Original Research

Reliability and Cost Analysis of Sachet Water Plant Using Copula Approach

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Abstract
The goal of this study is to provide an analytical framework for a standby serial sachet water plant. The plant is made up of five subsystems: a storage tank, filters, a tank, boosters, and a production machine, all of which are set up in a series-parallel configuration. The failure rates of both subsystems/units are continuous and supposed to tail exponential function, but repair is categories into the following; general repair for partially failed state while copula repair for complete failed state. A repair person is on-call 24 hours a day, seven days a week to repair failed units in the system. The transition diagram is used to obtain partial differential equations of degree one, which are resolved using supplementary variable procedure, Laplace conversion and MAPLE software, to derive expressions for numerous dependability agencies name; availability, reliability, MTTF, sensitivity MTTF, and cost of the sachet water plant. Arithmetical instances are delivered in order to demonstrate the achieved results and also to investigate the influence of parameters used. Tables and figures revealed that copula repair outperforms general repair in terms of returns.

Keywords – Availability; Reliability; Sachet Water; Serial System

1. Introduction

Typically, systems are examined in order to determine their reliability metrics. Process plants must be productive and earn a full income in order to survive. To accomplish this, the efficiency and dependability of the process's equipment must be of the highest caliber. More emphasis should be placed on operational management so as to increase the effectiveness and dependability of the related development curriculum. Our inability to pay adequate attention to process technology has been the most common weakness of our technical capabilities. Inputs used in the manufacturing phase include raw materials, electricity, machinery, information and technology, labor, and so on. To achieve quality and quantity, efficient plant management is required to monitor the conversion process and the variables influencing output. One method of plant management is the creation of a mathematical model. In the world of technology, the modeling method is widely used. This method is commonly used in the oil and milling industries, among others. Sachet water is a primary basis for water intake to developing countries for the lower and middle classes. Given that water is an essential resource for the survival of all animals, plus humans, an abundant provision of clean water is an absolute necessity for all. For this reasons, implementing the modeling method in the water sector would be critical to ensuring enough provision of clean water in the society.

The development of sachet water in developing countries began in the late 1990s, and sachet water advertising and consumption has skyrocketed. The majority of manufacturers are less concerned with increasing the availability, profitability, and dependability of their equipment. The continuous population growth and credulous intake of sachet water necessitate an increase in production, as it is difficult for the most disadvantaged citizens to obtain. The product is believed to be essential for the complement to similar sealed water which is purchased affordably. However, Minor problems experienced and resolved to avoid stoppage of production process through performing routing supervisions, servicing, replacement of wound unit among others. Repairs and replacements are typically performed only after a breakdown in most businesses. Furthermore, failure data is rarely available. This is a legend. However, additional maintenance attention is required to improve the equipment's availability and reliability. The development of sachet water in the late 1990s, and it is becoming increasingly clear that achieving high-quality maintenance requires mitigation at the source, as well as a focus on finding and preventing the cause of equipment degradation, rather than the more traditional method of either letting the equipment fail before fixing it or "firefighting" in the event of an emergency. Maintenance practice advances with the implementation of best-in-class benchmarks and the required training of staff committed to ongoing professional development. Some of the most important factors in any successful sachet water system are reliability, availability, and profit. As with other systems, sachet water systems are vulnerable to the following failures, including catastrophic, switch, partial, human among others. Proper maintenance planning is essential for achieving high system reliability, availability, and output. A sachet water system's availability and profit can be increased through proper maintenance planning, regular inspection, fault-tolerant units or subsystems.

Voluminous scholars have published their works in the subject of engineering, industrial, manufacturing system, dependability characteristic by assuming different failure rates, repair rates and other maintenance procedures. As a result, identifying small; Aliyu, Yusuf, and Ali (2015) looked at how a series-parallel system with a linear sequential cold standby unit could improve availability and profit. To examine the dependability, availability, and maintainability of industrial systems. Garg (2014) suggested PSO and fuzzy approaches. The use of credibility theory and several sorts of intuitionistic fuzzy numbers to analyze the dependability of a series-parallel system was a study work in respect of Garg (2016). Pourhassan MR, Raissi S, and Ha fezalkotob A conducted research to investigate into a simulation technique on reliability assessment of complex systems vulnerable to stochastic degradation and random shock (2020). Garg (2016) introduced a fuzzy kolmogrov's differential equations approach to evaluate the reliability of industrial systems. Based on the cost-free warranty policy, Niwas and Garg (2018) developed a method for measuring the dependability and profit of an industrial system. Investigation the performance of an industrial system employing a hybridized soft computing approach was carried out by Garg (2017). Kumar, Pant, and Singh (2018) used a hesitant fuzzy set to test the reliability of a system. Singh and Rawal (2015) presented a study on the availability, MTTF, and cost analysis of a complex system employing copula distribution under a proactive resume repair strategy. The reliability analysis of a linear sequential 2-out-of-3 system in the presence of a supporting device and a repairable service station was explored by Yusuf, Babagana, Yusuf, and Lawan (2014). Raissi, S., and Ebadi, Sh. (2018) centered their research on a Computer Simulation Model for Complex System Reliability Estimation. Yusuf (2016) investigated the reliability of a parallel system with a supporting device as well as two forms of preventative maintenance. The evaluation procedure Pourhassan, MR. Raissi, S., and Apornak, A. devised a simulation approach for modeling multi-state system reliability analysis in a power station under fatal and nonfatal shocks (2020). Analysis of Pure Water Production: A Case Study of Ten (10) Randomly Selected Pure Water Firms in Minna, Niger State was presented by Kontagora (2010). Contamination of Sachet Water in Nigeria: Assessment and Health Impact was discussed by Omalu, Eze, Olayemi, Gbesi, Ademiran, Ayanwale, Mohammad, and Khukwumeka (2010). Minner, Tagurum, Hassan, Afolaranmi, Bello, Dakhin, and Zoakah (2011) investigated Sachet Water: Prevalence of Use, Perception, and Quality in a community in Plateau state's Jos South local government area. Yusuf, Ismail, Lawan, Ali, and Nasir (2021) concentrated on the research of client–server system reliability modeling and analysis utilizing the Gumbel–Hougaard family copula. Besides, the aforementioned works and the present work have been conducted on dependability investigation for industrial, manufacturing and engineering systems among others. But none of the authors work exactly on the sachet water plant consists of five subsystems using copula method. A number of authors have used copula repair to study system performance, including Lado and Singh (2019), Ram and Kumar (2015), Ram, Singh, and Singh (2013), Rawal, Ramand Singh (2015), Gulati, Singh, Rawal, and Goel (2016), and many others. They have stated unequivocally that copula repair yields better results than general repair.

1.1 Notations

j	Time variable
ξ_k	Failure rate of storage tank, filter, tank, booster, production machine, $k = 1, 2, 3, 4, 5$
$W(r)/W(v)$	filter/booster repair rate

$\pi_\theta(m), \pi_\theta(r), \pi_\theta(n), \pi_\theta(v), \pi_\theta(e)$ Repair rates for storage tanks, filters, tanks, boosters, and production machines that have completely failed.

$A_c(j)$ For c = 0, 1, …, 9, Sc described the possible states of the sachet water plant

$\overline{A}(s)$ Laplace conversion of $A(r)$.

$A_c(r, j)$ For c = 1..., 9, described the states probability with repair and repair time.

$A_c(v, j)$ For c = 1..., 9, described the states probability with repair and repair time.

$E_g(j)$ Profit/gain expected within the interval [0, j].

B_1, B_2 The elements of cost are revenue and service costs.

$S_W(r)$ Function distribution $S_W(r) = W(r)e^{-\int_o^r W(r)dr}$ notation.

