Original Research

# Analysis of Mean Time to System Failure and Availability of a System with Cold Standby Unit

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**Abstract**
The reliability of a system with three components, A, B, and C, coupled in series and parallel is investigated in this study. System A is a two-out-of-two linear consecutive system, System B is a one-out-of-one linear consecutive system, and System C is a two-out-of-four linear consecutive system. The failure of system B, or the failure of all system A and C components, can cause the entire system to fail. The system was evaluated using the Markov birth–death process, which resulted in clear expressions for availability and mean time to system breakdown. The effect of failure and repair rates on mean time to failure and availability has been graphically explored based on numerical values presented in a table and graph and assigned to system characteristics to demonstrate the effect of failure and repair rates on mean time to failure and availability, The results reveal that system effectiveness indicators like availability and mean time to system failure rise with repair rates and fall with failure rates.

**Keywords** – Accessibility; Availability; Repair Rate; Failure Rate

## 1. Introduction

In power plants, backup systems, and engineering systems, system dependability is becoming increasingly critical. The systems must often maintain a high or needed level of reliability. In reliability analysis, the investigation of repairable systems is critical. In addition, repairmen are an important component of repairable systems and can have a direct or indirect impact on the system's economy. Reliability engineering's main goal is to boost a system's performance. The allocation of redundancy during the initial design phase is a direct way to improve any system's reliability. Active and standby redundancy are the two types of redundancy strategies. Active redundancy occurs when all redundant systems function simultaneously from time zero, even though the system only requires one at any one time. Standby redundancy, on the other hand, comes in three flavors: cold, warm, and hot. The component in the cold standby redundancy is more prone to failure before operation than the components in the cold standby redundancy. The failure pattern of a component in hot standby redundancy is independent of whether the component is active or not. Hot standby and active standby arrangements use the same mathematical models.

Diverse researchers have contributed to this field's research. Nakagawa and Osaki (1975) and Okumato (1997) investigated the behavior of a two-unit redundant system, assuming that when the operating unit fails, the system enters into repair mode. Gopalan and Naidu (1981) looked at the stochastic behavior of a two-unit repairable system under various inspection procedures, whereas Singh and Srinivasu (1987) looked at a two-unit cold standby system using the idea of repair preparation time. The three-unit subsystem was studied by Gupta and Bansal in 1990. Gupta and Sharma (1993) looked at a two-unit standby system that had two

different sorts of repairs. A standby system with a waiting for repair strategy was investigated by Ram et al. (2013). Muhammad (2007) employed fuzzy theory to add uncertainty into the alpha factor model, and a linear programming model to determine the mean time between failures of the system (MTBF).
In conducting out analysis in evaluating the reliability measures, Monika and Mangey (2013) used extra variables approaches and Laplace transformation. Ibrahim and Bashir (2014) created a probabilistic model for two dissimilar redundant systems with replacement for each common cause failure, analyzing and comparing some reliability characteristics for different parameter values. Uba et al. (2013) used the Kolmogorov forward equation approach to evaluate the reliability and availability of a two dissimilar unit cold standby system with three modes. Pravindra pankaj and Anil (2016) investigated the probabilistic analysis of a two-unit warm standby system with subject to hardware and human error failures . Pourhassan et al. (2020) presented simulation approach to reliability assessment of complex system under stochastic degradation and random shock. Raissi and Ebadi (2018) dealt with computer simulation model for reliability estimation of a complex system. Pourhassan (2021) analyzed the reliability of power station subject fatal and nonfatal shocks. Pourhassan et al. (2019) investigated the impact of fault in component reliability estimation on system designing. Attar et al. (2016) developed a simulation-based optimization model for free distributed repairable multi-state availability-redundancy allocation problems.
Reliability, availability, mean time to failure, and cost analysis are all standard system reliability measurements. Are the techniques for probabilistic risk assessment in system design, operation, and maintenance effective and efficient? The purpose of this article is to examine the reliability of a two-cold standby system using model formulation, availability analysis, reliability analysis, and mean time to failure.

