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Application of image-processing to crack propagation process identification in concrete under fatigue phenomenon

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ABSTRACT

Concrete is the most important material in geotechnical projects and constructions which that due to the cyclical loading and fatigues were always under failures. The fatigue is natural events were occurred in under cyclical loaded materials which causes very complex failures in under low stress and in elastic behavior. Although the concrete behavior and changes under axial loading are studied, but failure mechanism of fatigue and crack propagation on concrete (as the main factor to failure) is on obscurity. The investigation of crack propagation process from the beginning to failure under cyclic axial loading can be identified the fatigue mechanism. This study tired to monitoring and modeling the crack propagation mechanism under the cyclic uniaxial compressive strength (UCS_c) in axial loading of concrete specimens which help to understands of crack propagation and developed path in the body. For this purpose, the concrete specimens are tested by UCS_c, take photos repetitively (in 5 loading-unloading cycles) and used the image-processing techniques (IPT) for crack detection, propagation and simulation of the failure in concrete. The simulation results shown the IPT is good performance to crack propagation detections and propagation process is perfectly simulated by IPT.

1. Introduction

Geotechnical and civil engineering looking for materials which that had better engineering characteristics and higher strength properties (Azarafza et al., 2014). The concrete is the artificial material was playing an important role on construction (Haeri and Sarfarazi, 2016). Due to this importance and performance of concrete on constructions is always under consideration of many scholars were tried to evaluated the concrete behaviors under external loading (Simon and Chandra Kishen, 2017), matrixes changes (Moreno and Rubio, 2013), dynamical activity (Fajdiga and Sraml, 2009), stress-strain rate (Toyama et al., 2018), acoustic emission (Khandelwal and Ranjith, 2017) and crack propagation (Gasser, 2007) by using the various devices and modeling under the linear and non-linear fracture mechanics principles.

In the solid mechanics, concrete is classified as the elastic-brittle materials. If elastic-brittle materials loaded to specified point than unloaded, can be returns to primary condition. But if cross that point (yield point) by minimum extra pressure is broken. In other words, they don't have ductile or plastic behaviors (Khandelwal and Ranjith, 2013). This definition is modified by the fracture mechanic as the quasi-brittle materials. These type of materials (like concrete, some type of rocks) since 1960s till now is considered and discussed in different concepts specially tensile response, shear retention, tension-stiffness, tension-hardening, tension-softening, crack generation, crack propagation, etc. (de-Borst, 2002). As known, concrete has high compressive strength versus significantly lower in tensile strength. So, the tensile stresses failure is the main factor to concrete destruction (Eberhardt et al., 1999). The cyclical loading-unloading make the situation to tensile stresses failure and cracking (Fajdiga and Sraml, 2009). Generally two type of

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cracking is created in concrete, namely, shrinkage cracking and tension cracking. Shrinkage cracks are occurring when concrete members (cements and aggregates) endure restrained volumetric variation (shrinkage) as drying, autogenously shrinkage results or thermal effects. The tension cracking are occurring when concrete members is tolerates external loading (Bhowmik and Ray, 2018). The fatigue phenomenon is classified in tension cracking types in concrete.

Normally, concrete is loaded axially or all-round, the tensile stresses is increasing and passing through the elastic area to reach the yield point (YP). In this area, the micro-cracks generated and the existent cracks are grown then lead the material to final fracturing and in maximum value of σ_{ult}, the failure is occurred. This behavior is illustrated in Fig. 1(a). In concrete cyclical loading due to the loading-unloading cycles, the micro-cracks is generated and propagated in elastic area and tensile stresses passed the σ_{ult} and reached to the σ_r were decrypted as fatigue phenomenon. This behavior is illustrated in Fig. 1(b). Concrete fatigue is making the weakening behavior caused by repeatedly applied loads which cause the progressive and localized structural damage (cracking) much less than the maximum strength, σ_{ult} or ultimate tensile stress limit. Results of the loading-unloading is creating the microscopic sliding surfaces/cracks were effected the concrete behavior which eventually will reach a critical size and propagate suddenly then the structure will be fractured (Toyama et al. 2018). As seen in Fig. 1 the fatigue phenomenon is moved the maximum strength to residual strength (σ_r) were this effect makes failure under the σ_{ult} in concrete.

