Journal of Geotechnical Geology

Zahedan Branch,
Islamic Azad University

Journal homepage: geotech.iauzah.ac.ir

Comparison of Equivalent Continuum and Discontinuum Methods in Stability Analysis of a Natural Underground Karst

Mahnaz Laghaei*[1], Alireza Baghbanan[1], Masoud Torkan[1], Siavash Norouzi[2], Meysam Lak[3]

[1]*Department of Mining Engineering, Isfahan University of Technology, Isfahan, Iran*
[2]*Department of Mineral Engineering, New Mexico Tech, Socorro, NM 87801, USA*
[3]*Department of Mining and Metallurgy, Yazd University, Yazd, Iran*

ARTICLE INFORMATION

Received 16 April 2018
Revised 25 July 2018
Accepted 31 August 2018

KEYWORDS
Equivalent continuum method; Discontinuum method, Karst; Discrete Fracture Network; Numerical modeling.

ABSTRACT

Stability analysis of the underground karsts around excavated and supported tunnels is one of the most important tasks in terms of safety evaluation of tunnels and underground excavations. In this paper, the stability and the stabilization method of a natural hole which existed before excavation above a transportation tunnel were investigated. The hole was modeled in form of continuous and discontinuous media. Several realizations with different crack patterns were generated in order to eliminate random effect. The stability was analyzed numerically. The results show that the equivalent continuum method induces overestimation upon strength parameters and consequently results in higher uncertainty compared to the discontinuum method which is a better representation of the problem. The generated models in discontinuous medium were highly unstable and showed prominent signs of collapse. As per the agreement between the discontinuous models with the real problem's conditions, this instability and collapse of rock blocks may disturb the tunnel lining and yield to catastrophes. The results of using concrete foam as the filling and the whole support confirm the proper applicability of this approach alongside its practicability under the conditions specific to this project.

1. Introduction

Maintaining the safety of underground excavations and transportation tunnels in particular is of utmost importance in rock engineering related issues. Based upon the rock nature of the material surrounding these excavations, the investigation of the associated mechanical behavior and characteristics is necessary. Rock masses have inherently higher strength and stiffness than those of the other constituents of the earth's crust. However, this statement may not be true for weathered, fractured, or soft rocks (Bobet et al., 2009). In other words, presence of discontinuities is the source of difference between rocks and other materials of the Earth's crust and brings about a wider range of behaviors in them. Moreover, the constitutive model of fractures influences the mechanical behavior of rock mass and plays an important role in failure conditions and the stability of surroundings in underground excavations (Wu and Kulatilake, 2012). Underground spaces generally include excavations constructed and supported for a particular purpose and inherent openings whose instability might be dangerous. Karsts belong to the second category and refer to vast aquifers particularly developing in soluble rocks such as limestone, marble, and gypsum. Various numerical approaches have been used for stress analysis considering discontinuities. Numerical methods are generally categorized into discontinuum and equivalent continuum classes, both of which are utilized to assess the impact of fractures on the mechanical behavior of rock mass in stability analysis or failure status in the surroundings of underground excavations (Wu and Kulatilake, 2012). To date, studies using equivalent continuum method have involved stability of underground spaces by developing a continuous model in terms of micromechanics of rock mass (Yoshida and Horii, 2004) and also introducing cracks through joint factor and adopting an empirical method (Sitharam

*** Corresponding author.**
E-mail address: mahnaz.laghaei@mi.iut.ac.ir
Master of Rock Mechanics.

and Latha, 2002).Continuum approach represents a better tool to represent the overall behavior of fractured rock masses at large scales and implicitly takes into account the impact of fractures with regards to equivalent constitutive models and the associated characteristics (Min and Jing, 2003). By introducing numerical methods to rock engineering, methods such as Finite Element Method (FEM) and Boundary Element Method (BEM) were adopted in modeling with interface elements but their applications became gradually limited due to failure in computing logic in case of multitude interface elements, absence of an adequate algorithm to detect new collisions, limitation of formulations to small displacements and rotations, and inability to implement joint elements when large displacements along blocks boundaries are not the same size as the separation between required block elements (Jing and Stephansson, 2007). Due to the intrinsic limitations aforementioned, discrete element method, DEM (Cundall, 1971) and discontinuous deformation analysis, DDA (Shi and Goodman, 1985) with discontinuum basis appear to be better tools to perform stability analysis of underground excavations in rock mass compared to FEM, BEM, and finite difference method, FDM with the basis of continuum theory. DEM which was pioneered by Cundall (Cundall, 1971) and then developed by other researchers is a powerful tool for stress analysis in rock masses with elongated discontinuities. The method employs an explicit approach as the fracture network is explicitly introduced in details in discrete models, enabling the method to stand out against other tools for investigating the mechanical behavior of fractured rock mass and also large displacements, rotations, and complex constitutional behaviors are possible for intact rock and joints without the need for additional calculations. The merit thereupon compromises the problem dimensions since representing block and joints distinctly requires high computing capacity and speed. Hence, despite its comprehensiveness, its applicability in some cases is a challenge (Min and Jing, 2003). DEM has widely attracted attentions in the explicit modeling of fractures, due to which it has found broad spectrum of applications including backwards analysis of tunnel's response in fractured spaces and detecting and reinforcing unstable blocks (Vardakos et al., 2007), stability analysis of underground excavations by investigating the impact of constitutive model of fractures (Souley and Homand, 1996), fracture spacing and quantity and dimensions of the blocks surrounding the underground excavation (Bhasin and Høeg, 1998) and also in-situ stress state effect on stability and deformation of excavated spaces in rock mass (Solak, 2009). A research by Wu and Kulatilake (Wu and Kulatilake, 2012) was conducted to analyze the deformability and stability of the area surrounding a tunnel excavated in a fractured limestone by virtue of 3D stress analysis with equivalent continuous and discontinuous models bearing various joint sets. Their results showed that the continuum approach incompetently modeled the displacements in the roof and the walls of the tunnel being studied compared to the results of the discontinuum approach. Three dimensional modeling using DEM particularly is a time-consuming process and is challenging to adopt in common stability analyses (Vardakos et al., 2007). Therefore, two dimensional methods with the assumption of plane strain appear to suffice the purpose at a good accuracy. As stated in the foregoing remarks, the previous researches have investigated the stability of excavated spaces in rock mass with various conditions using different tools in particular continuum and discontinuum methods. Indeed, this is the excavations that distort the preexisting stability. In most of the related studies conducted thus far, discontinuities are considered as joint sets; however, a discontinuous medium is comprised of not only joint sets but also discontinuities in dimensions, orientations, and different positions. This method of modeling simplifies the medium to a large extent. Hence, discrete fracture network (DFN) may be used as the best tool to simulate fractured rock masses.