## 2. Material and Methods

### *2.1 Description of the Model and Its Premises*

The paper considered a repairable system which consists of four sub systems namely A, B, and C in series Parallel.
System B is a single unit, failure of it causes the complete failure of the system. Systems A consists of two units, one unit of subsystems A is active and other one unit in cold standby mode. Complete failure of the system will occur due to Subsystem A when one active unit and one standby unit of Subsystem A failed at a time, Subsystem C Consist of four units . The system can be repaired in both cases. For the failures, the repairs are done absolutely, so after the repair every system is as good as new. Failure and repair rates are assumed to be exponentially distributed.
Explicit availability and Mean Time to System Failure expressions are developed.

### *2.2. Notation*

t: Time scale.
$\beta_i$: Failure rates of the system where i=1, 2, 3
$\gamma_j$: Repair rates of the system where j=1, 2, 3
$s_0$: All Three units are in good working condition.,
$s_1$: Subsystem C one unit failed and one standby unit is working.
$s_2$: Subsystem C two units failed and two standby units are working.
$s_3$: Subsystem A one unit failed and other one unit is at ideal state therefore system failed.
$s_4$: Subsystem A all the two units are at ideal state, subsystem B failed ,subsystem C all the two units are at ideal state the system failed.
$s_5$: Subsystem A one unit failed and other one unit is at ideal state, subsystem B is at ideal state, subsystem C one unit failed and other unit is at ideal state therefore the system failed .
$s_6$: Subsystem A all the two units are at ideal state, subsystem B failed ,subsystem C all the two units are at ideal state the system failed state.
$s_7$: Subsystem A one unit is failed and other unit is at ideal state, subsystem B is at ideal state subsystem C all the two units failed and standby units are at ideal state the system failed.
$s_8$: Subsystem A all the two units are at ideal state subsystem B failed, subsystem C all the two units failed and standby units are at ideal state the system failed
$s_9$: Subsystem A all the two units are at ideal state subsystem B is at ideal state subsystem C failed and one standby unit failed and other unit is at ideal state the system failed.
$P_0, P_1, P_2$ Transition state probability of the state $S_0, S_1, S_2$: when all the system A, B, and C are in good condition.
$P_3, P_4, P_5, P_6, P_7, P_8, P_9$: Tran sition state probabiliies of the state
$S_3 S_{4,} S_{5,} S_{6,} S_7, S_8, S_9$: When system A, B , and C are in failed condition respectively.
The following assumptions are associated with the Model:
Initially the system is in good state, the System has two states, working and Failed states, the System has completely failed after the failure of system B, and failure of the unit of system of A and C , all failure and repair rates are constant, the System can be repaired when it is in complete failed mode, the repaired system works like a new one.