$\overline{S}_W(s)$ Laplace transforms of $S_W(r)$, i.e., $\overline{S}_W(s) = \int_0^\infty e^{-sr} W(r) e^{-\int_0^r W(r)dr} dr$

$\pi_\theta(r) = C_\theta(\pi_1(r), \pi_2(r))$ Copula repair distribution defined as:

$$c_\theta(\pi_1(r), \pi_2(r)) = \exp\left(r^\theta + \left\{\log\phi(r)^\theta\right\}^{\frac{1}{\theta}}\right), \quad 1 \le \theta \le \infty.$$

Where $\pi_1 = W(r)$, and $\pi_2 = e^r$

1.2 Assumptions

The subsequent hypotheses are made into account when validating the model:

(i) Firstly, storage tank, filters, tank, boosters and production machine are in perfect working condition.

(ii) storage tank, filter, tank, booster and production machine are essential for the system to operate.

(iii) The system would stop working if any of storage tank, filters, tank, boosters and production machine completely failed.

(iv) The sachet water plant's failing storage tank, filters, tank, boosters, and production machine can be fixed in either a functional or failed state.

(v) Failure rates are continuous and supposed tail exponential function.

(vi) General repair is used to fix partially failed states, while copula repair is used to repair completely failed states.

(vii) The factory performs very well after being repaired, and no damage was visible during the repair process.

(viii) The failing storage tank, filters, tank, boosters, and production machine are repaired immediately, and the machine is ready to complete the mission.

1.3 The sachet water plant is described as follows

Subsystems that make up this sachet water plant are: storage tank, filters, tank, boosters, and manufacturing machine, which are all grouped in a serial arrangement. The 1-out-of-2 G-policy applies to both filters and boosters. Initially, when a filter in the sachet water plant fails, the standby filter immediately turns on to function, system continues to works, so the failed filter under goes repair process. One booster fails, and for that reserve booster immediately turns on to function, the factory works while the abortive booster undergoes repair process. The factory stops working completely if any of storage tank, tank or production machine fails and it is assigned for repair. General repair is used to fix partially failed states, while copula repair is used to repair completely failed states.

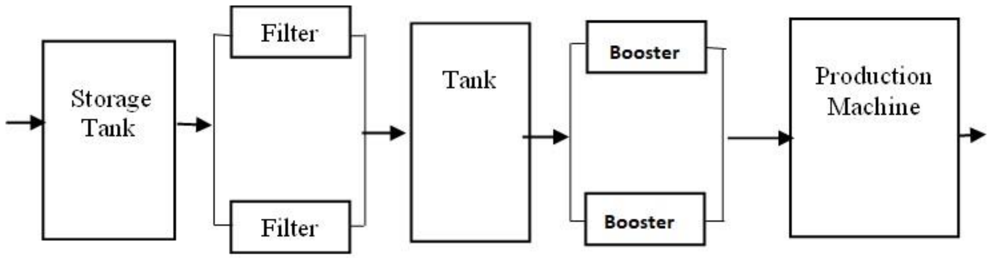

FIGURE 1
SHOWS SERIAL CONFIGURATION OF THE SACHET WATER PLANT

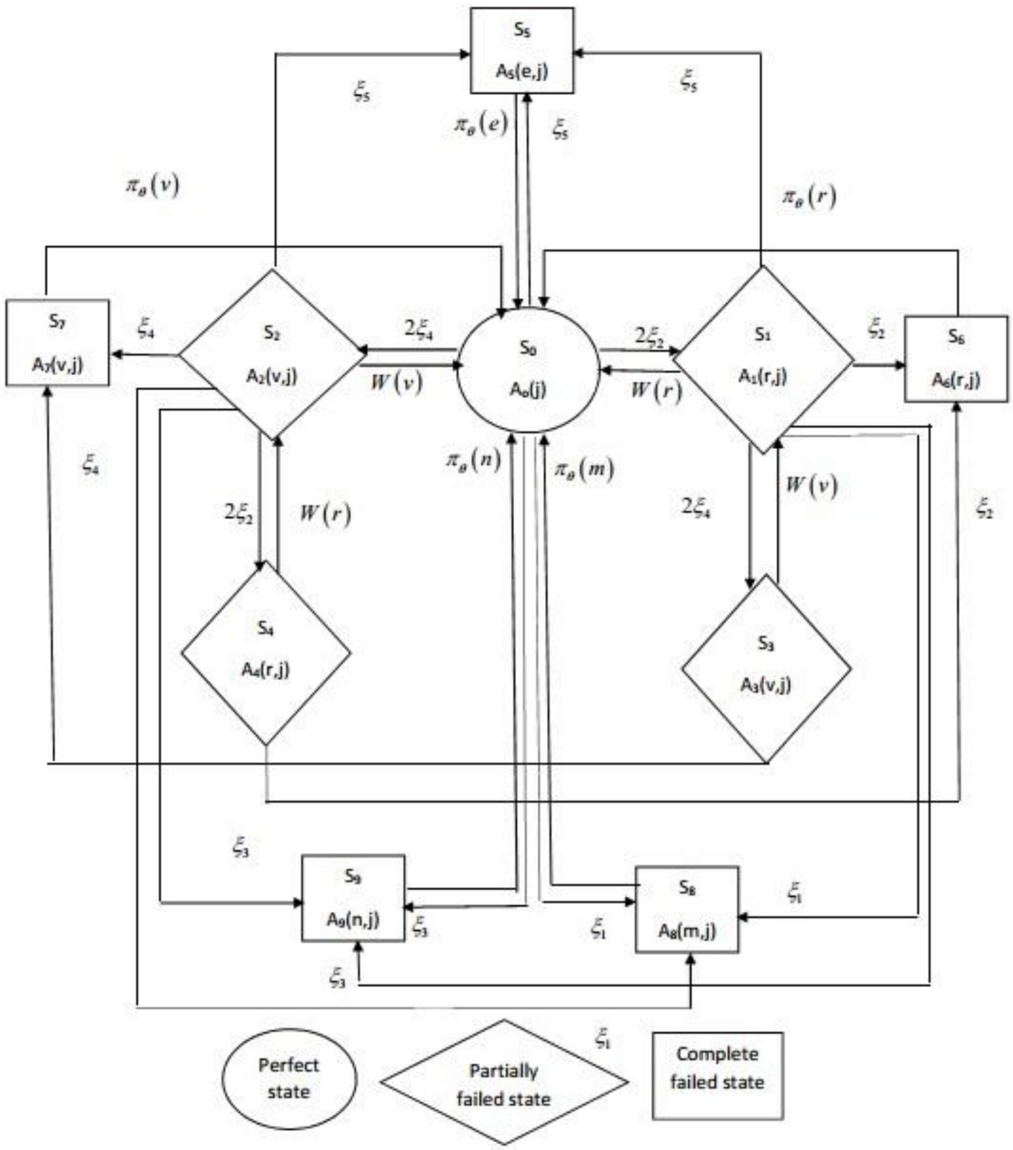

FIGURE 2
DEPICTS THE SACHET WATER PLANT'S TRANSITION PLAN.

1.4 Description of the State

S_o The storage tank, filters, tank, boosters, and manufacturing equipment are all in excellent condition.

S_1 / S_4 When one of the filters breaks, the standby mode filter shifts to operative mode, and the failed filter is allocated to the repair system.

S_2 / S_3 When one of the boosters fails, the system automatically shifts to operational mode, and the failed booster is assigned for repair.

S_5 If the storage tank fails, the sachet water production would shut down completely.

S_6 The entire sachet water will stop operating if the reserve filter fails a second time.

S_7 If the factory's second booster fails, the entire factory will shut down.

S_8 The breakdown of the sachet water plant's tank results in the entire system failing.

S_9 The entire system has failed as a result of a production machine breakdown.