Barenblatt (1962) and Dugdale (1960) are first man who presented the simple idealized approximation method to assessment of local fracture generation and growing based on stress-crack opening displacement relation for brittle and ductile fracture behaviors, respectively. Paris and Erdogan (1963) is one of the earlier scholar works on fatigue crack propagation model (FCP) and introduced the Paris law were able to accurately prediction of metal structures crack propagation behaviors. Hillerborg et al. (1976) used the cohesive crack method (CCM) to modeling of concrete softening damages.

Reinhardt et al. (1986) analyzed the failure mechanism on concrete based on tensile tests and mentioned the main factor of cracking in concrete is tensile stress mechanism. Alzos et al. (1976) is investigated the crack opening and crack propagation under cyclic loading relation. Sahasakmontri and Horii (1991) state that the cyclic loading stress degradation is causes the crack propagation in both experimental and analytical assessment. Bazant and Xu (1991) introduced the analytical assessment based on combination of Paris law and size effect law. This method describes the crack length increment per each cycle.

Bazant and Schell (1993) used the experimental method on high strength concrete to investigation of crack propagation. Feenstra and Borst (1995, 1996) is used the plastic model to crack propagation on concrete. Slowik et al. (1996) presented the FCP model based on changeable amplitude of load cycles and the Toumi et al. (1998) used the flexural cyclic loading to crack propagation in concrete. Bazant and Planas (1998) present the CCM parameters experimental determination. Kuhl and Ramm (1998) showed a model to detailed analysis of micro-plane linearization. de-Borst (2002) has introduced a comprehensive review on fracture mechanics in quasti-brittle materials like concrete. Subsequently, many scholars performed various tasks of crack propagation on concrete in the form of case studies and computer simulations like Kim et al. (2004), Tregger et al. (2006), Roesler et al. (2007), Ciavarella et al. (2008), Kumar and Barai (2009), Alshoaibi (2010), Kumar and Barai (2011), Paggi (2011), Ray and Chandra Kishen (2011; 2012), Le et al. (2014), Kirane and Bazant (2016), Azarafza et al. (2017), Simon and Chandra Kishen (2017), Zhao and Ye (2017), and Zhang et al. (2017).

As known by advent of computer analysis in geotechnical field, the computer based simulation and calculation have replaced the old empirical models. In recent years application of the dual models (computer based and empirical) shown good results and more detailed investigation with remarkable precision. In the meantime, artificial intelligence (AI) techniques have surpassed all approaches. One of the most usage AI techniques is the image processing techniques (IPT) has glanced up in computer based and empirical based dual models in geotechnics were used of this paper for crack detection, propagation monitoring and simulation of the failure progressive mechanism in concrete.

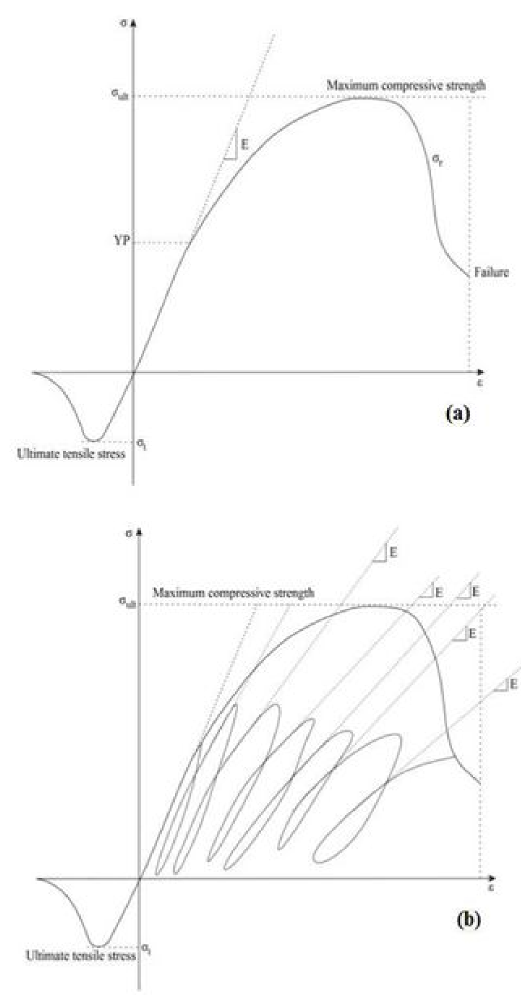

Figure. 1. The behavior of the elastic materials like concrete: (a) Topical behavior of concrete, (b) Fatigue behavior of concrete

