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Assessing the Value Efficiency of Each Unit in Relation to the Tangent Hyper Plane in DEA (Units Including Negative Data With Interval Scale)

Hossein Abbasiyan and seyyed ali kazemipour*
Department of Mathematics, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran

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Abstract

In data envelopment analysis, value efficiency is an efficiency concept that uses the decision maker's priorities to calculate it. In this article, data with an interval scale is the difference of two different data of inputs and outputs with a ratio scale, and one of the innovations of this research is that it calculates the value efficiency of units that include negative data with an interval scale. In real value efficiency, the indifference curve of the value function is used, which is unknown, and another innovation of this research is that we approximate this curve with the tangent hyper plane at the point with most preferences and with the proposal of the decision maker, we consider one of the technical efficiency units as the point with the most preferences. To find this tangent hyper plane, we use the dual problem of radial models, which have returns to variable scale. Finally, the distance of each decision-making unit to the tangent hyper plane shows the value efficiency of that unit. In the presented numerical example, the obtained results are very close to the results of Halme and his colleagues' models, and this method can provide a suitable approximation for value efficiency.

Keywords:
Relative efficiency
Radial Models in Data Envelopment Analysis
Interval Scale Data
Most Preference Solution

***Correspondence E-mail**: abbasiyan58@gmail.com

INTRODUCTION

Data Envelopment Analysis (DEA) was originally proposed by Charnes, Cooper and Rhodes (1978 and 1979) as a method for evaluating the Technical Efficiency of Decision Making Units (DMUs) essentially performing the same task. The basic DEA is value-free and it is considered an advantage of the DEA that no preference information is needed. But it is possible to incorporate into the analysis the Decision Maker's (DM's) judgments. To incorporate the DM's preferences into efficiency analysis, developed by Halme et al. (1998), Korhonen et al (2002), Joro et al (2003), on the interpretation of Value Efficiency (VE) by Korhonen et al (2005) and also the improved estimate of VE by Zohrehbandian (2011). Here we deal with the negative data which is derived often from observations of the variables measured on the Interval Scale (IS). In many applications from DEA, the IS variables like profit and changes in different variables (like sales, loans etc.) have been used as inputs and/or outputs. Data on the IS does not allow division (since the zero point is not defined and only distances can be calculated, Halme et al (2002)). The approach of Halme et al.'s measuring value efficiency of each Decision Maker Unit (DMU) as a distance to an approximated indifference contour of a DM's Value Function (VF) at Most Preference Solution (MPS). Different ways exist of obtaining an MPS. A way to introduction an MPS is to first to compute the technical efficiency of the unit (after decomposition of the IS variables) and then to make the choice from the set of efficient units. If the number of efficient units is more than one, the Decision Maker (DM) can choose one.

Our proposed method for measuring VE scores is so that first we approximate the indifference contour of the Value Function (VF) at MPS by the tangent hyper plane and then to calculate the value efficiency of an inefficient unit, we use the distance of that unit to the hyper plane tangent to the Production Possibility Set (PPS) in MPS, and to find this hyper plane, we do the following. Of course, here we may be dealing with negative data.

We use dual the proposed radial models for the IS data by Halme et al. (which the proposed procedure by maintains the applicability of the radial model after the decomposing IS variables) for introduce tangent hyper plane which approximates the indifference contour at MPS. The VE scores are calculated for each DMU, in the output direction without solving any linear programming problems, comparing the inefficient units having the same value as the MPS. The proposed method in this paper doesn't worse than the method of Halme et al. and dependence on a tangent hyper plane.

The rest of this paper is organized as follows. In section 2, we review the value efficiency analysis and the interval scale data. Our method of producing a measure of value efficiency is discussed in section 3. Numerical examples are presented in section 4 and finally, section 5 draws the conclusive remarks.

THEORETICAL FOUNDATIONS

Most Preference Solution (MPS) and Value Efficiency (VE)

When it is not necessary to emphasize the different roles of inputs and outputs, we denote $u=(y,-x)^T$ and $U=(Y,-X)^T$. Define the sets $\Lambda=\left\{\lambda \middle| \lambda\in\Re_+^n, A\lambda\le b\right\}$ and $T=\left\{u \middle| u=U\lambda, \lambda\in\Lambda\right\}$. All efficient DMUs lie on the efficient frontier, which is define as a subset of points of set T satisfying the efficiency condition above.

