Original Research

A new control chart based on discriminant analysis for simple linear profiles monitoring

Negin Khaksari, Mona Ayoubi

Received: 10 July 2020 / Accepted: 24 December 2020 / Published online: 31 December 2020

Department of Industrial Engineering, College of Engineering, West Tehran Branch, Islamic Azad University, Tehran, Iran
* Corresponding Author, Assistant Professor of Industrial Engineering, ayoubi.m@wtiau.ac.ir

Abstract
In many processes, quality characteristic is identified by the regression relationship between one or more dependent variables and one or more independent variables called profile. In this paper, a control chart based on discriminant analysis (DA) is proposed to monitor simple linear profiles in Phase II. A chi-square control chart joined with DA chart is also used to improve detecting variance shifts. Performance of the proposed method is evaluated in terms of average run length using Monte-Carlo simulations. Performance of the proposed control chart is compared to the basic methods in simple linear profile monitoring literature. Results present the desirable performance of the proposed method. The real case in shoes leather industry is also investigated to show the effectiveness of the proposed method. Results also confirm an acceptable performance of the real case, because the average run length of the proposed control chart is less than the average run length of the comparable method.

Keywords - Discriminant analysis; Phase II; Profile monitoring, Simple linear profiles, Statistical process control

1. INTRODUCTION

Discriminant analysis (DA) is a statistical tool that used for classification purpose. Two-group discriminant analysis is used for classifying observations in two groups. For this purpose, at first the discriminant function separating two groups must be constructed. Then, a new observation can be classified in one of the existing groups. Some researchers focused on monitoring a process or product quality characteristics using discriminant analysis. Maleki and Sahraeian (2015) chose to use linear DA and artificial neural network (ANN) to monitor the mean vector of a correlated multivariate-attribute process. Wang et al. (2016) also utilized distance discriminant analysis method for the stator winding single-phase grounding faults indicating that the distance discriminant analysis model has a great precision and less mistakes. Zhang et al. (2016) combined DA with kernel dictionary learning. They showed that sparse representation based classification (SRC) due to a linear algorithm, is not suitable for nonlinear data. They propose a new feature learning technique KDL-DA (Kernel dictionary learning based discriminant analysis) in order to assess a projection matrix and kernel dictionary at the same time. Hu and Li (2012) applied bayes discriminant analysis technique. linear DA is also used by Theophilou et al. (2016) who used principal component analysis (PCA), successive projection (SPA) and genetic algorithm (GA), followed by linear discriminant analysis (LDA) for diagnosing benign and malignant tumors as well as classifying different tumor subtypes. In addition, Lim et al. (2018) presented a new robust linear discriminant analysis in order to solve the classic LDA problems. Alfaro et al (2015), for testing the performance of linear discriminant analysis, neural networks, classification trees and boosting trees, used different simulated scenarios with different correlation structure and shift type. The findings demonstrate that Boosting tree method show better performance under the more feasible circumstances, while the performance of the other methods is related to the correlation structure and the class of change. Juefei-Xu and Savvides (2016) considered Multi-class Fukunaga Koontz Discriminant Analysis (FKDA) by utilizing the Fukunaga