2. Methodology

The aim of this study is monitoring and computer based investigation of crack propagation mechanism in concrete specimens under cyclic uniaxial compressive strength (UCS_c). For this purpose, the uniaxial compressive strength standard test (UCS) were described by ASTM is used. The Canon PowerShot SX610 camera which that horizontal fixed front the UCS is used to continuous shooting and taking photos from the specimens and recording the cracks generation and monitoring all stages of failure to final slip. Recorded images utilized by the MATLAB software (Mathwork, 2018) image-processing application. The mentioned methodology is used to step-by-step monitoring and modeling the crack generation and propagation mechanism under the UCS_c to achieve the final failure on concrete specimens.

2.1. Crack propagation mathematical formulation

The discontinuities are the weakness in elastic/homogeneous, continuous and isotopes materials like concretes, rocks which make very modest behavior change on it (Perez, 2016). Identification and characterization of fracture mechanics of these elements can be useful to stabilization and improvement of materials. Especially for concrete which is major ingredient in the engineering structures constructions around the world, the monitoring and analyzed is more efficient and sensitive. Thus, focus on this issue is increasing day-by-day among engineers involved in concrete projects like dams, retaining walls, roads, tunnels, building, etc. For this purpose, various approaches utilized such as numerical, analytical, fourier, integral transform, complex variables and knowledge-based methods (Azarafza et al., 2017). In these cases the knowledge-based is the newest approaches used in this field by application of AI techniques and tools to detailed close approximate estimation with high accuracy like data mining, deep-learning, machine learning, neural network, fuzzy logic, etc. However, the methods were used and developed for crack propagation were classified in two types; continuous and discrete methods which each type have own assumptions (Hofacker and Miehe, 2012, Kosteski et al., 2012, Braun and Fernández-Sáez, 2014; Toyama et al., 2018). Although any of these types have proponents and opponents who focus on the advantages and disadvantages modeling, but is still far from accurate definition of crack generation and propagation events especially fatigue phenomenon.

2.2. Uniaxial Compressive Strength

UCS is the capability of materials or structures to axial loading tending to reduce size, deformation and tolerability. UCS commonly used to assessment of the materials strength which accompanied can be evaluate the maximum axial stress/strain, tensile strength, yield point stress/strain, residual strength, Poisson's coefficient and elasticity modulus were is key value for structures design. UCS is often measured on a universal testing machine (UTM) or compression test machine (CTM) which presented various instructions (ISRM, 1981; ASTM D2938; ASTM D5102; ASTM D7012). According to the UCS, the maximum axial stress (σ_{ult}) is calculated based on the force-area ratio which is shown in Eq. (1).

$$\sigma_{ult} = \frac{F_{\max}}{A} \tag{1}$$

where, F_{max} maximum applied axial force and A cross-section area of specimen. To achieve appropriated measurements must load cell have small stiffness (in ideal condition), but in real cases this factor can be reached then the sample is prepared which make low error in test (Azarafza et al., 2017) and forces controlled by pressure transducer (Kuhinek et al., 2011). In this study, totally the 10 specimens of concrete is prepared in cubic shape under the ASTM instructions and the UCS_c test conducted on the specimens and the main mechanism of the failures is recorded by cameras. Figure 2 is presented the UTM devices where used in this work. The used device is applied hydraulic compressive pressure on the sample continuously and all deformation and forces recorded. The forces increases until the samples break which the pressures can be added manually or automatically.

2.3. Image-processing technique

The image-processing technique (IPT) is the processing of digital image to extraction of useful data from the image were utilizing in different purposes such as medical sciences, environmental sciences, geosciences, astronomy, engineering, social sciences and special issues (Gonzalez and Woods, 2017). Application of IPT is generally attributed digital processing of any two-dimensional images to extracting or detection of features from pixels were used computer algorithms to analysis the digital images (Solomon and Breckon, 2011). Generally the IPT is categorized in following techniques (Nixon, 2017):

- Image pre-processing,
- Image enhancement,
- Image segmentation,
- Feature extraction,
- Image classification,
- Multi-scale signal analysis,
- Pattern recognition,
- Projection,
- Anisotropic diffusion,
- Hidden Markov models,
- Image editing and restoration,
- Independent component analysis,
- Linear filtering,
- Partial differential equations,
- Pixelation,
- Principal components analysis,
- Self-organizing maps,
- Wavelets.