Definition 1: A point $U\lambda^*=u^*$ is efficient if and only if there does not exist another $u\in T$ such that $u\ge u^*$, and $u\ne u^*$.

Definition 2: A point $u^*\in T$ is weakly efficient if and only if there does not exist another $u\in T$ such that $u\ge u^*$.

We suggest that the DM's preferences are incorporated in efficiency analysis by explicitly locating his/her most preferred input-output vector on the efficient frontier. We call this vector the DM's Most Preferred Solution (MPS). It is a vector on the efficient frontier which he/she prefers to any other vector at the moment of the choice. (e.g., steuer 1986).

Using the knowledge of the MPS, the DM's (unknown) value function is approximated using so-called tangent cones at the MPS. The efficiency of each DMU is then determined with respect to this tangent cone. As a result we obtain scores that we call Value Efficiency (VE) scores, because the efficiency of each DMU is determined by means of an approximation of the indifference surface of an implicitly know value function at the MPS. (Halme et al.).

The MPS is a solution which is preferred by the DM to any other solution. Assuming a rational DM who prefers more of any output and less of any input, it is obvious that the MPS is efficient. Unfortunately defining the MPS in this way provides no assume that the DM is generally able to compare all possible solutions to the final solution at the end of the search. We assume that the choice of the MPS was based on the DM's value function $v(u)$, $u=(y,-x)^T \in \Re^{m+p}$, which is strictly increasing (i.e. strictly increasing in y and strictly decreasing in x) and with a (local) maximal value $v(u^*)$ over T, $u^*=(y^*,-x^*)^t \in \Re^{m+p}$. Furthermore, we assume that v is psedoconcave, because then its local optimum over a convex set is also global. (Bazzaraa and shetty 1979).

The purpose is to assess the efficiency of each unit in relation to the indifference contour of Decision Maker's (DM's) Value Function (VF) passing through the MPS. This assess could be done easily, if we explicitly knew the VF. The idea of VE is to incorporate the DM's preference information regarding a desirable combination of inputs and outputs into the analysis. The MPS is a (virtual or existing) DMU on the efficient frontier with the most desirable values of inputs and outputs. In practice, the VF is unknown and we cannot characterize the indifference curve precisely but we have to approximate it. Halme et al. (1999) assumed that the DM's (unknown) value function $v(u)$, $u=(y,-x)^T$ is pseudo concave, and strictly increasing in u (i.e. strictly increasing in y and strictly decreasing in x) and with a maximal value $v(u^*)$, $u^*=(y^*,-x^*)^T \in PPS$, at MPS u^*. Halme et al. (1998) explained that for approximating the tangent hyper planes of all possible pseudo concave value functions which obtaining their maximum at the MPS, can use the region containing all vectors u surely less than or equally preferred to the MPS and they also stated the following theorem.

THEOREM 1: let $u^*=(y^*,-x^*)^T \in T$ be the DM's most preferred solution. Then $u \in \Re^{m+p}$, an arbitrary point in the input/output space, is value Inefficient with respect to any strictly increasing pseudo concave value function v(u), $u=(y,-x)^T$ with a maximum at point u^*, if the optimum value z^* of the following primal problem is strictly positive.

Where $\lambda^* \in \Lambda = \left\{\lambda \middle| \lambda \in \Re^n_+, A\lambda \le b\right\}$, η^* correspond to the MPS: $y^*=Y\lambda^*, x^* \in X\lambda^*$, $T=\left\{u \middle| u=U\lambda, \lambda \in \Lambda\right\}$.

NOTE: For easy reference to the traditional DEA models we have give the output and input parts separately.