*2.3. State Transition and block diagrams of the model*

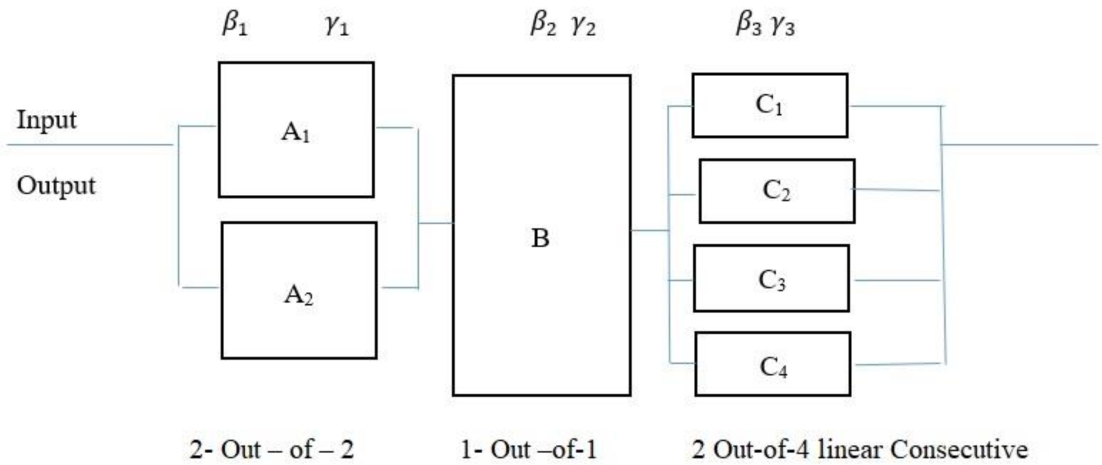


**Figure.1:** Block Diagram of the System

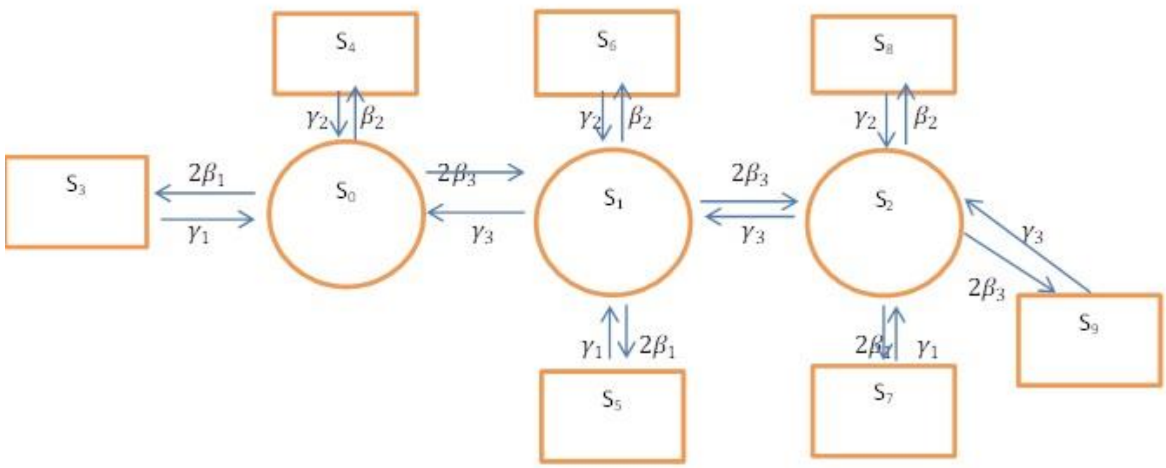


**Figure.2:** Schematic Diagram of the System

*2.4 Formulation and solution of the mathematical model*

We can derive the following set of differential equations regulating the current mathematical model from the likelihood of the considerations and continuity arguments.

$\frac{dp_0}{dt} = -(3\beta_3 + 2\beta_1 + \beta_2)P_0(t) + \gamma_3 P_1(t) + \gamma_1 P_3(t) + \gamma_2 P_4(t),$

$\frac{dp_1}{dt} = -(2\beta_1 + \beta_2 + 2\beta_3 + \gamma_3)P_1(t) + 2\beta_3 P_0(t) + \gamma_3 P_2(t) + \gamma_1 P_5(t) + \gamma_2 P_6(t),$

$$\frac{dp_2}{dt} = -(2\beta_1 + \beta_2 + 2\beta_3 + \gamma_3)P_2(t) + 2\beta_3 P_1(t) + \gamma_1 P_7(t) + \gamma_2 P_8(t) + \gamma_3 P_9(t),$$

$\frac{dp_3}{dt} = -\gamma_1 P_3(t) + 2\beta_1 P_0(t),$

$\frac{dP_4}{dt} = -\gamma_2 P_4(t) + \beta_2 P_0(t),$

$\frac{dP_5}{dt} = -\gamma_1 P_5(t) + 2\beta_1 P_1(t),$

$\frac{dP_6}{dt} = -\gamma_2 P_6(t) + \beta_2 P_1(t),$

$\frac{dP_7}{dt} = -\gamma_1 P_7(t) + 2\beta_1 P_2(t),$

$\frac{dP_8}{dt} = -\gamma_2 P_8(t) + \beta_2 P_2(t)$ ,
$\frac{dP_9}{dt} = -\gamma_3 P_9(t) + \beta_3 P_2(t).$ (1)

*2.3 Steady state availability Analysis for System*

In the availability instance depicted in Fig. The basic conditions for this system, according to El-said and El-Hamid (2006), are as follows:

P(0)=$[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0), P_8(0), P_9(0)]$

= [0,0,0,0,0,0,0,0,0,1] (2)

The differential equations in (1) above is transformed into matrix as $\dot{P} = AP(\infty)$ (3)