The IPT historical background is from 1960-1970s at the Jet Propulsion Laboratory (JPL), Massachusetts Institute of Technology (MIT), Bell Laboratories, University of Maryland research facilities where work on the IPT application to satellite imagery, remote sensing, wire-photo standards conversion, medical imaging, videophone, character recognition, and photograph enhancement (Gonzalez and Woods, 2017). Image pre-processing is the first and most important step in IPT which is used the algorithms and filters, removing the noise, sharpen of digital images tried to prepare a suitable image to analysis (Nixon, 2017). The Fig. 3 illustrated the IPT application process in this study. The main idea form this work is using the recognition of cracks and discontinuity network in concrete where created base on axial loading in UCS_c. Therefore, the IPT techniques of our work can be classified in following groups:

- Image pre-processing,
- Image enhancement,
- Image segmentation,
- Feature extraction,
- Image classification.

Figure. 2. The UTM devices where used in this work

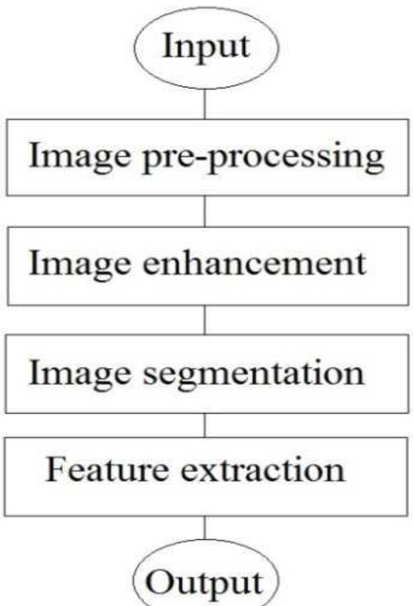

Figure. 3. IPT process flowcharts of the study

3. Empirical study on concrete crack generation and propagation

Before the UCS_c test conducted on specimens, there are molded under ASTM standard and cubic shape. These samples loaded-unloaded step-by-step in UCS_c which is illustrated in Fig. 4(a) and recorded data from the test is prepared as Fig. 4(b). Also, by using the camera, in all steps of loading-unloading process, the photos are taken. These steps and imaging is continues to concrete specimens failures and cycle done. Different stages and data of loading-unloading in the UCS_c tests form all specimens which used in this study is presented in Figs. 5 to 9. In this figure the staged changes, crack creation and generation in concrete and cyclic behavior (stress-strain curve) have been shown.

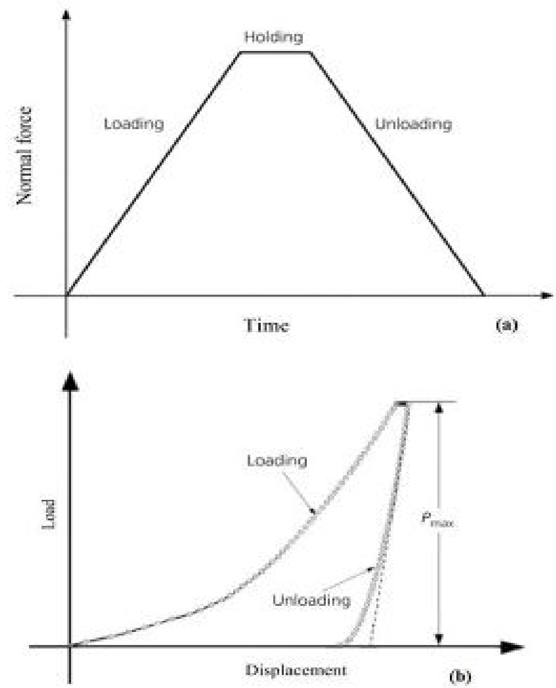

Figure. 4. UCS_c test work process: (a) loading-unloading in a cycle, (b) recorded data/results of cycle