2. Formulation of Sachet Water Mathematical Model

The transition diagram is used to generate the following sets of equations using the method used by the following authors: Lado et al (2019), Yusuf et al (2020), and Singh et al (2021).

$$\left(\frac{\partial}{\partial j}+\xi_1+2\xi_2+\xi_3+2\xi_4+\xi_5\right)A_0(j)=\int_0^\infty W(r)A_1(r,j)\,dr+\int_0^\infty W(v)A_2(v,j)\,dv+\int_0^\infty \pi_\theta(r)A_6(r,j)\,dr$$
$$+\int_0^\infty \pi_\theta(v)A_7(v,j)\,dv+\int_0^\infty \pi_\theta(e)A_5(e,j)\,de+\int_0^\infty \pi_\theta(m)A_8(m,j)\,dm+\int_0^\infty \pi_\theta(n)A_9(n,j)\,dn \quad (1)$$

$$\left(\frac{\partial}{\partial j}+\frac{\partial}{\partial r}+\xi_1+\xi_2+\xi_3+2\xi_4+\xi_5+W(r)\right)A_1(r,j)=0 \quad (2)$$

$$\left(\frac{\partial}{\partial j}+\frac{\partial}{\partial v}+2\xi_1+\xi_2+2\xi_3+\xi_4+\xi_5+W(v)\right)A_2(v,j)=0 \quad (3)$$

$$\left(\frac{\partial}{\partial j}+\frac{\partial}{\partial v}+\xi_4+W(v)\right)A_3(v,j)=0 \quad (4)$$

$$\left(\frac{\partial}{\partial j}+\frac{\partial}{\partial r}+\xi_2+W(r)\right)A_4(r,j)=0 \quad (5)$$

$$\left(\frac{\partial}{\partial j}+\frac{\partial}{\partial e}+\pi_\theta(e)\right)A_5(e,j)=0 \quad (6)$$

$$\left(\frac{\partial}{\partial j}+\frac{\partial}{\partial r}+\pi_\theta(r)\right)A_6(r,j)=0 \quad (7)$$

$$\left(\frac{\partial}{\partial j}+\frac{\partial}{\partial v}+\pi_\theta(v)\right)A_7(v,j)=0 \quad (8)$$

$$\left(\frac{\partial}{\partial j}+\frac{\partial}{\partial m}+\pi_\theta(m)\right)A_8(m,j)=0 \quad (9)$$

$$\left(\frac{\partial}{\partial j}+\frac{\partial}{\partial n}+\pi_\theta(n)\right)A_9(n,j)=0 \quad (10)$$

Boundary condition

$$A_1(0,j)=2\xi_2A_0(j) \quad (11)$$

$$A_2(0,j)=2\xi_4A_0(j) \quad (12)$$

$$A_3(0,j)=2\xi_4A_1(0,j) \quad (13)$$

$$A_4(0,j)=2\xi_2A_2(0,j) \quad (14)$$

$$A_5(0,j)=\xi_5\left(A_0(0,j)+A_1(0,j)+A_2(0,j)\right) \quad (15)$$

$$A_6(0,j)=\xi_2\left(A_1(0,j)+A_4(0,j)\right) \quad (16)$$

$$A_7(0,j)=\xi_4\left(A_2(0,j)+A_3(0,j)\right) \quad (17)$$

$$A_8(0,j)=\xi_1\left(A_0(j)+A_1(0,j)+A_2(0,j)\right) \quad (18)$$

$$A_9(0,j)=\xi_3\left(A_0(j)+A_1(0,j)+A_2(0,j)\right) \quad (19)$$

2.1 Mathematical model for sachet water solution

The following equations are produced using a Laplace conversion from equations (1) to (19) by the assist of boundary conditions.

$$(s+\xi_1+2\xi_2+\xi_3+2\xi_4+\xi_5)\overline{A}_0(s)=$$

$1+\int_0^{\infty\int} W(r)\overline{A}_1(r,s())+$
$\int_0^{\infty} W(v)\overline{A}_2(v,s)dv \int_0^{\infty} \pi_\theta(r)\overline{A}_6(r,s)dr$
$\int_0^{\infty} \pi_\theta(y)\overline{A}_7(v,s)dv + \int_0^{\infty} \pi_\theta(e)\overline{A}_5(e,s)de$
$\int_0^{\infty} \pi_\theta(m)\overline{A}_8(m,s)dm+\int_0^{\infty} \pi_\theta(n)\overline{A}_9(n,s)dn$ (20)

$\left(s+\frac{\partial}{\partial r}+\xi_1+\xi_2+\xi_3+2\xi_4+\xi_5+W(r)\right)\overline{A}_1(r,s())$ (21)

$\left(s+\frac{\partial}{\partial v}+\xi_1+2\xi_2+\xi_3+\xi_4+\xi_5+W(v)\right)\overline{A}_2(v,s())$ (22)

$\left(s+\frac{\partial}{\partial v}+\xi_4+W(v)\right)\overline{A}_3(v,s())$ (23)

$\left(s+\frac{\partial}{\partial r}+\xi_2+W(r)\right)\overline{A}_4(r,s)=0$ (24)

$\left(s+\frac{\partial}{\partial e}+\pi_\theta(e)+W(e)\right)\overline{A}_5(e,s)=0$ (25)

$\left(s+\frac{\partial}{\partial r}+\pi_\theta(r)\right)\overline{A}_6(r,s)=0$ (26)

$\left(s+\frac{\partial}{\partial v}+\pi_\theta(v)\right)\overline{A}_7(v,s)=0$ (27)

$\left(s+\frac{\partial}{\partial m}+\pi_\theta(m)\right)\overline{A}_8(m,s)=0$ (28)

$\left(s+\frac{\partial}{\partial n}+\pi_\theta(n)\right)\overline{A}_9(n,s)=0$ (29)

Boundary conditions

$\overline{A}_1(0,s)=2\xi_2\overline{A}_0(s)$ (30)

$\overline{A}_2(0,s)=2\xi_4\overline{A}_0(s)$ (31)

$\overline{A}_3(0,s)=2\xi_4\overline{A}_1(o,s)$ (32)

$\overline{A}_4(0,s)=2\xi_2\overline{A}_2(o,s)$ (33)

$\overline{A}_5(0,s)=\xi_5\left(\overline{A}_0(s)+\overline{A}_1(0,s)+\overline{A}_2(0,s)\right)$ (34)

$\overline{A}_6(0,s)=\xi_2\left(\overline{A}_1(0,s)+\overline{A}_4(0,s)\right)$ (35)

$\overline{A}_7(0,s)=\xi_4\left(\overline{A}_2(0,s)+\overline{A}_3(0,s)\right)$ (36)

$\overline{A}_8(0,s)=\xi_1\left(\overline{A}_0(s)+\overline{A}_1(0,s)+\overline{A}_2(0,s)\right)$ (37)

$\overline{A}_9(0,s)=\xi_3\left(\overline{A}_0(s)+\overline{A}_1(0,s)+\overline{A}_2(0,s)\right)$ (38)

Condition of Initials

$A_0(0)=1$, but other state transition probability is 0 at this time. (39)

The following solution can be obtained by resolving equations (21) to (29) with the use of boundary conditions.

