Original Research

Developing two 4-parameter and 5-parameter exponential smoothing methods with multiplicative trend for demand forecasting

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Abstract

Exponential smoothing methods, especially Holt-Winters family, have been extensively utilized to demand time series forecasting. Previous studies show that Extended Holt Winters methods, by adding a smoothing constant to the level equation of Holt Winters methods, can improve the forecasting performance significantly. The improvements gained by the Extended Holt Winters method with additive trend motivated this research to extend this idea to the Holt Winters method with multiplicative trends too.

In this paper, adding a smoothing constant to the Holt-Winters with multiplicative trend and also Holt-Winters with damped multiplicative trend was investigated, and the performance of these methods was compared with the classical methods. Quarterly and monthly time series of M3-Competition with the minimum length of ten years were used to measure the performance of proposed methods.

The results showed that the proposed methods, significantly outperformed the classical Holt Winters multiplicative trend methods.

Keywords - Demand forecasting; Exponential smoothing methods; Holt-Winters methods; Damped trend methods; Multiplicative Trend; M3-Competition; Symmetric relative efficiency measure

1. Introduction

Forecasting is extremely important issue for all executive decisions, from inventory management and scheduling to planning and strategic management (Petropoulos et al., 2014, Omidi et al., 2019). Exponential smoothing methods due to their robustness and accuracy of forecasting are among the most extensively used methods especially when there are large number of series (Taylor, 2003, Monfared et al., 2014).

Since their first introduction, in late 1960s when simple exponential smoothing method was introduced by Brown (1959), these methods have been significantly developed and evolved. Holt introduced trend corrected methods (Holt, 1957) which gained more accuracy when seasonality component was added to it by Winters (1960). This method is now known as Holt-Winters, in which the trend and seasonality components are considered in an additive form. Pegels (1969) presented a classification of Holt-Winters methods and presented the multiplicative trends methods.

Later on, a damping factor was applied to the additive trend Holt-Winters method by Gardner & McKenzie (1985) and Damped-trend Holt-Winters methods were introduced. Taylor (2003) introduced the damped multiplicative-trend Holt-Winters methods. Trater et al (2016) added a smoothing constant to the level equation and the EHW (Extended Holt-Winters) method was introduced that outperformed the classical Holt-Winters additive trend method.

The performance improvements provided by the EHW method motivated this study to extend the idea of EHW method to Holt-Winters method with multiplicative trends too.

Table 1 shows the nomenclature used to define different variants of HW method. Each method's name consists of three parts.

Table 1 – Nomenclature used in this paper for the name of the methods.

1. Method		2. Trend Type		3. Seasonality Type	
Abb.	Complete form	Abb.	Complete form	Abb.	Complete form
HW	Holt-Winters	-MT-	Multiplicative Trend	-MS	Multiplicative Seasonality
DHW	Damped HW	-AT-	Additive Trend	-AS	Additive Seasonality
EHW	Extended HW				
XHW	Extended HW in this paper				
XDHW	Extended DHW in this paper				

The purpose of this study is to extend the idea of EHW-AT-AS method proposed by Trater et al (2016) that considers the additive trend to the HW-MT-AS and DHW-MT-AS methods which have multiplicative trend components. The resulting methods are called XHW-MT-AS and XDHW-MT-AS. For brevity, we refer to these methods as EHW, XHW & XDHW instead of EHW-AT-AS, XHW-MT-AS, XDHW-MT-AS.

In section 2, the classical HW methods and their formulation are reviewed, and then the XHW and XDHW methods are introduced. Section 3 explains the research methodology and section 4 presents the results comparing the performance of the proposed methods with their base classical methods. Discussion of the results is presented in section 5.

2. HW METHODS

In this section, different variants of the Holt-Winters method in terms of trend components, are reviewed. Table 2 defines the terms used in formulas.

TABLE 2 – DESCRIPTIONS OF TERMS USED IN DIFFERENT METHODS.