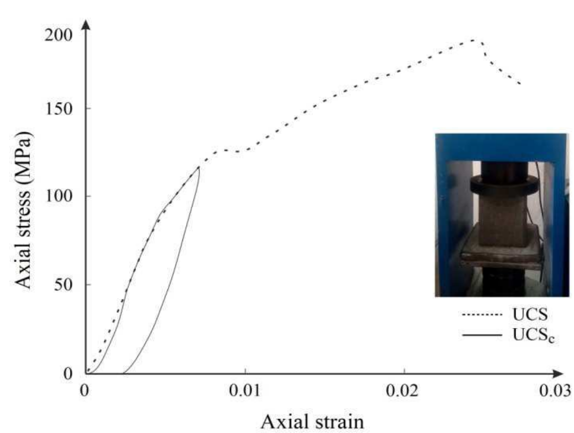

Figure. 5. UCS_c test steps and results (cycle 1)

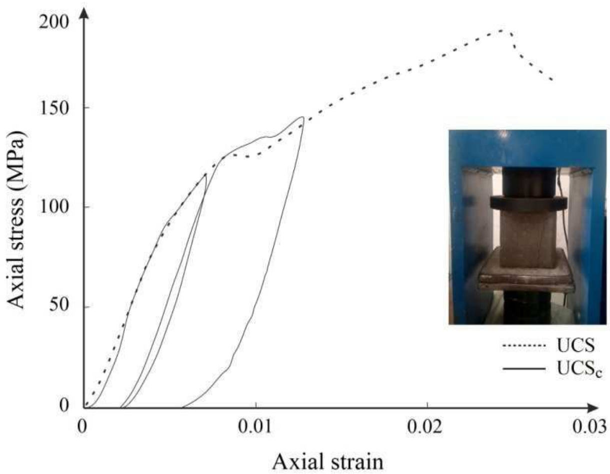

Figure. 6. UCS_c test steps and results (cycle 2)

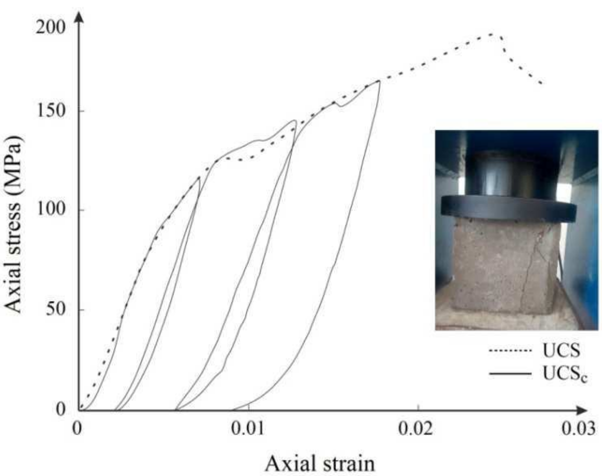

Figure. 7. UCS_c test steps and results (cycle 3)

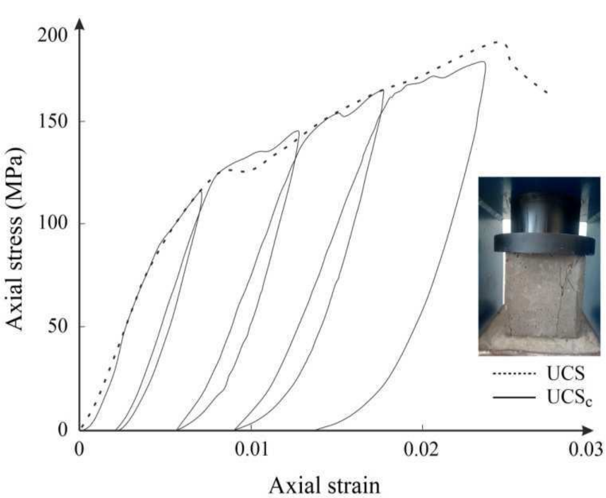

Figure. 8. UCS_c test steps and results (cycle 4)