$\overline{A}_0(s)=\frac{1}{D(s)}$ (40)

$\overline{A}_1(s)=\frac{2\xi_2}{D(s)}\left\{\frac{1-\overline{s}_\beta(s+\xi_1+\xi_2+\xi_3+2\xi_4+\xi_5)}{s+\xi_1+\xi_2+\xi_3+2\xi_4+\xi_5}\right\}$ (41)

$\overline{A}_2(s)=\frac{2\xi_4}{D(s)}\left\{\frac{1-\overline{s}_\beta(s+\xi_1+2\xi_2+\xi_3+\xi_4+\xi_5)}{s+\xi_1+2\xi_2+\xi_3+\xi_4+\xi_5}\right\}$ (42)

$\overline{A}_3(s)=\frac{4\xi_2\xi_4}{D(s)}\left\{\frac{1-\overline{s}_\beta(s+\xi_4)}{s+\xi_4}\right\}$ (43)

$\overline{A}_4(s)=\frac{4\xi_2\xi_4}{D(s)}\left\{\frac{1-\overline{s}_\beta(s+\xi_3)}{s+\xi_3}\right\}$ (44)

$\overline{A}_5(s)=\left(\frac{\xi_5+2\xi_2\xi_5+2\xi_4\xi_5}{D(s)}\right)\left\{\frac{1-\overline{s}_{\pi_\theta}(s)}{s}\right\}$ (45)

$\overline{A}_6(s)=\left(\frac{2\xi_2^2+4\xi_2^2\xi_4}{D(s)}\right)\left\{\frac{1-\overline{s}_{\pi_\theta}(s)}{s}\right\}$ (46)

$\overline{A}_7(s)=\left(\frac{2\xi_4^2+4\xi_2\xi_2^2}{D(s)}\right)\left\{\frac{1-\overline{s}_{\pi_\theta}(s)}{s}\right\}$ (47)

$\overline{A}_8(s)=\left(\frac{\xi_1+2\xi_1\xi_2+2\xi_1\xi_4}{D(s)}\right)\left\{\frac{1-\overline{s}_{\pi_\theta}(s)}{s}\right\}$ (48)

$\overline{A}_9(s)=\left(\frac{\xi_3+2\xi_2\xi_3+2\xi_3\xi_4}{D(s)}\right)\left\{\frac{1-\overline{s}_{\pi_\theta}(s)}{s}\right\}$ (49)

Where D(s) is defined as;

$$D(s)=\left\{s+\xi_1+2\xi_2+\xi_3+2\xi_4+\xi_5-\begin{pmatrix}2\xi_2\overline{s}_\beta(s+\xi_1+\xi_2+\xi_3+2\xi_4+\xi_5)+\\2\xi_4\overline{s}_\beta(s+\xi_1+2\xi_2+\xi_3+\xi_4+\xi_5)+\\\begin{bmatrix}(2\xi_2^2+4\xi_2^2\xi_4)+(2\xi_4^2+4\xi_3\xi_4^2)+\\(\xi_5+2\xi_2\xi_5+2\xi_4\xi_5)+\\(\xi_1+2\xi_1\xi_2+2\xi_1\xi_4)+\\(\xi_3+2\xi_2\xi_3+2\xi_3\xi_4)+\end{bmatrix}\overline{s}_{\pi_\theta}(s)\end{pmatrix}\right\} \quad (50)$$

The cumulative Laplace transformed state transition probabilities that the system is working are as follows:

$\overline{A}_{up}(s)=\left[\overline{A}_0(s)+\overline{A}_1(s)+\overline{A}_2(s)+\overline{A}_3(s)+\overline{A}_4(s)\right]$ (51)

$$\overline{A}_{up}(s)=\frac{1}{D(s)}\left\{\begin{matrix}1+2\xi_2\left(\frac{1-\overline{s}_\beta(s+\xi_1+\xi_2+\xi_3+2\xi_4+\xi_5)}{s+\xi_1+\xi_2+\xi_3+2\xi_4+\xi_5}\right)+\\2\xi_4\left(\frac{1-\overline{s}_\beta(s+\xi_1+2\xi_2+\xi_3+\xi_4+\xi_5)}{s+\xi_1+2\xi_2+\xi_3+\xi_4+\xi_5}\right)\\+4\xi_2\xi_4\left(\frac{1-\overline{s}_\beta(s+\xi_4)}{s+\xi_4}\right)+4\xi_2\xi_4\left(\frac{1-\overline{s}_\beta(s+\xi_3)}{s+\xi_3}\right)\end{matrix}\right\} \quad (52)$$

$\overline{A}_{down}(s)=1-\overline{A}_{up}(s))$ (53)

3. STUDY OF THE SACHET WATER PLANT FOR DIFFERENT RELIABILITY METRICS

3.1 Analysis of Availability with Copula Repair

Suppose that $S_{\pi_\theta}(s) = \bar{S}_{exp[r^\theta + \{log\, W(r)\}^\theta]^{1/\theta}}(s) = \frac{exp[r^\theta + \{log\, W(r)\}^\theta]^{1/\theta}}{s + exp[r^\theta + \{log\, W(r)\}^\theta]^{1/\theta}}$, $\bar{S}_W(s) = \frac{W}{s+W}$, assuming failure rates as $\xi_1 = 0.001, \xi_2 = 0.002, \xi_3 = 0.003, \xi_4 = 0.004, \xi_5 = 0.005$, $W(r) = W(v) = 1$. As a result of replacing those relations in equation (52) and using the inverse Laplace transform, the availability expression is:

$$\bar{A}_{up}(j) \begin{bmatrix} -0.000017e^{-1.00500j} - 0.0000170e^{-1.00400j} \\ +0.003376e^{-2.72751j} - 0.000288e^{-1.02947j} \\ -0.000001e^{-1.01825j} + 0.99694e^{-1.02947j} \end{bmatrix}_{Copula-R} \quad (54)$$

When you use time(t) as j = 0, 1...10 in equation (52), availability is determined as shown below.

TABLE 1
COMPUTED AVAILABILITY OVER TIME

j	0	1	2	3	4	5	6	7	8	9	10
Availability	1.00000	0.99699	0.99680	0.99675	0.99670	0.99665	0.99659	0.99653	0.99647	0.99641	0.99635

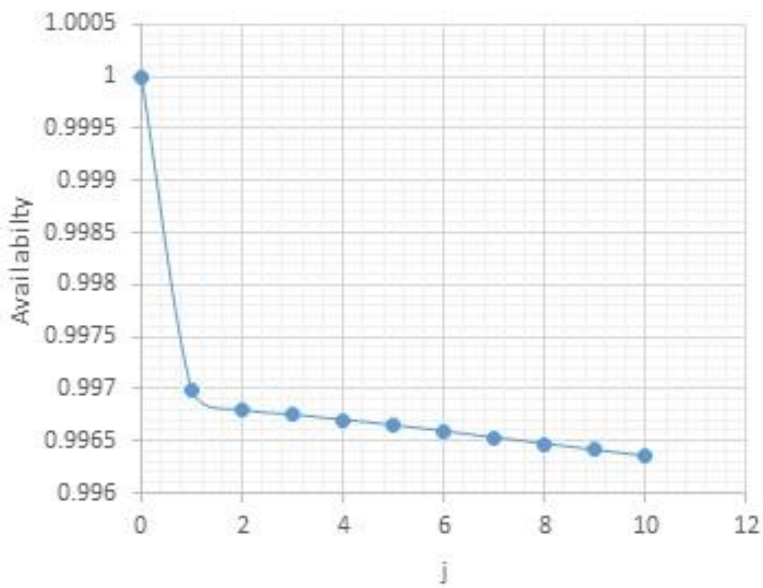

FIGURE 3
AVAILABILITY COUNTER J

3.2 Analysis of availability with general repair

Setting $\bar{S}_W(s) = \frac{W}{s+W}$, failure rates as $\xi_1 = 0.001, \xi_2 = 0.002, \xi_3 = 0.003, \xi_4 = 0.004, \xi_5 = 0.005$ and repair rates as $W(r) = W(v) = 1$ all in equation (52), after that, using the inverse Laplace transform, the availability expression is derived as:

$$\bar{A}(j) \begin{bmatrix} 0.000025e^{-1.00300j} + 0.000075e^{-1.00400j} \\ +0.004626e^{-1.03387j} + 0.000038e^{-1.01829j} \\ +0.003933e^{-1.00476j} + 0.991298e^{-0.00005j} \end{bmatrix}_{up_{General-R}} \quad (55)$$

Because equation (55) is a function of time, assuming j = 0, 1...10, availability is estimated as shown in Table 2.

