Original Research

A Robust Possibilistic Chance-Constrained Programming Model for Optimizing a Multi-Objective Aggregate Production Planning Problem under Uncertainty

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Abstract
Data uncertainty and multiple conflicting objectives are two crucial issues that the Decision Makers (DMs) must handle in making Aggregate Production Planning (APP) decisions in real practice. In order to address these two-mentioned issues, this study presents a multi-objective multi-product multi-period APP problem in an uncertain environment. The model strives to minimize the total costs of the APP plan, total changing rate in workforce levels, and total holding inventory and backorder quantities simultaneously through the Robust Possibilistic Chance-Constrained Programming (RPCCP) optimization approach. In this integrated approach, the RPCCP is applied for handling uncertain data. The RPCCP can not only handle any fuzzy position in the fuzzy model but also control the robustness of optimality and feasibility of the fuzzy model. Then, an Augmented Epsilon-Constraint (AUGMECON) technique is used to cope with multiple conflicting objectives. The AUGMECON technique can produce exact Pareto optimal solutions, which offer the DMs different selections to assess against conflicting objectives. Next, an industrial case study is provided to validate the applicability and effectiveness of the proposed methodology. The obtained outcomes indicate that the proposed RPCCP model outperforms the Possibilistic Chance-Constrained Programming (PCCP) model in terms of interested performance measurements (i.e., average and standard deviation of the objective function). In addition, a set of strong Pareto optimal solutions can be generated to accommodate alternative selections according to the DM's preferences. Finally, by applying the Max-Min method, the best compromised (trade-off) solution is determined through a comparison among the attained Pareto solutions.

Keywords - Aggregate production planning; robust possibilistic programming; chance-constrained; credibility measure; multiple-objective optimization; epsilon-constraint

Introduction

APP is defined as a medium to long range capacity planning process that determines the appropriate levels of production, holding inventory, subcontracting, backordering or lost sales, labor, capacity, and even pricing to satisfy market demand whilst minimizing total costs or maximizing total profit over a determined time horizon (from six to twelve, or even eighteen months) [1-4]. It helps to link the decisions of the short-term planning of operational and the long-term planning of strategic. Moreover, APP operating strategies play a vital role in enterprise resource planning and organizational integration. Hence, APP has become a great interest that attracts many academics and practitioners.

Since Holt et al. [5] proposed the HMMS (Holt, Modigliani, Muth, and Simon) rules, many researchers have proposed mathematical models to address APP problems. The HMMS model is an extensively applied scheme to deal with APP decision problems. The rules of its linear decision are useful to identify the suitable rate of production and the level of labor that minimizes total relevant costs, i.e., regular payroll, overtime, hiring, downsizing, and inventory holding costs. Nevertheless, when APP models are used, input data (demand and resources) are often considered to be deterministic or crisp values and with a single objective [6-14].

In an actual production system, the input data for APP problems, such as market demand, operation costs, available resources, machine capacities, etc., are generally acknowledged to be complex and vague due to the fact that some information is insufficient or impossible to acquire accurately. There are two possible causes of data uncertainty: (1) environmental uncertainty due to the disruption from both ends of the supply chain (i.e., from upstream suppliers and downstream customers) and (2) system uncertainty due to the instability of internal processes and operations in an organization [15]. This uncertainty has a significant impact on deciding the APP plan. Therefore, if the uncertainty is neglected in the planning process, it will lead to negative influences on the performance of the company. To cope with the uncertainty in planning, many approaches have been developed, e.g., Stochastic Programming (SP), Possibilistic Programming (PP), and Robust Programming (RP). SP is a modelling optimization method that uncertain parameters are described by probability and randomness theory – based on known probability distributions [16-18]. Nevertheless, there are two main weaknesses in the SP approach. First, it depends on the collection of large historical data, which are tough to obtain in case of an APP problem. Second, the requirement of a large amount of uncertain data will raise the model's complexity and result in a lack of computational efficiency. Also, probability theory might not provide us the right meaning to address several decision-making problems in real applications [19]. To improve these drawbacks, PP has been proposed as an efficient method that yields flexibility in dealing with imprecise/vague information since the uncertainty of data is represented by applying the possibilistic distributions that are according to the fuzzy set theory (incomplete available data and the subjective knowledge or experiences of the DMs/experts). Although this method has the ability to solve different types of uncertain parameters with high computational efficiency, the satisfaction and reliability levels of this method may not be guaranteed due to the fact that the obtained results are calculated by the expected or average value of imprecise data (the risk of the uncertain objective function cannot only be controlled by using imprecise parameter's expected or average value). In contrast, the DMs usually consider reducing the impact of risks as low as possible in the decision-making process. In this regard, RP is a method that can help the DMs to control risk-aversion levels of output results, which was recommended to incorporate with PP to overcome the deficiency of the PP method [20]. The Robust Possibilistic Programming (RPP) approach has been applied in various fields. However, to the best of the author's understanding, so far, such method has not been investigated yet in APP problems.

Furthermore, in developing process of an APP plan, multi-objective decision making is needed. This is due to the fact that such a plan is usually prepared by taking inputs from different departments within an organization. These departments usually establish their own expectation for the plan on the basis that it would improve their performance. Therefore, departmental expectations are considered as the objectives of an APP plan. Obviously, the expectation from a department is not always aligned with those from other departments. In other words, they are conflicting objectives. As a result, developing an APP plan with conflicting objectives falls into the domain of multi-objective optimization. Typically, a multi-objective model provides a set of efficient/compromised solutions, which represent the effective trade-offs among the conflicting objectives. These solutions are popularly known as Pareto solutions or non-dominated solutions [21]. From the literature, the conflicting objectives in the APP decision problems can be, for example, minimization of total costs, inventory investment, backlogged quantity, fluctuation of workforces, or maximization of total profits, utilization of facilities and their equipment, and customer service levels [22-28]. Taking into consideration of solving the APP model with many conflicting objective functions at the same time not only helps the DMs/planners to specify a bigger scope of these different options, but also makes the mathematical models of the APP problems much more practical.

Regarding the enumerated issues such as APP decisions, uncertain data, multi-objective decision making in APP problems, and the use of uncertain modelling approaches, this study, thus, aims to propose an integrated approach that supports the DMs or planners for solving these issues to achieve better solutions, as well as make the APP problems become more compatible with real-life situations. Also, to differentiate this study from the other existing studies, a summary of key features of APP problems on the literature review is shown in Table 1. In summary, a developed multi-objective RPP model for a multi-product multi-period APP decision problem where these parameters such as production times and costs, machine capacity, and customer demand are considered as fuzzy numbers with triangular possibility distribution to describe the uncertain property encountered in practical production planning systems. There are three objective functions of the proposed model, which are to minimize the total costs of APP plan, total changing rate in workforce

TABLE 1
SUMMARY OF THIS STUDY VS. THE RELEVANT STUDIES

References	Model specifications					Place of fuzzy parameters			Solution performance		Mathematical model	Solution approach						
	Products	Palming period	Number of objectives	Uncertainty level	Budget limitation	Objective function	One-side of constraint	Both sides of constraint	Satisfaction of solution	Pareto solution		FLP	FGP	FMOP	Others	AUGMECON	PCCP	RP
[22]	M	M	M	D							MILP		✓					
[6]	S	M	S	D							MILP				✓			
[23]	M	M	M	F		✓	✓		✓		FMOMILP	✓						
[1]	S	M	S	D							MILP				✓			
[24]	M	M	M	D					✓		MOMILP			✓				
[25]	M	M	M	D					✓		MOMILP		✓					
[7]	M	M	S	D							MILP				✓			
[26]	M	M	M	F	✓		✓		✓		FMOMILP		✓					
[8]	M	M	S	D							MILP				✓			
[9]	M	M	S	D							MILP				✓			
[10]	M	M	S	D							MILP				✓			
[11]	S	M	S	D							MILP				✓			
[12]	M	M	S	D							MILP				✓			
[28]	M	M	M	D							MOMILP		✓					
[3]	M	M	S	F		✓	✓	✓	✓		MILP	✓						
[4]	M	M	M	F		✓	✓	✓	✓		FMOMILP			✓			✓	
This study	M	M	M	F	✓	✓	✓	✓	✓	✓	FMOMILP					✓	✓	✓

Abbreviations: S: Single, M: Multiple, D: Deterministic, F: Fuzzy, MILP: Mixed-integer linear programming, MOMILP: Multi-objective mixed-integer linear programming, FMOMILP: Fuzzy multi-objective mixed-integer linear programming, FLP: Fuzzy linear programming, FGP: Fuzzy goal programming, FMOP: Fuzzy multi-objective programming, AUGMECON: Augmented epsilon-constraint technique, PCCP: Possibilistic chance-constrained programming, RP: Robust programming. Others: Linear decision rules, solver software (i.e., Lingo, Gam, Cplex), heuristic, etc.

levels, and total holding inventory and backorder quantities. By using the RPP method, the obtained results become more robust and reliable in terms of optimizing under uncertain environments. In addition, the proposed model may generate many new optimal solutions that can reflect the conflicting among objectives through applying the AUGMECON technique. This study will be helpful for the DMs or planners in identifying the optimal production and workforce levels in their APP problems, as uncertainty and multi-objective decision making are taken into consideration.

Consequently, the key contributions of this paper can be listed as follows:

- This study presents a mathematical model for a multi-product multi-period APP problem that simultaneously takes into account two important issues, which are uncertainty and multi-objective decision making.
- To cope with the uncertainty in the proposed optimization model, the RPCCP with credibility measure (the output results of the uncertain model are optimized based on the risk-averseness levels) is introduced. The combination of PCCP and RP approaches is capable of maintaining such a fuzziness property till the end of the optimization process, which can help to find the most appropriate fuzziness levels of the uncertain data. In addition, the robustness of both optimality and feasibility is taken into account. Thus, the proposed RPCCP can enable robust solutions.
- For the multi-objective decision making, the AUGMECON technique is applied to express the conflicting attribute of objectives, analyze the trade-off among considered objectives as well as provide more selections for the DMs through obtaining Pareto optimal solutions. Moreover, the AUGMECON technique can ensure that the obtained solutions are strong Pareto solutions (Pareto frontier). Then, in order to evaluate and determine the best compromised (trade-off) solution in a set of Pareto solutions, the Max-Min method is utilized.
- The integrated application of RPCCP and AUGMECON in this study is the first of its kind for solving the multi- objective APP problem under uncertain environment.
- The feasibility of the proposed model and its solution method are evaluated through a given industrial case study.

The remainder of this paper is arranged as follows. In the next Section "A mathematical formulation of problem", the description of interested APP problem and its assumptions, as well as associated model formulation, are presented. Subsequently, the proposed solving methodology (the background of Possibilistic Programming (PP), Robust

Programming (RP), and AUGMECON technique) and the solving procedures of algorithm are provided in Section "Solution approach". Then, the information of an industrial case study of considered APP problem for evaluating the applicability of the proposed solving methodology is described in Section "Case study". In Section "Results and discussions", the computational results, analysis, discussions, managerial implications of the APP problem, and its proposed solving methodology are reported. Finally, the conclusions, limitations, and future research directions are stated in Section "Conclusions".

MATHEMATICAL FORMULATION OF PROBLEM

I. Problem Description

A manufacturing company produces a quantity of product g, $g \in G$ to fulfill the customer demand in each period of time t, $t \in T$. The main target of the APP is to specify the most suitable production level for satisfying the customer demand based on the adjustment of production time (i.e., regular time, overtime), hiring and downsizing labor, backordering, subcontracting, inventory, and other controllable variables. The basic structure of the APP is depicted in Figure 1. The APP problem here is aimed to minimize the total costs of the APP plan, total change rate in workforce levels, and total holding inventory and backorder quantities simultaneously. In addition, the problem is considered in an uncertain environment. Hence, the parameters (such as customer demand, operation costs, labor level, machine capacity) can vary subject to triangular fuzzy numbers in each planning period.