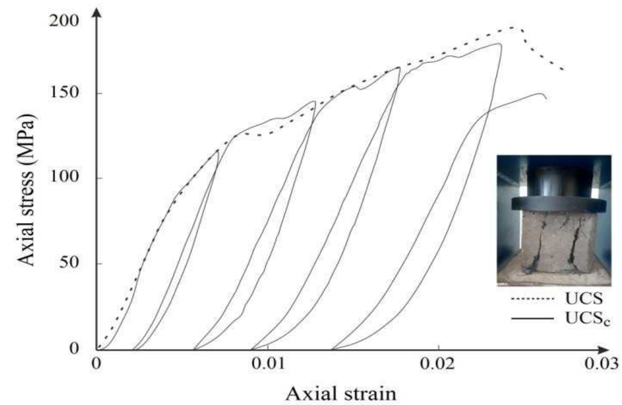

Figure. 9. UCS_c test steps and results (cycle 5)

4. Images assessment on concrete crack generation and propagation

The taken images from each step of UCS_c are used to feature extraction and classification by IPT. For this purpose, the continuous processes include pre-processing, main-processing, image-improvement and edge detection are used were conducted the filtering and sharpen application of MATLAB software (Mathwork, 2018). The filters which used to identification, enhancement, and reconstruction of imagery included the following applications:

- Gray-scale filtration,
- Prewitt filtration,
- Laplacian and Laplacian of Gaussian filtrations,
- Masking,
- Edge detection and Canny matrix.

The IPT result of a crack generation and propagations on a studied concrete specimen is presented on the Fig 10 (for all cycles). Also the simulation technique used to extraction of features form the crack generation and propagations form the UCS_c samples and detect the step to step changes on the specimens. These simulation results are shown in Figs. 11 and 12. According to the results of image processing were utilized on the crack generation of UCS_c tests, the crack propagations able to detect in different steps. The crack generation and propagation on concrete is make weak planes which causes the future loading cycles, at first the existing cracks closed and shown the significant changes in low stress (high strain). Also, tensions concentration within the existing cracks has led to reduction of concrete strength. When this stress is increasing (loading), the tension concentrations are at the crack tips were helped to propagation rapidly. Otherwise, by unloading, the concrete elasticity causes to reform materials to primary status which is forced the crack under tensile conditions. According to the Griffith's tensile theorem in solid materials (Griffith, 1920), these occurrences help to crack propagation much more than loading condition. During the next loading, the stresses are located at crack tip and make progressive failure and growing cracks. Which these cycles is continued, the cracks are interconnected and the critical sliding surface is created. Thus, concrete is failure occurred less than σ_{ult}.

