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Iterative random search heuristic for the Single-Source Capacitated Multi-Facility Weber Problem with Setup Costs

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Abstract

Here, we will study the Single-Source Capacitated Multi-Facility Weber Problem with Setup Costs (SSCMFWP-SC) to find location of certain numbers of facilities in continuous space so that demands by certain numbers of customers would be satisfied. This would be done in a way that total transportation cost between customers and facilities as well as total setup cost would be minimized. Facilities have limited capacity and each customer has to satisfy all of its demands just from one facility. Setup cost of facilities is variable and dependent on combination of machineries used by each facility. To solve the problem, two versions of the proposed heuristic method named iterative random search will be presented in which local search method and exact solution method are used. proposed method has been tested on a dataset available in the literature and the obtained solutions compared to the best of them in the literatures. The results show extraordinary performance of recommended methods. Moreover, best available solutions in the literature have been improved and the best obtained solutions can be used as a comparison source in future studies.

Keywords:
Location-Allocation
Weber problem
Setup cost
Heuristic algorithm
Random search

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INTRODUCTION

Suppose a number of customers, each with a certain demand are located in fixed and certain locations. Weber problem is going to find location of just one facility to satisfy customer demands in a way that total transportation cost between customers and it would be minimized. In another problem, it is going to find location of more than one facility to satisfy customer demands. The problem has been first set forth by Cooper (1963) and is recognized as Multi-Source Weber Problem (MSWP) and or Location-Allocation (LA) problem. It was shown by Cooper (1967) that objective function of the problem is neither convex nor concave; and, the main problem with non-convex non-linear programming problems is availability of local minimums. Moreover, it was proved by Megiddo and Supowit (1984) that the problem is NP-Hard. Also, complexity of the problem is dependent on non-linear behavior in the number of customers and facilities (Brimberg, Hansen, Mladenovi´c, & Taillard, 2000). Locating storages, distribution centers, communication centers, and production facilities are some examples of the problem (Cooper, 1963).

Like the other problems and with consideration of various limitations, different models of location-allocation problem would be available. The problem can be studied in continuous or discrete space or on the network. Rectilinear, Euclidean, and or squared Euclidean distances could be considered. Moreover, facilities can be limited in the problem in terms of capacity or there could be no limitation of capacity. Also, each customer can satisfy his own demand just from one facility and/or from several of them. Fixed and or variable costs of facility setting up can be considered as well. In MSWP, due to unlimited capacity of facilities, in optimum solution each customer will be just allocated to one facility and all his demands will be satisfied through the nearest facility. If capacity limitation of facilities would be added to MSWP, it would be recognized as CMSWP. In CMSWP and due to limited capacity of facilities, each customer can satisfy his own demand via different facilities. If in CMSWP, each customer would be obligated to satisfy all of its demands just from one facility; it would be considered as SSCMFWP. It has to be noted that with placement of facility locations and fixing them; CMSWP and SSCMFWP will be respectively reduced to Transportation Problem (TP) and Generalized Assignment Problem (GAP). Solving SSCMFWP is more complicated than MSWP and CMSWP due to binary variables (Irawan, Salhi, & Soemadi, 2020; Oncan¨, 2013). In real-world problems, the establishment of a new facility usually requires a setup cost. In the literature of location-allocation problems, some studies have considered a fixed cost for setting up new facilities. But the cost of setting up the facility in practice can depend on many factors and be a variable cost. As an example, the setup cost of opening a new facility can depend on the number and type of machines that will be used by that facility. Therefore, in this case, the setup cost of opening a new facility will be a variable cost, which will increase the number of variables in the model and make it more complicated. As a result, the problem will be closer to real world problems. Here, certain type of location-allocation problem would be considered as the Single-Source Capacitated Multi-Facility Weber Problem with Setup Costs (SSCMFWP-SC). One set of machineries is available; and, each machine type is recognized with such specifications like capacity, purchase cost, and the number of it available. Each facility requires to purchase and use of machineries to satisfy customer demand. So, in the problem setup cost of facility opening as a variable cost would be considered and its value for each facility is dependent to combination of type of machineries used by that facility. Facilities are limited in capacity and each customer has to satisfy its own demand just from one facility.

