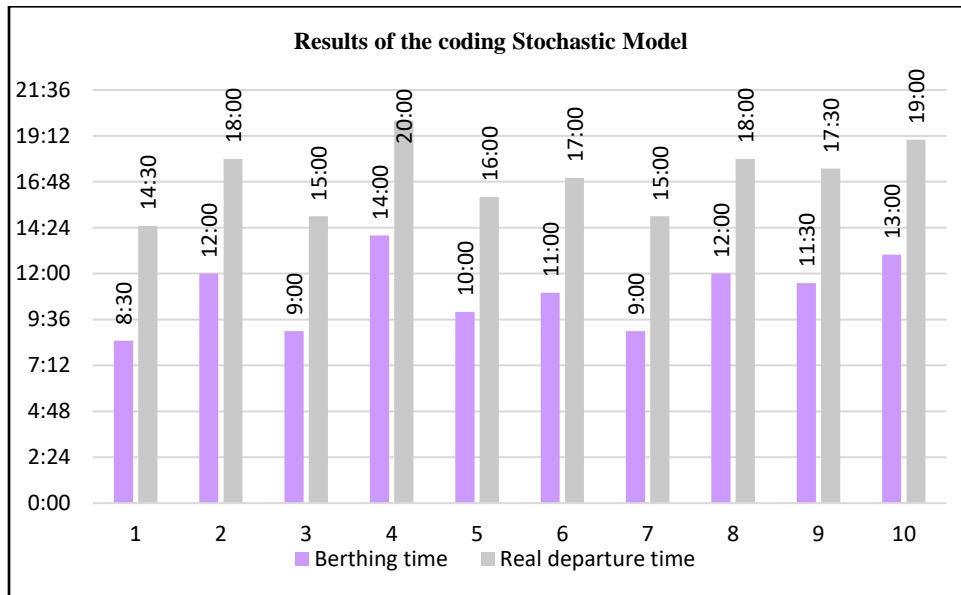


FIGURE 4  
RESULTS OF THE CODING STOCHASTIC MODEL(Z:0)

The results of the discrete probability models ( $Z_{WS}$ ,  $Z_{HN}$ ,  $Z_{EV}$ ) are in (Table 4, Table 5, Table 6).

TABLE 4  
 $Z_{WS}$  SCENARIOS

vessel	Berthing time	Real departure time
$Z_1=0$		
1	8:30	12:33
2	12:00	14:25
3	9:00	17:06
4	14:00	17:49
5	10:00	12:50
6	11:00	14:41
7	9:00	11:00
8	12:00	19:47
9	11:30	17:57
10	13:00	19:19
$Z_2=0$		
1	8:30	13:21
2	12:00	15:12
3	9:00	18:22
4	14:00	18:10
5	10:00	13:07
6	11:00	15:03
7	9:00	11:10
8	12:00	20:36
9	11:30	18:40
10	13:00	19:50

$Z_3=0$		
1	8:30	14:00
2	12:00	15:00
3	9:00	19:00
4	14:00	18:00
5	10:00	13:00
6	11:00	15:00
7	9:00	11:00
8	12:00	21:00
9	11:30	19:30
10	13:00	20:00
$Z_4:82720$		
1	8:30	14:00
2	12:00	15:00
3	9:00	19:00
4	14:00	18:00
5	10:00	13:00
6	11:00	15:00
7	9:00	11:00
8	12:00	21:00
9	11:30	19:30
10	13:00	20:00
$Z_5=191083$		
1	8:30	14:24
2	12:00	16:07
3	9:00	21:28
4	14:00	19:25
5	10:00	14:02
6	11:00	16:17
7	9:00	11:47
8	12:00	23:23
9	11:30	21:09
10	13:00	22:00
TOTAL $Z_{ws}=54760.6$		

As you see, the amount of ship arrival time and desired departure time are specified in Figure 2. Figure 3 shows the coding results of the deterministic model that  $z$  value is equal to 0. Figure 4 shows the coding results of the Stochastic Scheduling Model that  $z$  value is equal to 0. Which means that the ships have no delay in berthing and desired departure time. The results of the discrete probability models ( $Z_{WS}$ ,  $Z_{HN}$ ,  $Z_{EV}$ ) are in (Table 4, Table 5, Table 6).

In  $Z_{WS}$  according to the five scenarios defined, the value of  $z_{ws}$  is equal to 54760.6. In  $Z_{HN}$  according to the five scenarios defined, the value of  $z_{HN}$  is equal to

158552. In  $Z_{EV}$  according to the five scenarios defined, the value of  $z_{EV}$  is equal to 1630.33.

The reason that Scenario 3, Scenario 4 and Scenario 5 were infeasible is that the real departure time in the scenarios was assumed to be constant, and in Scenario 4 and 5 the time of berthing the ships should have occurred sooner than impossible arrival time. However, the best outcome is  $Z_{EV}$  because it focuses on uncertainty in the solution process, and  $Z_{HN}$  focuses only on probability in modeling, and  $Z_{WS}$  does not pay attention to uncertainty.