Koontz Transform for separating classes in Fisher's Linear Discriminant Analysis, Locality Preserving Projections (LPP) and Unsupervised Discriminant Projection (UDP) which are linear subspace learning methods used in face recognition. The advantages of represented method FKDA, in comparison with the past methods, are trying hard to find out optimal projection direction vectors which are orthogonal as well as detecting accurately answers to the "trace ratio" purpose in discriminant analysis problems. Ren et al. (2019) investigated the identification of asphalt fingerprints based on ATR-FTIR spectroscopy and principal component-linear discriminant analysis. They combined the attenuated total reflectance fourier transform infrared spectroscopy (ATR-FTIR) with the chemo metric methods. Some researchers used Fisher discriminant analysis to classification purpose. For nonlinear multivariate processes, Galiaskarov et al. (2017) proposed a new method by combining kernel principal component analysis with a moving window for monitoring. They also used fisher discriminant analysis to recognize the faults. Nor et al. (2015) integrated nonlinear kernel variation of Fisher discriminant analysis (FDA) with the wavelet analysis called multi-scale kernel Fisher discriminant analysis (MSKFDA). The advantage of this method is an efficient division of the deterministic and stochastic characteristics and considering time and frequency domain aspects which leads to extracting characteristics related to out-of-control circumstances. Pei et al. (2006) conducted a research in which an absolute-value based fisher discriminant analysis and the individuals and moving range chart are combined for creating a novel fault detection algorithm. They presented quantitative data classification method and projected trend analysis that are two kinds of fault detection approaches by applying the discriminant model. Zheng et al. (2019) worked on fault classification by the semi-supervised Fisher Discriminant Analysis for monitoring the industrial processes. Integrating the metric level outputs and the K Nearest Neighbor (KNN) algorithm gained by the sub-classifiers provided a method for finding the final classification outcome. Because of using additional information obtained from unlabeled data, semi-supervised learning is more likely to produce more efficient model. In another research conducted by Nor et al. (2017), three classifiers including wavelet analysis, Kernel Fisher discriminant analysis (KFDA), and support vector machine (SVM) for fault detection and diagnosis (FDD) in chemical process system were employed. In order to measure the performance of the suggested multi-scale KFDA-SVM techniques, the simulated Tennessee Eastman process was employed as a benchmark. One of the attractive use of discriminant analysis is in the classification of data obtained from space missions. In these circumstances, the massive volume of data is available that requires advanced statistical methodologies for extracting information. In order to deal with this problem, Nassar and Hussein (2015) proposed the new advanced learning algorithm by utilizing projection to latent structure discriminant analysis technique (PLS_DA). Their represented method is capable of modeling, analyzing, classifying data and detecting core contributors to abnormal events when many predictors and response variables are measured. Some researchers employed Partial Least Square Discriminant Analysis in their research. Maquina et al. (2017) applied infrared spectroscopy with partial least square discriminant analysis for characterizing the moringa, mafurra and cotton biodiesel mixes. Yan et al. (2015) developed a method for batch-to-batch quality control represented by using HPLC-MS fingerprint and process knowledgebase. They obtained the fingerprints of extract solutions generated by normal and abnormal operation conditions after conducting the HPLC-MS fingerprint analysis method in which fault detecting was conducted by multivariate statistical models and fault diagnosis was detected by a discriminant analysis model based on the probabilistic discriminant partial-least-squares method. Zhao et al. (2016) utilized the other kind of discriminant analysis called nested-loop Fisher discriminant analysis (NeLFDA) algorithm. Zhao and Gao (2015) also presented a novel discriminant analysis for improving the weaknesses of traditional Fisher discriminant analysis (FDA) by deploying a nested loop algebra. The new nested-loop Fisher discriminant analysis (NeLFDA) aims to eliminate three drawbacks of conventional model. The fault diagnosing performance and classification ability of this method show good performance as opposed to the old ones. Deng et al. (2017) proposed Fault discriminant enhance kernel principle component analysis (FDKPCA) which uses historical fault data for boosting the efficiency of fault detection. Nonlinear Kernel principal components (KPCs) and fault discriminant components (FDCs) which are two kinds of data characteristics were monitored at the same time by their represented method. In this method, Kernel local-nonlocal preserving discriminant analysis (KLNPDA) was used for detecting FDCs; and KPCA was applied for monitoring KPCs based on normal operation data.

One of the area that has many applications in industries is profile monitoring that can be defined as the relationship between one or more dependent variables and one or more independent variables. Profiles have different types one of which is simple linear profile. There can be found many conducted research regarding monitoring the profile processes. For example, for simple linear profile monitoring in Phase II focused in this paper, Kang and Albin (2000), Kim et al. (2003), Gupta et al. (2006), Zou et al. (2006), Zou et al. (2007), Zhang et al. (2009), Mahmoud et al. (2010), Li and Wang (2010), Zhu and Lin (2010), Hosseinifard et al. (2011) and Noorossana and Ayoubi (2012) proposed some monitoring methods. Narvand et al. (2013) used linear mixed model to monitoring auto-correlated linear profiles in Phase II. Kalaei et al. (2018) considered monitoring of standard deviations in multistage linear profiles in Phase I. Maleki et al. (2018) carried out a comprehensive review on the area of profile monitoring. Rahimi et al. (2019)

monitored mean vector and covariance matrix of multivariate simple linear profiles by considering within profiles correlation. Ahmadi et al. (2019) focused on monitoring multivariate simple linear profiles with estimated parameters in Phase II. Moheghi et al. (2020) considered monitoring GLM profiles using robust estimators. Haq et al. (2020) focused on monitoring simple linear profiles with one observation per sample.

Having studied the literature of discriminant analysis and simple linear profile monitoring, one can find out that the usage of discriminant analysis-based control charts in monitoring profile characteristics, can be considered as a gap in this scope. However, discriminant analysis was not utilized in the area of profile monitoring, so this method is opted for monitoring the simple linear profiles in Phase II, in this paper. In the next section, simple linear profile model will be presented. In section 3, the proposed method is introduced. Then, the findings of simulation results by using Mont Carlo method will be demonstrated in section 4. Section 5 deals with the real case of shoes leather industry. In the final section, concluding remarks are presented.

2. SIMPLE LINEAR PROFILES MODEL IN Phase II

In many practical situations, the quality of a product or process is defined by a relationship between one or more response variables and one or more independent variables entitled profile. One kind of profiles is simple linear profile in which a dependent variable is related to an independent variable. In simple linear profiles, j^{th} sample consists of n fixed x_i (independent variables), hence n observations of (x_i, y_{ij}) are available. Therefore, the in-control simple linear profile model is shown by the following equation:

$$y_{ij} = A_{0j} + A_{1j}x_i + \varepsilon_{ij} \text{ , } i = 1,2,\ldots,n \tag{1}$$

The matrix form of the above equation is modeled as:

$$\mathbf{y}_j = \mathbf{X}\mathbf{a}_j + \boldsymbol{\varepsilon}_j \tag{2}$$

Or

$$\begin{bmatrix} y_{1j} \\ y_{2j} \\ \vdots \\ y_{nj} \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \vdots \\ \varepsilon_{nj} \end{bmatrix}, \tag{3}$$

where y_{ij} is the dependent variable for i^{th} observation ($i = 1,2,\ldots,n$) in the j^{th} sample ($j = 1,2,\ldots$) and x_i is the independent variable for the i^{th} observation. Also, ε_{ij} 's are independent and identically distributed normal random errors with fixed variance of σ^2.