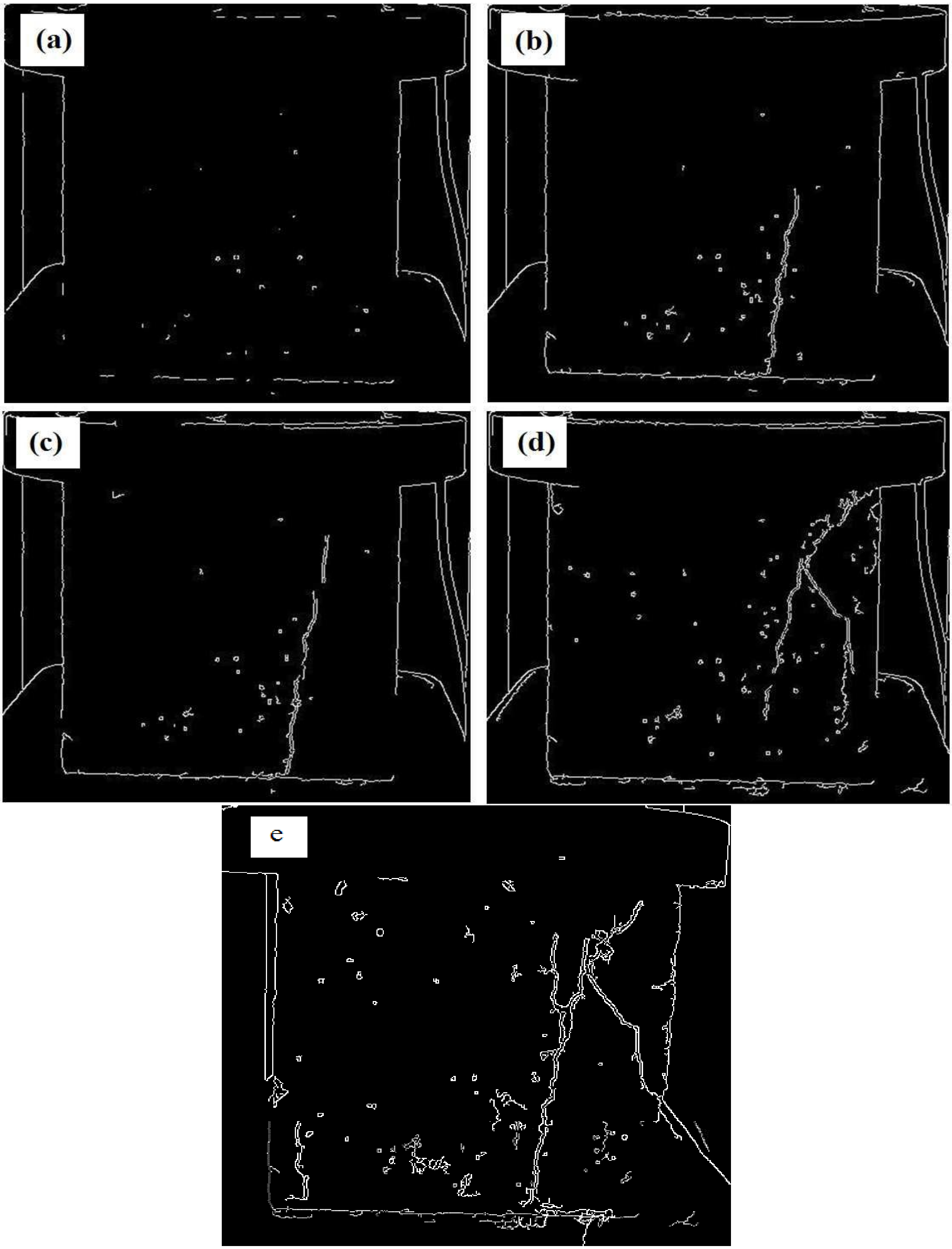

Figure. 10. Crack propagation by IPT, (a) cycle 1, (b) cycle 2, (c) cycle 3, (d) cycle 4, (e) cycle 5