The focus of this research is to assess the accuracy and applicability of continuum methods versus discontinuum methods in stability analysis of natural underground spaces which has not been investigated in details. The karst subject to this study was detected through geophysical surveys and located above a transportation tunnel in the west of Iran. Since the karst was on the verge of collapse on the tunnel lining, which results in significant disasters, it was attempted to numerically evaluate the possibility of supporting it. The modeling benefited from both continuum and discontinuum methods. FLAC^{2D} was utilized to simulate the continuous medium using the equivalent parameters of the fractured rock mass. Discontinuum modeling was performed based upon the two-dimensional surveys of the joints and generation of stochastic fracture networks with arbitrary patterns in UDEC.

2. Methodology

A transportation tunnel which is located in the west of Iran in Zagros Mountain was subjected to this study. After excavating and completely lining the tunnel, to investigate the risk of hazards, geophysical surveys were done in several sections of the tunnel. The tomography results indicated several karst formations in various sizes and dimensions above the tunnel. To prevent the possible perils of tunnel damage, it is best to explore the potential danger of collapse. The tunnel had a horseshoe profile and a portal with 10m height at the elevation of 1830m. The evaluated burden on the center of the tunnel was 50m from the topographical surface of the ground. The vicinity was comprised of multiple joint sets and karsts with variable dimensions. Due to the multiplicity of identified karsts, a section of the largest karst with the average width of 10m, approximate height of 16m, and length of 100m aligned with the tunnel's advancement was investigated. Fig. 1 shows the aforementioned section used for simulation. As a consequence of surrounding rocks collapse, the tunnel's safety was gradually endangered, which was likely to yield to disasters if not supported.

2.1. Physical and Mechanical Properties of Rock Mass

According to geological studies, the host rock mass was characterized as limestone. The corresponding physical and mechanical properties including density, Young's modulus, internal friction angle, cohesion, and Poisson's ratio were obtained from laboratory test results which were conducted according to ISRM suggested methods and used in the numerical study for comparison between the performances of continuum and discontinuum methods in the stability of the karst and the sensitivity analysis (Table 1). In this research work, the ratio of horizontal to vertical stress is considered according to the study area (medium depth in Zagros mountain area) and is approximated by 0.7. Table 2 contains the results of the tests conducted on the joints and field surveys including dilation angle and joint friction angle. The general information of the tunnel's lining is tabulated in Table 3.

Table 1 Properties of intact rock materials

Parameter	Unit	Value
Density	Kg/m^3	2200
Young's Modulus	GPa	28.6
Cohesion	MPa	9.4
Friction Angle	Degree	57.13
Poisson's Ratio	-	0.26

Table 2 Properties of discontinuities

Parameter	Unit	Value
Joint Friction Angle	Degree	34
Joint Cohesion	MPa	0
Normal Stiffness	GPa/m	2000
Shear Stiffness	GPa/m	200
Dilation Angle	Degree	5

Table 3 Properties of tunnel's lining

Parameter	Unit	Value
Density	Kg/m^3	2446
Young's Modulus	GPa	23.5
Poisson's Ratio	-	0.2
Thickness	m	0.5

2.2. Numerical Analysis Process

The plane strain assumption is valid in the two-dimensional modeling of a section perpendicular to the tunnel's length due to the karst development in this direction. FLAC^{2D} and UDEC were used to compare the karst stability in the continuous and discontinuous media, respectively. FLAC codes benefit from explicit finite difference method whereas UDEC is based on Distinct Element Method for designing discontinuous media and is able to model rock masses in form of discrete blocks and discontinuities with the capabilities of large displacements along discontinuities and rotations of blocks (Itasca, 2013a,b). Despite the competency of UDEC in modeling both media properly, FLAC^{2D} was selected owing to its user-friendly interface. In order to simulate the two media, the selection of an appropriate region was necessary. According to the karst section, the equivalent radius of the section was determined by 6.9m. The effective zone, which is assumed to be 4-10 times as much as the equivalent radius (Min et al., 2004), was assigned as 90m×90m in the equivalent continuous medium and 30m×40m in the discontinuous medium to model both karst and the tunnel and analyze their interactions.

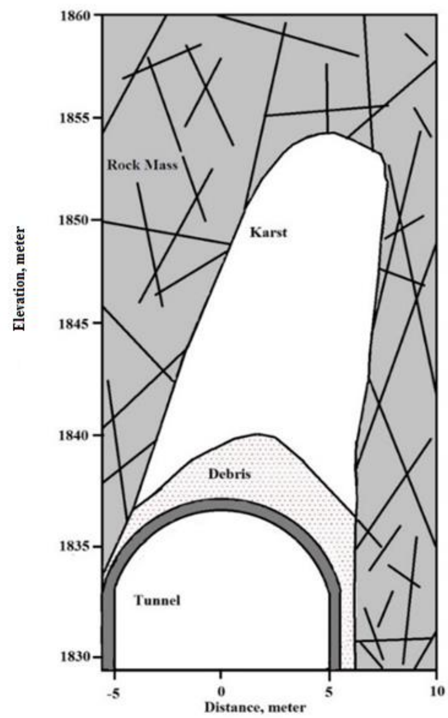

Figure 1. Schematic representation of model

The conducted two-dimensional surveys on the fracture networks in the field indicated four joint sets, with their respective dips and dip directions listed in Table 4. Due to the higher complexity in the nature of the fracture networks than that of the joint sets, the parameters of this Table sufficed the generation of the discrete fracture networks and no other joint set parameters are needed. Therefore, the same tabulated geometrical parameters were used to produce different DFN realizations. The statistical fit on the distribution functions associated with orientations indicated that Fisher and power law functions provide better fits on the orientation and the trace length of each of the four joint sets, respectively. Accordingly, the values corresponding to Fisher constant, fractal dimension, and maximum and minimum trace lengths were determined for the joint sets. The two-dimensional fracture intensity for each of the joint sets was computed as 0.55 m-2.