The profile parameters are also estimated by utilizing the following equations:

$$a_{1j} = \hat{A}_{1j} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})y_{ij}}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{4}$$
$$a_{0j} = \hat{A}_{0j} = \bar{y}_j - a_{1j}\bar{x}$$

3. PROPOSED DISCRIMINANT ANALYSIS-BASED CONTROL CHART FOR MONITORING SIMPLE LINEAR PROFILES IN PHASE II

In control charts, there are two types of data that should be separated which are in-control and out-of-control data sets. Discriminant analysis by using the special groups defined by the users can divide all data in the pre-specified in-control and out-of-control groups. Therefore, the purpose of control chart is analogous with discriminant analysis that justifies the use of DA in monitoring processes. For this purpose, at first the discriminant function must be derived using supervised procedure.

In this paper regarding to Zolfaghari and Amiri (2016), two statistics of T^2 and multivariate exponentially weighted moving average (MEWMA) are considered as the variables of discriminant function. Zolfaghari and Amiri (2016) used geometric rotation procedure to find the discriminant function. However, in this paper, the algebraic fisher discriminant function is applied to construct the discriminant function.

The DA statistic for sample j, which is a linear combination of MEWMA and T^2 statistics, is defined as follows:

$$DA_j = W_1 \times MEWMA_j + W_2 \times (T_j^2), \tag{5}$$

where the $MEWMA_j$ statistic is calculated as follows: (Note that to achieve the smaller value of the statistic, the mean-corrected vector of $\mathbf{e}_j = \begin{bmatrix} a_{0j} \\ a_{1j} \end{bmatrix} - \begin{bmatrix} A_0 \\ A_1 \end{bmatrix}$ is used. Also, θ is smoothing constant.)

$$\mathbf{z}_j = \theta\mathbf{e}_j + (1-\theta)\mathbf{z}_{j-1} \tag{6}$$

Hence, we have:

$$MEWMA_j = \mathbf{z}_j' \Sigma_{\mathbf{z}_j}^{-1} \mathbf{z}_j, \tag{7}$$

where the covariance matrix of $\mathbf{z}_j$ is:

$$\boldsymbol{\Sigma}_{\mathbf{z}_j} = \frac{\theta}{2-\theta}\boldsymbol{\Sigma}_e, \tag{8}$$

where,

$$\boldsymbol{\Sigma}_e=\sigma^2\begin{bmatrix}\frac{1}{n}+\frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2} & \frac{1}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\\ \frac{1}{\sum_{i=1}^{n}(x_i-\bar{x})^2} & \frac{-\bar{x}}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\end{bmatrix} \tag{9}$$

The T^2 part of the DA statistic is derived as:

$$T_j^2=\mathbf{e}'_j\,\Sigma_{\mathbf{e}}^{-1}\,\mathbf{e}_j\ , \tag{10}$$

The values of W_1 and W_2 in DA statistic of Equation (5) are also calculated using algebraic fisher discriminant analysis. Hence, matrices of sum of square within (**W**), sum of square between (**B**) and sum of square total (**T**) can be structured due to the following steps:

1. Produce 1000 in-control samples so that 1000 in-control MEWMA and T^2 statistics are calculated (i.e. samples 1 to 1000 are in-control).
2. Produce 1000 out-of-control samples so that 1000 out-of-control MEWMA and T^2 statistics are calculated. (i.e. samples 1001 to 2000 are out-of-control).
3. Consider 1000 in-control samples as the observations of group 1 and 1000 out-of-control samples as the observations of group 2. Each group has two variables of MEWMA and T^2.
4. Calculate the matrix of **W** such as $\mathbf{W}=\begin{bmatrix}SSW_{EWMA} & SSW_{EWMA-T^2}\\ SSW_{EWMA-T^2} & SSW_{T^2}\end{bmatrix}$,

where its elements are defined as follows:

$$SSW_{EWMA}=\sum_{j=1}^{1000}(EWMA_j-\overline{EWMA}_{1-1000})^2 + \sum_{j=1001}^{2000}(EWMA_j-\overline{EWMA}_{1001-2000})^2. \tag{11}$$

$$SSW_{T^2}=\sum_{j=1}^{1000}(T_j^2-\overline{T}^2_{1-1000})^2 + \sum_{j=1001}^{2000}(T_j^2-\overline{T}^2_{1001-2000})^2. \tag{12}$$

$$SSW_{EWMA-T^2}=\sum_{j=1}^{1000}(EWMA_j-\overline{EWMA}_{1-1000})\times(T_j^2-\overline{T}^2_{1-1000}) + \sum_{j=1001}^{2000}(EWMA_j-\overline{EWMA}_{1001-2000})\times(T_j^2-\overline{T}^2_{1001-2000}). \tag{13}$$

in the above equations, $\overline{EWMA}_{1-1000}=\frac{\sum_{j=1}^{1000}EWMA_j}{1000}$, $\overline{EWMA}_{1001-2000}=\frac{\sum_{j=1001}^{2000}EWMA_j}{1000}$, $\overline{T}^2_{1-1000}=\frac{\sum_{j=1}^{1000}T_j^2}{1000}$ and $\overline{T}^2_{1001-2000}=\frac{\sum_{j=1001}^{2000}T_j^2}{1000}$.