They carried out VE with standard DEA utilizing linear programming. In the following models, vectors $w^x>0$ and $w^y>0$ are the weighting vectors for inputs and outputs, respectively and the vector g^x consists of aspiration levels for inputs and g^y of aspiration levels for outputs. If a particular unit's efficiency has to be checked, vector ***g*** is replaced by its input/output vector. Here, vector $g=\left(g^y,g^x\right)^T \in PPS$ is value inefficient with respect to any strictly increasing pseudo concave value function $v(u)$, $u=\left(y,-x\right)^T$ with a maximum at point u^*, if the optimum value Z^* of the following problem is strictly positive:

Where λ^* and η^* correspond to the MPS: $y^*=Y\lambda^*$, $x^*=X\lambda^*$ and MPS is one of the efficient units that are selected by the decision maker if there are many of them.

The General VE primal Model	The General VE dual Model
$\max \quad Z=\sigma+\varepsilon(1^T s^+ + 1^T s^-)$ $subject\ \ to$ $Y\lambda-\sigma w^y - s^+ = g^y,$ $X\lambda+\sigma w^x + s^- = g^x,$ $F\lambda+\eta=d, \qquad (1)$ $\varepsilon>0,(Non-Archimedean)$ $\lambda_j \ge 0,\ if\ \ \lambda_j^*=0,\ j=1,\cdots,n$ $\eta_j \ge 0,\ if\ \ \eta_j^*=0,\ \ j=1,\cdots,k$	$\min \quad W=v^T g^x-\mu^T g^y+\rho^T d$ $subject\ \ to$ $-\mu^T y_j+v^T x_j+\rho^T F_j=0,\ \ j\in\{j \mid \lambda_j^*>0, j=1,\cdots,n\}$ $-\mu^T y_j+v^T x_j+\rho^T F_j\ge 0,\ \ j\in\{j \mid \lambda_j^*=0, j=1,\cdots,n\}$ $\mu^T w^y+v^T w^x=1 \qquad (2)$ $\mu,v\ge\varepsilon 1,\ \varepsilon>0,(Non-Archimedean)$ $\rho_j=0,\ \ if\ \ \ \eta_j^*=0,\ \ j=1,\cdots,k$ $\rho_j\ge 0,\ \ if\ \ \ \eta_j^*>0,\ \ j=1,\cdots,k$

Decomposing of Interval Scale Data

The Negative data values were observed frequently. We encountered as variables with negative observations that a result of a deduction of two Ratio Scale (RS) variables, for example: rate of growth of gross domestic product per capita, profit and taxes (profit = income - cost). We suggest that the original IS variable should be replaced with the two RS variables. However, even in the case when the values of the variable happen to be positive in the data we strongly suggest the approach among other things for the quite obvious reason that division on the interval scale is not allowed. We explain that who decomposing the IS variables into the RS variables as follows. Assume t inputs among the total of m, and s outputs among the total of p, have been measured on the IS. Replace each by two RS variables whose difference is the original variable. The new input matrix $X\in R_+^{(m+s)\times n}$ contains first the t new RS input variables originating from the IS input variable (minuends). Next come the s RS variables that originate from the IS output variable (subtrahends). As we arrange the new output variables originating from the IS input variables first (the subtrahends in the difference that corresponds to the IS input variable) and next the new output variables corresponding to the original IS outputs (minuends) for the output matrix $Y\in R_+^{(p+t)\times n}$. For further explanation, we assume that the inputs variables are $x_1,...,x_t,x_{t+1},...,x_m$ where $x_1,...,x_t$ are the IS input variables and the outputs variables are $y_1,...,y_s,y_{s+1},...,y_p$ where $y_1,...,y_s$ are the IS outputs variables. We define $x_i=x_i^{'}-x_i^{''}, i=1,...,t$ and $y_j=y_j^{'}-y_j^{''}, j=1,...,s$ where $x_i^{'}$ is the new RS input variable and $x_i^{''}$ is new the RS output variable and $y_j^{'}$ is the new RS output variable and $y_j^{''}$ is the new RS input variable. Fig. 1 illustrates the new input variables and the new output variables.

The coefficients of the new RS variables are set equal the dual formulation. Each resulting new constraint in the dual creates a new variable, denoted here by π, in the primal. The radial combined Banker Cooper Charnes (BBC) dual model after decomposing the IS variables into one input and output each is as following. In the BBC model, the Production Possibility Set (PPS) is a variable return scale.