In order to eliminate the random effect or, in other words, to ascertain the independency of modeling results of a specific model, a number of discrete fracture networks with similar geometrical parameters and arbitrary random seeds were generated using Monte Carlo simulation process by means of FracIUT^{2D} (Baghbanan and Joolaei, 2010). It should be noted that different geometrical parameters of the fracture networks are not interdependent as a result of random numbers selection. To efface the boundary effect, as it is recommended that the ratio of generation domain to discrete fracture network model be chosen above 4 (Samaniego and Priest, 1984), parent fracture networks were generated at 120m×120m dimensions and square models with 30m×40m side lengths were cut out from the center of generation domain and 5 of the aforementioned DFNs were randomly selected. Following the generation of discrete fracture networks and to analyze the stress in the discontinuous medium, the fracture networks were regularized, dead-end fractures were isolated, and singly connected joints (which shared only one contact with other joints and had no role in the block formation) were removed. Blocks can be considered rigid or deformable in discontinuous medium analysis. The first discrete element codes considered blocks as rigid; however, the effect of intact blocks deformation in stability problems is imperative. Thus, to solve the problem in the discontinuous medium, the intact blocks were zoned and assumed to be deformable. The discontinuous model was then tessellated with triangular meshes at the average 0.3m of length such that each side was of 100 zones. Averagely, the discontinuous models comprised 7000 blocks and 77000 zones. The meshing system in the equivalent continuous media was configured with 300 uniform meshes at 30cm in dimension in each direction. The boundary conditions in the models differed in terms of ground conditions, in-situ stresses, and the varying sizes of the models. It had to be determined whether the hole appeared before or after tunnel excavation. Therefore, the in-situ stresses had to be applied accordingly. Stability analysis of karst is performed considering the tunnel excavation in both disturbed and undisturbed conditions. When the domain is considered as a very disturbed medium, the disturbance factor (D) is assumed to be 0.8. Moreover, when the domain is not affected by excavating of the tunnel, the disturbance factor is considered as zero.

The constitutive model of elastic/plastic, Mohr-Coulomb failure was assigned to the intact rock and the equivalent rock mass and the area contact elastic/plastic with Coulomb slip failure was selected for the joint behavior. Roclab software was used to acquire the required parameters for the modeling of the equivalent continuous medium, with the rock mass parameters tabulated in Table 5. As the host rock was characterized as limestone, the average value of mi for this rock was considered as 10 and the disturbance factor of the rock mass was assumed to be 0 and 0.8 to make a comparison and investigate the influence of considering the tunnel excavation on the stability of underground karst in two different states. The value of GSI (Geological Strength Index) for this rock mass was considered as 45 and evaluated based on different patterns of discrete fracture networks according to the provided table in (Hoek et al. 1998) which is also entailed in Roclab manual.

One of the common techniques for stability analysis of underground excavations is to evaluate the displacements in the region concerned. In this paper, the Sakurai equation, as one of the widely applied methods in determining admissible displacements around underground continuous media (Sakurai, 1981) was adopted. Due to the fact that this criterion is an approximation of the displacements in a zone with a specific radius and the karst geometry does not precisely conform to any regular shape, the karst area was approximated and the corresponding equivalent radius was used in the Sakurai equation. To calculate the admissible displacements around underground karst, Sakurai criterion in the second warning level of the danger is used as follows (Eq. 1):

$$\mathrm{Log}(\varepsilon_c) = -0.25\ \mathrm{Log}E - 1.22 \qquad \text{(Eq.1)}$$

Also, allowable displacement (u) is determined by calculating critical strain (ε_c) and equivalent radius of opening (r) as follows (Eq.2):

$$u = \varepsilon_c \times r \qquad \text{(Eq.2)}$$

As an implication of the equation's dependency on deformability modulus, the allowable displacements for this hole in two disturbed and undisturbed situations were estimated by 2.9cm and 2.6cm, respectively. The load values induced by the overburden in the equivalent continuous and discontinuous media were different according to the model dimensions. As in equivalent continuum method, the model is 90×90m in dimensions, the overburden is considered to be zero. However, in discontinuum medium, the model is in 40 m height and as according to the location of the tunnel, the height of overburden in this model is considered as 20m. Hence, the amount of overburden pressure is determined to be 0.43 MPa. The continuous medium was of a 1.94 MPa in-situ vertical stress and a 1.36MPa in-situ horizontal stress according to the corresponding ratio of average horizontal to vertical stress. The vertical in-situ stress in the discontinuous medium increased in downwards direction and was equal to 0.86MPa and applied on top of the model. The horizontal in-situ stress was K0 times as much as the vertical stress and equal to 0.61MPa which increased at a specific downwards gradient. Both models had roller boundaries at the bottom and two sides in a fashion that displacement and velocity for these boundaries were only allowed on top of the models and confined on other sides (Fig. 2).