LITERATURE REVIEW

Here, those literatures related to location-allocation problems not considering the setup costs of facilities will not be studied. So, refer to Jahadi and Solimanpur (2021) please to review more in references of MSWP, CMSWP, and SSCMFWP problems with no consideration of setup cost of facilities. In location-allocation problem literature, there are a few studies with consideration of facility setup cost; so, here, literature of MSWP, CMSWP, and SSCMFWP would be briefly reviewed with consideration of setup cost of facilities. MSWP has been studied by Brimberg, Mladenovic, and Salhi (2004) supposing uncertain numbers of facilities and considering fixed costs of establishing facilities; and, a multi-phase heuristic algorithm to solve the problem has been developed. A zone-dependent fixed cost has been introduced for MSWP by Brimberg and Salhi (2005). It has been assumed by them that number of facilities are not specified and a number of heuristic algorithms have been developed to solve the problem. The uncapacitated continuous location-allocation problem in the presence of a zone-dependent fixed cost of opening a facility has been studied by Abdullah, Zainuddin, and Salim (2008) and a SA meta-heuristic algorithm has been developed by them to solve the problem. CMSWP has been studied by Hosseininezhad, Salhi, and Jabalameli (2015) with consideration of zone-dependent fixed cost of opening a facility and a three stage Cross Entropy based meta-heuristic algorithm has been developed by them to solve the problem. Three types of fixed cost for opening facilities have been introduced by Luis, Salhi, and Nagy (2015); and, two heuristic algorithms have been developed by them to solve the problem. With consideration of facility setup cost as a variable cost which its value for each facility depends on combination of type of machineries used by that facility, a non-linear model has been introduced by Irawan et al. (2020) known as SSCMFWP-SC. Both Rectilinear and Euclidean distances have been considered by them; and, two meta-heuristic algorithms have been provided by them based on VNS and SA to solve the problem. Also, a dataset has been developed by them for the problem.

MATHEMATICAL FORMULA

In this section, first, the mathematical model of the SSCMFWP-SC is presented. In the following, in order to be used within the proposed solution methods, two other models derived from the main model will be presented in a reduced form.

Mathematical model of the SSCMFWP-SC

Suppose that certain number of customers each with certain demand are located in a continuous space i.e. their spatial coordinates specified. SSCMFWP-SC is aimed at finding location of certain number of facilities with maximum capacity authorized for each being specified in convex space of customer locations so that their demands would be satisfied and total transportation cost between customers and facilities and total setup cost would be minimized. Each of the facilities requires to purchase and use of machineries with consideration of its own capacity so that customer demands would be satisfied. To do so, certain number of machine types are available. Cost of purchase as well as capacity and number of each machine type available is clear. Each customer can satisfy all of its own demand just from one facility. So, each customer must be allocated to just one facility. The problem is simultaneously looking for finding location of facilities, method of allocation of customers to facilities and number of each machine type used by each facility. To formulate the problem, following symbols can be used.

Parameters

n:	Numbers of customers.
m:	Numbers of facilities.
w_j:	Customer demand (j).
Q_i:	Maximum authorized capacity of each facility (i).
$A_j = (a_j, b_j)$:	Customer location coordinates (j).
u:	Cost of transportation unit against distance unit and one unit of customer demand.
p:	Numbers of machine types.
e_k:	Numbers of available machine type k.
q_k:	Capacity of machine type k.
c_k:	Purchase cost of machine type k.

Decision variables

$X_i = (x_i, y_i)$: Location coordinates of facility i.

z_{ij}: $\begin{cases} 1, & if\ customer\ j\ is\ allocated\ to\ facility\ i, \\ 0, & otherwise. \end{cases}$

L_{ik}: Numbers of machine type k used by facility i.

Euclidean distance between customer j and facility i is computed as follows:

$$d(X_i, A_j) = \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}$$

The mathematical model of the SSCMFWP-SC is formulated as bellow:

Model 1 :

$\boldsymbol{min}$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left(z_{ij} \times d(X_i, A_j) \times u \times w_j\right) + \sum_{i=1}^{m} \sum_{k=1}^{p} (L_{ik} \times c_k) \tag{1}$$