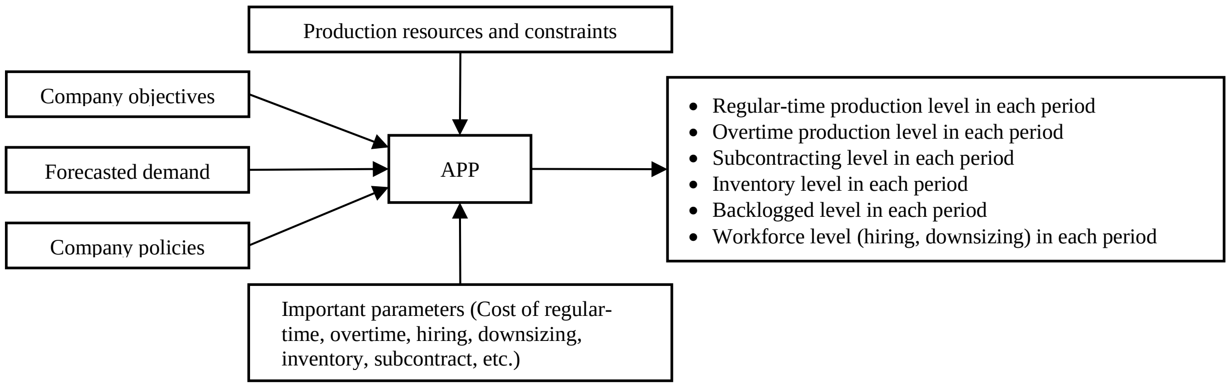

FIGURE 1
A BASIC APP PROCESS, ITS INPUTS AND OUTPUTS

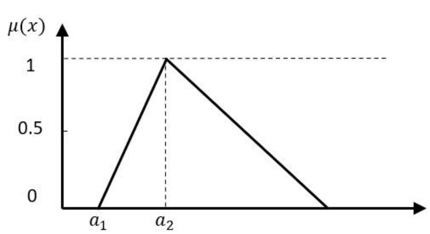

FIGURE 2
TRIANGULAR (POSSIBILITY) DISTRIBUTION

II. Assumptions of Mathematical Model

To solve the interested fuzzy multi-objective mathematical APP model, some assumptions should be adhered:

1. The demand for products is known but it is imprecise.
2. The demand for products could either be totally fulfilled or be backordered. However, the backordered quantities of products must be compensated in the following period.
3. The inventory quantities of products are specified at the beginning as well as the end of the planning horizon.
4. At the beginning of the first planning period, an initial labor level is known.
5. Overtime production and subcontracting are allowed.
6. Total budget, machine capacity, labor levels, warehouse space at the manufacturing plant are bounded by their equivalent maximum levels.
7. Subcontracting and backordering levels cannot exceed their maximum acceptable levels.
8. All objective functions and constraints are constructed by linear forms.
9. All imprecise parameters of the model are assumed to be triangular fuzzy numbers.

III. Notations and Mathematical Model

The notations and mathematical formulation are presented as below. It is noted that these fuzzy parameters are displayed with a tilde on.

Notations

Index sets:

g	Set of produced products, $g = 1,\ldots,G$
t	Set of planning horizon, $t = 1,\ldots,T$
j	Set of objective function, $j = 1,\ldots,J$
l	Set of fuzzy constraints, $l = 1,\ldots,L$

Objective functions:

$\tilde{Z}_1$	Total costs ($)
Z_2	Change in workforce level (persons)
Z_3	Holding and backordering quantities (units)

Parameters:

$\widetilde{De}_{gt}$	Forecasted demand for product g-th in period t (units)
$\tilde{r}_{gt}$	Manufacturing expense for product g-th in regular-time at period t ($/unit)
$\tilde{o}_{gt}$	Manufacturing expense for product g-th in overtime in period t ($/unit)
$\tilde{s}_{gt}$	Subcontracting expense for product g-th in period t ($/unit)
$\tilde{\iota}_{gt}$	Holding inventory expense for product g-th in period t ($/unit)
$\tilde{b}_{gt}$	Backordering charge for product g-th in period t ($/unit)
$\tilde{h}_t$	Hiring rate for a worker in period t ($/person-hour)
$\tilde{f}_t$	Downsizing rate for a worker in period t ($/person-hour)
$\widetilde{MaxL}_t$	Workforce level limit in period t (person-hours)
$\widetilde{MaxM}_t$	Capacity limit of machine in period t (machine-hours)
$\widetilde{mh}_{gt}$	Machine's required hours for manufacturing product g-th at period t (machine-hours/unit)
wh_{gt}	Worker's required hours for manufacturing product g-th at period t (person-hours/unit)
$MaxW_t$	Warehouse space limit in period t (ft^2/unit)
ws_{gt}	Storage spaces for a unit of product g-th in period t (ft^2/unit)
QS_{gt}^{max}	Limit for subcontracted amount of product g-th in period t (units)
QB_{gt}^{max}	Limit for backordered amount of product g-th in period t (units)
Z_{budget}	Available budget ($)

Decision variables:

QR_{gt}	Quantity of product g-th manufactured in regular-time in period t (units)
QO_{gt}	Quantity of product g-th manufactured in overtime at period t (units)
QS_{gt}	Quantity of product g-th subcontracted in period t (units)
QI_{gt}	Quantity of holding inventory product g-th in the end of period t (units)
QB_{gt}	Quantity of backordered product g-th in period t (units)
NH_t	Number of hired workers in period t (person-hours)
NF_t	Number of downsized workers in period t (person-hours)
α_l	Confidence levels (satisfaction levels) of fuzzy constraint l.

Mathematical model

Objective functions:

- *Minimize total costs*

In practice, the cost minimization is considered as a common decision that is made for solving the APP problem. Usually, the total costs of APP problem are the sum of the manufacturing cost, inventory cost, backordering cost, and costs of changing workforce levels over a time period T. However, the coefficients of costs in the objective function could be unclear (imprecise) due to some information being estimated, unobtainable or incomplete. Accordingly, the objective function of our APP problem is formulated in the following equations:

$$Min\ \widetilde{Z_1} = \sum_{g=1}^{G} \sum_{t=1}^{T} \left(\tilde{r}_{gt}QR_{gt} + \tilde{o}_{gt}QO_{gt} + \tilde{s}_{gt}QS_{gt} + \tilde{\iota}_{pt}QI_{gt} + \tilde{b}_{gt}QB_{gt}\right) + \sum_{t=1}^{T} \left(\tilde{h}_t NH_t + \tilde{f}_t NF_t\right) \quad (1)$$

The production costs are shown in the first five terms including regular-time and overtime production costs, subcontract cost, inventory holding cost, and backordering cost. The remaining portion indicates the costs of changing workforce levels, which are the costs of hiring and downsizing workers, where $\tilde{r}_{pt}$, $\tilde{o}_{pt}$, $\tilde{s}_{pt}$, $\tilde{\iota}_{pt}$, $\tilde{b}_{pt}$, $\tilde{h}_t$, and $\tilde{f}_t$ are fuzzy parameters with the triangular possibility distribution.

- *Minimize total changes in labor level*

Under realistic circumstances of APP, the requirement about workforce can be easily calculated by aggregating the forecasted demand beforehand. Nevertheless, companies would find it difficult to have a much varying workforce planning due to labor skills, labor regulations, and other restrictions associated to the social welfare of the workforce. Moreover, the fluctuations of hiring and downsizing labor also result in a negative influence on labor productivity. Thus, it is necessary to manage and maintain the fluctuation in workforce levels as low as possible. To have a smoother workforce planning and to eliminate the negative impact of hiring and downsizing, the second objective function is considered as follows:

$$Min\ Z_2 = \sum_{t}^{T}(NH_t + NF_t) \quad (2)$$

Equation (2) presents the second objective function that minimizes the difference between the downsizing and hiring number of workers.

- *Minimize total holding inventory and backordered quantities*

According to the philosophy of Just-in-Time (JIT) in manufacturing planning and control, JIT is an approach that is used to minimize waste in the production system. One outstanding waste in manufacturing is inventory. Holding too high amount of inventory causes the company excessively high operating costs such as storage cost, occupied space cost, management cost. etc. Besides, there is always the limitation of warehouse space as well as the budget for investing in warehouse facilities. Thus, there is an attempt to reduce the inventory as low as possible to eliminate this waste. However, when the inventory level becomes too low, it could conversely lead to backorders (if the backlog is allowed) when the demand cannot be fulfilled because of a lack of available products). Therefore, when minimizing the total holding inventory quantity, the backordered quantity should also take into consideration to be minimized along with inventory quantity. With a summation of both quantities, both inventory level and backorder quantities can be minimized otherwise the obtained plan can be twisted by pursuing too much either on minimizing inventory or backordered quantities.

$$Min\ Z_3 = \sum_{g}^{G}\sum_{t=1}^{T}\left(QI_{gt} + QB_{gt}\right) \quad (3)$$

The third objective function is minimizing the sum of holding inventory and backordered quantities of all products in all planning periods, as presented in Equation (3).

These above objective functions are subject to the following constraints.

Constraints:

- *Carrying inventory constraint*

$$\widetilde{De}_{gt} = IQ_{gt-1} - QB_{gt-1} + QR_{gt} + QO_{gt} + QS_{gt} - QI_{gt} + QB_{gt}; \quad \forall G, \forall T \quad (4)$$

$$QB_{gt} \le QB_{gt}^{max}; \quad \forall G, \forall T \quad (5)$$

The forecasted demand of a customer cannot be obtained exactly in the real world. Therefore, $\widetilde{De}_{gt}$ denotes for fuzzy estimated product demand g in each period t. Constraint (4) shows that the total amount of products, including inventory amount of products, regular time and overtime production amount of products, subcontracting quantities, and backordering quantities primarily must fulfill the forecasted demand. The demand can be either met or backordered in a specific period, but a backorder in the following period must be fulfilled. Constraint (5) presents that the backordering quantities of product g in period t that are limited by the allowed maximum backordered level.

- *Labor level constraints*

$$\sum_{g=1}^{G} wh_{gt-1}\left(QR_{gt-1} + QO_{gt-1}\right) + NH_t - NF_t - \sum_{g=1}^{G} wh_{gt}\left(QR_{gt} + QO_{gt}\right) = 0; \quad \forall T \quad (6)$$

$$\sum_{g=1}^{G} wh_{gt}\left(QR_{gt} + QO_{gt}\right) \le \widetilde{MaxL}_t; \quad \forall T \quad (7)$$

where Constraint (6) reflects the net changes, i.e., hiring and downsizing workers, of the level of workforce in the current period t from that of the previous t-*1*. Constraint (7) shows that the actual level of labor in period t is limited by the maximum available labor level. The maximum available labor level could be inaccurate because of the uncertain conditions of supply, demand, and labor skills in the market.

- *Machine capacity constraint*

$$\sum_{g=1}^{G} \widetilde{mh}_{gt}\left(QR_{gt} + QO_{gt}\right) \le \widetilde{MaxM}_t; \quad \forall T \quad (8)$$

$$QS_{gt} \le QS_{gt}^{max}; \quad \forall G, \forall T \quad (9)$$

where $\widetilde{mh}_{gt}$ and $\widetilde{MaxM}_t$ are imprecise data of the machine hour usage per a product g, and the maximum capacity limit of the machine in each period, respectively. Constraint (8) is set to limit the available machine capacity, where the hours of machine usage to produce all types of product in period t must not exceed the machine capacity limit. Likewise, this capacity limit could be fuzzy (in reality) since the machine's available hours could be influenced by the availability as well as operating conditions of machines at any one time. Constraint (9) indicates that the subcontracted amount of product g in each period t that is limited by the maximum allowable subcontracted level.

- *Warehouse capacity constraint*

$$\sum_{g=1}^{G} ws_{gt}QI_{gt} \le MaxW_t; \quad \forall T \quad (10)$$

Constraint (10) presents the storage restriction of the warehouse in period t. That is, the amount of space for storing all units in each period t must not surpass the respective maximum available warehouse space.