For the system above, the system of differential equations in (1) can be written in matrix form as:

$$\begin{bmatrix} \dot{P_0} \\ \dot{P_1} \\ \dot{P_2} \\ \dot{P_3} \\ \dot{P_4} \\ \dot{P_5} \\ \dot{P_6} \\ \dot{P_7} \\ \dot{P_8} \\ \dot{P_9} \end{bmatrix} = \begin{bmatrix} -K_0 & \gamma_3 & 0 & \gamma_1 & \gamma_2 & 0 & 0 & 0 & 0 & 0 \\ 2\beta_3 & -K_1 & \gamma_3 & 0 & 0 & \gamma_1 & \gamma_2 & 0 & 0 & 0 \\ 0 & 2\beta_3 & -K_2 & 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 \\ 2\beta_1 & 0 & 0 & -\gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & 0 & 0 & -\gamma_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_1 & 0 & 0 & 0 & -\gamma_1 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & 0 & -\gamma_2 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_1 & 0 & 0 & 0 & 0 & -\gamma_1 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\gamma_2 & 0 \\ 0 & 0 & 2\beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma_3 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \end{bmatrix}$$

$K_0 = (2\beta_1 + \beta_2 + 2\beta_3)$,$K_1 = (2\beta_1 + \beta_2 + 2\beta_3 + \gamma_3)$, $K_2 = (2\beta_1 + \beta_2 + 2\beta_3 + \gamma_3)$

The solutions for P i (t)= 0,1,2, ..., 9 can be used to calculate the system availability. 1,2,3 are the values of i. The system's functioning states are zero, one, and two. The steady-state availability is given by El-Said (2008), Haggag (2009), and Wang et al (2006):

$AP(\infty) = 0$ (5)

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -K_0 & \gamma_3 & 0 & \gamma_1 & \gamma_2 & 0 & 0 & 0 & 0 & 0 \\ 2\beta_3 & -K_1 & \gamma_3 & 0 & 0 & \gamma_1 & \gamma_2 & 0 & 0 & 0 \\ 0 & 2\beta_3 & -K_2 & 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 \\ 2\beta_1 & 0 & 0 & -\gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & 0 & 0 & -\gamma_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_1 & 0 & 0 & 0 & -\gamma_1 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & 0 & -\gamma_2 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_1 & 0 & 0 & 0 & 0 & -\gamma_1 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\gamma_2 & 0 \\ 0 & 0 & 2\beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma_3 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \end{bmatrix}$$

$K_0 = (2\beta_1 + \beta_2 + 2\beta_3)$,$K_1 = (2\beta_1 + \beta_2 + 2\beta_3 + \gamma_3)$, $K_2 = (2\beta_1 + \beta_2 + 2\beta_3 + \gamma_3)$

using the normalizing condition

$P_0(0) + P_1(0) + P_2(0) + P_3(0) + P_4(0) + P_5(0) + P_6(0) + P_7(0) + P_8(0) + P_9(0) = 1$ (7)

AV$(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty)$ (4)

The derivatives of the state probabilities become 0 in the steady state, hence the left hand side of (4) is set to zero.

we substitute (7) in the last row of (5) following [5,6,7]. The resulting matrix is

$$\begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\1 \end{bmatrix} = \begin{bmatrix} -K_0 & \gamma_3 & 0 & \gamma_1 & \gamma_2 & 0 & 0 & 0 & 0 & 0 \\ 2\beta_3 & -K_1 & \gamma_3 & 0 & 0 & \gamma_1 & \gamma_2 & 0 & 0 & 0 \\ 0 & 2\beta_3 & -K_2 & 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 \\ 2\beta_1 & 0 & 0 & -\gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & 0 & 0 & -\gamma_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_1 & 0 & 0 & 0 & -\gamma_1 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & 0 & -\gamma_2 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_1 & 0 & 0 & 0 & 0 & -\gamma_1 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\gamma_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty)\\P_1(\infty)\\P_2(\infty)\\P_3(\infty)\\P_4(\infty)\\P_5(\infty)\\P_6(\infty)\\P_7(\infty)\\P_8(\infty)\\P_9(\infty) \end{bmatrix}$$

To get the state probabilities, we need to solve the set of linear equations above.