TABLE 5  
Z<sub>HN</sub> SCENARIOS

vessel	Berthing time	Real departure time
Z <sub>HN</sub> =158552		
1	8:30	14:45
2	12:00	16:08
3	9:00	21:28
4	14:00	19:25
5	10:00	14:02
6	11:00	16:17
7	9:00	11:47
8	12:00	23:23
9	11:30	21:09
10	13:00	22:01

TABLE 6  
Z<sub>EV</sub> SCENARIOS

vessel	Berthing time	Real departure time
1	8:30	13:22
2	12:00	15:30
3	9:00	19:24
4	14:00	18:35
5	10:00	13:25
6	11:00	15:28
7	9:00	11:22
8	12:00	21:31
9	11:30	19:30
10	13:00	20:33

First step Z<sub>3</sub>:3007.99, Scenarios 1= z<sub>1</sub>:2436.47, Scenarios 2= z<sub>2</sub>:2707.19, Scenarios 3= z<sub>3</sub>: infeasible  
Scenarios 4= z<sub>4</sub>: infeasible, Scenarios 5= z<sub>5</sub>: infeasible

### 5. CONCLUSION AND FUTURE RESEARCH

Unusual layouts in the BAP is a real-world feature in terminals. In real world, the servicing and handling times of ships (unloading and loading) depending on the conditions of the port, personnel, and equipment of the ship and the berth may not be known and certain, or there is fluctuation in the cost of waiting for berthing or delay in the departure of ships. This may dramatically affect the BAP solutions. In this paper a stochastic version the BAP in which the time and cost were

assumed to be uncertain was developed. The proposed BAP was investigated in both continuous and discrete situations using SCCP and scenario based mathematical programming, respectively. Numerical example was provided to present the applicability of the proposed modeling in both continuous and discrete situations.

Future research for this paper may consider the arrival time of the vessels as uncertain and see its impact on the cost of a delayed departure from ships and the cost of waiting for the berthing.

### REFERENCES

- Adarang, H., Bozorgi-Amiri, A., Khalili-Damghani, K., Tavakkoli-Moghaddam, R. (2020) 'Planning for Medical Emergency Transportation Vehicles during Natural Disasters', *Journal of Optimization in Industrial Engineering*, Vol. 13, No. 2, pp.185-197.
- Adarang, H., Bozorgi-Amiri, A., Khalili-Damghani, K. (2020) 'A robust bi-objective location-routing model for providing emergency medical services',