- *Total budget constraint*

$$\widetilde{Z_1} \le Z_{budget} \quad (11)$$

Constraint (11) ensures that the total costs of the aggregate production plan are not allowed to exceed the total budget set by the company.

- *Non-negativity constraint*

$$QR_{gt}, QO_{gt}, QS_{gt}, QI_{gt}, QB_{gt}, NH_t, NF_t \ge 0; \quad \forall G, \forall T \quad (12)$$

Constraint (12) indicates that all decision variables are positive numbers.

SOLUTION APPROACH

This section demonstrates how the interested fuzzy multi objective mathematical APP model to address with uncertainty and multi-objective decision making. The controllable variables in APP model are theoretically assumed as deterministic and known in advance, but this is not true in the real world. Due to unobtainable or incomplete information or conditions, APP input parameters, e.g., forecasted market demand, relevant operation costs, and machine or warehouse capacities, are normally uncertain. Besides, these parameters have a significant influence on the strategy of the entire APP plan. Thus, the uncertainty of input data is one of the difficult issues that the DMs must face in planning.

In order to handle these uncertain parameters, Robust Possibilistic Programming (RPP), which is a combination of Possibilistic Chance-Constrained Programming (PCCP) and Robust Programming (RP), is adopted. PCCP is capable of controlling the confidence level of satisfaction of possibilistic chance constraints (risk violation of constraints). However, the main drawback of PCCP is that it cannot control the risk of obtaining a worse objective function because the objective function value is computed by the expected (average) value of uncertain parameters. In this regard, RP is introduced to incorporate with PCCP to solve the drawback of PCCP. Generally, in the solving process of RPP, the uncertainty can cause two concerns: feasibility aspect and optimality aspect. The feasibility aspect concerns managing the relationship between the possibilistic fuzzy in both two sides of constraints while the optimality aspect concerns finding the optimal value of the fuzzy objective function. Having gained the advantages of both PCCP and RP, the proposed RPP can yield a solution with both aspects of feasibility and optimality (robust solution).

Next, to handle the issue of multiple objectives, the AUGMECON technique is applied. This technique is well-known as a posteriori method that is capable of producing efficient compromised solutions (a set of Pareto solutions) for the managers and DMs before making their final decision. From Pareto optimal solutions, the managers and DMs can then select the most suitable solution according to their aspiration level or they can select the best compromised (trade-off) solution by using the Max-Min method. A brief explanation of the proposed RPP, AUGMECON, and Max-Min approaches is presented in the next sub-section.

I. Robust Programming (RP)

RP is well-known as an efficient approach with the capability of providing risk-averse results in optimization problems under uncertain environments. The concept of robustness in a mathematical programming model is defined based on two separate parts, which are "robustness of feasible solution" and "robustness of optimal solution". The robustness of a feasible solution implies that the solution of all the possible values of the uncertain parameters should be feasible. It makes an effort to minimize the violation and infeasibility of soft constraints in the mathematical programming model under uncertain conditions. The robustness of the optimal solution implies that the solution of the objective function should be near to the optimal ideal solution. In other words, it minimizes the maximum distance from the optimal ideal solution to the obtained solution of the objective function for most of the possible values of uncertain parameters. Once a solution is simultaneously satisfied with these two types of robustness, it is called a robust solution [29]. So, they are considered to be two main elements that are used to adjust the level of risk in output decisions.

As stated in the study of Pishvaee et al. [20], RP approaches are categorized into three different types, which consist of (1) hard worst situation, (2) soft worst situation, and (3) realistic situation RP. In the hard worst situation RP approach, since the worst values of uncertain parameters are used to calculate for both objective function and constraints of the mathematical model, this approach can yield the solution with the highest level of safety to cope with any change in uncertain condition. For the soft worst situation robust programming, this approach is considered as a flexible form of the hard-worst situation approach. Although this approach uses the worst value of uncertain parameters for finding the minimum value of the objective function in the mathematical model, some constraints of the model are allowed to violate with a specified acceptable level (constraint violation concepts). The realistic robust programming is an approach that can institute a rational trade-off for the obtained robustness level of solution in the relationship between objective function and constraints, which is the robustness level of optimality and feasibility. It should be noted that both hard and soft worst situations of RP are special situations of the realistic RP situation. The advantages and disadvantages of these RP situations have been discussed and pointed out by the study of Pishvaee et al. [20]. Also, throughout this study, it is shown that the realistic situation of the robust possibilistic programming outperforms the others. Since this approach can not only generate feasible solutions corresponding to the possible value of uncertain parameters (from the most likely situation to the worst situation) but also attempt to find the solution that has the minimum deviation from the ideal optimal solution. Therefore, this approach is considered as a rational compromise between the robustness of feasibility solution and optimality solution.

II. Possibilistic Programming (PP) Model

In practical applications, collected input data are largely stained by epistemic uncertainty. To effectively control epistemic uncertainty, the PP model has been introduced. PCCP model is an improved form of the PP model, which is appropriate for dealing with uncertain problems. PCCP model is an immensely effective approach in that the DMs can manipulate the confidence levels of chance constraints in relation to the given situation [30-33].

Considering a basic mathematical formulation of optimization model under uncertainty as follows:

$$\begin{aligned} Min \quad & z = \tilde{c}x \\ S.t \quad & Ax \le \tilde{k} \\ & Sx = 0 \\ & \tilde{B}x \le \tilde{h} \\ & Dx = \tilde{g} \\ & x \ge 0 \end{aligned} \qquad (13)$$

where c denotes the parameter of objective function. A, S, B, D, k, h, and g represent the parameters on the left-hand and right-hand sides of the constraints. The parameters with the symbol (~) above the relevant letters are imprecise parameters that follow the triangular possibility distribution. x represents a continuous decision variable. To formulate a corresponding PCCP version of the model presented in Equation (13), the Expected Value (*EV*) operator is used for handling fuzzy parameters at the objective function while Credibility (*Cr*) measurement is used for handling fuzzy constraints. An explanation of *Cr* and *EV* is presented below.

$$\begin{aligned} Min \quad & EV[z] = EV[\tilde{c}]x \\ S.t \quad & Cr\{Ax \le \tilde{k}\} \ge \alpha_1 \\ & Sx = 0 \\ & Cr\{\tilde{B}x \le \tilde{h}\} \ge \alpha_2 \\ & Cr\{Dx = \tilde{g}\} \ge \alpha_3 \\ & x \ge 0;\ 0.5 \le \alpha_1, \alpha_2, \alpha_3 \le 1 \end{aligned} \qquad (14)$$

Expected value (EV) operator

Assume that a triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ is specified by the DMs, whereby a_1, a_2, a_3 denote the optimistic, the most likely, and the pessimistic values of the triangular fuzzy number, correspondingly (see Figure 2). The membership function of fuzzy number $\tilde{c}$ can be determined by the following equations:

$$\mu_{\tilde{a}}(x) = \begin{cases} f_a(x) = \dfrac{x - a_1}{a_2 - a_1}, & if\ a_1 \le x \le a_2 \\ 1 & if\ x = a_2 \\ g_a(x) = \dfrac{a_3 - x}{a_3 - a_2}, & if\ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases} \qquad (15)$$

The EV operator is deployed for converting the uncertain objective function to an equivalent crisp form. According to [34] and [35], if $\tilde{a}$ is a triangular fuzzy number, the expected interval of fuzzy number $\tilde{a}$, denoted $EI(\tilde{a})$, the expected value $EV(\tilde{a})$ of $\tilde{a}$ can be defined as follows:

$$\begin{aligned} EI(\tilde{a}) &= [E_1^a, E_2^a] \\ &= \left[\int_0^1 f_a^{-1}(x)dx, \int_0^1 g_a^{-1}(x)dx\right] \\ &= \left[\tfrac{1}{2}(a_1 + a_2), \tfrac{1}{2}(a_2 + a_3)\right] \end{aligned} \qquad (16)$$

$$EV(\tilde{a}) = \frac{E_1^a + E_2^a}{2} = \frac{a_1 + 2a_2 + a_3}{4}$$

Credibility measure

In PCCP models, possibility and necessity reflect the two extremes of an uncertain parameter. Possibility measure is used to indicate the most optimistic value, whereas necessity measure is used to represent the most pessimistic value [36]. Let $\tilde{a}$ be a fuzzy variable with membership function μ and let *r* and *b* be real numbers. The possibility of a fuzzy event, denoted by *b*, can be specified by:

$$Pos\{\tilde{a} \le b\} = \sup_{r \le b} \mu(r) \qquad (17)$$

The necessity degree of occurrence of this fuzzy event is specified by the following equation:

$$Nec\{\tilde{a} \le b\} = 1 - Pos\{\tilde{a} \le b\} = 1 - \sup_{r > b} \mu(r) \qquad (18)$$

The measures of possibility and necessity, i.e., $Pos\{\cdot\}$ and $Nec\{\cdot\}$, reflect the extreme optimistic and pessimistic attitudes, respectively. However, the extreme attitudes are not preferable in practices. Thus, to reflect an intermediate attitude of the DMs between these two extremes, another measure, known as the credibility measure, is introduced. The credibility measure (*Cr*) is computed by averaging the possibility and necessity measurements as shown below:

$$\begin{aligned} Cr\{\tilde{a} \le b\} &= \frac{1}{2}(Pos\{\tilde{a} \le b\} + Nec\{\tilde{a} \le b\}) \\ &= \frac{1}{2}\left(\sup_{r \le b}\mu(r) + 1 - \sup_{r > b}\mu(r)\right) \end{aligned} \qquad (19)$$

According to the above equations, the possibility, necessity of $\{\tilde{a} \le b\}$ can be shown as follows:

$$\begin{aligned} Pos\{\tilde{a} \le b\} &= \begin{cases} 0, & b \le a_1 \\ \dfrac{b - a_1}{a_2 - a_1}, & a_1 \le b \le a_2 \\ 1, & b \ge a_2 \end{cases} \\ Nec\{\tilde{a} \le b\} &= \begin{cases} 0, & b \le a_2 \\ \dfrac{b - a_2}{a_3 - a_2}, & a_2 \le b \le a_3 \\ 1, & b \ge a_3 \end{cases} \end{aligned} \qquad (20)$$

Credibility is the power of being able to be believed or trustworthy. When the credibility value is 1, an event is almost certain to occur [37]. The credibility of $\{\tilde{a} \leq b\}$ is presented by:

$$Cr\{\tilde{a} \leq b\} = \begin{cases} 0, & b \leq a_1 \\ \dfrac{b - a_1}{2(a_2 - a_1)}, & a_1 \leq b \leq a_2 \\ \dfrac{a_3 - 2a + b}{2(a_3 - a_2)}, & a_2 \leq b \leq a_3 \\ 1, & b \geq a_3 \end{cases} \quad (21)$$

Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$. Following the definition of credibility and fuzzy operations, the credibility of a fuzzy event characterized by $\{\tilde{a} \leq \tilde{b}\}$ can be demonstrated as follows:

$$Cr\{\tilde{a} \leq \tilde{b}\} = \begin{cases} 1, & a_3 \leq b_1 \\ \dfrac{a_3 - 2a_2 + 2b_2 - b_1}{2(a_3 - a_2 + b_2 - b_1)}, & a_2 \leq b_2, a_3 > b_1 \\ \dfrac{b_3 - a_1}{2(b_3 - b_2 + a_2 - a_1)}, & a_2 > b_2, a_1 < b_3 \\ 0, & a_1 \geq b_3 \end{cases} \quad (22)$$

Based on Equations (21) and (22), it can be shown that for $(0 \leq \alpha \leq 0.5)$:

$$\begin{aligned} & Cr\{\tilde{a} \leq b\} \geq \alpha \Leftrightarrow b \geq (1 - 2\alpha)a_1 + (2\alpha)a_2 \\ & Cr\{\tilde{a} \leq \tilde{b}\} \geq \alpha \Leftrightarrow (1 - 2\alpha)a_1 + (2\alpha)a_2 \leq (2\alpha)b_2 + (1 - 2\alpha)b_3 \end{aligned} \quad (23)$$

It can also be demonstrated that for $(0.5 \leq \alpha \leq 1)$:

$$\begin{aligned} & Cr\{\tilde{a} \leq b\} \geq \alpha \Leftrightarrow b \geq (2 - 2\alpha)a_2 + (2\alpha - 1)a_3 \\ & Cr\{\tilde{a} \leq \tilde{b}\} \geq \alpha \Leftrightarrow (2 - 2\alpha)a_2 + (2\alpha - 1)a_3 \leq (2\alpha - 1)b_1 + (2 - 2\alpha)b_2 \end{aligned} \quad (24)$$

Lastly, according to the proposed operators of the *EV* and *Cr* measure, an analogous crisp form of the provided uncertain optimization model in Equation (14) is derived as follows:

$$\begin{aligned} Min \quad & EV[\tilde{c}]x = \left(\frac{c_1 + 2c_2 + c_3}{4}\right)x \\ S.t \quad & Ax \leq (2\alpha_1 - 1)k_1 + (2 - 2\alpha_1)k_2 \\ & Sx = 0 \\ & [(2 - 2\alpha_2)B_2 + (2\alpha_2 - 1)B_3]x \leq (2\alpha_2 - 1)h_1 + (2 - 2\alpha_2)h_2 \\ & Dx = (2 - 2\alpha_3)g_2 + (2\alpha_3 - 1)g_3 \\ & x \geq 0;\ 0.5 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1 \end{aligned} \quad (25)$$

in which α_l, $l \in [1, L]$ are the minimum acceptable confidence levels (satisfaction levels) of chance constraints that could be specified based on the preferences of DMs. The uncertain objective function is calculated by expected value in which the attribute of the uncertain objective function cannot be interpreted. As a result, the DMs could face with high risks in some practical situations. This is also the main deficiency of the PCCP model. To overcome this deficiency, the RPP model is applied.