Notation	Description
Y_t	Observed values in period t
L_t	Estimate of level at the end of period t
b_t	Estimate of trend at the end of period t
S_t	Estimate of seasonal factor for period t
F_{t+h}	Forecasted value for period t+h
h	Index for forecast horizon. h=1, 2, 3 or 4
s	Index for season s=1, 2, 3 or 4
α, β, γ	HW smoothing constants for level, trend and seasonal components
ϕ	Trend damping parameter in damped trend methods
δ	Additional smoothing constant in EHW, XHW, XDHW methods
E_t	Forecast error at in period t

2.1. HW-MT-AS

Smoothing methods use historical data or previous observations to forecast the variable of interest. Each observed data point can be broken down into two parts: systematic and random. The systematic part consists of three components which are level, trend and seasonality. Level is the expected value of the variable of interest, trend demonstrates the rate of growth or decline and seasonality is the predictable seasonal fluctuation (Roehrich, 2008). Exponential smoothing methods give more weights to recent data and exponentially decaying weights to past data (Makridakis et al., 1997).
The HW method is used to forecast the systematic component, when trend and seasonality are present. In each period after observing the new data, the following equations (1-4) are used to update level, trend, seasonality and forecast the future periods, respectively.

$$L_t = \alpha(Y_t - S_{t-s}) + (1-\alpha)(L_{t-1} + b_{t-1}) \quad (1)$$

$$b_t = \beta\left(L_t \big/ L_{t-1}\right) + (1-\beta)b_{t.1} \quad (2)$$

$$S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-s} \quad (3)$$

$$F_{t+h} = L_t.b_t^h + S_{t+h-s} \quad (4)$$
$$0 \le \alpha, \beta, \gamma \le 1$$

As it can be seen in the forecasting equation (4), the trend is multiplied by level and the seasonal component is added to the result.

2.2. DHW-MT-AS

The DHW methods have an additional parameter (φ) that is used to dampen the trend component. The damped Holt-Winters with multiplicative trend, uses the following equations (5-8) to update level, trend, seasonality and forecast the future periods.

$$L_t = \alpha(Y_t - S_{t-s}) + (1-\alpha)(L_{t-1}.b_{t-1}{}^{\varphi}) \quad (5)$$

$$b_t = \beta\left(L_t \big/ L_{t-1}\right) + (1-\beta).b_{t.1}{}^{\varphi} \quad (6)$$

$$S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-s} \quad (7)$$

$$F_{t+h} = L_t.b_t^{\varphi_h} + S_{t+h-s} \quad (8)$$
$$0 \le \alpha, \beta, \gamma, \varphi \le 1$$
$$\varphi_h = \varphi + \varphi^2 + ... + \varphi^h$$

2.3. EHW-AT-AS

The idea behind the original EHW method is adding a smoothing parameter (δ) to the level equation in order to separate the S_{t-s} from being multiplied by α. In standard HW method, the term (Y_t-S_{t-s}) is multiplied by α to deseasonalise the observed values. Trater's idea is to control the amount of S_{t-s} in the level equation by using the δ parameter (Trater et al., 2016). δ also is a constant between 0 and 1. Equations (9-12) are used to update level, trend, seasonality and forecast the future periods.

$$L_t = \alpha Y_t - \delta S_{t-s} + (1-\alpha)(L_{t-1} + b_{t-1}) \quad (9)$$
$$b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t.1} \quad (10)$$
$$S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-s} \quad (11)$$
$$F_{t+m} = L_t + m.b_t + S_{t+m-s}$$
$$0 \le \alpha, \beta, \gamma, \delta \le 1 \quad (12)$$

The EHW method is proposed in additive form of trend and seasonality (-AT-AS). In sections 2.4 and 2.5, we have applied this idea to HW-MT-AS and DHW-MT-AS methods.