- Journal of Humanitarian Logistics and Supply Chain Management.
- Bacalhau, E.T., Casacio, L., Tavares de Azevedo, A. (2021) 'New hybrid genetic algorithms to solve dynamic berth allocation problem', *Expert Systems with Applications*, Vol. 167, 114198.
  - Barbosa, F., Berbet Rampazzo, P., Yamakami, A., Camanho, A.S. (2019) 'The use of frontier techniques to identify efficient solutions for berth allocation problem solved with a hybrid evolutionary algorithm', *Computers and Operations Research*, Vol. 107, pp. 43-60.
  - Bierwirth, C., & Meisel, F. (2010) 'A survey of berth allocation and quay crane scheduling problems in container terminals', *European Journal of Operational Research*, Vol. 202, No. 3, pp.615-627.
  - Bierwirth, C., & Meisel, F. (2015) 'A follow-up survey of berth allocation and quay crane scheduling problems in container terminals', *European Journal of Operational Research*, Vol. 244, No. 3, pp. 675-689.
  - Budipriyanto, A., Wirjodirdjo, B., Pujawan, N., & Gurning, S. (2015) 'Berth allocation problem under uncertainty: A conceptual model using collaborative approach', *Industrial Engineering and Service Science*, No. 4, pp.429-437.
  - Carlo, H. J., Vis, I. F., & Roodbergen, K. J. (2015) 'Seaside operations in container terminals: literature overview, trends, and research directions', *Flexible Services and Manufacturing Journal*, Vol. 27, No. (2-3), pp. 224-262.
  - Charnes, A. and Cooper, W. (1963) 'Deterministic equivalent for optimizing and satisficing under chance constraints', *Operations Research*, Vol. 11, No.1, pp.18-39.
  - Chargui, K., Zouadi, T., Fallahi, A., Reghioui, M., Aouam, T. (2021) 'Berth and quay crane allocation and scheduling with worker performance variability and yard truck deployment in container terminals', *Transportation Research Part E: Logistics and Transportation Review*, Vol.154, 102449.
  - Correcher, J. F., Van den Bossche, T., Alvarez-Valdes, R., & Berghe, G. V. (2019) 'The berth allocation problem in terminals with irregular layouts', *European Journal of Operational Research*, Vol. 272, No. 3, pp.1096-1108.
  - Ghasemi, P., Khalili-Damghani, K., Hafezalkotob, A., S Raissi., (2019) 'Uncertain multi-objective multi-commodity multi-period multi-vehicle location-allocation model for earthquake evacuation planning', *Applied Mathematics and Computation*, Vol. 350, pp.105-132.
  - Ghasemi, P., Khalili Damghani, K., Hafezalkotob, A., Raissi, S. (2019) 'Multi-Objective Mathematical Model for Location, Allocation and Distribution of Relief Commodities under Uncertainty', *Industrial Management Studies*, Vol.16, No.51, pp.107-144.
  - Ghasemi, P., Khalili-Damghani, K., Hafezalkotob, A., Raissi, S. (2020) 'Stochastic optimization model for distribution and evacuation planning (A case study of Tehran earthquake) ', *Socio-Economic Planning Sciences*, Vol.71, 100745.
  - Ghasemi, P., & Khalili-Damghani, K. (2021) 'A robust simulation-optimization approach for pre-disaster multi-period location-allocation-inventory planning', *Mathematics and Computers in Simulation (MATCOM)*, Elsevier, Vol.179(C), pp.69-95.
  - Guan, Y., Xiao, W. Q., Cheung, R. K., & Li, C. L. (2002) 'A multiprocessor task scheduling model for berth allocation: heuristic and worst-case analysis', *Operations Research Letters*, Vol.30, No.5, pp. 343-350.
  - Guan, Y., & Cheung, R. K. (2004) 'The berth allocation problem: models and solution methods', *Or Spectrum*, Vol. 26, No.1, pp.75-92.
  - Hosseini, A., Sahlin, T. (2019) 'An optimization model for management of empty containers in distribution network of a logistics company under uncertainty', *Journal of Industrial Engineering International*, Vol.15, No. 4, pp.585-602.
  - Guo, L., Jun, W., Zheng, J. (2021) ' Berth allocation problem with uncertain vessel handling times considering weather conditions', *Computer and Industrial Engineering*, Vol.158, 107417.
  - Imai, A., Nagaiwa, K. I., & Tat, C. W. (1997) 'Efficient planning of berth allocation for container terminals in Asia', *Journal of Advanced transportation*, Vol.31, No.1, pp.75-94.
  - Imai, A., Nishimura, E., & Papadimitriou, S. (2001) 'The dynamic berth allocation problem for a container port', *Transportation Research Part B: Methodological*, Vol. 35, No. 4, pp. 401-417.