AV(∞) thus is

$AV(\infty)=\frac{D_1}{N_1}$,

Where

$D_1 = \gamma_1\gamma_2\gamma_3^3 + 2\beta_3\gamma_1\gamma_2\gamma_3^2 + 4\beta_3^2\gamma_1\gamma_2\gamma_3$,

$N_1 = 2\beta_1\gamma_2\gamma_3^3 + \beta_2\gamma_1\gamma_3^3 + 8\beta_3^3\gamma_1\gamma_2 + \gamma_1\gamma_2\gamma_3^3 + 4\beta_1\beta_3\gamma_2\gamma_3^2 + 8\beta_1\beta_3^2\gamma_2\gamma_3 + 2\beta_1\beta_3\gamma_1\gamma_3^2 + 4\beta_2\beta_3^2\gamma_1\gamma_3 + 2\beta_3\gamma_1\gamma_2\gamma_3^2 + 4\beta_3^2\gamma_1\gamma_2\gamma_3$ .

*Mean Time System Failure for the System*

MT

$$P(0)(-Q_1^{-1})\begin{bmatrix}1\\1\\1\end{bmatrix} \quad (8)$$

Which in matrix form is:

$$\text{MTSF}=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix} -K_0 & 2\beta_3 & 0 \\ \gamma_3 & -K_1 & 2\beta_3 \\ 0 & \gamma_3 & -K_2 \end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$\text{MTSF}=\frac{D_2}{N_1} = where\ D_2 = 2\beta_1\gamma_2\gamma_3^3 + 4\beta_1\beta_3\gamma_2\gamma_3^2 + 2\beta_2\beta_3\gamma_1\gamma_3^2 + 8\beta_1\beta_3^2\gamma_2\gamma_3 + 4\beta_2\beta_3^2\gamma_1\gamma_3 + 8\beta_1^3\gamma_1\gamma_2,\ N_1 = 2\beta_1\gamma_2\gamma_3^3 + \beta_2\gamma_1\gamma_3^3 + 8\beta_3^3\gamma_1\gamma_2 + \gamma_1\gamma_2\gamma_3^3 + 4\beta_1\beta_3\gamma_2\gamma_3^2 + 8\beta_1\beta_3^2\gamma_2\gamma_3 + 2\beta_1\beta_3\gamma_1\gamma_3^2 + 4\beta_2\beta_3^2\gamma_1\gamma_3 + 2\beta_3\gamma_1\gamma_2\gamma_3^2 + 4\beta_3^2\gamma_1\gamma_2\gamma_3$

## 3. RESULTS AND DISCUSSION

It is obvious from Fig. 1 and Fig. 3 that System Availability and MTSF decline as the number of users increases $\beta_i$ While the System Availability and MTSF increase as the MTSF increases, as seen in Figs. 2 and 4, the System Availability and MTSF decrease when the MTSF decreases. $\gamma_j$ decreases.