2.4. XHW-MT-AS

As mentioned in previous section in this approach, two smoothing constants, α and δ, are used in the level equation. This method considers the trend in

$$L_t = \alpha Y_t - \delta S_{t-s} + (1-\alpha)(L_{t-1} + b_{t-1}) \quad (13)$$
$$b_t = \beta({L_t}/{L_{t-1}}) + (1-\beta)b_{t.1} \quad (14)$$
$$S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-s} \quad (15)$$
$$F_{t+h} = L_t.b_t^h + S_{t+h-s} \quad (16)$$
$$0 \le \alpha, \beta, \gamma, \delta \le 1$$

multiplicative form while EHW considers it in additive form. The updating equations (13-16) are as follows.

2.5. XDHW-MT-AS

The Damped methods outperform other methods in most cases, and as shown by Fildes (2001) and Fildes et al (2008), Damped methods can be considered as benchmark methods among the family of HW forecasting methods.

Therefore, in this study, the EHW idea was further investigated to check if it could improve the DHW-MT-AS method. So the smoothing constant (δ) was applied in the level equation of DHW-MT-AS method and XDHW-MT-AS method is introduced

$$L_t = \alpha Y_t - \delta S_{t-s} + (1-\alpha)(L_{t-1}.b_{t-1}{}^{\varphi}) \quad (17)$$
$$b_t = \beta({L_t}/{L_{t-1}}) + (1-\beta).b_{t.1}{}^{\varphi} \quad (18)$$
$$S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-s} \quad (19)$$
$$F_{t+h} = L_t.b_t^{\varphi_h} + S_{t+h-s} \quad (20)$$

as below.

$$0 \le \alpha, \beta, \gamma, \varphi, \delta \le 1$$

$$\varphi_m = \varphi + \varphi^2 + \ldots + \varphi^m$$

3. Data & Methodology

Quarterly and monthly time series of M-3 Competition were used in our simulations with two programming restrictions:

1. We always started the simulation process on input series from their the 1st quarter or month. For example, if a quarterly series started on it's 3rd quarter, we ignored the two initial values (the 3rd and the 4th quarter values) and started the simulation from the third value which is the 1st quarter value of the next period.
2. We needed to consider a fixed length for all of the series. In order to include as many as possible, only the series with at least 10 years of data which equals having 40 values in quarterly and 120 values in monthly series were considered. The data after the tenth year was truncated and not used. The tenth year of data is forecasted, therefore the actual values of the tenth year are only used to calculate errors.

Totally, 665 out of 756 quarterly series and 1017 out of 1428 monthly series of M3 Competition data set span ten years. Applying the above restrictions left us with 77% of the total series for the analyses. The series used for simulation lie in five categories shown in the table 3. No preprocessing is performed on the data.

TABLE 3 – NUMBER OF QUARTERLY AND MONTHLY TIME SERIES USED IN THIS STUDY.

Category	Quarterly	Monthly
Micro	203	197
Industry	75	332
Macro	279	284
Finance	53	114
Demographic	55	90
Total	**665**	**1017**

To compare the performance of XHW-MT-AS versus HW-MT-AS and also XDHW-MT-AS versus DHW-MT-AS, we implemented all of the four methods in Microsoft Excel 2016 using the Solver add-in and Microsoft Visual Basic editor.
Simulation of each forecasting method consists of three steps: 1. Initialization, 2. Optimization 3. Forecasting. All input time series have 10 years of quarterly or monthly data. We used the first two years in Initialization step. Also, the first 9 years are used in optimization step to find the best smoothing constant values. The objective function was to minimize mean squared error (MSE) or mean absolute error (MAE) as defined in equations (25) and (26).
In the Forecasting step, the tenth year of data which includes 4 values for quarterly and 12 values for monthly time series were forecasted. The real observed values of the tenth year of series were used for model validation by calculating the forecast errors, i.e. (MAE), or mean absolute percentage error (MAPE) defined by in equations (26) and (30).