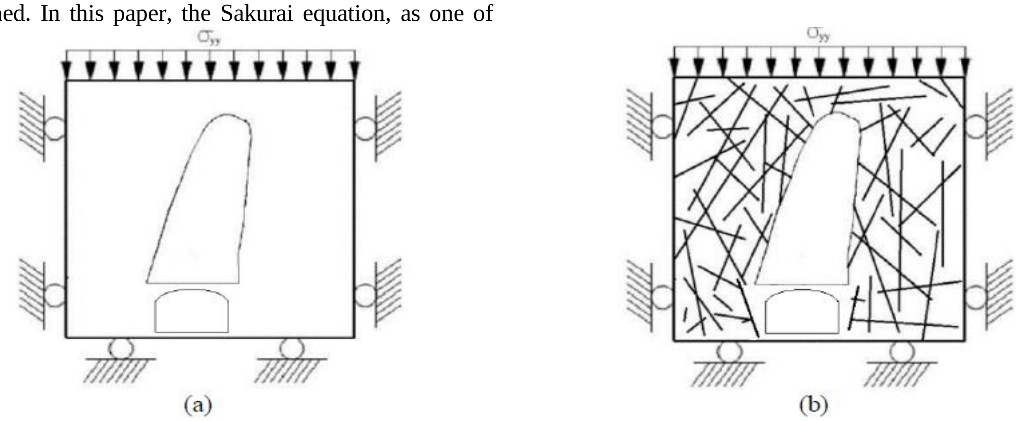

Figure 2. Boundary conditions for numerical modeling (a) continuous medium (b) discontinuous medium

Table 4 Geometrical Properties of Discrete Fracture Network

Join set ID	Orientation (D/DD)	Fisher Constant	Fracture Intensity (m^{-2})	Length Distribution function parameters		
				Fractal Dimension	L_{min} (m)	L_{max} (m)
1	20/150	3.2	0.55	1.35	1.8	61.7
2	85/162	7.2	0.55	1.35	1.8	61.7
3	70/34	8.0	0.55	1.35	1.8	61.7
4	63/90	8.0	0.55	1.35	1.8	61.7

Table 5 Rock mass properties in two different conditions

Parameter	Unit	Value	
Disturbance factor	-	0.8	0.0
Friction Angle	Degree	46.4	55.7
Cohesion	MPa	0.26	0.51
Deformability modulus	GPa	3.9	6.5

3. Results and discussions

3.1. Investigation of stability around the karst with equivalent continuum approach

Following the simulation of the concerned zone alongside the corresponding boundary conditions in the equivalent continuous medium, the vertical and horizontal displacements were compared in two disturbed and undisturbed situations. Fig. 3 shows the vertical displacements surrounding the hole, with the maximum 1cm in both disturbed and undisturbed mode which is at the bottom of the karst. In Fig. 4 the horizontal displacements are 5.5mm and 9 mm in undisturbed and disturbed medium, respectively. According to the displacement plots of the model, the displacement values in the equivalent continuous medium even in very disturbing situation were noticeably below the critical convergence of the hole.

Fig. 5 illustrates the stress distribution in y-direction around the tunnel and karst above. From Figs. 5(a) and 5(b), the concentration of S_{yy} around the karst is obvious in disturbed and undisturbed states, which is about 1 to 2 MPa around the karst walls. It can be seen that there are no remarkable differences between stresses states in two conditions considered for D. Therefore, as observed, only a small region has experienced a trivial displacement and no major instability exists. Furthermore, according to the computed allowable displacement in the equivalent discontinuous medium, the displacements occurred within the safe zone and the karst is acceptably stable.

3.2. Investigation of stability around the karst with discontinuum approach

As previously aforementioned, the simulation of the discontinuous medium was performed using discrete fracture network. A number of DFN models with the same geometrical parameters and various random seeds were generated to neutralize the random effect. Afterwards, by eliminating the dead-end fractures and introducing discrete blocks, the discontinuous model was zoned and subjected to the boundary conditions. According to the similarity of results acquired from different fracture networks, in the following, only two DFNs are investigated. It is observed that despite similar input parameters in models, the probability of stratification and other discontinuities exists in some of the models as presented in Fig.6. Figs. 6a and 6b show the displacements of the blocks surrounding the karst. The red vectors denote displacement and the blue circles show maximum block collapse. The maximum displacements in Figs. 6a and 6b are greater than 1.6 m and 2 m, respectively. As observed, the displacements in both models are higher than critical displacement and the clues of falling rock blocks are obvious. Figs. 7 and 8 respectively illustrate vertical and horizontal displacements in the discontinuous model. Maximum displacement generally occurred at the karst roof in vertical direction and was greater than 1.6m in model 'a' and 2 m in model 'b'. The horizontal displacement evaluated in Fig. 8exceeds 16cm on the left wall. The distribution of stresses in y-direction around the karst is illustrated in Fig 9 and it ranges from 0 to 6MPa.

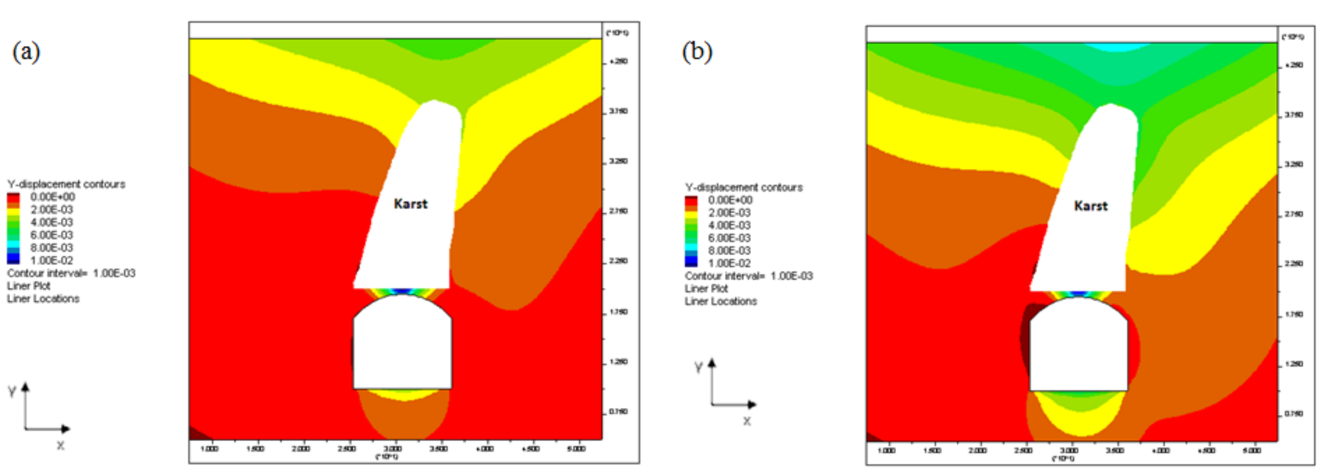

Figure 3. The Y-displacement contours in continuous medium when: (a) D=0, (b) D=0.8