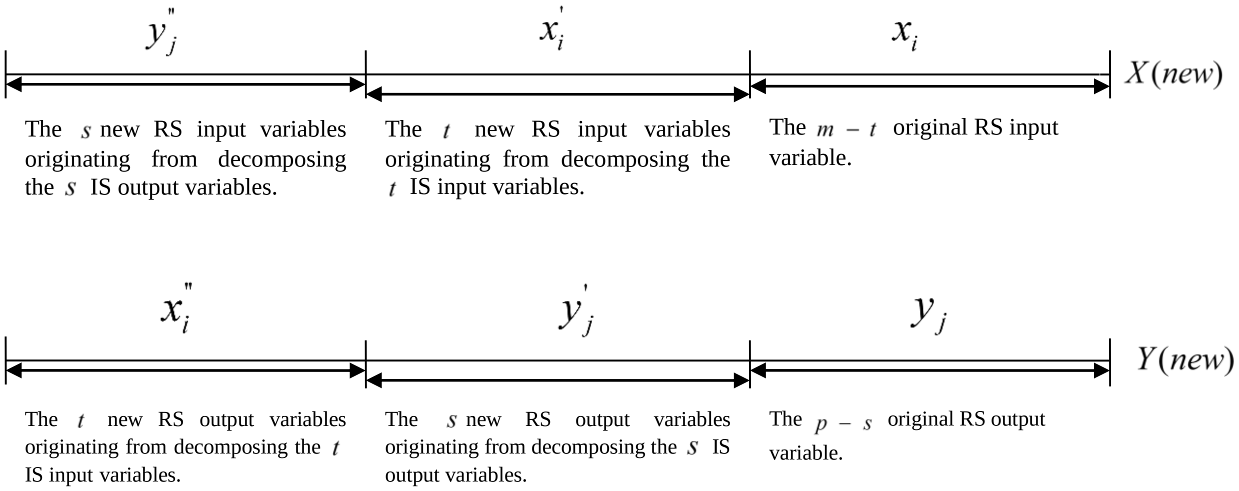

Fig. 1. New output and input variables

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m+s} v_i x_{io} - \sum_{r=1}^{p+t} \mu_r y_{ro} + u \\
\textit{subject to} \quad & \sum_{i=1}^{m+s} v_i x_{ij} - \sum_{r=1}^{p+t} \mu_r y_{rj} + u \geq 0, \quad j=1,\dots,n \\
& \sum_{r=1}^{p+t} \mu_r y_{ro} + \sum_{i=1}^{m+s} v_i x_{io} = 1 \qquad (3) \\
& \mu_r - v_i = 0, \quad i=r=1,\dots,t+s \\
& \mu_r, v_i \geq 0, \quad \forall r, \forall i
\end{aligned}$$

The dual of model (3) is the following radial combined BCC model.

$$\begin{aligned}
\max \quad & \sigma \\
\textit{subject to} \quad & \sum_{j=1}^{n} y_{rj}\lambda_j - \sigma y_{ro} - \pi_r \geq y_{ro}, \quad r=1,\dots,t+s \\
& \sum_{j=1}^{n} y_{rj}\lambda_j - \sigma y_{ro} \geq y_{ro}, \quad r=t+s+1,\dots,p+t \\
& \sum_{j=1}^{n} x_{ij}\lambda_j + \sigma x_{io} - \pi_i \leq x_{io}, \quad i=1,\dots,t+s \qquad (4) \\
& \sum_{j=1}^{n} x_{ij}\lambda_j + \sigma x_{io} \leq x_{io}, \quad i=t+s+1,\dots,m+s \\
& \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0, \quad j=1,\dots,n
\end{aligned}$$

Naturally apart from the above model, input or output oriented models can be considered. If we set $x_{io}, i=1,\dots,m+s,$ to zero in (4) we get the output oriented formulation. The input oriented model is derived analogously. Note that efficient units remain efficient after decomposition. Increasing of variables in DEA means also in this case that inefficient units may become efficient and in fact only the scores of the inefficient units change (for more explain see the paper of Halme et al. (1998), Dealing with interval scale data in DEA).