3.1. INITIALIZATION

All HW methods' components need to start at their initial values which are (L_0, b_0, $S_{1\ldots4}$). Various heuristic methods have been proposed to determine suitable initial values, such as optimizing the initial values along with the smoothing parameters (Rasmussen, 2004). In this study, the following method is adopted to calculate the initial values of level, trend and seasonal components.

Initial Level: $$L_0=\frac{1}{4}\sum_{1}^{4} Y_t \tag{21}$$

Initial Trend: $$B_0=\sqrt[4]{\sqrt[4]{Y_5/Y_1}+\sqrt[4]{Y_6/Y_2}+\sqrt[4]{Y_7/Y_3}+\sqrt[4]{Y_8/Y_4}} \tag{22}$$

Initial Seasonal: $$S_1=Y_1\text{-}L_0,\ S_2=Y_2\text{-}L_0,\ S_3=Y_3\text{-}L_0,\ S_3=Y_3\text{-}L_0 \tag{23}$$

The initial value for all smoothing constants at the beginning of simulation was set as 0.2.

3.2. OPTIMIZATION

In this step, a constrained optimization problem was solved to find the optimal values of the smoothing constants. As mentioned before, the objective function is to minimize MSE or MAE. The smoothing constants should be bounded to the interval of 0 to 1 which forms the constraints of the problem. Microsoft Excel Solver was used to solve the optimization problem using the GRG nonlinear method.

In most studies, the objective function is minimizing MSE or MAE. To check the accuracy in both aspects, we used both of them, one at a time. The MSE and MAE objective functions are computed over the 9-years span of data using the following formulas.

$$E_t = F_t - Y_t \tag{24}$$

$$MSE_n = \frac{1}{n}\sum_{t=1}^{n} E_t^2 \tag{25}$$

$$MAE_n = \frac{1}{n}\sum_{t=1}^{n} |E_t| \tag{26}$$

3.3. Forecasting

In this step the all variants of HW methods were used to forecast the tenth year (4 quarterly values or 12 monthly values) of the input series. All of the above steps are executed using Microsoft Excel Visual Basic Code. The code repeats all steps for each series in each method.

3.4. Accuracy analysis

3.4.1 Optimization Accuracy

After calculating the MSE(MAE) values in the optimization step, we used the symmetric relative efficiency measure (SREM) defined below to effectively compare MSEs(MAEs) of an extended method against the MSEs(MAEs) of the classic methods.

MSE values are substituted by MAE values for methods with MAE objective function.

The SREM is a measure to compare the extended method's performance with its classic counterpart. SREM is defined as 27 and 28.

$$SREM_{XHW/HW}\ (MSE) = \begin{cases} \left(1 - \dfrac{MSE_{XHW}}{MSE_{HW}}\right) \times 100 & ; \quad MSE_{XHW} < MSE_{HW} \\ \left(\dfrac{MSE_{HW}}{MSE_{XHW}} - 1\right) \times 100 & ; \quad MSE_{XHW} \geq MSE_{HW} \end{cases} \tag{27}$$

$$SREM_{XDHW/DHW}\ (MSE) = \begin{cases} \left(1 - \dfrac{MSE_{XDHW}}{MSE_{DHW}}\right) \times 100 & ; \quad MSE_{XDHW} < MSE_{DHW} \\ \left(\dfrac{MSE_{DHW}}{MSE_{XDHW}} - 1\right) \times 100 & ; \quad MSE_{XDHW} \geq MSE_{DHW} \end{cases} \tag{28}$$

3.4.2 Forecasting Accuracy

Absolute Percentage Error (APE) is one of the most common error measures used for evaluating the accuracy of forecasts. APE and Mean Absolute

$$APE(\%) = \frac{E_t}{Y_t} \times 100$$

$$MAPE_n(\%) = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{E_t}{Y_t} \right| \times 100$$

Percentage Error (MAPE) reflect the accuracy as a ratio defined by the following formulas.

To compare the forecasting accuracy in the tenth year for standard and extended methods, we calculated MAPE of the tenth year for each series and compared the performance of extended vs classic methods using the SREM.