Subject to

$$\sum_{i=1}^{m} z_{ij} = 1, \forall j = 1,2, \dots, n \tag{2}$$

$$\sum_{k=1}^{p} (L_{ik} \times q_k) \leq \left(Q_i + \max_{k=1}^{p} q_k\right), \ \forall i = 1,2, \dots, m \tag{3}$$

$$\sum_{j=1}^{n} \left(w_j \times z_{ij}\right) \leq Q_i, \forall i = 1,2, \dots, m \tag{4}$$

$$\sum_{j=1}^{n} \left(w_j \times z_{ij}\right) \leq \sum_{k=1}^{p} (L_{ik} \times q_k), \ \forall i = 1,2, \dots, m \tag{5}$$

$$\sum_{i=1}^{m} L_{ik} \leq e_k, \ \forall k = 1,2, \dots, p \tag{6}$$

$$z_{ij} \in \{0,1\}, \ \forall i = 1,2, \dots, m; \ j = 1,2, \dots, n \tag{7}$$

$$L_{ik} \geq 0, \ \text{integer}, \ \forall i = 1,2, \dots, m; \ k = 1,2, \dots, p \tag{8}$$

$$X_i \in \mathbb{R}^2, \ \forall i = 1,2, \dots, m \tag{9}$$

Total transportation cost between customers and facilities as well as total setup cost of facilities is minimized by objective function (1). It is guaranteed by constraints (2) that each customer can be just allocated to one facility. It is stated by constraints (3) that total capacity of machines used by each facility can in maximum exceed to the maximum capacity of type of machines ($max_{k=1}^{p} q_k$) from maximum authorized capacity of the same facility (Q_i). It is assured by constraints (4444444444444444) and (55) that total customer demand satisfied by each facility cannot be exceeded the maximum authorized capacity of the same facility (Q_i) and the total capacity of machines used by that facility. It is stated by constraints (6) that total number of each type of machine used by all of the facilities has not to exceed available number of that type of machine (e_k). constraints (777) state that variables of allocation of customers to facilities are binary. It is shown by constraints (88888) that number of each location coordinates of facilities are continuous variables. It is shown by constraints (9) that location coordinates of facilities are continuous variables.

Mathematical model of simultaneous allocation of customers and machineries to facilities

If in SSCMFWP-SC, location of facilities would be specified; assume that certain numbers of facilities with maximum authorized capacity have been established on predefined locations. So, distances between facilities and customers would be clear. The problem is simultaneously trying to find allocation of customers to facilities and number of each type of machine used by each facility. Mathematical model of the problem of simultaneous allocation of machineries and customers to the facilities is a reduced order linear model of the main model. To formulate the problem, symbols used in the main model is used and the mathematical model is as follows:

Parameters added

d_{ij}: Distance between customer j and facility i.

Model 2:

$\boldsymbol{min}$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left(z_{ij} \times d_{ij} \times u \times w_j\right) + \sum_{i=1}^{m} \sum_{k=1}^{p} (L_{ik} \times c_k) \tag{10}$$

Subject to

$$\sum_{i=1}^{m} z_{ij} = 1,\ \forall j = 1,2,\dots,n \tag{11}$$

$$\sum_{k=1}^{p} (L_{ik} \times q_k) \le \left(Q_i + \max_{k=1}^{p} q_k\right),\ \forall i = 1,2,\dots,m \tag{12}$$

$$\sum_{j=1}^{n} \left(w_j \times z_{ij}\right) \le Q_i,\ \forall i = 1,2,\dots,m \tag{13}$$

$$\sum_{j=1}^{n} \left(w_j \times z_{ij}\right) \le \sum_{k=1}^{p} (L_{ik} \times q_k),\ \forall i = 1,2,\dots,m \tag{14}$$

$$\sum_{i=1}^{m} L_{ik} \le e_k,\ \forall k = 1,2,\dots,p \tag{15}$$

$$z_{ij} \in \{0,1\},\ \forall i = 1,2,\dots,m;\ j = 1,2,\dots,n \tag{16}$$

$$L_{ik} \ge 0,\ integer,\ \forall i = 1,2,\dots,m;\ k = 1,2,\dots,p \tag{17}$$

Mathematical model of discrete problem related to the main problem

This problem is exactly the same as the main problem with the difference that there is a discrete space of facility location. To do so, there are certain number of potential locations to open facilities; and, the distance between location of customers and potential locations are also clear due to location coordinates of each of them being clear. The problem is aimed at selecting m location from among potential locations to open facilities so that customer demands would be satisfied. That is, total transportation cost between customers and facilities and total setup cost of facilities would be minimized. If one facility with maximum authorized capacity has been opened in one potential location; then, numbers of each machine type used by that facility as well as allocation of customers to that facility have to be specified. In addition to the symbols of the main model, a number of symbols as mentioned below is used in the model:

Parameters

R: Numbers of potential locations to construct facilities.

r: The index related to potential locations.

d_{rj}: The distance between potential location r and customer j.