- Imai, A., Nishimura, E., Hattori, M., & Papadimitriou, S. (2007) 'Berth allocation at indented berths for mega-containerships', *European Journal of Operational Research*, Vol.179, No. 2, pp.579-593.
- Imai, A., Nishimura, E., & Papadimitriou, S. (2013) 'Marine container terminal configurations for efficient handling of mega-containerships', *Transportation Research Part E: Logistics and Transportation Review*, Vol. 49, No.1, pp.141-158.
- Javid, N., Khalili-Damghani, K., Makui, A., Abdi, F. (2020) 'multi-objective flexibility-complexity trade-off problem in batch production systems using fuzzy goal programming', *Expert Systems with Applications*, Vol. 148, 113266.
- Khalili-Damghani, K., Tavana, M., Santos-Arteaga, F.J., Ghanbarzad-Dashti, M. (2017) 'A Customized Genetic Algorithm for Solving Multi-Period Cross-Dock Truck Scheduling Problems', *Measurement*, Vol.108, pp.101-118.
- Khalili-Damghani, K., Poortarigh, M., Pakgozar, A. (2017) 'A new model for probabilistic multi-period multi-objective project selection problem', *24th International Conference on Production Research (ICPR)*, pp.598-603.
- Khalili-Damghani, K., Shahrokh, A., Pakgozar, A. (2017) 'Stochastic multi-period multi-product multi-objective Aggregate Production Planning model in multi-echelon supply chain', *International Journal of Production Management and Engineering*, Vol. 5, No. 2, pp.85-106.
- Khalili-Damghani, K., Tavana, M. & Ghasemi, P. (2021) 'A stochastic bi-objective simulation-optimization model for cascade disaster location-allocation-distribution problems', *Ann Oper Res*.
- Lashgari, M., Sadjadi, S.J., Sahihi, M. (2020) 'A multi-product, multi-period model to select supplier for deteriorating products while considering uncertainty as well as backorder', *Journal of Industrial Engineering International*, Vol. 16, No.3, pp.93-101.
- Li, C. L., Cai, X., & Lee, C. Y. (1998). Scheduling with multiple-job-on-one-processor pattern', *IIE transactions*, 30(5), 433-445.
- Liu, B., Li, Zh-Ch., Sheng, D., Wang, Y. (2021) 'Integrated planning of berth allocation and vessel sequencing in a seaport with one-way navigation channel', *Transportation Research Part B: Methodological*, Vol. 143, pp.23-47.
- Nishimura, E., Imai, A., & Papadimitriou, S. (2001) 'Berth allocation planning in the public berth system by genetic algorithms', *European Journal of Operational Research*, Vol. 131, No. 2, pp. 282-292.
- Lozkins, A., Krasilnikov, M., and Bure, V. (2019) 'Robust uncapacitated multiple allocation hub location problem under demand uncertainty: minimization of cost deviations', *Journal of Industrial Engineering International*, Vol. 15, No.1, pp. S199–S207.
- Monaco, M. F., & Sammarra, M. (2007) 'The berth allocation problem: a strong formulation solved by a Lagrangean approach', *Transportation Science*, Vol. 41, No. 2, pp.265-280.
- Nourzadeh, F., Ebrahimnejad, S., Khalili-Damghani, K., Hafezalkotob, A. (2020) 'Chance constrained programming and robust optimization approaches for uncertain hub location problem in a cooperative competitive environment', *Scientia Iranica*.
- Nourzadeh, F., Ebrahimnejad, S., Khalili-Damghani, K., Hafezalkotob, A. (2020) 'Development of a Model for Locating Hubs in a Competitive Environment under Uncertainty: A Robust Optimization Approach', *International Journal of Engineering*, Vol. 33, No. 1, pp.124-133.
- Reza-Pour, F., Khalili-Damghani, K. (2017) 'A New Stochastic Time-Cost-Quality Trade-Off Project Scheduling Problem Considering Multiple-Execution Modes, Preemption, and Generalized Precedence Relations', *Industrial Engineering & Management Systems*, Vol. 16, No. 3, pp.271-287.
- Rodrigues, F., Agra, A. (2021) 'An exact robust approach for the integrated berth allocation and quay crane scheduling problem under uncertain arrival time', *European Journal of Operational Research*, Vol. 295, No. 2, pp.499-516.
- Sajedi, S., Sarfaraz, AH., Bamdad, S., Khalili-Damghani, K. (2020) 'Designing a Sustainable Reverse Logistics Network Considering the Conditional Value at Risk and Uncertainty of Demand under Different Quality and Market Scenarios', *International Journal of Engineering*, Vol. 33, No.11, pp.2252-2271.

- Schepler, X., Absi, N., Feillet, D., Sanlaville, E. (2019) 'The Stochastic discrete berth allocation problem. EURO Journal on Transportation and Logistics, Vol. 8, No. 4, pp.363-396.
- Shahabi, A., Raissi, S., Khalili-Damghani, K., Rafei, M. (2019) 'Designing a resilient skip-stop schedule in rapid rail transit using a simulation-based optimization methodology', Operational Research, pp.1-31.
- Tavana, M., Khalili-Damghani, K., Santos-Arteaga, FJ., Zandi, MH. (2017) 'Drone shipping versus truck delivery in a cross-docking system with multiple fleets and products', Expert systems with applications, Vol. 72, pp.93-107.
- Thanos, E., Toffolo, T., Santos, H.G., Vancroonenburg, W., Berghe, G. (2021) 'The tactical berth allocation problem with time-variant specific quay crane assignments', Computers and Industrial Engineering, Vol. 155, 107168.
- Wawrzynaik, J., Drozdowski, M., Sanlaville, E. (2020) 'Selecting algorithms for large berth allocation problems', European Journal of Operational Research, Vol. 283, No. 3, pp.844-862.
- Xiang, X., Liu, Ch. (2021) 'An expanded robust optimization approach for the berth allocation problem considering uncertain operation time', Omega, Vol. 103, 102444.
- Xi, X., Changchun, L., Lixin, M. (2017) 'A bi-objective robust model for berth allocation scheduling under uncertainly', Transportation Research Part E: Logistics and Transportation Review, Vol. 106, pp.294-319.
- Zhen, L., Hay, L., and Peng, E. (2011) 'A decision model for berth allocation under uncertainty', Eur. J. Oper. Res, Vol. 212, No.1, pp.54-68.
- Zhen, L., Liang, Z., Zhuge, D., Lee, L. H., & Chew, E. P. (2017) 'Daily berth planning in a tidal port with channel flow control', Transportation Research Part B: Methodological, Vol. 106, pp.193-217.