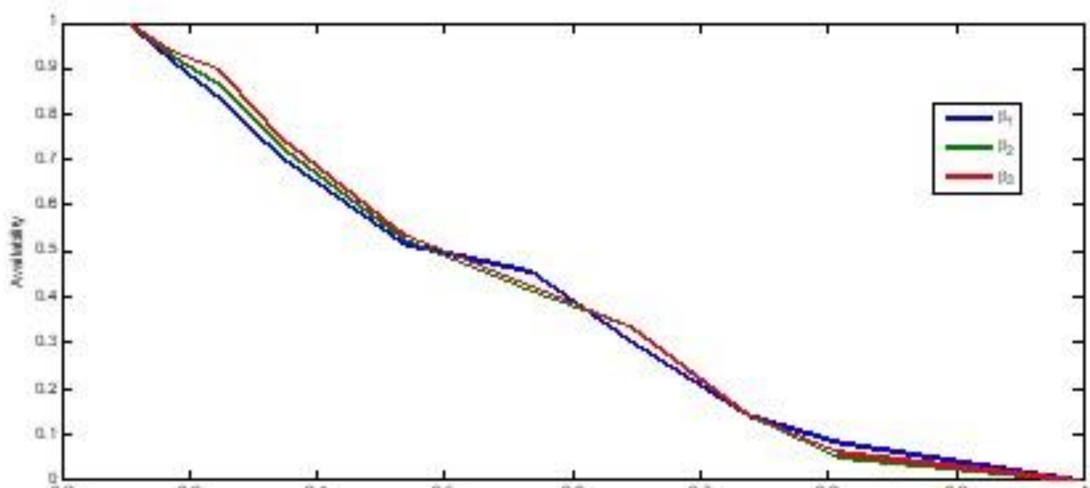


Figure 3: Availability against failure rates

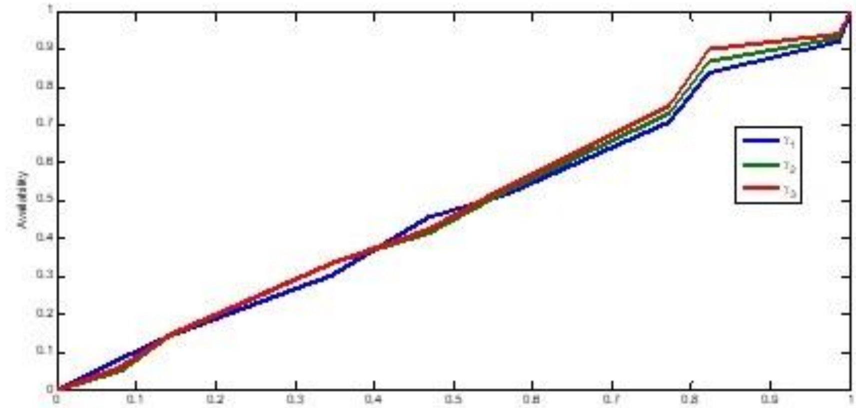


Figure 4: Availability against Repair Rates

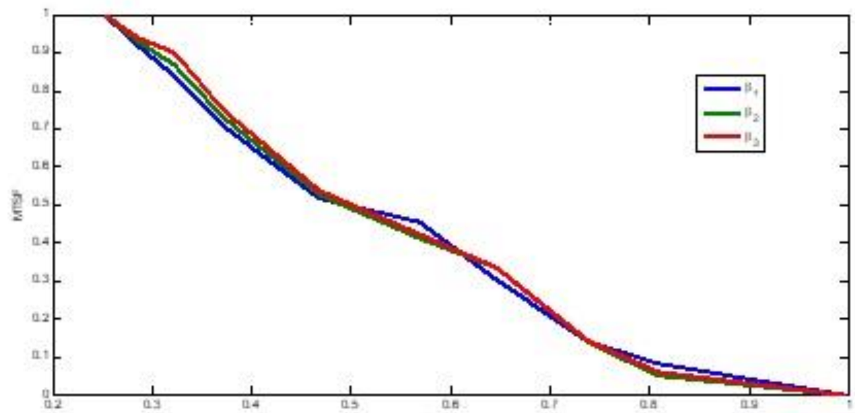


Figure 5: Availability against Failure Rates

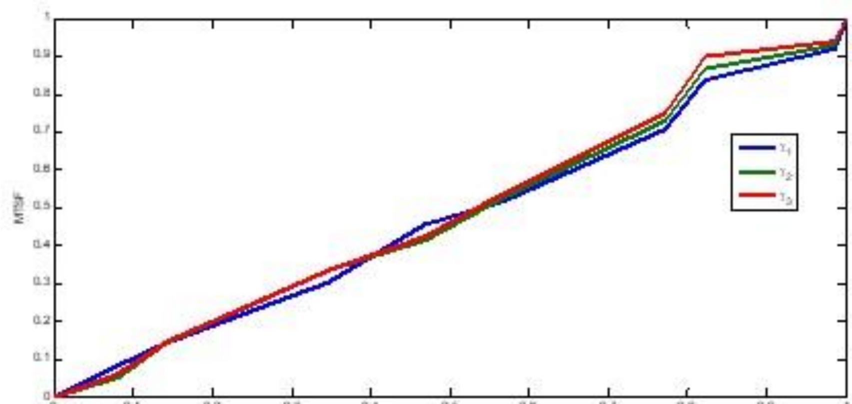


Figure 6: Availability against Repair Rates

## 4. CONCLUSION

For the considered system, this study generated precise terms for metrics of system effectiveness such as Mean time to system breakdown and availability analysis. Important findings were highlighted via graphs. The results reveal that system effectiveness indicators like availability and mean time to system failure rise with repair rates and fall with failure rates.

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