Also in each extended method the median APEs of each forecasting horizon (h) were compared with the median APEs of the classic method. Therefore, a good insight on forecasting accuracy is provided.

4. Results

4.1. Smoothing

For all the methods of HW-MTAS, XHW-MT-AS, DHW-MT-AS & XDHW-MT-AS, the input series of monthly and quarterly periods are smoothed under both MAE and MSE objective functions. We then calculated the SREM values to compare the smoothing accuracy of the methods. Table 4 shows the mean of SREM values for monthly data in each category and in total. Table 5 shows the mean of SREM values for quarterly data in each category and in total.

Table 4. Smoothing accuracy of methods in monthly data

SREM By Category	Mean SREM of MSE	Mean SREM of MAE	Mean SREM of MSE	Mean SREM of MAE
	MSE Minimized	MAE Minimized	MSE Minimized	MAE Minimized
	XHW.MTAS/ HW.MTAS	XHW.MTAS/ HW.MTAS	XDHW.MTAS/ DHW.MTAS	XDHW.MTAS/ DHW.MTAS
Micro	21.36	14.98	8.85	5.19
Industry	15.17	13.32	11.64	7.40
Macro	17.47	23.19	33.34	19.57
Finance	15.52	19.38	26.34	12.42
Demographic	10.73	23.56	40.03	27.43
Total	**16.66**	**17.98**	**21.32**	**12.71**

TABLE 5. SMOOTHING ACCURACY OF METHODS IN QUARTERLY DATA

SREM By Discipline	Mean SREM of MSE	Mean SREM of MAE	Mean SREM of MSE	Mean SREM of MAE
	MSE Minimized	MAE Minimized	MSE Minimized	MAE Minimized
	XHW.MTAS/ HW.MTAS	XHW.MTAS/ HW.MTAS	XDHW.MTAS/ DHW.MTAS	XDHW.MTAS/ DHW.MTAS
Micro	14.78	9.67	12.11	8.17
Industry	16.84	10.88	18.16	9.45
Finance	23.80	27.02	32.63	18.48
Macro	27.57	20.96	23.13	14.37
Demographic	11.25	19.79	24.49	14.35
Total	**19.53**	**18.82**	**23.30**	**13.64**

As it can be seen from the tables (4) and (5), the total mean SREM value is positive which indicates the better overall smoothing performance of XHW vs HW and XDHW vs DHW method. In other words, the XHW and XDHW fits the data better than HW and DHW.

Relying solely on mean measure could sometimes misleads us about the distribution of the data points. To ensure that the SREM values mostly fall on the positive side we used boxplots to simply check SREM values distribution in the range of (-100 to +100) percentage.

Fig. 1 shows the total distribution of SREM values over all methods. The mean SREM of the values is also shown using a "+" sign. It can be seen that the distribution of SREM values is mostly positive and even the first quartile is above zero which indicates that at least 75 percent of the values are positive confirming that in more than 75% percent of series XHW and XDHW have higher smoothing accuracy than HW and DHW.

In 16 series the GRG nonlinear could not find the optimal value of MSE value. Using solver's GRG nonlinear method in exponential smoothing problems where the error function might be non-convex or have multiple local optimal solutions, can lead to incorrect answers (Ravinder, H. V., 2016). In the following cases we used the evolutionary algoriyhm of Solver: {N781, N1705, N1790, N1793, N1800, N1801, N1891, N1985, N2090, N2093, N2105, N2117, N2598, N2601, N2735, N2768}.