TABLE 2
COMPUTED AVAILABILITY OVER TIME

j	0	1	2	3	4	5	6	7	8	9	10
Availability	1.00000	0.99699	0.99680	0.99675	0.99670	0.99665	0.99659	0.99653	0.99647	0.99641	0.99635

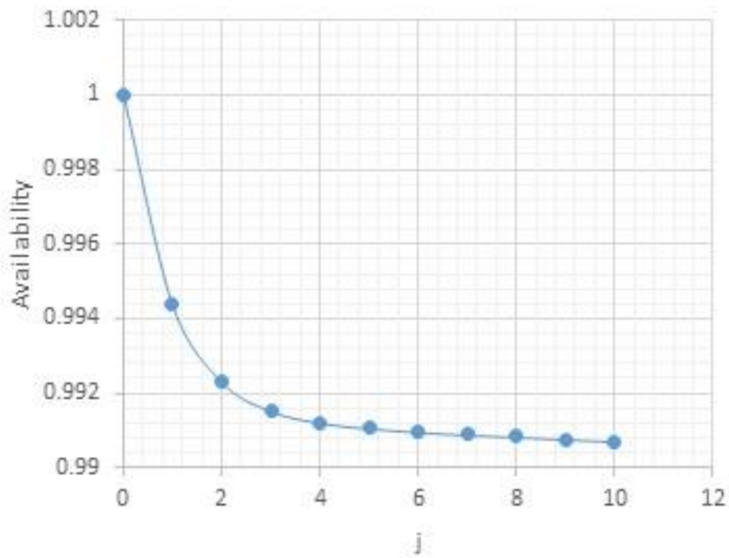

FIGURE 4
AVAILABILITY COUNTER J

3.3 *Analysis of Reliability*

If W, π are taking to zero, $\xi_1 = 0.001, \xi_2 = 0.002, \xi_3 = 0.003, \xi_4 = 0.004, \xi_5 = 0.005$ and applying inverse Laplace transformed, expression obtained is reliability function:

$$Re(j) = \begin{bmatrix} 0.001882e^{-0.00400j} - 3.003660e^{-0.02100j} + \\ 0.001777e^{-0.00300j} + 2e^{-0.01700j} + 2e^{-0.19000j} \end{bmatrix} \quad (56)$$

Reliability, on the other hand, is a function of time. Reliability is computed with j = 0, 1...10 in equation (56), as shown below.

TABLE 3
COMPUTED RELIABILITY OVER TIME

j	0	1	2	3	4	5	6	7	8	9	10
Reliability	1.00000	0.99105	0.98208	0.97310	0.96410	0.95509	0.94608	0.93707	0.92806	0.91905	0.91006

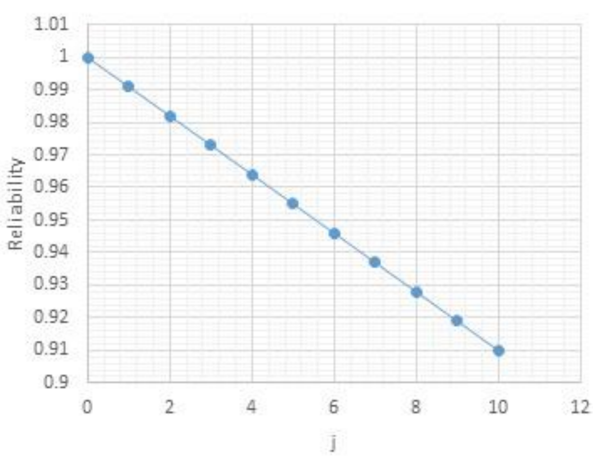

FIGURE 5
RELIABILITY COUNTER J

3.4 *Analysis of MTTF*

Allowing repairs to zero while s tends zero in equation (52), MTTF expression is obtained as:

$$MTTF = \lim_{s \to 0} \overline{A}_{up}(s) = \frac{1}{\xi_1+2\xi_2+\xi_3+2\xi_4+\xi_5} \begin{Bmatrix} 1 + \frac{2\xi_2}{\xi_1+\xi_2+\xi_3+2\xi_4+\xi_5} + \\ \frac{2\xi_4}{\xi_1+2\xi_2+\xi_3+\xi_4+\xi_5} \\ + \frac{4\xi_2\xi_4}{\xi_4} + \frac{4\xi_2\xi_4}{\xi_3} \end{Bmatrix} \quad (57)$$

Assuming $\xi_1 = 0.001, \xi_2 = 0.002, \xi_3 = 0.003, \xi_4 = 0.004, \xi_5 = 0.005$, to calculate MTTF for ξ_1 ,all other failure rates are kept constant while ξ_1 is varied as 0.001, 0.002...0.009 in equation (57). The MTTF for rest failure rates is calculated in the same method, the outcomes are displayed in the subsequent Table 2.

TABLE 5
CALCULATED MTTF USING FAILURE RATE

Failure Rate	$MTTF\ \xi_1$	$MTTF\ \xi_2$	$MTTF\ \xi_3$	$MTTF\ \xi_4$	$MTTF\ \xi_5$
0.001	80.94196	87.04093	95.19091	97.41440	111.80693
0.002	75.59595	80.94196	87.31111	90.98039	102.22751
0.003	70.87806	76.04576	80.94196	85.59442	94.06811
0.004	66.68686	71.96952	75.47474	80.94196	87.04444
0.005	62.94128	68.48289	70.69255	76.84195	80.94196
0.006	59.57575	65.43928	66.46464	73.17967	75.59595
0.007	56.53676	62.74074	62.69747	69.87654	70.87806
0.008	53.78021	60.31905	59.31934	66.87538	66.68686
0.009	51.26947	58.12492	56.27337	64.13267	62.94128

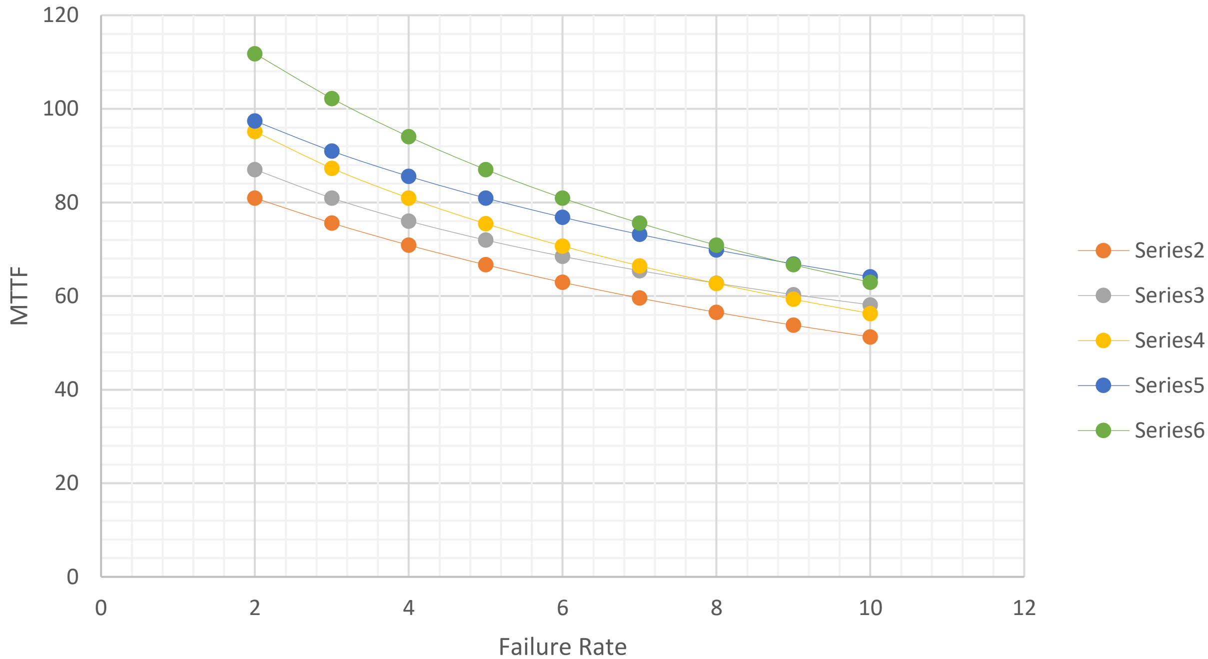

FIGURE 6
MTTF COUNTER FAILURE RATE

3.5 Analysis of Sensitivity

MTTF as failure rate parametric expression, is differentiated partially to obtained sensitivity expression. After then, to calculate sensitivity for ξ_1 ,all other failure rates are constant while ξ_1 is varied as 0.001, 0.002...0.009. However, sensitivity calculations for rest failure rates are carried out in the same way, with the results presented in the table below.