5. Calculate the matrix of **T** such as $\mathbf{T}=\begin{bmatrix}SST_{EWMA} & SST_{EWMA-T^2}\\ SST_{EWMA-T^2} & SST_{T^2}\end{bmatrix}$, where its elements are defined as follows:

$$SST_{EWMA}=\sum_{j=1}^{2000}(EWMA_j-\overline{EWMA}_{1-2000})^2 \tag{14}$$

$$SST_{T^2}=\sum_{j=1}^{2000}(T_j^2-\overline{T}^2_{1-2000})^2 \tag{15}$$

$$SST_{EWMA-T^2}=\sum_{j=1}^{2000}\Big[(EWMA_j-\overline{EWMA}_{1-2000})\times(T_j^2-\overline{T}^2_{1-2000})\Big] \tag{16}$$

in the above equations, $\overline{EWMA}_{1-2000}=\frac{\sum_{j=1}^{2000}EWMA_j}{2000}$ and $\overline{T}^2_{1-2000}=\frac{\sum_{j=1}^{2000}T_j^2}{2000}$.

6. Compute the matrix of **B** as follows:

$$\mathbf{B}=\mathbf{T}-\mathbf{W} \tag{17}$$

7. Calculate the eigenvalues and eigenvectors of $\mathbf{W}^{-1}\mathbf{B}$ matrix by solving the following equation:

$$\left|\mathbf{W}^{-1}\mathbf{B}-\gamma\mathbf{I}_{2\times2}\right|=0\,, \tag{18}$$

where, $\mathbf{I}_{2\times2}$ is an identity matrix. (Note that for 2 by 2 matrices, two distinct values of γ are calculated as eigenvalues. For two-group DA, only the greatest value of γ is required to compute one discriminant function. Solving Equation (18) is similar to choosing the first principal component of $\mathbf{W}^{-1}\mathbf{B}$ matrix. Coefficients of the first principal component are considered as W_1 and W_2 of Equation (5)).

In this paper, the upper control limit of the DA statistic of Equation (5) is obtained using 10000 Monte-Calro simulation runs to achieve the desired in-control average run length (ARL).

To improve the performance of the proposed method in detecting variance shifts, the chi-square control chart introduced by Noorossana et al. (2010) is used in conjunction with the DA control chart as bellow:

$$\chi_j^2 = \sum_{i=1}^{n} \frac{\varepsilon_{ij}^2}{\sigma^2}\,, \tag{19}$$

where, χ_j^2 follows chi-square distribution with n degrees of freedom. Hence, the upper control limit of the chi-square control chart is $UCL_{\chi^2} = \chi^2_{n,\alpha}$ which is $100(1-\alpha)$ percentile of chi-square distribution with n degrees of freedom.

4. PERFORMANCE EVALUATION OF THE PROPOSED METHOD

In this paper, for evaluating the performance of the proposed control chart, 10000 Monte Carlo simulation runs are used. In this paper, the underlying in-control model used by Kang and Albin (2000) is also considered which is $y_{ij} = 3 + 2x_i + \varepsilon_{ij}$ (i.e., $\begin{bmatrix} A_0 \\ A_1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$).

Also, $\varepsilon_{ij} \overset{i.i.d}{\sim} N(0, \sigma^2 = 1)$. The constant values of 2, 4, 6 and 8 (i.e. $n = 4$) are considered for x variable. The starting vector of $\mathbf{z}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is also used in Equation (6). The value of 0.2 is considered for the smoothing constant of θ. To calculate the discriminant function, the 1000 in-control samples are generated from the above-mentioned in-control model. To calculate the 1000 out-of-control samples, four different shifts are considered so that 250 samples are generated when A_0 is shifted to the value of 4, 250 samples are generated when A_1 is shifted to the value of 3, 250 samples are generated when both A_0 and A_1 are changed to the values of 5.5 and 1.5, respectively. Finally, 250 samples are generated when σ^2 is changed to the value of 2. Due to these samples, the discriminant function of $DA_j = 0.9931 \times MEWMA_j + 0.1177 \times T_j^2$ is obtained by finding the first principal component of $\mathbf{W}^{-1}\mathbf{B}$ matrix. The overall in-control ARL of 200 ($\alpha = 0.005$) is considered so that $\alpha_1 = \frac{3}{4}\alpha = \frac{3}{4} \times 0.005$ (i.e., in-control ARL of 267) is devoted to DA control chart, and $\alpha_2 = \frac{1}{4}\alpha = \frac{1}{4} \times 0.005$ (i.e., in-control ARL of 800) is dedicated to the chi-square control chart. The upper control limit of the DA control chart is 10.84 obtained by simulations to achieve the in-control ARL of 267. The upper control limit of the chi-square control chart is $UCL_{\chi^2} = \chi^2_{4,(\frac{0.005}{4})} = 17.9715$.