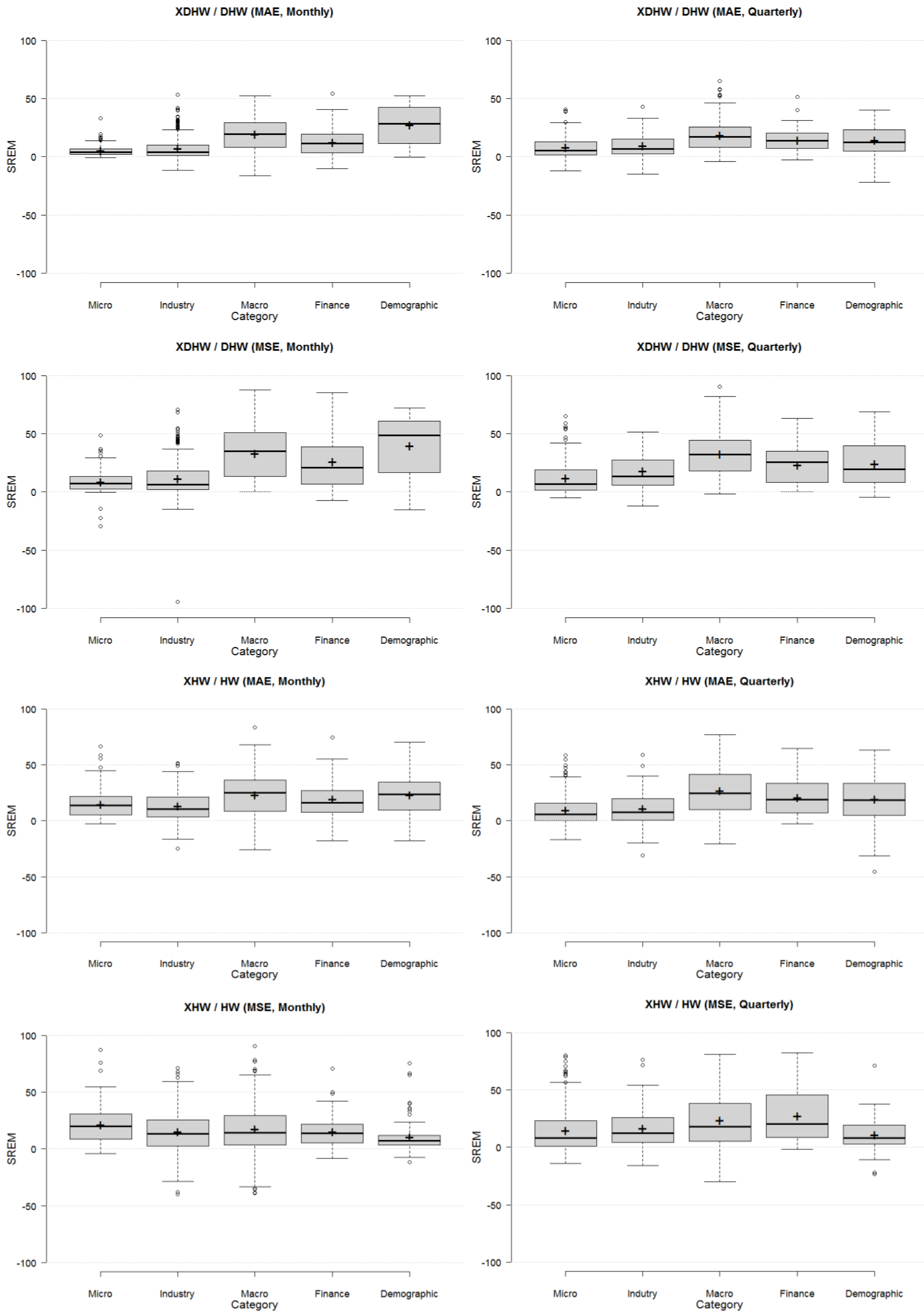

FIG 1. BOXPLOTS OF SREM VALUES IN METHODS FOR MONTHLY AND QUARTERLY DATA.

4.2 Forecasting

As mentioned in section 3.4.2., two approaches were utilized to measure the forecasting accuracy. In the first approach we calculated the MAPE value over the forecasted year for each input series and used SREM formula to compare the MAPEs obtained by classic methods. Table 6 shows the mean of SREM values for monthly data in each category and in total.