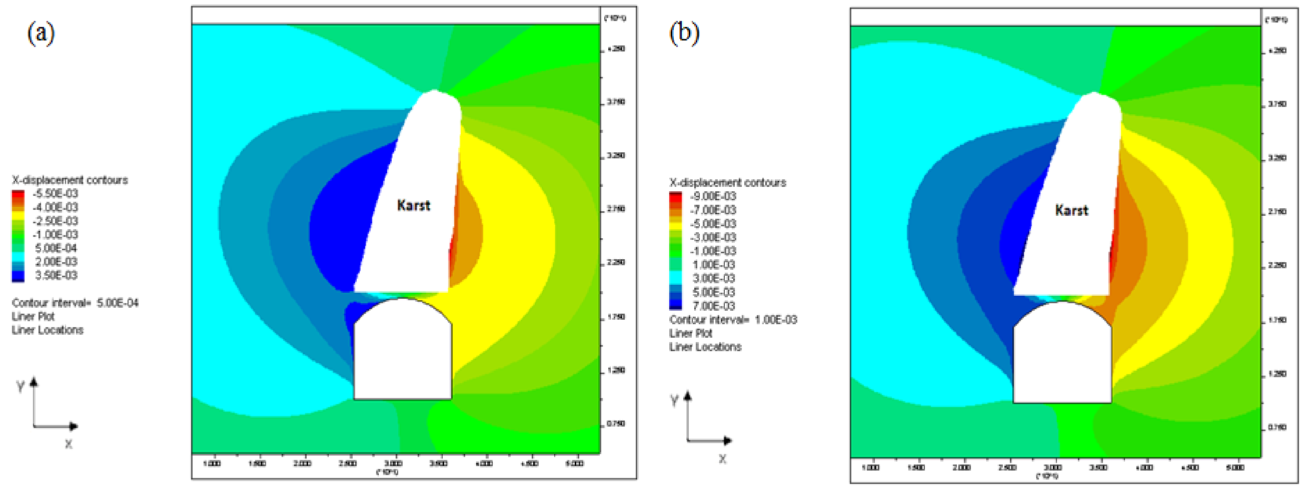

Figure 4. The X-displacement contours in continuous medium when: (a) D=0, (b) D=0.8