Simulation results are reported in Tables 1-3. In the Tables, the performance of the proposed method is compared to the performance of the EWMA-R and T^2 proposed by Kang and Albin (2000) and EWMA3 method suggested by Kim et al. (2003). Tables 1, 2 and 3 present ARL values for shifts in intercept, slope and variance of the model, respectively.

TABLE 1. ARL VALUES OF THE PROPOSED CONTROL CHART WHEN A_0 SHIFTS TO $A_0 + \lambda\sigma$

	λ									
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2
Proposed DA- χ^2	57.3	16.2	8.1	5.2	3.9	3.1	2.6	2.3	2	1.8
EWMA-R	66.5	17.7	8.4	5.4	3.9	3.2	2.7	2.3	2.1	1.9
T^2	137.7	63.5	28.0	13.2	6.9	4.0	2.6	1.8	1.5	1.2
EWMA3	59.1	16.2	7.9	5.1	3.8	3.1	2.6	2.3	2.1	1.9

TABLE 2. ARL VALUES OF THE PROPOSED CONTROL CHART WHEN A_1 SHIFTS TO $A_1 + \beta\sigma$

	β									
	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
Proposed DA- χ^2	95.9	33.6	15.7	9.4	6.5	5.0	4.1	3.5	3.0	2.7
EWMA-R	119.0	43.9	19.8	11.3	7.7	5.8	4.7	3.9	3.4	3.0
T^2	166.0	105.6	60.7	34.5	20.1	12.2	7.8	5.2	3.7	2.7
EWMA3	101.6	36.5	17.0	10.3	7.2	5.5	4.5	3.8	3.3	2.9

TABLE 3. ARL VALUES OF THE PROPOSED CONTROL CHART WHEN σ SHIFTS TO $\eta\sigma$

	η									
	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
Proposed DA- χ^2	37.2	12.7	6.4	3.9	2.8	2.2	1.8	1.6	1.5	1.4
EWMA-R	34.3	12.0	6.1	3.9	2.9	2.3	1.9	1.7	1.5	1.4
T^2	39.6	14.9	7.9	5.1	3.8	3.0	2.5	2.2	2.0	1.8
EWMA3	33.5	12.7	7.2	5.1	3.9	3.2	2.8	2.5	2.3	2.1

Results of Tables 1-3 show that the proposed method performs much better than the existing method in detecting mean shifts. The proposed control chart has also desirable performance in detecting variance shifts and its performance becomes better when shift size increases.

5. A REAL CASE

Amiri et al. (2011) used a real case of simple linear profile in shoes leather industry. One of the most important quality characteristics in the dyeing process is the relationship between color effluent (response variable) and temperature (predictor variable) in shoes leather industry. Amiri et al. (2011) gathered 11 profile samples including color effluent at 5 equally-spaced temperatures of 25, 32, 39, 46 and 53ºc. Their dataset is reported in Table A of the Appendix. They monitored the process in Phase I and concluded that all the collected samples are in-control. Hence, using the 11 in-control profile samples the underlying in-control model is:

$$y_{ij} = \text{-}0.0509 + 0.0034 x_i + \varepsilon_{ij}\,, \tag{20}$$

Where $\varepsilon_{ij} \sim N(0, 0.00057)$. For calculating the linear discriminant function, 1000 in-control data and using 0.5σ shift in each parameter, 1000 out-of-control samples are generated. Hence, the linear discriminant function is obtained as $DA_j = 0.9936 \times MEWMA_j + 0.1132 \times T_j^2$.

The performance of the proposed DA- χ^2 control chart is compared to the performance of EWMA3 method performing better than T^2 and EWMA-R methods. Also, $+0.02\sigma$ shift is imposed to the profile slope. The overall in-control ARL of 200 is also considered in this case. The value of $\theta = 0.2$ is also chosen. In the proposed DA- χ^2 control chart, the UCL of DA and χ^2 control charts are 10.84 and 19.9995, respectively. For EWMA3 method to achieve an overall in-control ARL of 200, the values of 40.00465, 3.012 and 0.16117 are chosen for L_I, L_S and L_E, respectively. (For more details about the EWMA3 method, refer to kim et al. (2003)).

Since, we consider only the shift in slope to conduct our comparison, only DA in the proposed DA- χ^2 control chart, and only EWMA chart for slope in EWMA3 method are considered. Figures 1-2 show the results of DA and slope EWMA control charts demonstrating the DA and slope EWMA charts issue an out of control signal at sample 4 and sample 9, respectively. Hence, the DA control chart outperforms the EWMA3 methods.