THE NEW APPRIACH OF VE SCORES

New Models

We purpose using the tangent hyper plane on PPS in Most Preferred Solution (MPS) to approximate the indifference contour of unknown Value Function (VF). If we rewrite model (1) according to the new input and output variables (after the decomposing IS variables) we get the following model.

$$\begin{aligned}
\max \quad & \sigma + \varepsilon\left(\sum_{i=1}^{m+s} s_i^- + \sum_{r=1}^{p+t} s_r^+\right) \\
\textit{subject to} \quad & \sum_{j=1}^{n} y_{rj}\lambda_j - \sigma y_{ro} - \pi_r - s_r^+ = y_{ro}, \quad r=1,\dots,t+s \\
& \sum_{j=1}^{n} y_{rj}\lambda_j - \sigma y_{ro} - s_r^+ = y_{ro}, \quad r=t+s+1,\dots,p+t \\
& \sum_{j=1}^{n} x_{ij}\lambda_j + \sigma x_{io} - \pi_i + s_i^- = x_{io}, \quad i=1,\dots,t+s \qquad (5) \\
& \sum_{j=1}^{n} x_{ij}\lambda_j + \sigma x_{io} + s_i^- = x_{io}, \quad i=t+s+1,\dots,m+s \\
& \sum_{j=1}^{n} \lambda_j = 1, \ s_i^-, s_r^+ \geq \varepsilon, \forall i, r \\
& \varepsilon > 0, (Non-Archimedeam) \\
& \lambda_j \geq 0, \quad \text{If} \ \lambda_j^* = 0, j=1,\dots,n
\end{aligned}$$

Where $(\lambda,\sigma,\pi,s^+,s^-,\varepsilon)$ is an optimal solution and λ^* is corresponds to the MPS. The dual model of the above model obtains weights of output/input variables as the normal vector of the tangent hyper plane on PPS. The dual model of the above model is as following:

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m+s} v_i x_{io} - \sum_{r=1}^{p+t} \mu_r y_{ro} + u \\
\text{subject to} \quad & \sum_{i=1}^{m+s} v_i x_{ij} - \sum_{r=1}^{p+t} \mu_r y_{rj} + u = 0, \quad j \in \{j \mid \lambda_j^* > 0, j = 1,\ldots,n\} \\
& \sum_{i=1}^{m+s} v_i x_{ij} - \sum_{r=1}^{p+t} \mu_r y_{rj} + u \geq 0, \quad j \in \{j \mid \lambda_j^* = 0, j = 1,\ldots,n\} \\
& \sum_{r=1}^{p+t} \mu_r y_{ro} + \sum_{i=1}^{m+s} v_i x_{io} = 1 \qquad (6) \\
& \mu_r - v_i = 0, \quad i = r = 1,\ldots,t+s \\
& \mu_r, v_i \geq \varepsilon \quad \forall r, \forall i;\ \varepsilon > 0, (Non-Archimedeam)
\end{aligned}$$

The obtained hyperplane from the model (6) i.e. $\sum_{i=1}^{m+s} v_i^* x_{io} - \sum_{r=1}^{p+t} \mu_r^* y_{ro} + u^*$ is tangent on PPS at DMU_j that $j \in \{j \mid \lambda_j^* > 0, j = 1,\ldots,n\}$ and in fact these are the reference DMUs of MPS. Since the MPS is efficient, so the set $\{j \mid \lambda_j^* > 0, j = 1,\ldots,n\}$ so that $\sum_{i=1}^{m+s} v_i x_{ij} - \sum_{r=1}^{p+t} \mu_r y_{rj} + u = 0$, contain MPS. Hence, this hyper plane passes among MPS.

A New Approach to Approximating of VE

First, we decompose the IS inputs and outputs variables. Then, according to the new inputs and outputs (according to Figure (1)), for each unit, the technical efficiency model will be obtained as follows (the model 7), so that by solving this model, the effective or not of each unit is determined. The efficient unit is selected as MPS. If the number of efficient units is large, then the decision maker considers one as the most preferred solution (MPS). Of course, it should be noted that the negative data is among the IS data, and in the analysis of the data, it has been converted into the difference between two positive data.