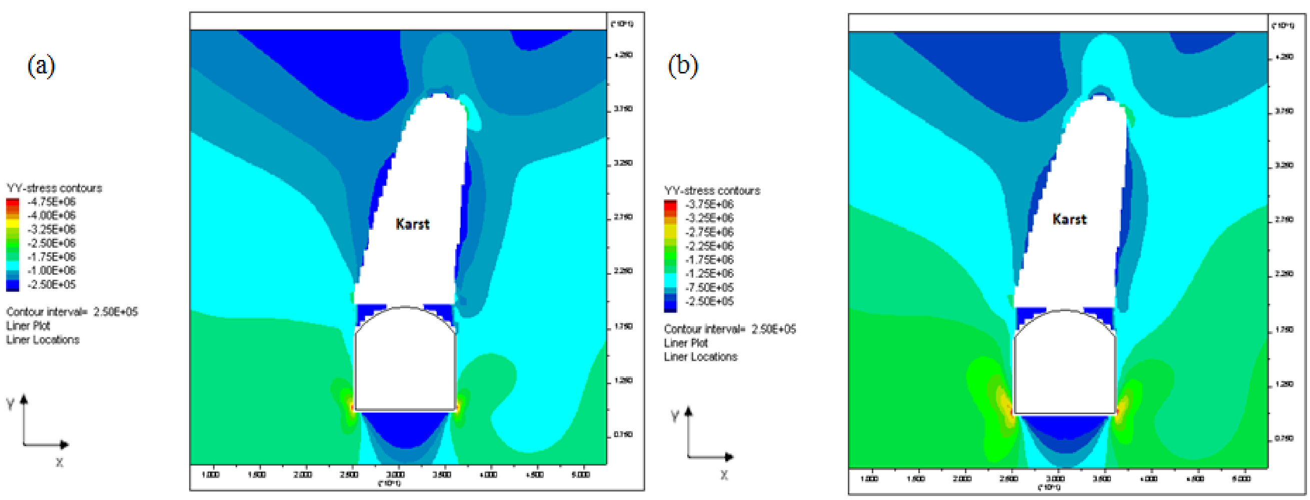

Figure 5. The YY-stress contours in continuous medium when: (a) D=0, (b) D=0.8

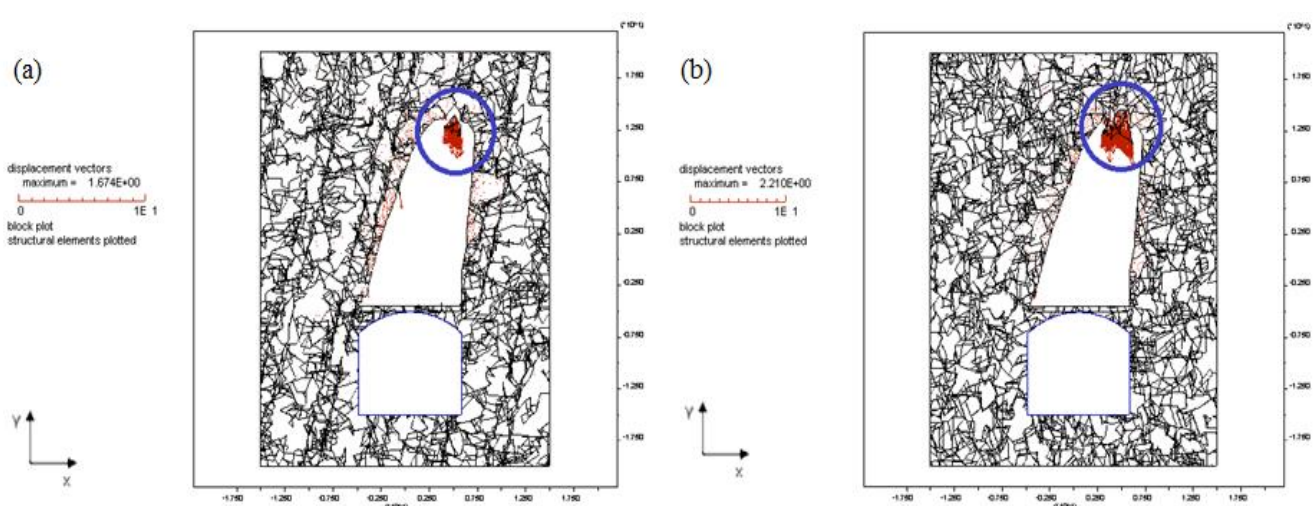

Figure 6. Blocks displacements after applying boundary conditions, the area with the highest block collapse is discerned with blue circle. The maximum displacement in (a) DFN1 and (b) DFN2 was greater than 1.6m and 2 m respectively a) D=0 , b) D=0.8