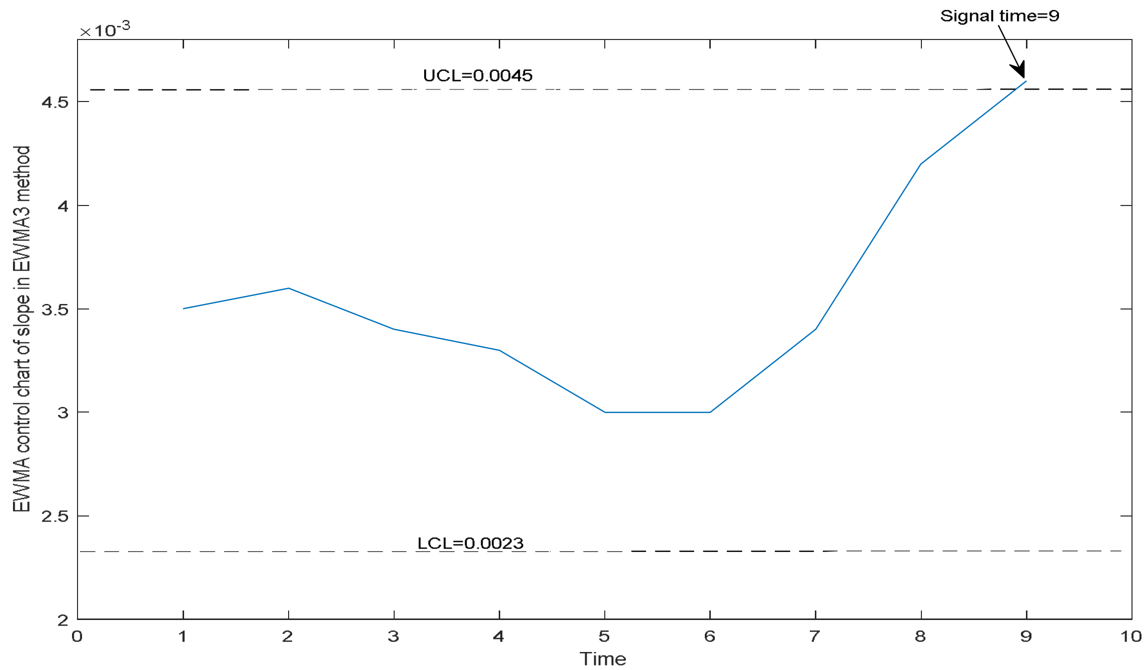

FIGURE 1. PERFORMANCE OF THE SLOPE EWMA CHART IN EWMA3 METHOD UNDER $+0.02\sigma$ SHIFT IN SLOPE PARAMETER

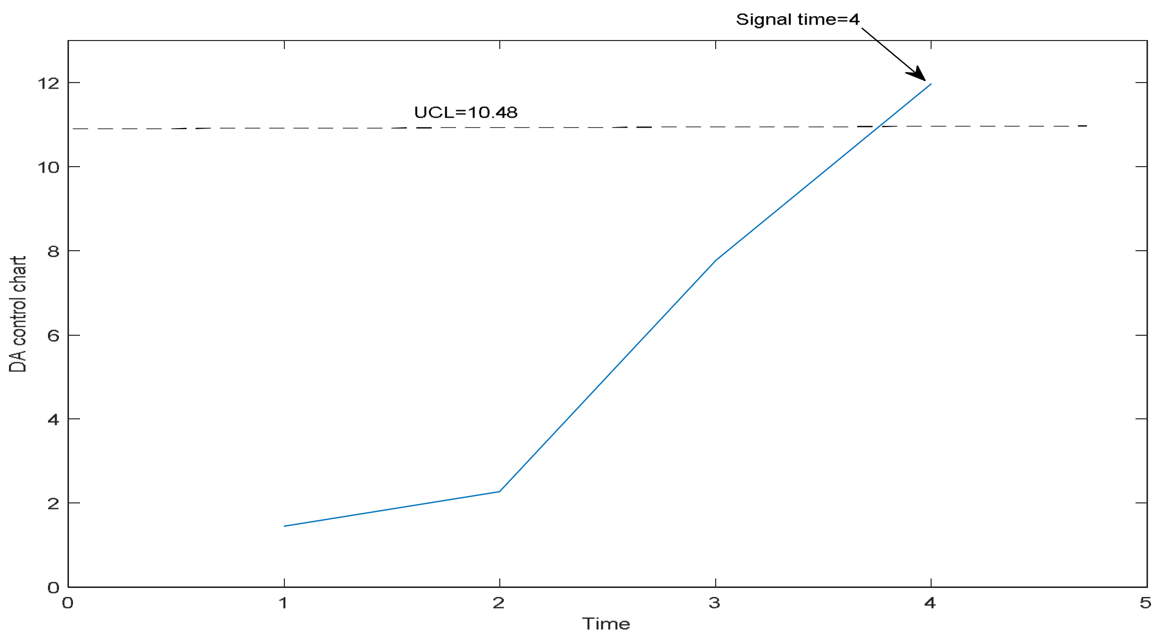

FIGURE 2. PERFORMANCE OF THE DA CONTROL CHART IN DA- χ^2 METHOD UNDER $+0.02\sigma$ SHIFT IN SLOPE PARAMETER

6. CONCLUDING REMARKS

In this paper, a control chart based on discriminant analysis was proposed to monitor simple linear profiles in Phase II. The chi-square control chart was also used in conjunction with the DA control chart to improve detecting variance shifts. Simulation results confirmed the ideal performance of the proposed method in detecting mean shifts. In detecting the variance shifts, the proposed method performs better in moderate to large shifts, and its performance in small variance shifts is also acceptable. The performance of the proposed method was evaluated in shoes leather industry to show its acceptable performance in a real case.