$$\begin{aligned}
\max \quad & \sigma \\
\text{subject to} \quad & \sum_{j=1}^{n} y_{rj} \lambda_j - \sigma y_{ro} \geq y_{ro}, \quad r = 1,\ldots,p+t \\
& \sum_{j=1}^{n} x_{ij} \lambda_j + \sigma x_{io} \leq x_{io}, \quad i = 1,\ldots,m+s \qquad (7) \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0, j = 1,\ldots,n.
\end{aligned}$$

We want to approximate the value θ such that $(x_j, y_j) + \theta(W^x, W^y) = (\bar{x}, \bar{y})$, where $(\bar{x}, \bar{y})$ is the projected point of DMU_j on the indifference contour VF at MPS which we utilizing the tangent hyperplane at MPS, obtained from solving the model (6), instead of it. Suppose that (v^*, μ^*, u^*) is an optimal solution of the model (6). So the equation of the tangent hyperplane of VF at MPS is $v^{*^T} x - \mu^{*^T} y + u^* = 0$. Hence, we have: $v^{*^t}(x_j + \theta W^x) - \mu^{*^t}(y_j + \theta W^y) + u^* = 0$. In other words, $\theta = -\dfrac{v^{*^t} x_j - \mu^{*^t} y_j + u^*}{v^{*^t} W^* - \mu^{*^t} W^y}$.

We purpose to obtain VE scores only in output orientation. So, we use from the output oriented direction $(W^x, W^y) = (0, y_j)$, that thus we have: $\theta = \dfrac{v^{*^t} x_j + u^*}{\mu^{*^t} y_j} - 1$. Note that we consider the case when both the two new variables in decomposing the IS variable into two ratio scale variables as objectives and don't consider the case when one of the new variables is non-discretionary by character. The basic idea of VEA is illustrated in Fig. 2. We assume that the DMUs produce two outputs and all consume the same amount of one input. To evaluate point A, we would like to assess the ratio $\dfrac{OA^4}{OA}$. Because the VF is unknown, we can assume that the tangent hyper plane at MPS is tangent on the indifference contour and if we use from the tangent hyper plane then we obtain the VE score as the ratio $\dfrac{OA^2}{OA}$ (equal to the result of the method of Halme et al.) or $\dfrac{OA^3}{OA}$ (this the VE score is better than the method of Halme et al.). The VE score θ_j for evaluating DMU_j is as $\theta_j = \dfrac{v^{*^T} x_j + u^*}{\mu^{*^T} y_j} - 1 = \dfrac{OA^2}{OA} - 1 = \dfrac{AA^2}{OA}$ or $\theta_j = \dfrac{OA^3}{OA} - 1 = \dfrac{AA^3}{OA}$. In fact, since line ℓ is the appropriate line for selection the tangent hyper plane at MPS, as a result, we have $\frac{OA^2}{OA} \le \theta_j \le \frac{OA^3}{OA}$.