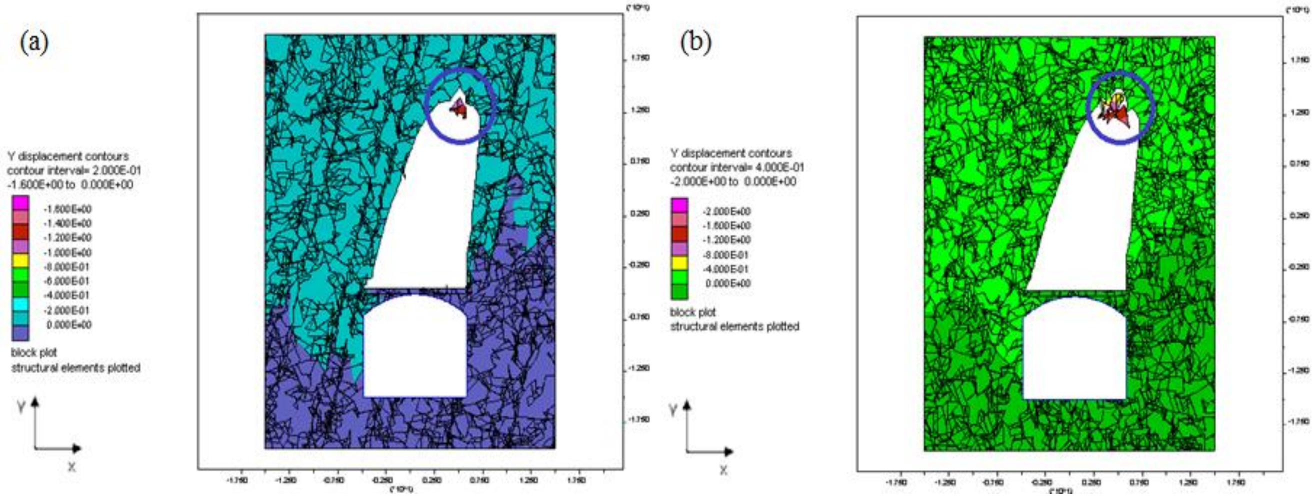

Figure 7. The Y-displacement contours in discontinuous media: (a) DFN1, (b) DFN2

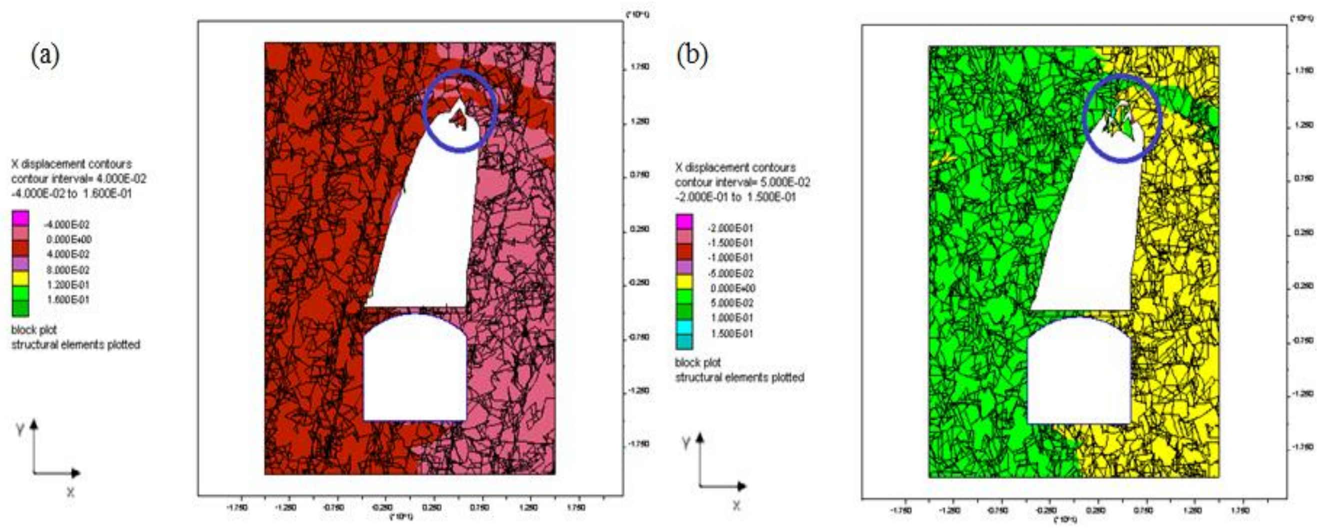

Figure 8. The X displacement contours in discontinuous media (a) DFN1, (b) DFN2

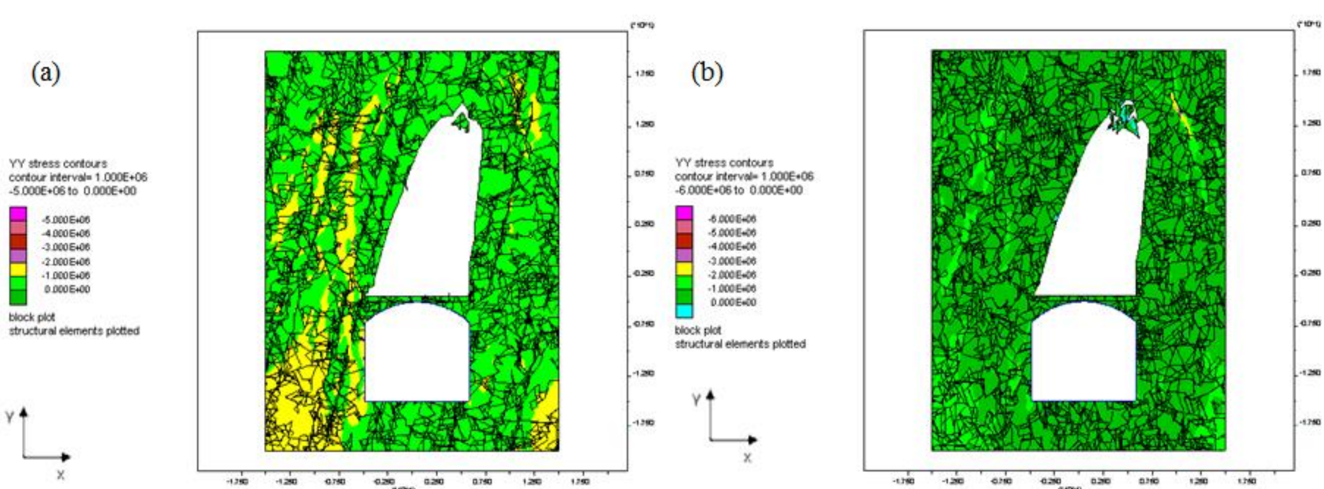

Figure 9. The YY-Stress contours in discontinuous media: (a) DFN1, (b) DFN2

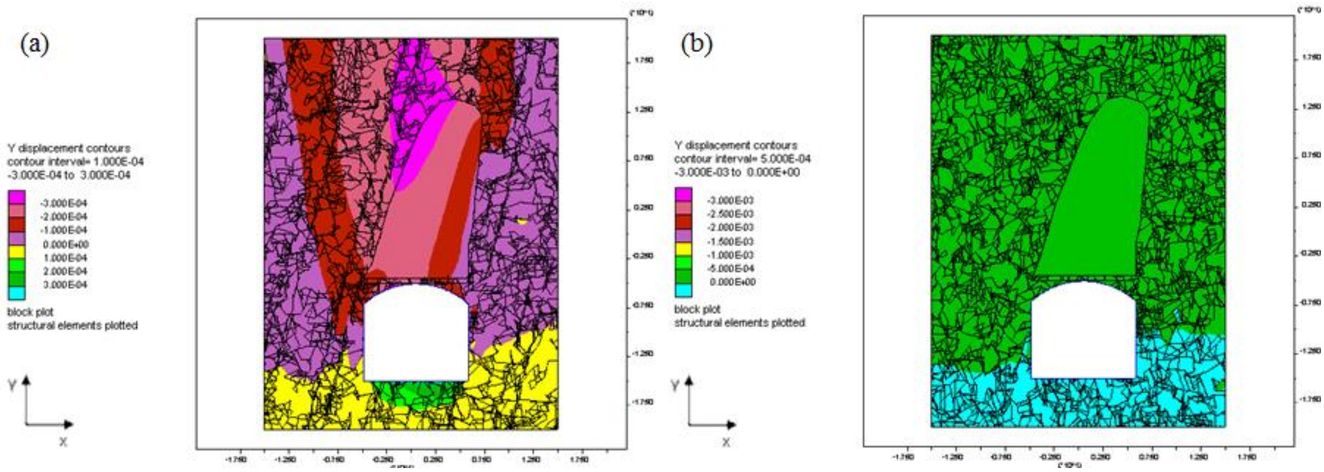

Figure 10. Y displacement after support implementation in two different model realizations: (a) DFN1, (b) DFN2

The concentration of stresses is not uniform in the vicinity of karst. The displacements computed using discontinuum method are significantly beyond allowable values. Due to the conditions of the tunnel and the lining, access to the karst is practically impossible. Nor are support systems such as rock bolts implementable. There exist various methods to supply support for the situations as such as proposed in (Gularte et al., 1993) and (Gómez et al., 2010). Nonetheless with respect to the large extension of this hole and the multiplicity of the existing karsts throughout the tunnel, the suggested method to avoid danger to the tunnel's lining is to use concrete foam to fill the karst. A light concrete foam with the density of 1200 kg/m^3, elastic module of 4 GPa, and Poisson's ration of 0.2 was selected for this purpose in the numerical modeling (Foamed Concrete, 2010). Contour displacements in Y direction after support implementation for aforementioned discrete models are depicted in Fig. 10. The average of Y-displacements in discontinuous models was 3 mm which indicates the decline of displacement magnitude and efficiency of suggested support system.