Decision Variables

y_{rj}: 1, if customer j is allocated to opened facility in location r; 0, otherwise.

s_{ir}: 1, if facility I is opened with maximum authorized capacity Q_i in location r; 0, otherwise.

L_{kr}: Numbers of machine type k used by the facility opened in location r.

Model 3:

min

$$\sum_{r=1}^{R} \sum_{j=1}^{n} \left(y_{rj} \times d_{rj} \times w_j \times u\right) + \sum_{k=1}^{p} \sum_{r=1}^{R} (L_{kr} \times c_k) \tag{18}$$

Subject to

$$\sum_{r=1}^{R} y_{rj} = 1,\ \forall j = 1,2,\dots,n \tag{19}$$

$$\sum_{i=1}^{m} s_{ir} \le 1,\ \forall r = 1,2,\dots,R \tag{20}$$

$$\sum_{r=1}^{R} s_{ir} = 1,\ \forall i = 1,2,\dots,m \tag{21}$$

$$\sum_{k=1}^{p} (L_{kr} \times q_k) \le \sum_{i=1}^{m} \left(s_{ir} \times \left(Q_i + \max_{k=1}^{p} q_k\right)\right),\ \forall r = 1,2,\dots,R \tag{22}$$

$$\sum_{j=1}^{n} \left(w_j \times y_{rj}\right) \le \sum_{i=1}^{m} (s_{ir} \times Q_i),\ \forall r = 1,2,\dots,R \tag{23}$$

$$\sum_{j=1}^{n} \left(w_j \times y_{rj}\right) \le \sum_{k=1}^{p} (L_{kr} \times q_k),\ \forall r = 1,2,\dots,R \tag{24}$$

$$\sum_{r=1}^{R} L_{kr} \le e_k,\ \forall k = 1,2,\dots,p \tag{25}$$

$$y_{rj} \in \{0,1\},\ \forall r = 1,2,\dots,R; j = 1,2,\dots,n \tag{26}$$

$$L_{kr} \ge 0,\ integer,\ \forall r = 1,2,\dots,R;\ k = 1,2,\dots,p \tag{27}$$

$$s_{ir} \in \{0,1\},\ \forall i = 1,2,\dots,m;\ r = 1,2,\dots,R \tag{28}$$

Solution methods

There are usually different methods and algorithms used to solve a problem. Exact methods can achieve optimal solutions; However, in the case of NP-Hard, they are not efficient enough and their computational time increase according to the dimensions of the problem. Heuristic and meta-heuristic algorithms produce solutions at a very high speed that are close to the optimal solution, but often fall into the trap of local optima and don't guarantee achieving the optimal solution. One of the methods used to solve a problem is to turn that method into a search process. It is possible that the initial solution would be generated by a constructive algorithm or even obtained randomly. In each iteration, the neighbors of the current solution are searched for perhaps a better solution. The structure and how to create neighborhoods in these algorithms is crucial. In this section, two versions of the proposed heuristic method named iterative random search are presented to solve the SSCMFWP-SC. The proposed method sues an iterative search process based on reduced variable neighborhood search in which the neighborhood structure is formed by a random approach as well as an exact solution method to generate new solutions in the neighborhood. Therefore, all the neighbors of the current solution are not visited based on the neighborhood structure, and only one neighbor of the current solution will be visited in each iteration in order to find a better solution. Within the proposed solution method, a method for generating an initial solution and a method for generating new solutions in the neighborhood, as well as methods to improve the quality of the initial solution and new solutions generated in each iteration, will be used. In the continuation of this section, first, the methods that will be used in the main proposed methods will be described, and at the end, the main steps of the main proposed methods will be presented.