TABLE 6. FORECASTING ACCURACY OF METHODS IN MONTHLY DATA

SREM By Category	Mean SREM of MAPE	Mean SREM of MAPE	Mean SREM of MAPE	Mean SREM of MAPE
	MSE Minimized	MAE Minimized	MSE Minimized	MAE Minimized
	XHW.MTAS/ HW.MTAS	XHW.MTAS/ HW.MTAS	XDHW.MTAS/ DHW.MTAS	XDHW.MTAS/ DHW.MTAS
Micro	30.69	24.46	4.05	4.08
Industry	9.30	6.77	0.56	3.11
Macro	31.06	28.91	3.07	15.79
Finance	12.01	8.05	2.65	9.33
Demographic	39.83	34.83	16.77	30.50
Total	**22.53**	**19.00**	**3.60**	**9.96**

Table 7 shows the mean of SREM values for quarterly data in each category and in total.

As it can be seen, the total positive mean SREMs in Tables 6 and 7 indicate better forecasting performance in XHW and XDHW compared to HW and DHW. Boxplots have also been checked and the results are similar to the previous section.

TABLE 7. FORECASTING ACCURACY OF METHODS IN QUARTERLY DATA

SREM By Category	Mean SREM of MAPE	Mean SREM of MAPE	Mean SREM of MAPE	Mean SREM of MAPE
	MSE Minimized	MAE Minimized	MSE Minimized	MAE Minimized
	XHW.MTAS/ HW.MTAS	XHW.MTAS/ HW.MTAS	XDHW.MTAS/ DHW.MTAS	XDHW.MTAS/ DHW.MTAS
Micro	2.03	1.35	3.64	1.48
Industry	4.79	4.84	-5.16	2.64
Finance	22.90	21.00	12.68	10.02
Macro	11.45	14.20	6.10	3.89
Demographic	18.82	20.87	7.98	6.70
Total	**13.24**	**12.63**	**6.99**	**5.82**

The Mean SREM of MAPEs shows minor underperformances in industry quarterly series in XDHW method only when the objective function was to minimize MSE. Therefore, we used the second approach in measuring forecast accuracy to gain a deeper insight into the forecasting accuracy.

In the second approach, for each method, the median APE of all forecasted values in each single forecasting horizon was calculated. This way we can compare how the studied methods performed in each forecast step. Figure 2 show the median APEs. Similar results were obtained for the mean of APEs too. As can be seen, the errors of XHW and XDHW are almost always lower than those of HW & DHW, indicating lower error generation in each forecasting step by XHW and XDHW.