REFERENCES

[1] Alfaro, E., Alfaro, J. L., Gámez, M., & García, N. (2015). A comparison of different classification techniques to determine the change causes in Hotelling's T2 control chart. Quality and Reliability Engineering International, 31(7), 1255-1263.

[2] Amiri, A., Zand, A., & Soudbakhsh, D. (2011, January). Monitoring simple linear profiles in the leather industry (a case study). In Proceedings of the 2nd International Conference on Industrial Engineering and Operations Management, Kuala Lumpur, Malaysia (pp. 22-24).

[3] Deng, X., Tian, X., Chen, S., & Harris, C. J. (2017). Fault discriminant enhanced kernel principal component analysis incorporating prior fault information for monitoring nonlinear processes. Chemometrics and Intelligent Laboratory Systems, 162, 21-34.

[4] Galiaskarov, M. R., Kurkina, V. V., & Rusinov, L. A. (2017). Online diagnostics of time‐varying nonlinear chemical processes using moving window kernel principal component analysis and Fisher discriminant analysis. Journal of Chemometrics, 31(8), e2866.

[5] Gupta, S., Montgomery, D. C., & Woodall, W. H. (2006). Performance evaluation of two methods for online monitoring of linear calibration profiles. International Journal of Production Research, 44(10), 1927-1942.

[6] Haq, A., Bibi, M., & Shah, B. A. (2020). A novel approach to monitor simple linear profiles using individual observations. Communications in Statistics-Simulation and Computation, DOI: 10.1080/03610918.2020.1799229.

[7] Hosseinifard, S. Z., Abdollahian, M., & Zeephongsekul, P. (2011). Application of artificial neural networks in linear profile monitoring. Expert Systems with Applications, 38(5), 4920-4928.

[8] HU, Y. X., & LI, X. B. (2012). Bayes discriminant analysis method to identify risky of complicated goaf in mines and its application. Transactions of Nonferrous Metals Society of China, 22(2), 425-431.

[9] Juefei-Xu, F., & Savvides, M. (2016). Multi-class Fukunaga Koontz discriminant analysis for enhanced face recognition. Pattern Recognition, 52, 186-205.

[10] Kalaei, M., Soleimani, P., Niaki, S. T. A., & Atashgar, K. (2018). Phase-I monitoring of standard deviations in multistage linear profiles. Journal of Industrial Engineering International, 14(1), 133-142.

[11] Kang, L., & Albin, S. L. (2000). On-line monitoring when the process yields a linear profile. Journal of quality Technology, 32(4), 418-426.

[12] Kim, K., Mahmoud, M. A., & Woodall, W. H. (2003). On the monitoring of linear profiles. Journal of Quality Technology, 35(3), 317-328.

[13] Li, Z., & Wang, Z. (2010). An exponentially weighted moving average scheme with variable sampling intervals for monitoring linear profiles. Computers & Industrial Engineering, 59(4), 630-637.

[14] Lim, Y. F., Yahaya, S. S. S., & Ali, H. (2018). Robust Linear Discriminant Analysis with Highest Breakdown Point Estimator. Journal of Telecommunication, Electronic and Computer Engineering (JTEC), 10(1-11), 7-12.

[15] Mahmoud, M. A., Morgan, J. P., & Woodall, W. H. (2010). The monitoring of simple linear regression profiles with two observations per sample. Journal of Applied Statistics, 37(8), 1249-1263.

[16] Maleki, M. R. (2015). Online monitoring and fault diagnosis of multivariate-attribute process mean using neural networks and discriminant analysis technique. International Journal of Engineering, 28(11), 1634-1643.

[17] Maleki, M. R., Amiri, A., & Castagliola, P. (2018). An overview on recent profile monitoring papers (2008–2018) based on conceptual classification scheme. Computers & Industrial Engineering, 126, 705-728.

[18] Máquina, A. D. V., Souza, L. M. D., Gontijo, L. C., Santos, D. Q., & Borges Neto, W. (2017). Characterization of Biodiesel by Infrared Spectroscopy with Partial Least Square Discriminant Analysis. Analytical Letters, 50(13), 2117-2128.

[19] Moheghi, H. R., Noorossana, R., & Ahmadi, O. (2020). GLM profile monitoring using robust estimators. Quality and Reliability Engineering International. DOI: 10.1002/qre.2755.

[20] Narvand, A., Soleimani, P., & Raissi, S. (2013). Phase II monitoring of auto-correlated linear profiles using linear mixed model. Journal of Industrial Engineering International, 9(1), 12.

[21] Nassar, B., & Hussein, W. (2015, March). State-of-health analysis applied to spacecraft telemetry based on a new projection to latent structure discriminant analysis algorithm. In 2015 IEEE Aerospace Conference (pp. 1-11). IEEE.

[22] Noorossana, R., & Ayoubi, M. (2012). Profile monitoring using nonparametric bootstrap T 2 control chart. Communications in Statistics-Simulation and Computation, 41(3), 302-315.

[23] Noorossana, R., Eyvazian, M., & Vaghefi, A. (2010). Phase II monitoring of multivariate simple linear profiles. Computers & Industrial Engineering, 58(4), 563-570.

[24] Nor, N. M., Hussain, M. A., & Hassan, C. R. C. (2015). Process monitoring and fault detection in non-linear chemical process based on multi-scale kernel Fisher discriminant analysis. In Computer Aided Chemical Engineering (Vol. 37, pp. 1823-1828). Elsevier.