Applying the PCCP model shown in Equation (25), the crisp corresponding formulation of the PCCP model for the Multi-Objective Aggregate Production Planning (MOAPP) problem can be derived as shown below:

$$\begin{aligned} MinE[\widetilde{Z_1}] = \sum_{g=1}^{G} \sum_{t=1}^{T} & \left[\left(\frac{r_{gt}^1 + 2r_{gt}^2 + r_{gt}^3}{4}\right) QR_{gt} + \left(\frac{o_{gt}^1 + 2o_{gt}^2 + o_{gt}^3}{4}\right) QO_{gt} + \left(\frac{s_{gt}^1 + 2s_{gt}^2 + s_{gt}^3}{4}\right) QS_{gt} \right. \\ & \left. + \left(\frac{i_{gt}^1 + 2i_{gt}^2 + i_{gt}^3}{4}\right) QI_{gt} + \left(\frac{b_{gt}^1 + 2b_{gt}^2 + b_{gt}^3}{4}\right) QB_{gt}\right] \\ & + \sum_{t=1}^{T} \left[\left(\frac{h_t^1 + 2h_t^2 + h_t^3}{4}\right) NH_t + \left(\frac{f_t^1 + 2f_t^2 + f_t^3}{4}\right) NF_t\right] \end{aligned}$$

$$Min\ Z_2 = \sum_{t}^{T} (NH_t + NF_t)$$

$$Min\ Z_3 = \sum_{g}^{G} \sum_{t=1}^{T} (QI_{gt} + QB_{gt})$$

Subject to:

$$(2 - 2\alpha_1)De_{gt}^2 + (2\alpha_1 - 1)De_{gt}^3 = IQ_{gt-1} - QB_{gt-1} + QR_{gt} + QO_{gt} + QS_{gt} - QI_{gt} + QB_{gt};\ \forall G, \forall T$$

$$QB_{gt} \leq QB_{gt}^{max};\ \forall G, \forall T$$

$$\sum_{g=1}^{G} wh_{gt-1}(QR_{gt-1} + QO_{gt-1}) + NH_t - NF_t - \sum_{g=1}^{G} wh_{gt}(QR_{gt} + QO_{gt}) = 0;\ \forall T$$

$$\sum_{g=1}^{G} wh_{gt}(QR_{gt} + QO_{gt}) \leq (2\alpha_2 - 1)MaxL_t^1 + (2 - 2\alpha_2)MaxL_t^2;\ \forall T$$

$$\sum_{g=1}^{G} [(2 - 2\alpha_3)mh_{gt}^2 + (2\alpha_3 - 1)mh_{gt}^3](QR_{gt} + QO_{gt}) \leq (2\alpha_3 - 1)MaxM_t^1 + (2 - 2\alpha_3)MaxM_t^2;\ \forall T$$

$$QS_{gt} \leq QS_{gt}^{max};\ \forall G, \forall T$$

$$\sum_{g=1}^{G} ws_{gt} QI_{gt} \leq MaxW_t;\ \forall T$$

$$E[\widetilde{Z_1}] \leq Z_{budget}$$

$$QR_{gt}, QO_{gt}, QS_{gt}, QI_{gt}, QB_{gt}, NH_t, NF_t \geq 0;\ \forall G, \forall T$$

$$0.5 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$$

III. Robust Possibilistic Programming (RPP) Models

According to [20], RPP model is divided into six forms (RPP-I, RPP-II, RPP-III, MRPP, HWRPP, and SWRPP) in which RPP-II is considered to be the most efficient approach. The effectiveness of this approach has also been verified through the studies of Mousazadeh et al. [38] and Fazli-Khalaf et al. [39]. Therefore, the form of RPP-II is applied in this study.

Following the PCCP model, the Robust Possibilistic Programming model (RPP-II) can be presented as follows:

$$\begin{aligned} Min \quad & EV[z] + \gamma(Z_{max} - EV[z]) \\ & +\delta[(2\alpha_1 - 1)k_1 + (2 - 2\alpha_1)k_2 - k_1] \\ & + \pi[B_3 - (2 - 2\alpha_2)B_2 + (2\alpha_2 - 1)B_3]x \\ & + \rho[(2\alpha_2 - 1)h_1 + (2 - 2\alpha_2)h_2 - h_1] \\ & + \sigma[g_3 - (2 - 2\alpha_3)g_2 + (2\alpha_3 - 1)g_3] \\ S.t \quad & Ax \leq (2\alpha_1 - 1)k_1 + (2 - 2\alpha_1)k_2 \\ & Sx = 0 \\ & [(2 - 2\alpha_2)B_2 + (2\alpha_2 - 1)B_3]x \\ & \qquad \leq (2\alpha_2 - 1)h_1 + (2 - 2\alpha_2)h_2 \\ & Dx = (2 - 2\alpha_3)g_2 + (2\alpha_3 - 1)g_3 \\ & x \geq 0;\ 0.5 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1 \end{aligned} \tag{26}$$

where Z_{max} presents the objective function's value in the pessimistic case (worst case). It is calculated as follows:

$$Z_{max} = c_3 x \tag{27}$$

From Equation (26), it can be seen that there are four more terms added along with the first term in the objective function. While the first term minimizes the expected value (average) of uncertain parameters, the second term represents the distance from the expected value to the worst value of the objective function. This term is used to control the robustness of the solution (optimality) through minimizing the maximum deviation of the expected and worst value. Besides, to compensate against the other terms, it is multiplied by the important weight γ. The remaining terms are related to the robustness of feasibility, which are in an effort for specifying the appropriate confidence levels of fuzzy constraints (chance constraints). These terms indicate the distance from the worst-case value of uncertain parameters to the current value that are used in the equivalent chance constraints. In these terms, δ, π, ρ and σ correspond to the penalty unit of possible violated constraints that contain uncertain parameters. They can be determined appropriately based on the practical application situation. Differently from the PCCP model, the confidence levels (α_l) of chance constraints in the RPCCP model now become variables and they are able to be determined automatically by the model.

RPP linearization

As it can be realized that the chance constraints and objective function in Equation (26) become nonlinear with the imprecise technological coefficients. Hence, to solve the nonlinear model, it is required to be transformed into a linear one with an auxiliary variable $M = \alpha_2 x$. Then, the above nonlinear model can be transformed into the corresponding linear one as follows [40]:

$$\begin{aligned} Min \quad & EV[z] + \gamma(Z_{max} - EV[z]) \\ & + \delta[(2\alpha_1 - 1)k_1 + (2 - 2\alpha_1)k_2 - k_1] \\ & + \pi[B_3 x - (2x - 2M)B_2 + (2M - x)B_3] \\ & + \rho[(2\alpha_2 - 1)h_1 + (2 - 2\alpha_2)h_2 - h_1] \\ & + \sigma[g_3 - (2 - 2\alpha_3)g_2 + (2\alpha_3 - 1)g_3] \\ S.t \quad & Ax \leq (2\alpha_1 - 1)k_1 + (2 - 2\alpha_1)k_2 \\ & Sx = 0 \\ & [(2 - 2\alpha_2)B_2 + (2\alpha_2 - 1)B_3]x \\ & \qquad \leq (2\alpha_2 - 1)h_1 + (2 - 2\alpha_2)h_2 \\ & Dx = (2 - 2\alpha_3)g_2 + (2\alpha_3 - 1)g_3 \\ & 0.5x \leq M \leq x \\ & x \geq 0;\ 0.5 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1 \end{aligned} \tag{28}$$

IV. Multi-Objective Robust Possibilistic Programming (MORPP) Model

The MORPP can be presented as follows:

$$\begin{aligned} Min \quad & \psi_1(x) = EV[z] + \gamma(Z_{max} - EV[z]) \\ & + \delta[(2\alpha_1 - 1)k_1 + (2 - 2\alpha_1)k_2 - k_1] \\ & + \pi[B_3 x - (2x - 2M)B_2 + (2M - x)B_3] \\ & + \rho[(2\alpha_2 - 1)h_1 + (2 - 2\alpha_2)h_2 - h_1] \\ & + \sigma[g_3 - (2 - 2\alpha_3)g_2 + (2\alpha_3 - 1)g_3] \\ Min \quad & \psi_2(x) \\ & \ldots \\ Min \quad & \psi_j(x) \\ S.t \quad & Ax \leq (2\alpha_1 - 1)k_1 + (2 - 2\alpha_1)k_2 \\ & Sx = 0 \\ & [(2 - 2\alpha_2)B_2 + (2\alpha_2 - 1)B_3]x \\ & \qquad \leq (2\alpha_2 - 1)h_1 + (2 - 2\alpha_2)h_2 \\ & Dx = (2 - 2\alpha_3)g_2 + (2\alpha_3 - 1)g_3 \\ & 0.5x \leq M \leq x \\ & x \geq 0;\ 0.5 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1 \end{aligned} \tag{29}$$

The epsilon-constraint technique is utilized in this study to handle the conflicting multi-objective mathematical model. This technique is well-known as the most efficient approach for multi-objective problems. The highlighted feature of the epsilon-constraint technique is that it can seek to produce Pareto optimal solutions (non-dominated optimal solutions) in relation to the conflicting objective functions. With a multi-objective mathematical model, the concept of the epsilon-constraint technique is to optimize one objective function considered as the main one or has the highest priority while the other objective functions are converted to be corresponding constraints and bounded with an amount of epsilon.

When using the epsilon-constraint technique, two crucial things need to be regarded. Firstly, the range of each objective function must be defined over an efficient set, which is typically between two extreme points of each objective function (Negative Ideal Solution - NIS, and Positive Ideal Solution - PIS). Normally, the value of NIS and PIS of each objective function can be determined based on the pay-off table (the table with the PIS value that is obtained by separately optimizing each objective function while the NIS value of each objective function is estimated by the minimum or maximum value in the equivalent column) [41,42]. Secondly, the Right-Hand Side (RHS) of epsilon-constraints (epsilon values), where these values should be changed systemically in each objective function's range to generate various Pareto solutions, must be determined. One of the deficiencies in the basic epsilon-constraint technique is that it could provide weak Pareto optimal solutions [43]. In order to improve this deficiency and ensure the efficiency of solutions, Mavrotas [43] proposed a new form of epsilon-constraint technique, which is called the AUGMECON technique. The advantages and disadvantages of this method have been discussed in the study of Mavrotas [43]. The definitions of dominated, weak and strong Pareto solutions are also presented in Appendix.