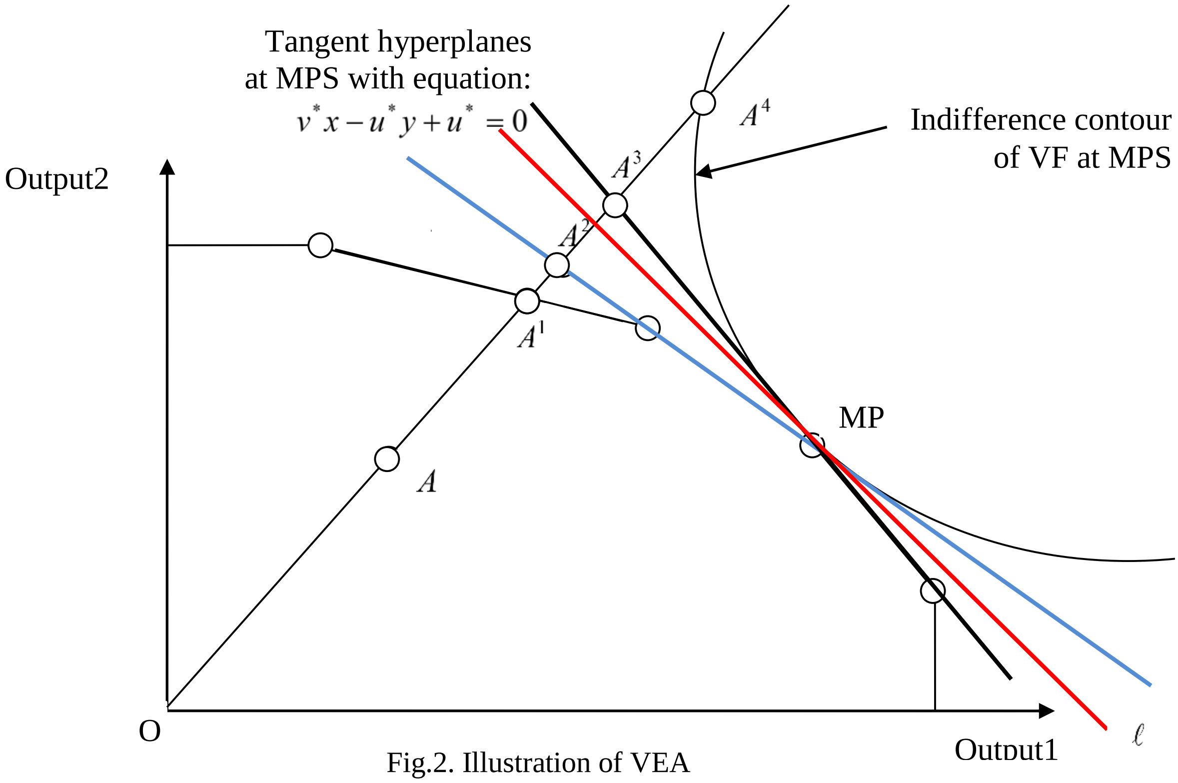

Fig.2. Illustration of VEA

ILLUSTRATIVE EXAMPLE

In this section, we use the data recorded in table 1 to illustrate how the approach revised in this work perform. The value efficiency of 14 DMUs each consuming one input (I) to produce two outputs (O1 and O2) in to be assessed. O2 data have an interval scale that some units have negative data in this output.

Table1: Decision making units and their input/output values.

DMUs	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14
I	48	49	49	48	50	47	45	48	50	47	19	23	47	35
O1	48	45	35	34	25	25	16	15	58	25	4	4	14	13
O2	-17	-6	5	4	-12	3	2	-4	-16	-14	3	-5	1	1
y	32	33	36	35	19	31	30	31	38	26	7	6	21	19
x	49	39	31	31	31	28	28	35	54	40	4	11	20	18

We need to decompose the IS output variable O2 into two ratio scale (RS) variables which O2 generated by the difference between two RS values (O2=y-x). Here, we have $p=2$, $m=1$, $t=0$, $s=1$ and the x data of O2 is considered as first input (x_1) and the y data of O2 as first output (y_1). The new inputs and outputs are listed in the Table below.

Table 2: The new variables values after decomposing O2.

DMUs	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14
x_1	49	39	31	31	31	28	28	35	54	40	4	11	20	18
x_2	48	49	49	48	50	47	45	48	50	47	19	23	47	35
y_1	32	33	36	35	19	31	30	31	38	26	7	6	21	19
y_2	48	45	35	34	25	25	16	15	58	25	4	4	14	13

To compute the technical efficiency for each DMU with the new variables values, utilizing the data of table 2. We write model (7) for each DMU and solve it and according to the obtained results, units D4, D9, and D11 are efficient. We picked the unite D4 as MPS and also $\lambda^*_{D4}=1$. For finding output/input weights, we use from model (6) and we have:

$$\begin{aligned}
\min \quad & 31v_1+48v_2-35\mu_1-34\mu_2+u \\
s.t. \quad & 31v_1+48v_2-35\mu_1-34\mu_2+u=0 \\
& 49v_1+48v_2-32\mu_1-48\mu_2+u\geq 0 \\
& \vdots \\
& 18v_1+35v_2-19\mu_1-13\mu_2+u\geq 0 \\
& 35\mu_1+34\mu_2+31v_1+48v_2=1 \\
& -v_1 \qquad +\mu_1 \qquad =0 \\
& v_1, v_2, \mu_1, \mu_2 \geq \varepsilon, \ \varepsilon>0, (Non-Archimedeam)
\end{aligned} \tag{8}$$

The optimal weights of the model (8) are $v_1^*=0.0050$, $v_2^*=0.0052$, $\mu_1^*=0.0050$, $\mu_2^*=0.0048$, $u^*=-0.0066$. Halme et al use from the model (1) and (2) and we have used of the output oriented direction i.e. $(W^x,W^y)=(0,y_j)$. Definitely, if the DM changes the MPS, the tangent hyper plane will change and as a result, the value efficiency scores will change.