[25] Nor, N. M., Hussain, M. A., & Hassan, C. R. C. (2017). Fault diagnosis and classification framework using multi-scale classification based on kernel Fisher discriminant analysis for chemical process system. Applied Soft Computing, 61, 959-972.

[26] Pei, X., Yamashita, Y., Yoshida, M., & Matsumoto, S. (2006). Discriminant analysis and control chart for the fault detection and identification. In Computer Aided Chemical Engineering (Vol. 21, pp. 1281-1286). Elsevier.

[27] Rahimi, S. B., Amiri, A., & Ghashghaei, R. (2019). Simultaneous monitoring of mean vector and covariance matrix of multivariate simple linear profiles in the presence of within profile autocorrelation. Communications in Statistics-Simulation and Computation, DOI: 10.1080/03610918.2019.1588314.

[28] Ren, R., Han, K., Zhao, P., Shi, J., Zhao, L., Gao, D., ... & Yang, Z. (2019). Identification of asphalt fingerprints based on ATR-FTIR spectroscopy and principal component-linear discriminant analysis. Construction and Building Materials, 198, 662-668.

[29] Theophilou, G., Lima, K. M., Martin-Hirsch, P. L., Stringfellow, H. F., & Martin, F. L. (2016). ATR-FTIR spectroscopy coupled with chemometric analysis discriminates normal, borderline and malignant ovarian tissue: classifying subtypes of human cancer. Analyst, 141(2), 585-594.

[30] Wang, Y., Zhou, J., Wei, G., Dong, Z., & Chen, H. (2016). Stator winding single-phase grounding faults protective scheme based on discriminant analysis for Powerformers with selectivity. International Journal of Electrical Power & Energy Systems, 77, 145-150.

[31] Yan, B., Fang, Z., Shen, L., & Qu, H. (2015). Root Cause Analysis of Quality Defects Using HPLC–MS Fingerprint Knowledgebase for Batch‐to‐batch Quality Control of Herbal Drugs. Phytochemical analysis, 26(4), 261-268.

[32] Yazdi, A. A., Hamadani, A. Z., & Amiri, A. (2019). Phase II monitoring of multivariate simple linear profiles with estimated parameters. Journal of Industrial Engineering International, 15(4), 557-570.

[33] Zhang, J., Li, Z., & Wang, Z. (2009). Control chart based on likelihood ratio for monitoring linear profiles. Computational statistics & data analysis, 53(4), 1440-1448.

[34] Zhang, G., Sun, H., Ji, Z., Xia, G., Feng, L., & Sun, Q. (2016). Kernel dictionary learning based discriminant analysis. Journal of Visual Communication and Image Representation, 40, 470-484.

[35] Zhao, C., & Gao, F. (2015). A nested-loop Fisher discriminant analysis algorithm. Chemometrics and Intelligent Laboratory Systems, 146, 396-406.

[36] Zhao, C., Wang, W., & Gao, F. (2016). Probabilistic fault diagnosis based on Monte Carlo and nested-loop fisher discriminant analysis for industrial processes. Industrial & Engineering Chemistry Research, 55(50), 12896-12908.

[37] Zheng, J., Wang, H., Song, Z., & Ge, Z. (2019). Ensemble semi-supervised Fisher discriminant analysis model for fault classification in industrial processes. ISA transactions, 92, 109-117.

[38] Zhu, J., & Lin, D. K. (2009). Monitoring the slopes of linear profiles. Quality Engineering, 22(1), 1-12.

[39] Zolfaghari, S., & Amiri, A. (2016). Monitoring multivariate-attribute quality characteristics in two-stage processes using discriminant analysis based control charts. Scientia Iranica. Transaction E, Industrial Engineering, 23(2), 757.

[40] Zou, C., Zhang, Y., & Wang, Z. (2006). A control chart based on a change-point model for monitoring linear profiles. IIE transactions, 38(12), 1093-1103.

[41] Zou, C., Zhou, C., Wang, Z., & Tsung, F. (2007). A self-starting control chart for linear profiles. Journal of Quality Technology, 39(4), 364-375.

Appendix

The secondary dataset of leather color effluent:

TABLE A. LEATHER COLOR EFFLUENT FOR 11 PROFILES IN 5 DIFFERENT TEMPERATURES (Amiri et al.; 2011)

Temerature / Sample Number	25	32	39	46	53
1	0.0218	0.02878	0.09083	0.10111	0.12566
2	0.0302	0.05422	0.07183	0.11716	0.13127
3	0.0288	0.02868	0.08575	0.09310	0.13549
4	0.0306	0.07571	0.01011	0.11624	0.12850
5	0.0488	0.02806	0.08549	0.11812	0.11880
6	0.0310	0.09438	0.07157	0.11922	0.14965
7	0.0231	0.07626	0.08093	0.13988	0.15714
8	0.0455	0.09253	0.15109	0.08746	0.14101
9	0.0209	0.04746	0.10231	0.12651	0.12299
10	0.0578	0.02227	0.11557	0.11261	0.09202
11	0.0463	0.06435	0.08679	0.07877	0.10632