Based on the efficient simple AUGMECON technique, the aforementioned MORPP model is transformed into a single objective one as presented below:

$$
\begin{aligned}
Min\quad & \psi_1(x) = EV[z] + \gamma(Z_{max} - EV[z]) \\
& - \varphi\left(\frac{s_2(x)}{r_2} + \frac{s_3(x)}{r_3} + \ldots + \frac{s_j}{r_j}\right) \\
& + \delta[(2\alpha_1 - 1)k_1 + (2 - 2\alpha_1)k_2 - k_1] \\
& + \pi[B_3x - (2x - 2M)B_2 + (2M - x)B_3] \\
& + \rho[(2\alpha_2 - 1)h_1 + (2 - 2\alpha_2)h_2 - h_1] \\
& + \sigma[g_3 - (2 - 2\alpha_3)g_2 + (2\alpha_3 - 1)g_3] \\
S.t\quad & \psi_2(x) = \varepsilon_2 - s_2 \\
& \psi_3(x) = \varepsilon_3 - s_3 \\
& \ldots \\
& \psi_j(x) = \varepsilon_j - s_j \\
& Ax \leq (2\alpha_1 - 1)k_1 + (2 - 2\alpha_1)k_2 \\
& Sx = 0 \\
& [(2 - 2\alpha_2)B_2 + (2\alpha_2 - 1)B_3]x \\
& \qquad \leq (2\alpha_2 - 1)h_1 + (2 - 2\alpha_2)h_2 \\
& Dx = (2 - 2\alpha_3)g_2 + (2\alpha_3 - 1)g_3 \\
& 0.5x \leq M \leq x \\
& x \geq 0;\ 0.5 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1;\ s_j \geq 0
\end{aligned}
\tag{30}
$$

where $\varepsilon_2, \varepsilon_3, ..\varepsilon_j$ are the RHS values of the constrained objective functions in epsilon-constraint. φ is a very small number commonly determined between [10^{-3}; 10^{-6}]. In order to ensure the efficiency of obtained results, the rational surplus and slack variables are added into the constraints of the objective functions and set as equality constraints. Besides which, these variables are integrated as a second term into the objective function with a lower importance. Then, to eliminate the scaling problem in the second term, the surplus variables of respective objective functions, denoted as $s_2, s_3, \ldots s_j$, are divided by the range of respective objective functions, denoted as $r_2, r_3, \ldots r_j$.

In the AUGMECON technique, the value of epsilon of objective functions is needed to be varied systemically so that new Pareto optimal solutions can be generated over the efficient set. To do that, after the range of each objective function is determined, it will be divided into equal portions (p_j) and then the total grid points ($p_j + 1$) are utilized for varying the epsilon value of objective functions. With the range of objective function r_j $(2, ..j)$, the discretization step for the respective objective function can be computed by:

$$Step_j = (r_j / p_j) \tag{31}$$

The RHS of the respective constraint in the *f*-th iteration in a particular objective function is computed as follows:

$$\varepsilon_j^l = \psi_j^{min} + (f \times Step_j); \quad f = 0, \ldots, p_j \tag{32}$$

in which ψ_j^{min} is the minimum value of the *j*-th objective function (NIS) that is determined by the pay-off table and l is the counter of a particular objective function.

The surplus variable corresponding to the innermost objective function will be checked after each iteration. For instance, considering the second objective function (j = 2), the bypass coefficient can be computed by the following equation:

$$bp = int\ (s_2 / Step_2) \tag{33}$$

where the $int()$ function is used to return the integer value of a real number. Once the value of $Step_2$ is smaller than the surplus variable s_2, this means that a similar solution will be achieved in the next iteration. The difference is that the value of the surplus variable will be $s_2 - Step_2$. This leads to redundant iterations due to there is no new generated Pareto optimal solution. Hence, these redundant iterations should be bypassed in the solving process. The number of consecutive iterations that can be skipped is indicated by the bypass coefficient bp.

V. The Max-Min Method

The Max-Min method is implemented to choose the best compromised (trade-off) solution among objectives, the formulation of selecting the final solution can be set as follows:

$$BTS = \sum_{j=1}^{J} \frac{Z_j}{Z_j^{PIS}} \tag{34}$$

where Z_j^{PIS} $(j = 1, \ldots, J)$ is the PIS of objective function.

VI. Performance Measures

Usually, the performance of a multiple-objective model is evaluated by the actual values of objectives. However, their performances are not able to directly compare with each other if they have different units or scales. Hence, the objective functions should be normalized into a common scale (from 0.0 to 1.0) by using the membership function, which is called a satisfaction level. The satisfaction levels of objective functions can be presented by the following equations.

The satisfaction level of minimizing objective function:

$$\mu_{z_j} = \begin{cases} 1 & , z_j \le z_j^{PIS} \\ \frac{z_j^{NIS} - z_j}{z_j^{NIS} - z_j^{PIS}} & , z_j^{PIS} \le z_j \le z_j^{NIS} \\ 0 & , z_j \ge z_j^{NIS} \end{cases} \quad (35)$$

The satisfaction level of maximizing objective function:

$$\mu_{z_j} = \begin{cases} 1 & , z_j \ge z_j^{PIS} \\ \frac{z_j - z_j^{NIS}}{z_j^{PIS} - z_j^{NIS}} & , z_j^{NIS} \le z_j \le z_j^{PIS} \\ 0 & , z_j \le z_j^{NIS} \end{cases} \quad (36)$$

VII. Multi-Objective Robust Possibilistic Programming (MORPP) Model for APP Problems

Following above descriptions, the final equivalent MOAPP model into the formulation of proposed RPCCP incorporating AUGMECON technique is presented as follows:

$$\begin{aligned} Min \quad & EV[\widetilde{Z_1}] - \varphi\left(\frac{s_2}{r_2} + \frac{s_3}{r_3}\right) + \gamma\left(Z_1^{max} - EV[\widetilde{Z_1}]\right) \\ & + \sigma \textstyle\sum_g^G \sum_t^T \left[De_{gt}^3 - (2 - 2\alpha_1)De_{gt}^2 - (2\alpha_1 - 1)De_{gt}^3\right] \\ & + \delta \textstyle\sum_t^T [(2\alpha_2 - 1)MaxL_t^1 + (2 - 2\alpha_2)MaxL_t^2 - MaxL_t^1] \\ & + \pi\big[\textstyle\sum_g^G \sum_t^T \big(mh_{gt}^3 QR_{gt} - (2QR_{gt} - 2M_{gt}^1)mh_{gt}^2 \\ & \qquad + (2M_{gt}^1 - QR_{gt})mh_{gt}^3\big) \\ & \quad + \textstyle\sum_g^G \sum_t^T \big(mh_{gt}^3 QO_{gt} - (2QO_{gt} - 2M_{gt}^2)mh_{gt}^2 \\ & \qquad + (2M_{gt}^2 - QO_{gt})mh_{gt}^3\big)\big] \\ & + \rho \textstyle\sum_t^T [(2\alpha_3 - 1)MaxM_t^1 + (2 - 2\alpha_3)MaxM_t^2 - MaxM_t^1] \end{aligned}$$

Subject to:

$$\sum_t^T (NH_t + NF_t) = \varepsilon_2 - s_2$$

$$\sum_g^G \sum_{t=1}^T \left(QI_{gt} + QB_{gt}\right) = \varepsilon_3 - s_3$$

$$\begin{aligned}(2 - 2\alpha_1)De_{gt}^2 + (2\alpha_1 - 1)De_{gt}^3 = IQ_{gt-1} - QB_{gt-1} \\ + QR_{gt} + QO_{gt} + QS_{gt} - QI_{gt} + QB_{gt}; \quad \forall G, \forall T\end{aligned}$$

$$QB_{gt} \le QB_{gt}^{max}; \quad \forall G, \forall T$$

$$\begin{aligned}\sum_{g=1}^G wh_{gt-1}\left(QR_{gt-1} + QO_{gt-1}\right) \\ + NH_t - NF_t - \sum_{g=1}^G wh_{gt}\left(QR_{gt} + QO_{gt}\right) = 0; \quad \forall T\end{aligned}$$

$$\begin{aligned}\sum_{g=1}^G wh_{gt}\left(QR_{gt} + QO_{gt}\right) \\ \le (2\alpha_2 - 1)MaxL_t^1 + (2 - 2\alpha_2)MaxL_t^2; \quad \forall T\end{aligned}$$

$$\begin{aligned}\sum_{g=1}^G \left[(2 - 2\alpha_3)mh_{gt}^2 + (2\alpha_3 - 1)mh_{gt}^3\right]\left(QR_{gt} + QO_{gt}\right) \\ \le (2\alpha_3 - 1)MaxM_t^1 + (2 - 2\alpha_3)MaxM_t^2; \quad \forall T\end{aligned}$$

$$QS_{gt} \le QS_{gt}^{max}; \quad \forall G, \forall T$$

$$\sum_{g=1}^G ws_{gt} QI_{gt} \le MaxW_t; \quad \forall T$$

$$E[\widetilde{Z_1}] \le Z_{budget}$$

$$\frac{QR_{gt}}{2} \le M_{gt}^1 \le QR_{gt}; \quad \forall G, \forall T$$

$$\frac{QO_{gt}}{2} \le M_{gt}^2 \le QO_{gt}; \quad \forall G, \forall T$$

$$QR_{gt}, QO_{gt}, QS_{gt}, QI_{gt}, QB_{gt}, NH_t, NF_t \ge 0; \quad \forall G, \forall T$$

$$0.5 \le \alpha_1, \alpha_2, \alpha_3 \le 1; \; s_2, s_3 \ge 0$$

According to the DM's preferences, the first objective function (total costs of APP plan) has a higher priority than the second and third objective functions (total changing rate in workforce levels and total holding inventory and backorder quantities). Thus, the first objective function is minimized while the two remaining objective functions are converted to be the respective constraints.

VIII. Solution Procedures

In summary, the solution procedure to deal with the MOMILP model for APP decision problem under uncertain environment can be described as follows:

- ***Step 1***: Formulate the multi-objective mathematical model with uncertain parameters for APP problems.
- ***Step 2***: Convert the fuzzy multi-objective mathematical model into a corresponding crisp model by applying the PCCP (the fuzzy objective functions are defuzzified based on fuzzy expected value method while the fuzzy constraints are defuzzified based on the possibilistic chance-constrained programming method with the credibility measure).
- ***Step 3***: Solve the PCCP model with different confidence levels of fuzzy constraints.
- ***Step 4***: Integrate robust optimization into the PCCP model.
- ***Step 5***: Make a comparison between the PCCP and RPCCP models to evaluate the efficiency and robustness of the obtained results from these models.
- ***Step 6***: Determine the NIS and PIS of objective functions by using a pay-off table.
- ***Step 7***: Apply the AUGMECON technique to solve the MOAPP model.
- ***Step 8***: Present a set of various Pareto optimal solutions with their respective performances.
- ***Step 9***: Use the Max-Min method for finding the best compromised (trade-off) solution.

CASE STUDY

An industrial case of a ball screw manufacturing company is used to illustrate the effectiveness of the proposed method. The company plans to produce two types of standard ball screws, which are external and internal recirculation (Product 1 and Product 2, respectively). The planning horizon is assumed to be 4 months, from January to April. The input parameters for the planning process are presented in Tables 2 – 5, including forecasts of the product demand, production costs, and capacity information. It should be noted that some of these parameters, i.e., forecasts, workforce levels, machine capacities, are presented as triangular fuzzy numbers on monthly basis. Other related data are described in Table 6.