Initial solution

In this section, a proposed constructive method for generating an initial solution is developed, in which a celling approach and an exact solution method are used. The proposed method consists of two main steps, so that in the first step, a number of potential locations for placement of facilities should be determined, so that their number is greater than the number of facilities. the solution space of the problem is divided into a number of cells with equal sizes. The center point of each cell is selected as a potential location for facilities. Therefore, Continuous solution space of the problem becomes a discrete solution space by forming a set of potential locations, thus removing the constraint of continuity of variables and the initial locations of facilities will be selected from potential locations. In the second step, As the discrete problem can be solved more easily compared to the continuous problem and its computational time is relatively less, by using the CPLEX solver the model of the discrete problem related to the main problem (Model 3) is solved exactly and as a result of which an initial solution is produced. The quality of the solution improves with the increase in the number of cells, but the running time of CPLEX solver will also increase; therefore, to obtain a good solution at an acceptable computational time, The running time of CPLEX solver is limited to t_1. Using the outputs of CPLEX execution, the location coordinates of facilities (X) and the allocation of customers to facilities (z) and the number of each machine type used by each facility (L) is determined. You can find its main steps in Algorithm 1.

IMPROVEMENT

In this section, two local search methods are developed. These methods are used to improve the quality of the initial solution and new solutions that are produced as neighbors of the current solution in each iteration of the proposed main methods.

Algorithm 1 Initial solution

Require: $cn,\ t_1,\ A_j,\ w_j,\ u,\ Q_i,\ q_k,\ c_k,\ e_k,\ \forall i = 1,2,\ldots,m; j = 1,2,\ldots,n; k = 1,2,\ldots,p.$

1: Determine:

$mina$: The smallest longitudinal coordinate among customer locations

$maxa$: The largest longitudinal coordinate among customer locations

$minb$: The smallest latitudinal coordinate among customer locations

$maxb$: The largest latitudinal coordinate among customer locations

2: Divide the solution space of the problem by cn^2 cells, so that the size of the length and width of each cell is calculated as follows:

$$\frac{(maxa - mina)}{cn} = \text{length of each cell}$$

$$\frac{(maxb - minb)}{cn} = \text{width of each cell}$$

3: Determine the spatial coordinates of the center point of each cell.

4: Select cell centers as potential locations for facilities.
5: Calculate distances between potential locations and customer locations.
6: Solve the discrete problem related to the main problem (**Model 3**) using CPLEX in t_1 seconds.
7: Using the outputs of CPLEX execution, determine coordinates of facility locations (X) and the assignment of customers to facilities (z) and the number of each machine type used by each facility (L).
8: Find the distances between facility locations and customer locations ($d(X_i, A_j)$).
9: Calculate the value of the objective function (F).
Return X_i, z_{ij}, L_{ik}, F; $\forall i = 1,2,\dots,m; j = 1,2,\dots,n; k = 1,2,\dots,p$.

Iterative location improvement approach

In this section, an iterative method is developed as a local search to use within the main methods with the aim of improving new solutions. This method is a modified version of the Cooper's ALA algorithm (Cooper, 1963) in which location of facilities and allocation of customers to facilities are alternatively improved to the extent that the improvement rate is small. The proposed method keeps the variables related to the allocation of customers to facilities (z_{ij}) and the number of each type of machine used by each facility (L_{ik}) constant each time is repeated, and new locations are established. It finds the new location of facilities using Weiszfeld (Weiszfeld, 1937) equations. Therefore, the new location coordinates of the facility are calculated by Eq. 29 as below:

$$x_i' = \frac{\sum_{j=1}^{n} \frac{z_{ij} \times w_j \times a_j}{d(X_i, A_j)}}{\sum_{j=1}^{n} \frac{z_{ij} \times w_j}{d(X_i, A_j)}}; \quad y_i' = \frac{\sum_{j=1}^{n} \frac{z_{ij} \times w_j \times b_j}{d(X_i, A_j)}}{\sum_{j=1}^{n} \frac{z_{ij} \times w_j}{d(X_i, A_j)}} \tag{29}$$

In which $X_i' = (x_i', y_i')$ is the new location coordinates of the facility i. The objective function of the SSCMFWP-SC consists of two parts, so let: f_t be the representation of the first part, i.e., the cost of transportation between facilities and customers; f_s be the representation of the second part, i.e. the cost of setting up facilities; and $F = f_t + f_s$ be the representation of the total cost.

Therefore, the proposed method by holding the second part of the objective function (f_s) improves iteratively just the value of the first part of the objective function (f_t) until the improvement value reaches less than ϵ_1. The main steps of this method are presented in Algorithm 2.