TABLE 6
CALCULATED SENSITIVITY IN RESPECT OF FAILURE RATE

Failure Rate	$\frac{\partial(MTTF)}{\xi_1}$	$\frac{\partial(MTTF)}{\xi_2}$	$\frac{\partial(MTTF)}{\xi_3}$	$\frac{\partial(MTTF)}{\xi_4}$	$\frac{\partial(MTTF)}{\xi_5}$
0.001	-5700.18838	-6890.59881	-9294.06865	-7123.09106	-10407.17162
0.002	-5013.65989	-5415.76611	-6945.33950	-5840.83045	-8814.93554
0.003	-4439.52280	-4438.24041	-5869.50055	-4981.12361	-7550.76302
0.004	-3956.30548	-3751.57551	-5098.45934	-4352.97741	-6532.00617
0.005	-3545.73239	-3245.56565	-4487.10943	-3865.69257	-5700.18838
0.006	-3194.19721	-2857.69019	-3984.08325	-3471.73621	-5013.05989
0.007	-2891.09785	-2550.58852	-3562.10246	-3143.94147	-4439.52280
0.008	-2891.09785	-2300.90127	-3203.56605	-2865.55618	-3956.30548
0.009	-2628.08101	-2093.42779	-2895.97515	-2625.55340	-3545.73239

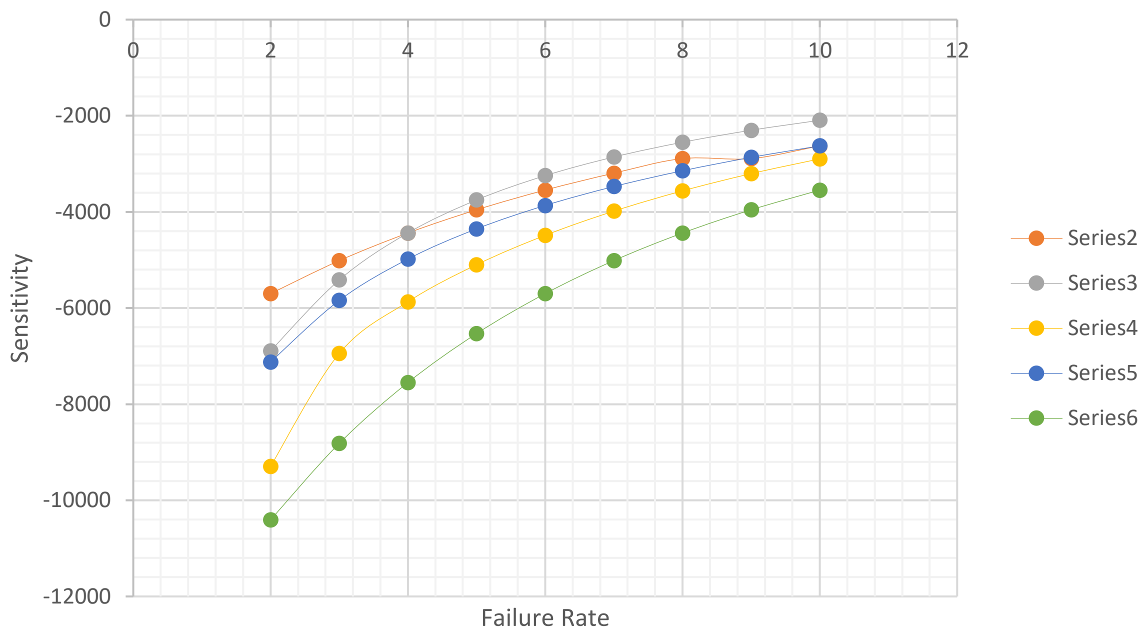

FIGURE 7
SENSITIVITY COUNTER FAILURE RATE

3.6 Analysis of MTTF for Minor, Medium and Major failure rate

Most failure of manufacturing and industrial systems can be minor, medium or major, each failure may tend to increase maintenance cost, because of this management work out modality in order to reduce the occurrence of each failure, so as to maximizes revenue mobilization. From equation (57) letting

$\xi_1 = 0.001, \xi_2 = 0.002, \xi_3 = 0.003, \xi_4 = 0.004, \xi_5 = 0.005$ and then computing the MTTF for all the failure rates aforementioned. The results were displayed in the table 6.

3.7 Analysis of Cost with Copula Repair

$$E_g(j) = B_1 \int_0^t A_{up}(j)dj - B_2 j$$

The cost analysis of a system is usually done to look at the financial consequences in terms of revenue and service cost over a specific time period. Because availability and cost have a close relationship, integrating availability with respect to time at a specific interval offers an expression for expected gain/profit while keeping other cost factors constant.

$$E_{g_{(Copula)}}(j) = B_1 \begin{Bmatrix} 0.000017e^{-1.00300j} + 0.000016e^{-1.00400j} \\ -0.001238e^{-2.72751j} + 0.00280e^{-1.02947j} \\ +0.000001e^{-1.01825j} - 16831.27521e^{-0.00005j} \\ +16831.27613 \end{Bmatrix} - B_2(j) \quad (59)$$

TABLE 6
COMPUTED MTTF FOR MINOR, MEDIUM, AND MAJOR FAILURE RATES

STATUS	Failure Rate	$MTTF\ \xi_1$	$MTTF\ \xi_2$	$MTTF\ \xi_3$	$MTTF\ \xi_4$	$MTTF\ \xi_5$
Minor failure	0.001	80.94196	87.04093	95.19091	97.41440	111.80693
Medium failure	0.005	62.94128	68.48289	70.69255	76.84195	80.94196
Major failure	0.009	51.26947	58.12492	56.27339	64.13267	62.94128

Fixing B_1 to one and allowing B_2 to be 0.1, 0.2..., 0.5 within a time interval, such as j = 0, 1...10 in equation (59). The predicted gain/profit of the system is calculated as shown in the table below.