4. Conclusion

Numerical modeling of a fractured rock mass was performed by means of equivalent parameters in continuous medium and generation of discrete fracture networks in a completely discontinuous medium. The respective results on the selection of both numerical simulation methods were studied. The results obtained from the displacements around the karst approximated the displacement greater than 2m at discontinuous medium whereas this amount was estimated over 1cm in equivalent continuum method. This significant disparity in the estimation of the displacements at a specific region indicates the overestimation produced in equivalent continuum method for approximation of deformability in fractured rock masses. The analyses upon the displacements and stresses in continuum method considering the domain as totally disturbed and as undisturbed show that the area is relatively stable and there is no need to use support system even if the domain is affected by tunnel excavation. However, due to several detected collapses and also the agreement between the discontinuum method results and observations, the importance of employing this method is highlighted. Assuming that DFN-DEM represents the medium geometry in a close agreement with the actuality and discontinuum numerical method provides more accurate results, it can be suggested that utilizing continuum and equivalent continuum methods lead to high levels of uncertainty in the stability analysis of such underground structures. Neglecting this issue may lead to massive catastrophes. Comparison between results of karst with and without tunnel excavation in this medium shows that the modeling method has more importance than excavation sequence. Moreover, the results show that by considering the close agreement between discontinuum modeling and the real problem, uncertainty in simulations of the continuous models in this research work is prominent.

Acknowledgements

The authors wish to thank the Department of Mining Engineering, Isfahan University of Technology for preparing requirement of studies.

References

Baghbanan A., Joolaei A. (2010). The generation of 2D and 3D stochastic fracture networks. In: *Proceedings of the 14th symposium of geological society of Iran*, Urmia, Iran, Feb. 2010. [In Persian]

Bhasin R., Høeg K. (1998). Parametric study for a large cavern in jointed rock using a distinct element model (UDEC—BB). *International Journal of Rock Mechanics and Mining Sciences*, 35: 17-29.

Bobet A., Fakhimi A., Johnson S., Morris J., Tonon F., Yeung M.R. (2009). Numerical models in discontinuous media: review of advances for rock mechanics applications. *Journal of geotechnical and geoenvironmental engineering*, 135: 1547-1561.

Cundall P.A. (1971). A computer model for simulating progressive, large-scale movement in blocky rock system. In: *Proceedings of the International Symposium on Rock Mechanics*, Nancy, France, October 1971.

Cement and Concrete Institute (2010). *Foamed Concrete Report*. Cement and Concrete Institute press, Midrand, South Africa.

Gómez J., Baxter A., Dirdal M. (2010). Hollow Core versus Solid Bar Soil Nails for Support Applications in Karst Terrain: What We Learned!. In:

Proceedings of the Earth Retention Conference 3, Washington, USA, August 2010.

Gularte F.B., Griffis R.A., Kasunich J.E., Best T.C. (1993). Case study of compaction grouting for foundation support in karst terrain: Earth and Marine Sciences Building, University of California at Santa Cruz. *Environmental Geology*, 22: 291-295.

Hoek E., Marinos P., Benissi M. (1998). Applicability of the Geological Strength Index (GSI) classification for very weak and sheared rock masses. The case of the Athens Schist Formation. *Bulletin of Engineering Geology and the Environment*, 57: 151-160.

Itasca (2013). *FLAC^{2D}-Fast Lagrangian Analysis of Continua (Version 7 Users Manuals).* Itasca Consulting Group Minneapolis, NM.

Itasca (2013). *UDEC-Universal Distinct Element Code (Version 6 Users Manuals).* Itasca Consulting Group Minneapolis, NM.

Jing L., Stephansson O. (2007). *Fundamentals of Discrete Element Methods for Rock Engineering: Theory and Applications*. Elsevier Science, 562 p.

Min K-B., Jing L. (2003). Numerical determination of the equivalent elastic compliance tensor for fractured rock masses using the distinct element method. *International Journal of Rock Mechanics and Mining Sciences*, 40: 795-816.

Min K-B., Jing L., Stephansson O. (2004). Determining the equivalent permeability tensor for fractured rock masses using a stochastic REV approach: method and application to the field data from Sellafield. *UK Hydrogeology Journal*, 12: 497-510.

Sakurai S. (1981). Direct strain evaluation technique in construction of underground opening. In: *Proceedings of the 22nd US Symposium on Rock Mechanics (USRMS)*, Cambridge, Massachusetts, July 1981.

Samaniego J., Priest S. (1984). The prediction of water flows through discontinuity networks into underground excavations. In: *Proceedings of the Design and Performance of Underground Excavations: ISRM Symposium* , Cambridge, UK, Sep. 1984.

Shi G.H., Goodman R.E. (1985). Two dimensional discontinuous deformation analysis. *International Journal for Numerical and Analytical Methods in Geomechanics*, 9: 541-556.

Sitharam T., Latha G.M. (2002). Simulation of excavations in jointed rock masses using a practical equivalent continuum approach. *International Journal of Rock Mechanics and Mining Sciences*, 39: 517-525.

Solak T. (2009). Ground behavior evaluation for tunnels in blocky rock masses. *Tunnelling and Underground Space Technology*, 24: 323-330.

Souley M., Homand F. (1996). Stability of jointed rock masses evaluated by UDEC with an extended Saeb-Amadei constitutive law. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 3: 233-244.

Vardakos S.S., Gutierrez M.S., Barton N.R. (2007). Back-analysis of Shimizu Tunnel No. 3 by distinct element modeling. *Tunnelling and Underground Space Technology*, 22: 401-413.

Wu Q., Kulatilake P. (2012). Application of equivalent continuum and discontinuum stress analyses in three-dimensions to investigate stability of a rock tunnel in a dam site in China. *Computers and Geotechnics*, 46: 48-68.

Yoshida H., Horii H. (2004). Micromechanics-based continuum model for a jointed rock mass and excavation analyses of a large-scale cavern. *International Journal of Rock Mechanics and Mining Sciences*, 41: 119-145.