Algorithm 2 Iterative location improvement algorithm (ILI)

Require: ϵ_1, A_j, X_i, w_j, z_{ij}, f_t, u, $\forall i = 1,2,\dots,m; j = 1,2,\dots,n; k = 1,2,\dots,p$.
repeat
1: Calculate new location coordinates of facilities using Eq. (29) (X_i').
2: Calculate distances between new facility locations and customer locations ($d(X_i', A_j)$).
3: Calculate the new value of the objective function (f_t').
4: $dif = f_t - f_t'$
5: **if** $f_t' < f_t$ **then** $f_t \leftarrow f_t'$ **end if**
until $dif > \epsilon_1$
return $f_t, X_i, \forall i = 1,2,\dots,m$.

Alternate location allocation approach

In this section, an iterative local search procedure is developed to be used within the main proposed methods with the aim of improving new solutions. Every time this procedure is repeated, it consists of two main steps, so that these two steps are repeated alternately. In the first step of each iteration, the location of facilities is improved using the ALI approach (see Algorithm 2) by fixing the variables related to customer allocation to facilities (z_{ij}) and the number of each machine type used by each facility (L_{ik}). In the second step, the variables of facility location ($X_i = (x_i, y_i)$) are fixed, and the reduced problem of simultaneously allocating customers and machinery to the facilities (Model 2) is solved in a running time of t_3 seconds. The variables of facility locations ($X_i = (x_i, y_i)$) are considered as inputs, and variables related to the allocation of customers to facilities (z_{ij}) and the number of each machine type used by each facility (L_{ik}) are considered as the outputs to the exact solution (CPLEX) of the problem. The new solution is compared with the current solution, and if it is better, it will be replaced with the current solution.

This procedure is iterated until the resultant improvement is smaller than ϵ_2. you can find its main steps in Algorithm 3.

Algorithm 3 Alternate location allocation algorithm (ALA)

Require: ϵ_2, t_3, X_i, z_{ij}, l_{ik}, F, u, $\forall i = 1,2,\dots,m; j = 1,2,\dots,n; k = 1,2,\dots,p$.
repeat
1: Apply **Algorithm 2**(ILI).
2: Solve **Model 2** with CPLEX solver in t_3 seconds.
3: Using the outputs of CPLEX, determine allocation of customers to facilities (z'_{ij}) and the number of each machine type used by each facility (L'_{ik}) and specify the value of the objective function (F').
4: $dif = F - F'$
5: **if** $F' < F$ **then** $F \leftarrow F'$ **end if**
until $dif > \epsilon_2$
Return X_i, z_{ij}, l_{ik}, $\forall i = 1,2,\dots,m; j = 1,2,\dots,n; k = 1,2,\dots,p$.

Generating a neighbor of the current solution

The exact solution of the discrete problem related to the main problem is used to generate a neighbor of the current solution. For this purpose, several potential locations are selected for facilities. The process of potential locations selection is carried out in two steps. First, a set of candidate locations is built for each facility. Then, some locations are selected randomly among the candidate locations for each facility and added to the set of potential locations. The set of candidate locations for each facility includes the current location of the facility and the location of a number of closest customers to the same facility. The customer locations are chosen among customers who are within the shortest distance from each facility. Let $pn^\star$ denotes the number of closest customers whose location are added to the set of candidate locations of each facility. Then, the set of candidate locations for each facility has $pn^\star + 1$ members. The potential locations of facilities should be specified to obtain a new solution in the neighborhood of the current solution using an exact solution of the discrete problem related to the main problem. For this purpose, a number of locations are selected among candidate locations for each facility randomly and added to the set of potential locations. Let sn denotes the number of locations added to the set of potential locations from the set of candidate locations for each facility. Then, the set of potential locations includes $sn \times m$ potential locations for locating facilities. Note that a larger number of potential locations leads to increased quality of the solution obtained by the exact solution of the discrete problem using the CPLEX solver, but the program running time also increases. Therefore, the running time of the CPLEX solver is bounded to t_2 to generate a new solution in reasonable computational time. By determining potential facility locations, the model of the discrete problem related to the main problem (Model 3) is solved exactly by the CPLEX solver in a running time of t_2 seconds. location coordinates of facilities ($X_i = (x_i, y_i)$), customers allocated to facilities (z_{ij}), and the number of each machine type used by each facility (L_{ik}), are specified using the outputs of CPLEX solver and the value of the objective function of the new produced solution is calculated. You can find its main steps in Algorithm 4.