TABLE 7
COMPUTED PROFIT WITH RESPECT TO TIME

j	$B_2 = 0.1$	$B_2 = 0.2$	$B_2 = 0.3$	$B_2 = 0.4$	$B_2 = 0.5$
0	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.89787	0.79787	0.69787	0.59787	0.49787
2	1.79474	1.59474	1.39474	1.19474	0.99474
3	2.69151	2.39151	2.09151	1.79151	1.49151
4	3.58825	3.18825	2.78825	2.38825	1.98825
5	4.48492	3.98492	3.48492	2.98492	2.48492
6	5.38155	4.78155	4.18155	3.58155	2.98155
7	6.27811	5.57811	4.87811	4.17811	3.47811
8	7.17461	6.37461	5.57461	4.77461	3.97461
9	8.07106	7.17106	6.27106	5.37106	4.47106
10	8.96745	7.96745	6.96745	5.96745	4.96745

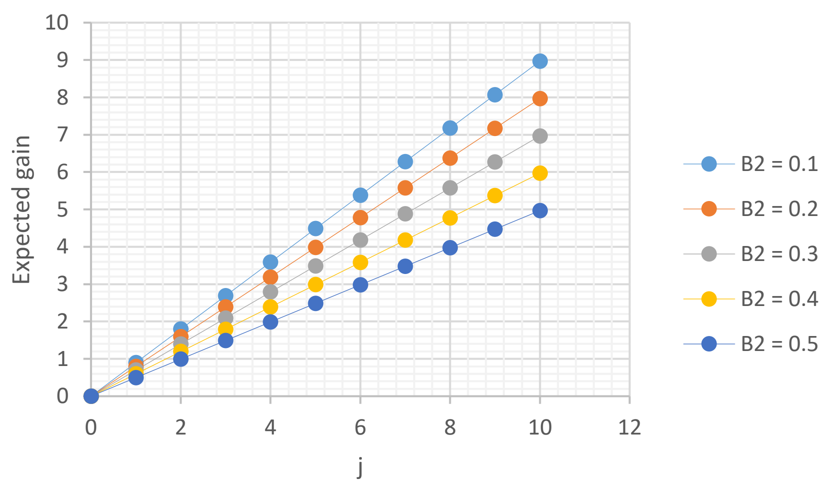

FIGURE 8 EXPECTED GAIN/PROFIT COUNTER J

3.8 Analysis of Cost with General Repair

$$E_g(j) = B_1 \int_0^t A_{up}(j)dj - B_2 j \qquad (60)$$

$$E_{g(General)}(j) = B_1 \begin{Bmatrix} -0.000025e^{-1.00300j} - 0.000075e^{-1.00400j} \\ -0.004475e^{-1.03387j} - 0.000038e^{-1.01829j} \\ -0.003915e^{-1.00476j} - 16831.26796e^{-0.00005j} \\ +16831.27649 \end{Bmatrix} - B_2(j) \qquad (61)$$

Fixing B_1 to one and allowing B_2 to be 0.1, 0.2..., 0.5 within a time interval, such as j = 0, 1...10 in equation (61). The predicted gain/profit of the system is calculated as shown in the table below.

TABLE 8
COMPUTED GAIN OVER TIME

j	$B_2 = 0.1$	$B_2 = 0.2$	$B_2 = 0.3$	$B_2 = 0.4$	$B_2 = 0.5$
0	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.89672	0.79672	0.69672	0.59672	0.49672
2	1.78990	1.58990	1.38990	1.18990	0.98990
3	2.68176	2.38176	2.08176	1.78176	1.48176
4	3.57312	3.17312	2.77312	2.37312	1.97312
5	4.46424	3.96424	3.46424	2.96424	2.46424
6	5.35525	4.75525	4.15525	3.55525	2.95525
7	6.24618	5.54618	4.84618	4.14618	3.44618
8	7.13705	6.33705	5.53705	4.73705	3.93705
9	8.02786	7.12786	6.22786	5.32786	4.42786
10	8.91860	7.91860	6.91860	5.91860	4.91860

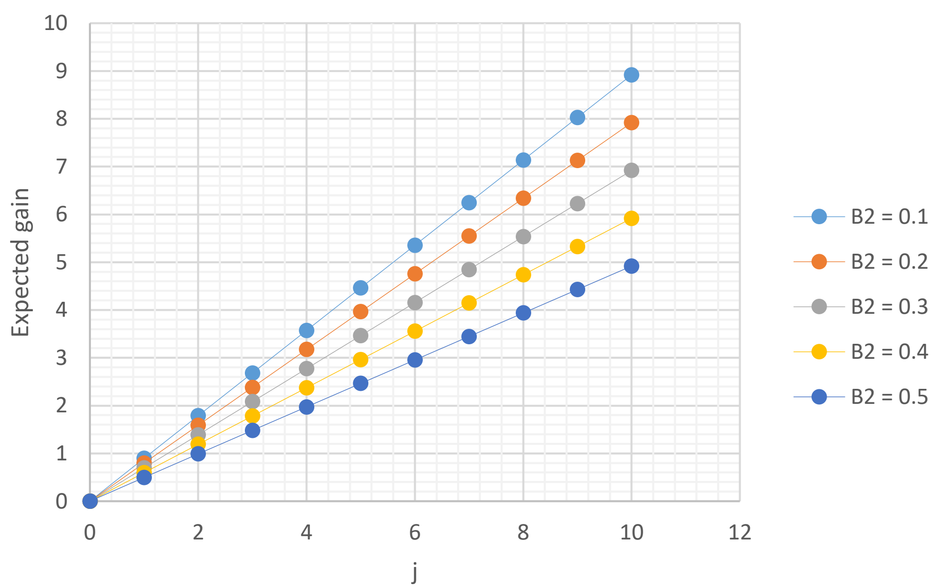

FIGURE 9
EXPECTED GAIN/PROFIT COUNTER J

3.9 Analysis of Cost with Copula Repair

$$E_g(j) = B_1 \int_0^t A_{up}(j)\, dj - B_2 j \quad (62)$$

$$E_{g(Copula)}(j) = B_1 \begin{Bmatrix} 0.000017e^{-1.00300j} + 0.000016e^{-1.00400j} \\ -0.001238e^{-2.72751j} + 0.00280e^{-1.02947j} \\ +0.000001e^{-1.01825j} - 16831.27521e^{-0.00005j} \\ +16831.27613 \end{Bmatrix} - B_2(j) \quad (63)$$

On the other hand, keeping B_2 at one, and assuming $B_1 = 2$, 3, …,6, within particular time interval as j = 0, 1, ...10 in equation (63). The system's expected gain/profit was obtained as captured in the Table below.

TABLE 9
CALCULATED PROFIT/GAIN OVER TIME, WHEN SERVICE COST REMAIN CONSTANT

j	$B_1 = 2$	$B_1 = 3$	$B_1 = 4$	$B_1 = 5$	$B_1 = 6$
0	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.99574	1.99361	2.99148	3.98935	4.98710
2	1.98945	3.98420	5.97893	7.97366	9.96840
3	2.98302	5.97453	8.96604	11.95755	14.94910
4	3.97649	7.96473	11.95298	15.94121	19.92940
5	4.96984	9.95476	14.93970	19.92462	24.90950
6	5.96309	11.94464	17.92618	23.90773	29.88930
7	6.95622	13.93432	20.91243	27.89054	34.86860
8	7.94923	15.92384	23.89846	31.87307	39.84770
9	8.94212	17.91318	26.88424	35.85530	44.82630
10	9.93489	19.90234	29.86979	39.83723	49.80470