TABLE 2
FORECASTED DEMAND OF BOTH PRODUCTS

Month	Product	
	$\widetilde{D}_{1t}$ (units)	$\widetilde{D}_{2t}$ (units)
1	(900, 1000, 1080)	(900, 1000, 1080)
2	(2750, 3000, 3200)	(450, 500, 540)
3	(4600, 5000, 5300)	(2750, 3000, 3200)
4	(1850, 2000, 2100)	(2300, 2500, 2650)

TABLE 3
RELATED OPERATING COSTS OF BOTH PRODUCTS

	Product 1	Product 2
$\tilde{r}_{pt}$ (\$/unit)	(17, 20, 22)	(8, 10, 11)
$\tilde{o}_{pt}$ (\$/unit)	(26, 30, 33)	(12, 15, 17)
$\tilde{s}_{pt}$ (\$/unit)	(22, 25, 27)	(10, 12, 13)
$\tilde{\imath}_{pt}$ (\$/unit)	(0.27, 0.30, 0.32)	(0.13, 0.15, 0.16)
$\tilde{b}_{pt}$ (\$/unit)	(35, 40, 44)	(16, 20, 23)

TABLE 4
MAXIMUM LABOR, MAXIMUM MACHINE, AND WAREHOUSE SPACE DATA

Month	$\widetilde{MaxL}_t$ (person–hours)	$\widetilde{MaxM}_t$ (machine–hours)	$MaxW_t$ (ft^2)
1	(175, 300, 320)	(360, 400, 430)	10,000
2	(175, 300, 320)	(450, 500, 540)	10,000
3	(175, 300, 320)	(540, 600, 650)	10,000
4	(175, 300, 320)	(450, 500, 540)	10,000

TABLE 5
MAXIMUM SUBCONTRACTING AND BACKORDERING LEVELS

Month	$MaxQS_{gt}$ (Units)		$MaxQB_{gt}$ (Units)	
	Product 1	Product 2	Product 1	Product 2
1	400	500	500	500
2	400	500	500	500
3	400	500	500	500
4	400	500	500	500

TABLE 6
OTHER RELEVANT DATA

Product	Initial holding inventory level (units)	Ending holding inventory level in the fourth month (units)	Storage space of products in warehouse (ft^2)	Labor hours for producing products (person–hours /unit)	Machine hours for producing products (machine–hours/unit)
1	400	300	2	0.05	(0.09, 0.10, 0.11)
2	200	200	3	0.07	(0.07, 0.08, 0.09)
Hiring cost (\$/person/hour)				(8, 10, 11)	
Downsizing cost (\$/person/hour)				(2.0, 2.5, 3.2)	
Initial labor level (persons)				300	
Total available budget (\$)				400,000	

RESULTS AND DISCUSSIONS

By using the information from the industrial case discussed in the previous section, the mathematical model of the multi-objective APP decision problem is coded and solved by IBM ILOG Cplex Optimization Software on the computer with an Intel(R) Core (TM) i5-8250U CPU @ 1.60GHz with 16GB. The outcomes of the model are assessed and discussed in this section.

I. Results of PCCP Model with Different Confidence Levels

To assess and investigate the performance of the PCCP model for the MOAPP problem, it will be solved with various confidence levels from 0.5 to 1 as a single objective optimization model. The objective function's value of each objective under different confidence levels is presented in Table 7 and depicted in Figure 3.

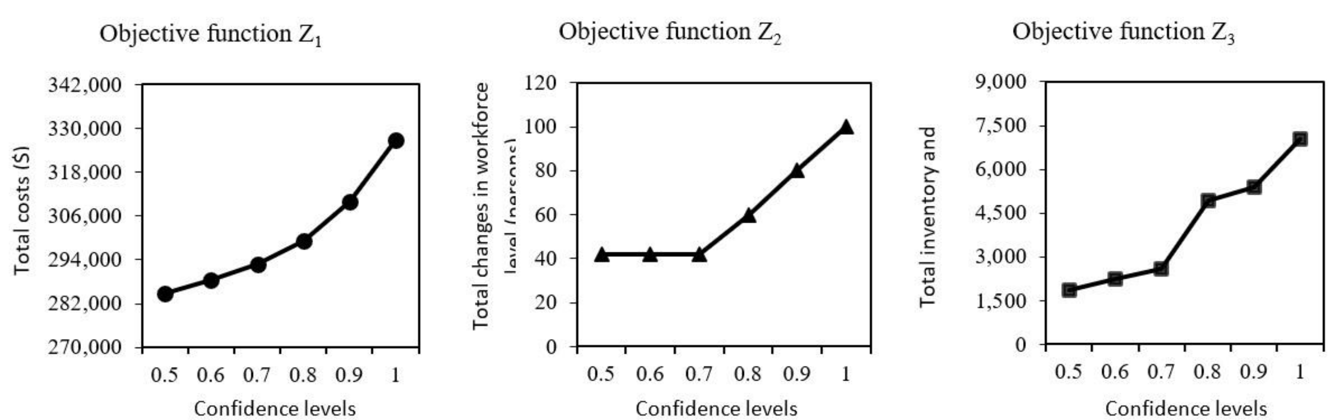

FIGURE 3
OBJECTIVE FUNCTION VALUE OF PCCP MODEL

TABLE 7
RESULTS OF PCCP MODEL WITH DIFFERENT CONFIDENCE LEVELS

Confidence levels	Objective function		
	Z_1 ($)	Z_2 (persons)	Z_3 (units)
0.5	284,843	42	1,858
0.6	288,457	42	2,226
0.7	292,726	42	2,595
0.8	299,276	60	4,941
0.9	310,049	80	5,398
1	326,742	100	7,030

From the obtained results in Table 7 and Figure 3, they highlight that increasing the confidence level will result in enhancing the values of all objective functions. This also means that these objective functions become worse or less desirable. The reasons for having such a worse solution can be interpreted as follows. There is a compromise or trade-off between the optimal value of objectives and the satisfaction of constraints (the risk of constraint violation). The set of feasible solutions will be smaller when the level of constraint satisfaction is higher and in reverse. At this point, the confidence level is represented for the level of the constraint satisfaction. Therefore, when the confidence level is set highly, the optimal objective values become worse. In fact, the PCCP model would operate in a risk-averse manner as the confidence level is increased.

In the PCCP approach, the confidence levels of a fuzzy constraint are usually pre-determined and relying on the experiences, knowledge, or subjective analyzes of the DMs. In fact, the solutions with respect to different confidence levels are evaluated in accordance with the objective function value. However, this objective function is determined based on the average (expected) value of fuzzy numbers. So, some disadvantages of this approach can be figured out as follows:

- It cannot guarantee the best possible solution for a pre-determined confidence level because of the DM's subjective adjustment. In addition, the number of experiments, needed for finding suitable confidence levels, dramatically increases with the number of chance constraints. Consequently, it is time-consuming.
- The differences between the objective function and its expected value are not considered. Thus, this can put the DMs in high risks and the outcomes could not be trustable.

To solve these problems of the PCCP approach, a RPCCP approach, which is a combination of PCCP and RP, is developed and modified from the previous study of Pishvaee et al. [20].

II. Results of RPCCP Model

Typically, fuzzy parameters in a mathematical model need to be defuzzified before it is solved. The purpose of this defuzzification process is to convert the values of these parameters from fuzzy to crisp numbers such that the model can be solved as a deterministic model. However, applying the defuzzification at the early stage of the optimization process results in the lost fuzziness property, with which a better solution can be found. Therefore, the fuzziness of the model's parameters should be maintained as long as possible. To facilitate such a strategy, the RPCCP approach is implemented in this study. By using this method, the confidence levels of constraints, i.e., customer demand constraint (α_1), labor level constraint (α_2), and machine capacity constraint (α_3), are considered as decision variables and optimized to find the most appropriate levels. The obtained results of objective functions from the RPCCP model of APP problem are numerically presented in Table 8.

TABLE 8
OBTAINED RESULTS OF RPCCP MODEL

Objective functions	Expected value of objective function	Deviation from the expected value to the worst value of the objective function	Optimal minimum confidence levels		
			Customer demand constraint (α_1)	Labor level constraint (α_2)	Machine capacity constraint (α_3)
Total costs (Z_1)	$ 284,830	$33,415	55%	67%	100%
Total changes in workforce level (Z_2)	29 persons	-	100%	64.5%	100%
Total inventory and backordered quantities (Z_3)	1,859 units	-	50%	50%	100%

The important weight ($\gamma = 0.15$). Penalty unit of possible violated constraints ($\delta = \pi = \rho = \sigma = 0.001$).

The objective functions are solved separately to find their own best solution. According to the presented results in Table 8, the minimum expected value of total costs (Z_1) is $ 284,830, the deviation from the expected value to the worst value of total costs is $33,415. In relation to the value of total costs, the optimal minimum confidence levels for restrictions of demand, labor level, and machine capacity are 55%, 67%, and 100%, respectively. The minimum value of total changes in workforce level (Z_2) is 29 persons. Regarding the value of Z_2, the optimal minimum confidence levels for restrictions of demand, labor level, and machine capacity are 100%, 64.5%, and 100%, respectively. The minimum value of total inventory and backordered quantities (Z_3) is 1,859 units. Relating to the value of Z_3, the optimal minimum confidence levels for restrictions of demand, labor level, and machine capacity are 50%, 50%, and 100%, respectively. The deviations of the second (Z_2) and third (Z_3) objective functions are not considered since these two objective functions do not contain uncertain parameters.

A comparison of objective functions between the PCCP and RPCCP model is graphically presented in Figure 4. The results indicate that across different confidence levels, the results of the RPCCP model are much better than those of the PCCP model in terms of the objective function values.

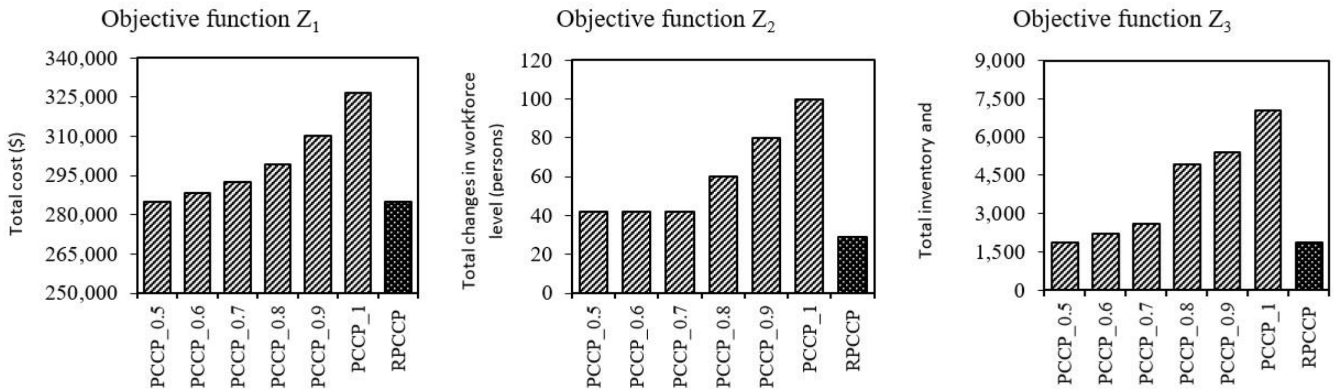

FIGURE 4
COMPARISON OF PCCP AND RPCCP MODEL

Based on Figure 4 and the obtained results from Table 8, the aforementioned disadvantages of the PCCP model can be solved by the RPCCP model. In this RPCCP model, the terms of optimality robustness (the distance from the worst to the mean values of the objective function), and the feasibility robustness (violation of constraints) are included in the objective function and optimized simultaneously. Thus, the RPCCP model can yield a balanced solution between the robustness of risks and feasibility of the expected value, which can provide enough information for an obtained solution under the impact of uncertainties.