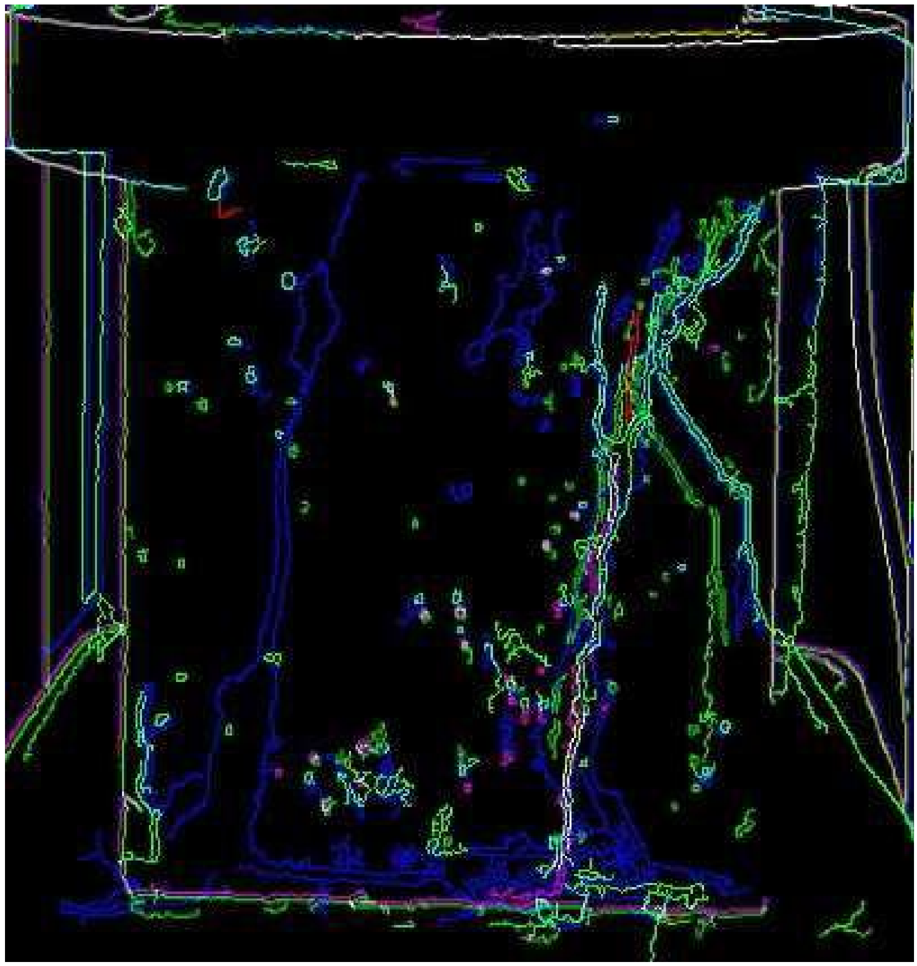

Figure. 11. The model of crack propagation in concrete specimens under UCS_c

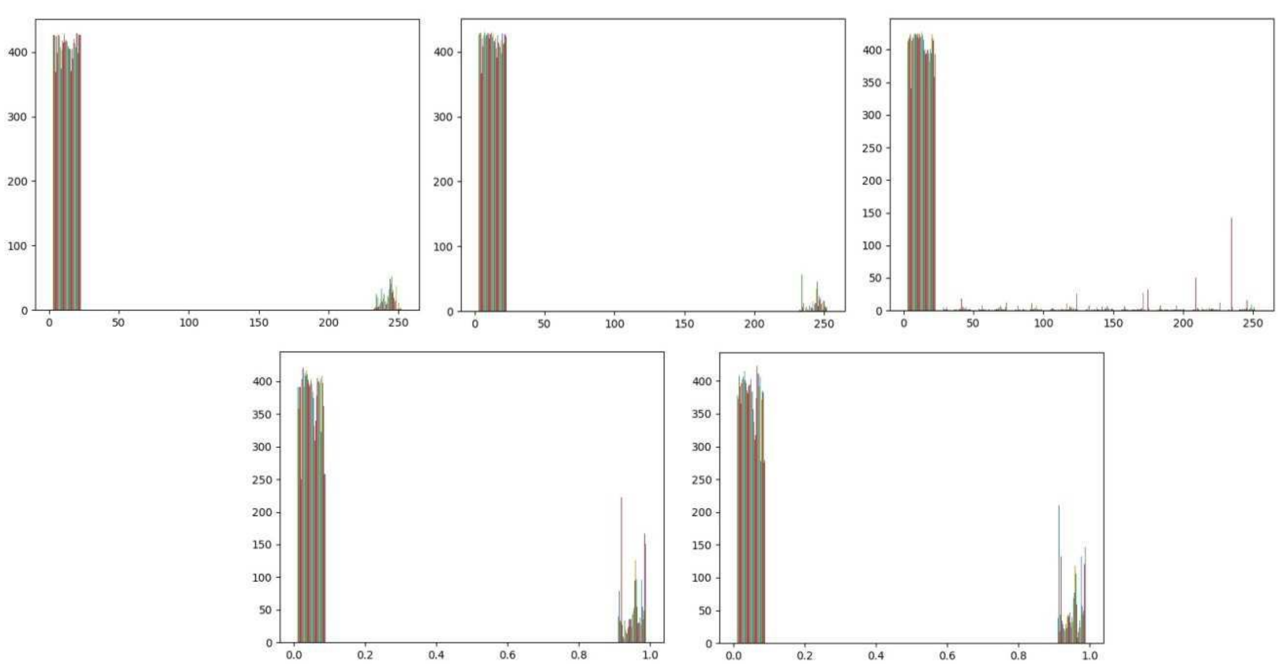

Figure. 12. The histograms of crack propagation in concrete specimens under UCSc

5. Conclusion

The aim of this study is investigation of crack generation and propagation on concrete based on fatigue loading. The artificial intelligence (AI) based image-processing technique (IPT) utilized for monitoring and modeling the crack propagation mechanism. For this purpose, the concrete specimens is tested by cyclic uniaxial compressive strength (UCS_c) were described by ASTM and the Canon PowerShot SX610 camera which horizontally fixed front the UTM is used to continuous shooting and taking photos from the specimens and recording the cracks generation and monitoring all stages of failure to final slip. By using the MATLAB software to detection, feature extraction and classification of images is tried achieve the detailed simulation of crack growing mechanism in concrete specimens. The taken images from each step of UCS_c are continuously processed including pre-processing, main-processing, image-improvement and edge detection which lead to create a model of discontinuity network on samples. The simulation results shown the IPT is good performance to crack propagation detections.

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