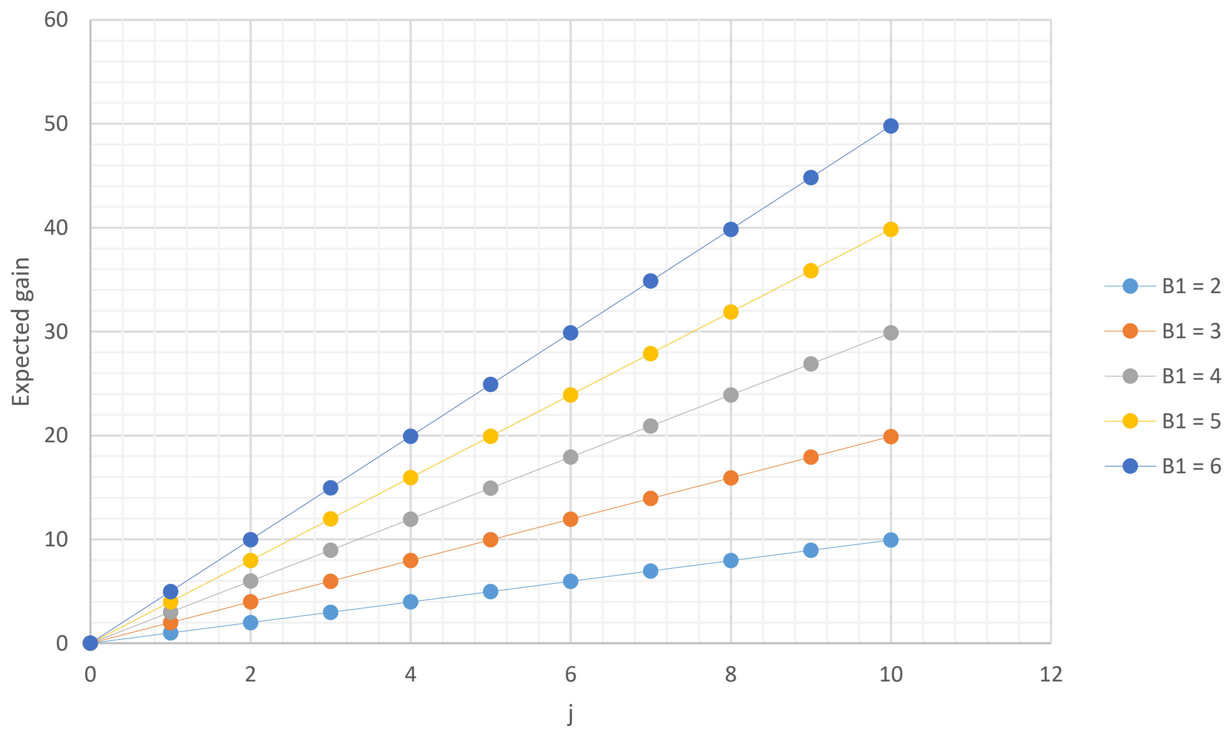

FIGURE 10
EXPECTED PROFIT/GAIN COUNTER J

3.10 Analysis of Cost with General Repair

$$E_g(j) = B_1 \int_0^t A_{up}(j) dj - B_2 j \quad (64)$$

$$E_{g(General)}(j) = B_1 \left\{ \begin{array}{l} -0.000025e^{-1.00300j} - 0.000075e^{-1.00400j} \\ -0.004475e^{-1.03387j} - 0.000038e^{-1.01829j} \\ -0.003915e^{-1.00476j} - 16831.26796e^{-0.00005j} \\ +16831.27649 \end{array} \right\} - B_2(j) \quad (65)$$

Likewise, keeping B_2 at one, and assuming $B_1 = 2, 3, \ldots, 6$, within particular time interval as j = 0, 1, ...10 in equation (65). The system's expected gain/profit is obtained as captured in the Table below.

TABLE 10
CALCULATED PROFIT/GAIN OVER TIME WHEN SERVICE COST REMAIN CONSTANT

j	$B_1 = 2$	$B_1 = 3$	$B_1 = 4$	$B_1 = 5$	$B_1 = 6$
0	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.99345	1.99017	2.98690	3.98362	4.98020
2	1.97980	3.96970	5.95960	7.94950	9.93930
3	2.96353	5.94529	8.92706	11.90882	14.89050
4	3.94623	7.91934	11.89245	15.86557	19.83860
5	4.92849	9.89272	14.85697	19.82121	24.78540
6	5.91051	11.86576	17.82101	23.77627	29.73150
7	6.89237	13.83856	20.78474	27.73093	34.67710
8	7.87411	15.81115	23.74820	31.68526	39.62220
9	8.85571	17.78357	26.71143	35.63928	44.56710
10	9.83720	19.75580	29.67440	39.59300	49.51150

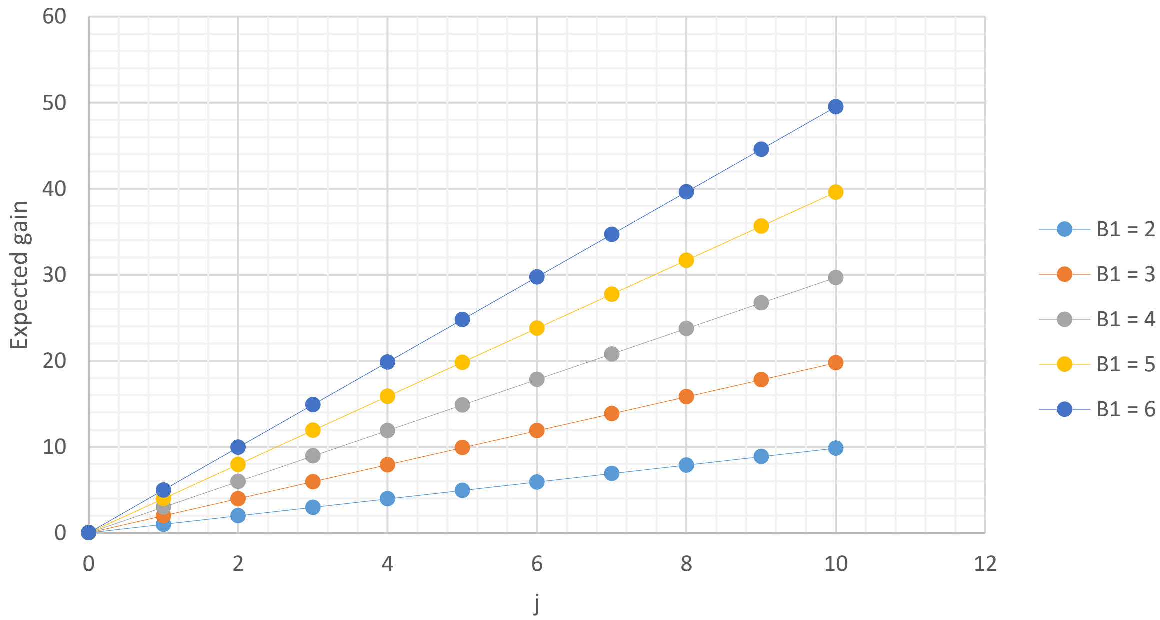

FIGURE 11
EXPECTED PROFIT/GAIN COUNTER J

4. Discussion and Conclusory Remarks

This paper considered sachet water plant consist of five subsystems arranged in series-parallel configuration. Expressions of dependability procedures of testing the sachet water plant's performance which include: availability, reliability, MTTF, sensitivity and cost were obtained using general and copula repair. The impact of repair and failure rate is captured in Table 1, 2, 7, 8, 9 and 10 which is also demonstrated by Figure 3, 4, 8, 9, 10 and 11, careful observations clearly support that copula repair produce more result on availability and cost over universal repair. Although, both repairs are good but the results obtained indicate that copula repair is better and efficient, and therefore recommended repair for the system's performance. However, this leads to maximizing revenue to the management. Table 5 shows the system's MTTF in terms of variation in failure rates while other are constant and this saying is also supported by Figure 6. The MTTF of the sachet water displays that continuous change of failure rate increasingly reduces the MTTF of the sachet water system. The fact that the system's MTTF reduces steadily with each failure rate, so to avoid this phenomenon, regular inspecting should be conducted. However, failure of manufacturing and industrial system may be minor, medium or major, the result obtained from Table 6 shows that $MTTF_{(min\,or)} > MTTF_{(madium)} > MTTF_{(major)}$. Table 6 shows that the strength of failure rates determined the performance of the corresponding sensitivity model, this result is also illustrated in Figure 7. From Table and Figure, it is evident that as failure rate of each subsystem increases so also the corresponding sensitivity. Table and Figure have shown that the strength of each failure rate determined the strength of the corresponding sensitivity. The influence of sensitivity on production; if a failure is severe enough, it will have an impact on both the production process and profit; as a result, management should endeavor to avoid each failure from occurring. Plant management, industrial production management, operational sustainability management, engineering management and dependability engineers, among others, will benefit from the study's findings.

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