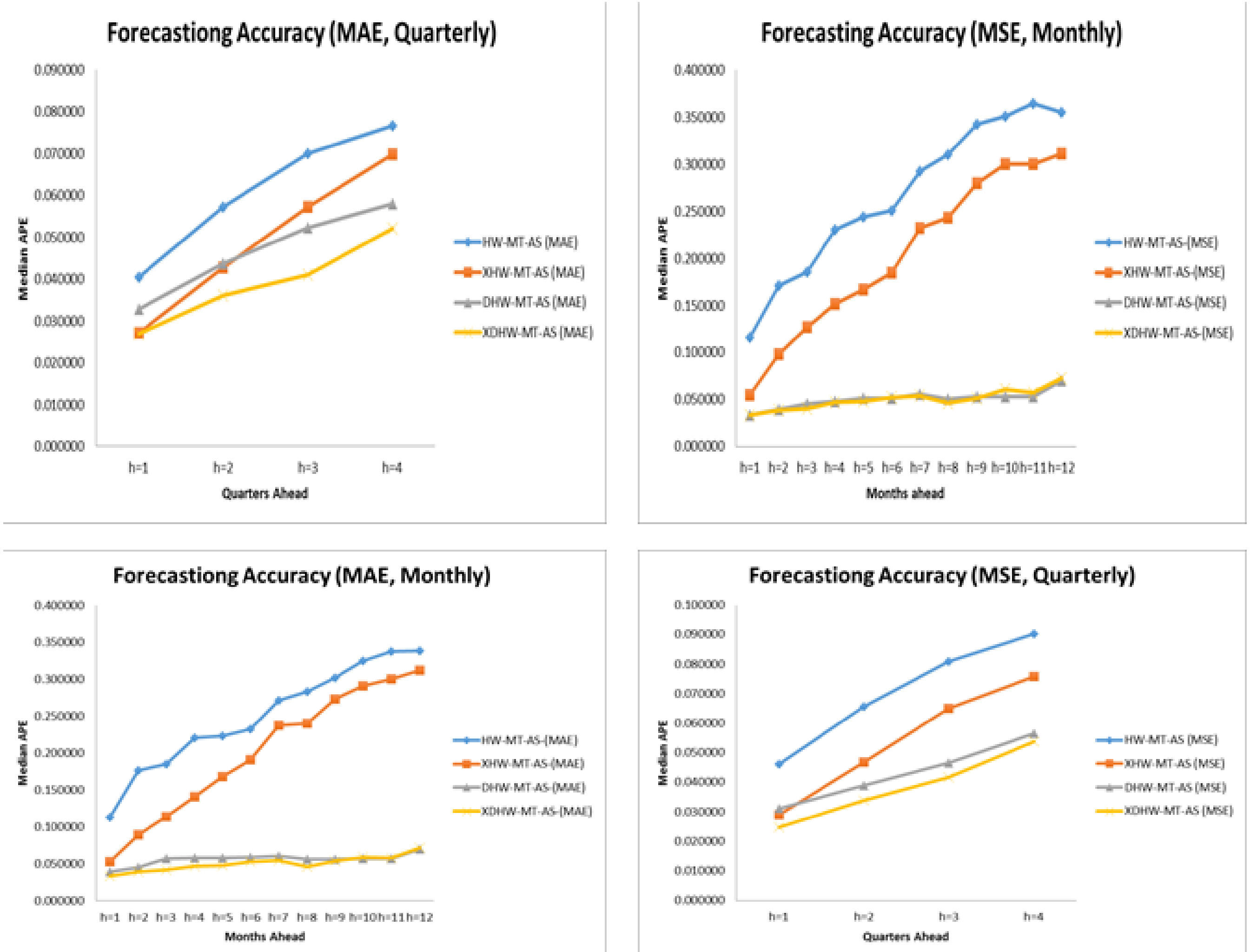

FIG 2 MEDIAN APE OF ALL METHODS IN EACH FORECAST HORIZON FOR MONTHLY DATA.

5. DISCUSSION

In this paper a new approach of time series forecasting for multiplicative trend was presented. To this end, we extended the EHW additive trend method proposed by Trater and introduced XHW method. XDHW method also was introduced for damped multiplicative trends. The proposed models were applied on M3 competition series and the results were compared with the classic methods. The results demonstrated that our proposed methods outperform the classic methods in almost all input series of M3 competition.

The smoothing accuracy of XHW and XDHW methods is significantly higher both in monthly and quarterly data as can be seen in tables 4 and 5.

The mean SREM varies between 6.96% to 39.22%. XHW and XDHW provided more accurate forecasts, on monthly data compared to HW method.

Forecasting performance of XHW method was significantly better than HW method on quarterly data, but the difference between the performance of XDHW and DHW was insignificant, as negative SREM was resulted in forecasting industry and micro quarterly data. Therefore, we can conclude that, overall, XHW and XDHW methods are more accurate compared to HW and DHW methods in both monthly and quarterly data.

EHW and XHW form a set of improved methods to be considered by forecasters due to their better smoothing and forecasting results. In this study we focused on the multiplicative trend and additive seasonality component. Extending this method for series with multiplicative seasonality or multiple seasonality can be considered as a future work. Similarly, the performance of these methods can be further examined in forecasting series with nonlinear trends.

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