The efficiency and reliability of the proposed RPCCP model are also evaluated. Specifically, its solutions are compared with the solutions of the PCCP model under nominal data. The obtained solutions of these two approaches are examined under 10 scenarios of data (realization), which are randomly generated from the range of corresponding uncertain parameters. For instance, a fuzzy number following the triangular possibility distribution is denoted as $\tilde{a} = (a_1, a_2, a_3)$. Hence, the realization will be formed by generating uniform random numbers between the pessimistic and optimistic values of fuzzy parameters (e.g., $a_{real} = [a_1, a_3]$). Then, the obtained solutions (x^*) from solving the RPCCP model under nominal data, which are altered in the realization model, as presented below:

$$
\begin{aligned}
Min \quad & \psi_1(x) = c_{real}x^* + \delta R^k + \pi R^B + \rho R^h + \sigma R^g \\
S.t \quad & Ax^* \le k_{real} + R^k \\
& Tx^* = 0 \\
& B_{real}x^* - R^B \le h_{real} + R^h \\
& Dx^* + R^g = g_{real} \\
& R^k, R^B, R^h, R^g \ge 0
\end{aligned}
$$

In the above realization model, δ, π, ρ and σ represent the penalty values of constraint's violation and are assumed to be equal. R^k, R^B, R^h and R^g are deviation variables that determine the value of violation of the chance constraints under different random realizations. Based on the Average and Standard Deviation (SD) of the objective function, the performance of the RPCCP model will then be measured and evaluated.

In this study, only the first objective function (total costs of APP plan) involves uncertain parameters. Therefore, only this objective function is used to test under this realization. It is assumed that the penalty value of the constraint's violation is equal to \$25. The measurement and evaluation of the results obtained by the RPCCP and PCCP model with different confidence levels are shown and illustrated in Table 9 and Figure 5, respectively.

TABLE 9
THE PERFORMANCE OF OBTAINED RESULTS UNDER REALIZATION OF PCCP AND RPCCP MODEL

No. Realization	Total costs of the realization model ($)						
	PCCP model solutions with different confidence levels						RPCCP
	0.5	0.6	0.7	0.8	0.9	1	
1	270,971.89	267,398.16	264,057.40	261,943.39	262,441.73	266,914.63	257,352.72
2	280,534.92	276,961.24	273,622.15	271,534.84	271,837.15	275,911.05	266,917.10
3	293,426.52	289,852.85	286,513.88	284,424.38	284,625.32	288,583.55	279,808.84
4	300,645.63	297,071.98	293,710.47	291,527.65	291,737.14	295,863.40	287,028.99
5	302,970.85	299,397.23	296,016.96	293,752.23	294,149.60	298,895.52	289,354.46
6	307,401.63	303,827.99	300,326.92	297,496.33	297,478.34	302,167.59	293,785.92
7	312,010.17	308,436.55	304,882.42	301,813.67	302,339.22	308,850.58	298,395.14
8	315,670.92	312,097.34	308,635.73	306,031.28	306,905.45	313,430.70	302,057.31
9	316,540.07	312,966.48	309,637.14	307,665.93	309,054.67	315,856.96	302,926.53
10	326,635.08	323,061.47	319,844.90	318,407.11	319,911.69	326,117.90	313,021.52
Average	302,680.77	299,107.13	295,724.80	293,459.68	294,048.03	299,259.19	289,064.85
SD	17,093.59	17,093.63	17,086.06	17,087.20	17,404.89	18,303.31	17,095.48

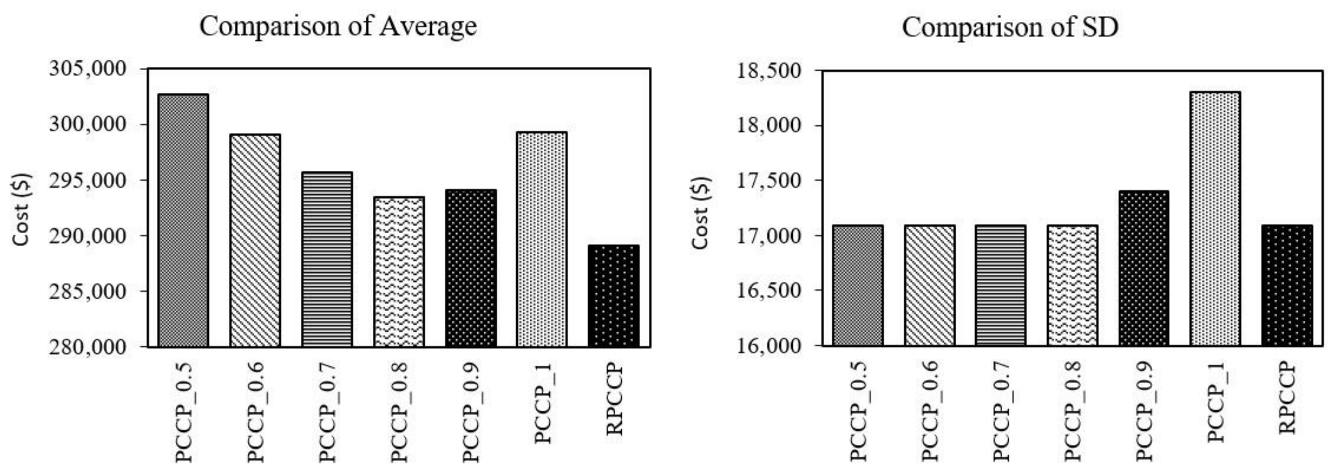

FIGURE 5
COMPARISON OF PCCP AND RPCCP MODEL

According to Table 9 and Figure 5, the average values of the objective function of the RPCCP model under different realizations are lower than the average values of the objective function of all the PCCP models with different confidence levels. Besides that, the SD of the RPCCP model is also in an acceptable range. Throughout these obtained results, the robustness and application of the RPCCP model can be justified. It could also say that the RPCCP is a more efficient approach to cope with uncertain situations.

Additionally, a sensitivity analysis on different penalty values of the violated constraints for the RPCCP model and PCCP model with different confidence levels under the realization model is also conducted. The diffident penalty values of violated constraints are presented in Table 10.

TABLE 10
DIFFERENT PENALTY VALUES IN THE REALIZATION MODEL ($)

Number	1	2	3	4	5	6
Penalty values ($\delta = \pi = \rho = \sigma$)	10	20	30	40	50	60

The value of the objective function (total costs) with various penalty values of violated constraints is used to compare the RPCCP model and PCCP model with different confidence levels, which are illustrated in Figure 6.

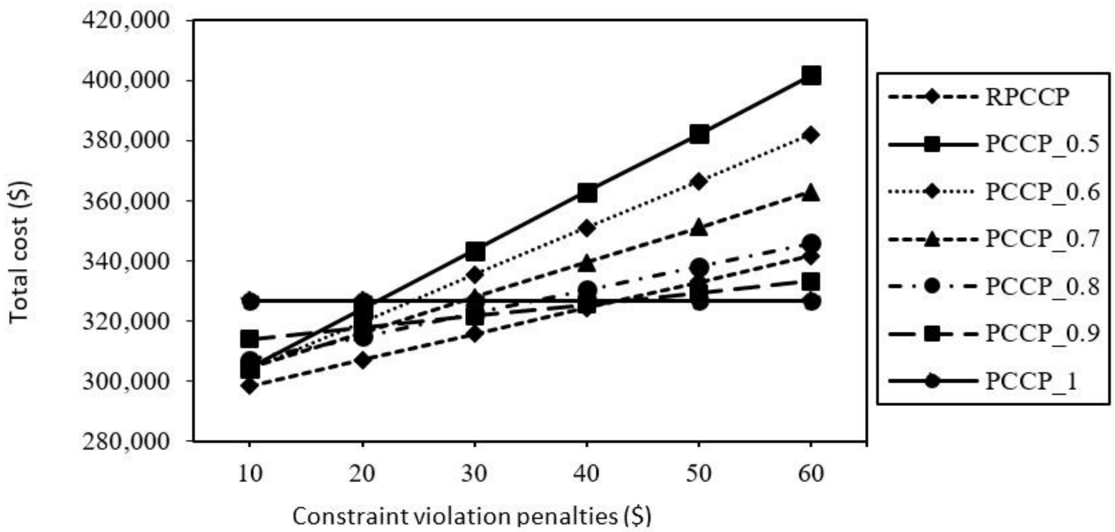

FIGURE 6
THE TOTAL COSTS OF RPCCP AND PCCP MODEL UNDER DIFFERENT PENALTY VALUES OF CONSTRAINT VIOLATION

According to Figure 6, the penalty value of chance-constraints violation has an important impact on the performance of the RPCCP models. As the penalty value of chance-constraints violation is high, the PCCP model with the confidence levels of 0.8, 0.9, and 1 (optimization under the worst situation: risk-averse models) has better performances. In contrast, as the penalty value of chance-constraints violation is low, the RPCCP model shows to outperform the others.

III. Results of Multi-Objective RPCCP Model by Using Augmented Epsilon-Constraint (AUGMECON) Technique

To cope with the conflicting multiple objective problem, the AUGMECON technique is employed. To apply this method, firstly, the NIS and PIS of each objective are needed to be determined. As aforementioned, the pay-off table method is used. Therefore, a pay-off table for identifying the NIS and PIS of each objective function is generated and presented in Table 11.

functions are then estimated by the maximum or minimum value in the equivalent column (it depends on the type of objective function). If the objective is minimization, the NISs will be determined by the following equation:

$$Z_j^{NIS} = max\{Z_j(v_q^*);\ j \neq q\} \tag{37}$$

where v_q^* is the PISs for respective objective functions Z_j.

As the objective function is maximization, the NIS is determined by the equation as follows:

$$Z_j^{NIS} = min\{Z_j(v_q^*);\ j \neq q\} \tag{38}$$

Based on Equation 37 (as all objective functions are minimization) and the results as shown in Table 11, the NIS and PIS of each objective function can be easily obtained, as numerically presented in Table 12.

After the bound of each objective function is determined by the pay-off table, the bound of the second and third objective functions will be separated into 20 equal parts (see Table 13). In addition, the second and third

TABLE 11
PAYOFF TABLE FOR OBTAINING PISS AND NISS

Objective functions	Symbol	Type	Unit	v_1^*	v_2^*	v_3^*
Total costs	Z_1	Min	($)	**284,830**	332,576	310,121
Total changes in workforce level	Z_2	Min	(Persons)	63	**29**	503
Total inventory and backordered quantities	Z_3	Min	(Units)	6,760	12,958	**1,859**

TABLE 12
OBTAINED PISS AND NISS FOR EACH OBJECTIVE FUNCTION

Objective functions	Symbol	Unit	PIS		NIS	
			Type	Value	Type	Value
Total costs	Z_1	($)	Min	284,830	Max	310,121
Total changes in workforce level	Z_2	(Persons)	Min	29	Max	503
Total inventory and backordered quantities	Z_3	(Units)	Min	1,859	Max	12,958

According to Table 11, the bold values are the PISs of the objective functions, which are obtained by optimizing each objective function. The NIS values of the objective functions are transformed to be the constraints. Therefore, the multi-objective model becomes a single objective model and then being solved regarding to the first objective function.

According to the pay-off table, the bound of the second objective function r_2 = 474 persons and the bound of the third objective function r_3 = 11,009 units. These two bounds are divided into 20 equal intervals with $Step_2$ = 24 and $Step_3$ = 555. The AUGMECON process is demonstrated as follows:

$For\ v = 0 \ldots 20$
$\quad \varepsilon_3 = 12{,}958 - v \times 555$
$\quad\quad For\ u = 0 \ldots 20$
$\quad\quad\quad \varepsilon_2 = 509 - u \times 24$
$\quad\quad\quad Solve\ Model\ (30)$
$\quad\quad Next\ u$
$Next\ v$

It is assumed that we are in the 11th innermost ($u = 10$) and the 6th outermost ($v = 5$) iteration, where $\varepsilon_2 = 269$ and $\varepsilon_3 = 10.183$ (these values are bold in Table 13). Having solved the model, the obtained value of s_2 and s_3 are 207 and 3,392, respectively. This means that the second and third objective functions' values in this iteration will be presented as Table 13.