Table 3: Value efficiency scores. (Based on percentage)

DMUs	The combined model of Halme et al	The output oriented model of Halme et al	The our proposed method
D1	4.77	10.86	10.86
D2	0.84	1.851	1.851
D3	0.10	0.22	1.34
D4	0.00	0.00	0.00
D5	21.79	62.48	64.34
D6	4.41	9.76	17.24
D7	3.10	7.35	37.47
D8	5.74	13.97	59.77
D9	0.00	0.00	0.00
D10	13.03	33.07	52.98
D11	0.00	0.00	0.00
D12	15.86	83.38	124.35
D13	19.34	60.19	63.98
D14	6.65	18.19	32.71

CONCLUSION

We have used the tangent hyper plane on the MPS to find the value efficiency and according to the results obtained in the above example, it can be said that the method mentioned in this article gives a suitable approximation for this efficiency. The presented radial models are for interval scale data which is the difference between two relative data and this model cannot be used to use the interval data itself. On the other hand, negative data has the same distance scale. As a suggestion, we can use common weights in finding this hyper plane, which may be able to obtain more realistic values for the value efficiency scores of decision making units, and for this purpose, multi-objective problem solving methods such as goal programming methods are used. We can also use the original data (without decomposing it) which may be used by the semi-natural radial model

(SORM). Finally, it should be noted that we have used models with variable returns to scale, which makes us have more efficient units. As another suggestion, the tangent hyper plane can be used to approximate the value efficiency of units with positive data. One of the limitations in this research is that negative data cannot be used directly in the presented model.

REFERENCES

Bazaraa, M., Shetty C.M. (1979). Nonlinear programming: theory and algorithms. John Wiley & Sons, New York.

Gerami J., Mozafari M. R., Wanke P. F. (2022). Improving information reliability of non-radial value efficiency analysis: An additive slacks based measure approach. 298(3): 967-978.

Gerami J. (2019). An interactive procedure to improve estimate of value efficiency in DEA. Expert Systems with Applications. 137: 29-45.

Halme, M., Joro, T., Korhonen, P., Salo S. and Wallenius J. (1999). A value efficiency approach to incorporating preference information in data envelopment analysis. Management Science, 45(1): 103-115.

Hosseinzadeh Lotfi, F., Ebrahimnejad A., Vaez-Ghasemi M., Moghaddas Z. (2020). Data envelopment analysis with R, Springer International Publishing.

Korhonen, P., Syrjanen, M. J. (2005). On the interpretation of value efficiency. Journal of Productivity Analysis. 24: 197-201.

Pastor, J Ruiz, J (2007). Variables with negative values in DEA. Modeling Data Irregularities and Structural Complexities in Data Envelopment Analysis. Springer, 63-84.

sharp, J A, Liu, W B, Meng, W. (2007). A modified slacks-based measure model for data envelopment analysis with 'natural' negative putputs and inputs. Journal of the Operational Research Society, 58, 1672-1677.

Steuer, R.E. (1986). Multiple Criteria Optimization: Theory, Computation and Application. John Wiley & Sons, New York.

Tajik Yabr A. H., Najafi S. E., Moghaddas Z., Shahnazari Shahrezaei P (2022). Interval Cross Efficiency Measurement for General Two-Stage Systems. Mathematical Problems in Engineering. Hindawi.

Vaez-Ghasemi M., Moghaddas, Z., Farzipoor Saen R. (2021). Cost efficiency evaluation in sustainable supply chains with marginal surcharge values for harmful environmental factors: a case study in a food industry. Operational Research (Springer Berlin Heidelberg). 1-16.

Zohrabandan, M. (2011). Using Zionts-Wallenius method to improve estimate of value efficiency in DEA. Applied Mathematical Modeling. 35 (2011) 3769-3776.