This can be concluded that it is redundant to solve the next iterations with $u = 12, 13, \ldots, 19$ (italic values in Table 13). This is since the same Pareto optimal solutions will be obtained with $Z_2 = 62$. The only difference between these solutions is the surplus variables. For example, with $u = 12$, the surplus variable s_2 will be $207 - 24 = 183$ or with $u = 19$, the surplus variable s_2 will be $207 - (8 \times 24) = 15$. Therefore, the redundant iterations can be bypassed and directly come to $u = 20$ with $\varepsilon_2 = 53$ from $u = 11$ (see Table 13). The bypass coefficient (denoted by bp) is computed as follows: $bp = int\ (204/24) = 9$. By using the bypass coefficient, the redundant iterations can be eliminated. This helps to save time in the solving process of the algorithm.

TABLE 13
GRID POINTS OF THE SECOND AND THIRD OBJECTIVE FUNCTIONS

Objective function		Z_2	Z_3
Counter		*u*	*v*
	0	509	12,958
	1	485	12,403
	2	461	11,848
	3	437	11,293
	4	413	10,738
	5	389	**10,183**
	6	365	*9,628*
	7	341	*9,073*
	8	317	*8,518*
	9	293	*7,963*
Grid points	10	**269**	*7,408*
	11	*245*	*6,854*
	12	*221*	6,299
	13	*197*	5,744
	14	*173*	5,189
	15	*149*	4,634
	16	*125*	4,079
	17	*101*	3,524
	18	*77*	2,969
	19	53	2,414
	20	29	1,859

$Z_2 = \varepsilon_2 - s_2 = 269 - 207 = 62$
and $Z_3 = \varepsilon_3 - s_3 = 10{,}138 - 3{,}392 = 6{,}746$

By applying the AUGMECON technique to solve the MOAPP model, the Pareto solutions can be obtained and illustrated in Table 14.

Based on Table 14, there is a trade-off among objective functions. Throughout many given solutions, the DMs can realize how much the other objectives must be sacrificed from varying or improving one objective. Having known that, it is very helpful for the DMs as they can choose the most satisfied solution according to their preferences.

These results are further assessed by using the Max-Min approach. The purpose of using this method is to obtain the best Pareto solution with the smallest deviation from each objectives' ideal solution. By applying the method to the solutions in Table 14, the second solution (BTS = 6.7668) is chosen as the best Pareto solution. The detailed production plan of this best solution by the Max-Min method can be displayed in Table 15.

TABLE 14
A SET OF PARETO OPTIMAL SOLUTIONS.

No.	Satisfaction levels			Objective function values			BTS
	μ_{z_1}(%)	μ_{z_2}(%)	μ_{z_3}(%)	Z_1 ($)	Z_2 (persons)	Z_3 (units)	
1	99.33	95.15	50.05	285,151	52	7,403	6.7765
2	100	93.04	55.97	284,830	62	6,746	**6.7668**
3	99.73	87.13	60.10	284,959	90	6,288	7.4864
4	99.49	81.65	65.09	285,074	116	5,734	8.0853
5	99.19	75.53	70.02	285,216	145	5,186	8.7910
7	98.81	66.88	75.02	285,399	186	4,632	9.9075
8	97.98	50.42	80.04	285,796	264	4,074	12.2983
9	97.09	35.65	85.01	286,220	334	3,523	14.4172
10	95.68	37.13	90.20	286,894	327	2,947	13.8684

TABLE 15
OPTIMAL PRODUCTION PLAN IN THIS INDUSTRIAL CASE

Items		Symbol	Unit	Month			
				1	2	3	4
Product 1	Regular–time production	QR_{1t}	units	630	2,975	4,999	2,296
	Overtime production	QO_{1t}	units	0	0	0	0
	Subcontracted	QS_{1t}	units	0	0	0	0
	Inventory	QI_{1t}	units	30	5	4	300
	Backordered	QB_{1t}	units	0	0	0	0
Product 2	Regular–time production	QR_{2t}	units	3,150	1,475	215	2,160
	Overtime production	QO_{2t}	units	0	0	0	0
	Subcontracted	QS_{2t}	units	0	0	0	0
	Inventory	QI_{2t}	units	2,350	3,325	540	200
	Backordered	QB_{2t}	units	0	0	0	0
Total hired workers		NH_t	persons	0	0	13	1
Total downsized workers		NF_t	persons	48	0	0	0
Machine capacity			machine-hours	353	450	540	447
Warehouse space			ft^2	7,110	9,985	1,628	1,200
Total costs		Z_1	$	284,830			
Total changes in workforce level		Z_2	persons	62			
Total inventory and backordered quantities		Z_3	units	6,746			

According to Table 15, the produced quantities of Product 1 in the regular–time production from the first month to the fourth month are 630, 2,975, 4,999, and 2,296 units, respectively. It can also be seen that the quantities of ending inventory of product 1 from the first month to the fourth month are 30, 5, 4, and 300 units, respectively, and there is no subcontracting unit. Product 2 shows 3,150, 1,475, 215, and 2,160 units of regular–time production from the first month to the fourth month. Product 2 also indicates a higher level of ending inventory with 2,350, 3,325, 540, and 200 units from the first month to the fourth month, respectively. Subcontracting is also not required. The number of dismissed workers is 48 persons in the first month. Thirteen new workers are employed in the third month and one person is hired in the fourth month. 7,110, 9,985, 1,628, and 1,200 ft^2 of warehouse space to keep Product 1 and Product 2 from the first month to the fourth month are required, respectively. With this plan, the total costs, the total fluctuation of workforce level, and the total inventory and backordered quantities are $284,830, 62 persons, and 6,746 units, respectively.

According to the above obtained results and discussions, it can be inferred that the proposed approach in this study can provide some significant contributions as follows:

1) Dealing with the uncertainty
    - RPCCP are divided into three types: (1) hard worst situation, (2) soft worst situation, and (3) realistic situation. Therefore, RPCCP provides a degree of flexibility for the DMs to make a choice of any points within the range of fully optimistic and pessimistic under the impact of uncertainties.
    - RPCCP can be able to deal with uncertainties in both objective functions and constraints.
    - As compared to other methods so-called the traditional defuzzification methods (e.g., the weighted average method, the ranking method), these methods do not provide any information regarding to the likelihood of violating constraints (feasibility concept). In contrast, having relied

upon the fuzzy relation between fuzzy numbers, the defuzzification method (chance-constrained) of the proposed RPCCP approach can not only help avoid being defuzzified earlier in the process of defuzzification but also seek the most appropriate confidence levels (fuzziness levels) of fuzzy constraints. This is one of the outstanding features of this proposed approach.

- The deviation from the worst value to the expected value and constraint violation penalty are optimized simultaneously in the objective function of the model. So, the approach can yield a solution with both aspects of feasibility and optimality (robust solution).

2) Dealing with the multi-objective decision making
   - The approach can generate various Pareto optimal solutions (strong Pareto solutions), which offer the DMs alternative choices to select when dealing with the multiple conflicting objectives.
   - The satisfaction level of the DMs from solutions is taken into consideration, which helps the DMs to evaluate the efficiency of the obtained solutions.

IV. Managerial Implications

In order to create an effective and realistic production plan, it is necessary for companies to have a suitable and efficient approach to deal with these two issues of dealing with uncertain data and multi-objective decision making problems. Through this study, some significant implications for companies can be indicated as follows:

- In real-world applications, the credibility level is usually employed to reflect the occurrence of a fuzzy event and often represent parameters with unknown values in a mathematical programming model. By specifying an appropriate credibility level, uncertain parameters are converted into crisp analogous parameters. This creates a deterministic scenario. If multiple credibility levels are given, each level determines a scenario in which a set of optimal results (operational decision variables) are obtained. Therefore, having a series of scenarios ranging from optimistic to pessimistic situations provides valuable inputs for the management to develop effective operational and strategic plans against future uncertainties.
- Practically, it is difficult for the company to control the fuzziness levels of constraints such as workforce level and maximum machine capacity or even customer demand cannot be controlled. However, having known the optimal fuzziness level of these constraints will help the company in making effort to run its operations toward the obtained fuzziness level. For example, if the optimal fuzziness level of the maximum machine capacity is relatively on the RHS of the maximum available machine capacity, the company can spend more budget on buying more machines to enhance the machine capacity and vice versa. Thus, the argument between increased expenditure and gained benefits can be assessed by their worthiness.
- From the aspect of making decisions as multiple conflicting objectives are considered simultaneously, this study can generate a set of Pareto optimal solutions (different compromised solutions). Therefore, it is extremely beneficial to managers or planners in choosing appropriate solutions following each company's policy.

CONCLUSIONS

To deal with two important issues in APP decision problems, which are the uncertainty of input data and multi-objective decision making, this study proposed a multi-objective multi-product multi-period APP decision problem in an uncertain environment. The considered uncertain parameters (following the triangular possibility distribution) of the APP model in the study included customer demand, operation time, operation cost, and machine capacity. Our APP model strived to simultaneously minimize total costs, holding inventory and backordered quantities, and variation in workforce levels with regard to the restriction of available budget, workforce levels, and machine and warehouse capacities.

In handle the fuzzy multiple objective APP model, a RPP, which is an integration of PCCP, RP, and AUGMECON technique has been developed. The PCCP and RP approaches were used for handling the uncertain data while the AUGMECON technique was used for handling multiple conflicting objectives.

The applicability and efficiency of the proposed methodology for the fuzzy multi-objective APP problem have been evaluated via a case study. Throughout the obtained results from the RPCCP model, as compared to the basic PCCP model, it was shown that the RPCCP model outperformed the basic PCCP model in terms of the measurement of average and SD of the objective function under the realization. Additionally, the RPCCP can not only maintain the fuzziness of data (avoid being defuzzified too early) but also seek the most appropriate (confidence) levels of fuzzy constraints. For multi-objective decision making, by applying the AUGMECON technique, the proposed approach had the ability to produce a set of Pareto optimal solutions, which made a rational trade-off among objective functions, as well as provided the DMs different alternative selections. Besides, the best compromised (trade-off) solution was also defined through a comparison by using the Max-Min method.

The primary limitation of the proposed approach is that all uncertain parameters in the proposed model are represented by the triangular possibility distribution. In fact, based on incomplete available data, subjective knowledge, or experiences of the DMs/experts, other appropriate distribution forms can be generated and then applied for the proposed model. In addition, there were only a few parameters (i.e., labor level, machine capacity, and customer demand) that are considered to be uncertain or imprecise parameters in the problem. With the proposed approach, more numbers of parameters could be considered to be fuzzy. The following may also be considered to be possible further studies.

- From the aspect of developing the model, the APP model could be extended by adding other important issues such as machine utilization, multiple manufacturing plants, labor skills, varying lead time, etc. Additionally, when the APP models are developed or expanded with the consideration of multiple different objectives, the business and sustainability aspects can also be embedded.
- From the aspect of solving approach, once APP models become very large and too complex to be solved by IBM ILOG CPLEX software (as it was used in this study), it is necessary to investigate the suitability of using metaheuristic algorithms such as Genetic Algorithm (GA), Ant Colony, and so on for any possibility to obtain optimal or closed to optimal results within acceptable solving time.

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APPENDIX

In Multi-Objective Linear Programming (MOLP) problems, a solution is called a "Pareto optimal solution" when none of the objective functions is possible to be enhanced without worsening, at least, one other objective function. Consequently, the Pareto frontier, can be constructed by a set of Pareto optimal solutions to provide the DMs a whole picture of compromise solutions from which they can select the most satisfied one based on their preferences.

A MOLP problem with j objective functions of minimization can be described in a mathematical form as follows:

$$\begin{aligned} &Min \quad [\psi_1(x), \psi_2(x), \dots, \psi_j(x)] \\ &S.t. \quad x \in X \end{aligned}$$

Now, a point $\hat{x} \in X$ is called:

- A dominated solution if there exist $x \in X$ such that $\psi_j(x) < \psi_j(\hat{x})\ \forall\ i$;
- A weak Pareto optimal solution if and only if there does not exist $x \in X$ such that $\psi_j(x) < \psi_j(\hat{x})\ \forall\ i$;
- A strong Pareto optimal solution if and only if there does not exist $x \in X$ such that $\psi_j(x) \leq \psi_j(\hat{x})\ \forall\ i$;