Algorithm 4 Generating a neighbor of the current solution

Require: $pn^\star$, sn, t_2, A_j, w_j, X_i, z_{ij}, l_{ik}, F, u, $\forall i = 1,2,\dots,m; j = 1,2,\dots,n; k = 1,2,\dots,p$
1: calculate distances between facility locations and customer locations ($d(X_i, A_j)$).
2: **for** $i = 1$ to m **do**
(I) Add the current location of facility i to the set of facility i candidate locations.
(II) Find the $pn^\star$ number of customers closest to facility i and add their locations to the set of facility i Candidate locations.
(III) Select random sn locations from the set of facility i candidate locations and add them to the set of potential facility locations.
3: **end for**
4: Calculate distances between potential facility locations and customer locations.
5: Solve the discrete problem related to the main problem (**Model 3**) exactly using CPLEX solver in t_2 seconds.
6: Regarding the outputs of CPLEX, determine coordinates of facility locations (X'_i) and the assignment of customers to facilities (z'_{ij}) and the number of each machine type used by each facility (L'_{ik}).
7: Calculate distances between facility locations and customer locations ($d(X'_i, A_j)$.

8: Calculate the value of the objective function of the new generated solution (F').

Return $X_i',\ z_{ij}',\ L_{ik}',\ F',\ \forall i = 1,2,\dots,m; j = 1,2,\dots,n; k = 1,2,\dots,p.$

Iterative random search heuristic approach

In this section, in order to solve the SSCMFWP-SC, a proposed solution method called iterative random search heuristic is developed, which is an iterative method based on reduced variable neighborhood search. Readers can refer to Hansen, Mladenovi´c, and Moreno P´erez (2010) to read more about VNS. First, an initial solution is generated using Algorithm 1, and then the neighbors of the current solution are searched in an iterative process in order to find a better solution. In each iteration, potential locations for facility deployment are selected from customer locations and facility locations using a random approach, and then using an exact solution method, a new solution is generated in the neighborhood of the current solution, and then the new Produced solution's quality is improved. Therefore, in each iteration, in order to generate a new solution in the neighborhood of the current solution, Algorithm 4 and to improve the new generated solution's quality, Algorithm 3 is applied. If the new generated solution is better than the current solution, the new solution is replaced as the current solution and the algorithm goes to the next iteration, otherwise the algorithm goes to the next iteration without replacing the new solution. The search process continues until the stop condition is met. Two versions of the proposed method have been developed, so that the general structure and main steps of both versions are similar. Let pn be the maximum number of customer locations that can be added to the set of candidate locations for each facility and the symbol $pn^\star$ denote the number of customer locations that are added to the set of candidate locations for each facility. In this case, the only difference between the two versions of the proposed method is the value of $pn^\star$. In the first version of the proposed method, which will be shown with symbol IRS1, the value of $pn^\star$ remains constant in all iterations and its value is equal to pn. But in the second version of the proposed method, which will be shown with the symbol IRS2, the value of $pn^\star$ will be different in each iteration from the previous iteration, so that the value of $pn^\star$ will change from 1 to pn. That is, in the first iteration, the value of $pn^\star$ is equal to 1, and in the second iteration, it is equal to 2, and in subsequent iterations, one unit will be added to its value in each iteration, and if its value is equal to pn, in the next iteration, its value will be equal to 1. This process will continue until the stop condition is met. The reason for changing the value of $pn^\star$ in each iteration is to prevent the generation of similar solutions and also to better search the solution space, and therefore it will be possible to produce diverse candidate location sets and potential location sets. To stop the random search process, we set the maximum iteration number. Let the symbol $maxiter$ represents the maximum number of iterations. Since the search algorithm is a random process, the more the number of iterations is, the better the quality of solutions. However, an increased number of iterations raises the running time. Therefore, to create a balance between the quality of the solution and the running time, some initial trials were performed based on which the value of $maxiter$ was determined. The main steps of Algorithms of IRS1 and IRS2 are presented as flowcharts in **Error! Reference source not found.** and **Error! Reference source not found.**, respectively.

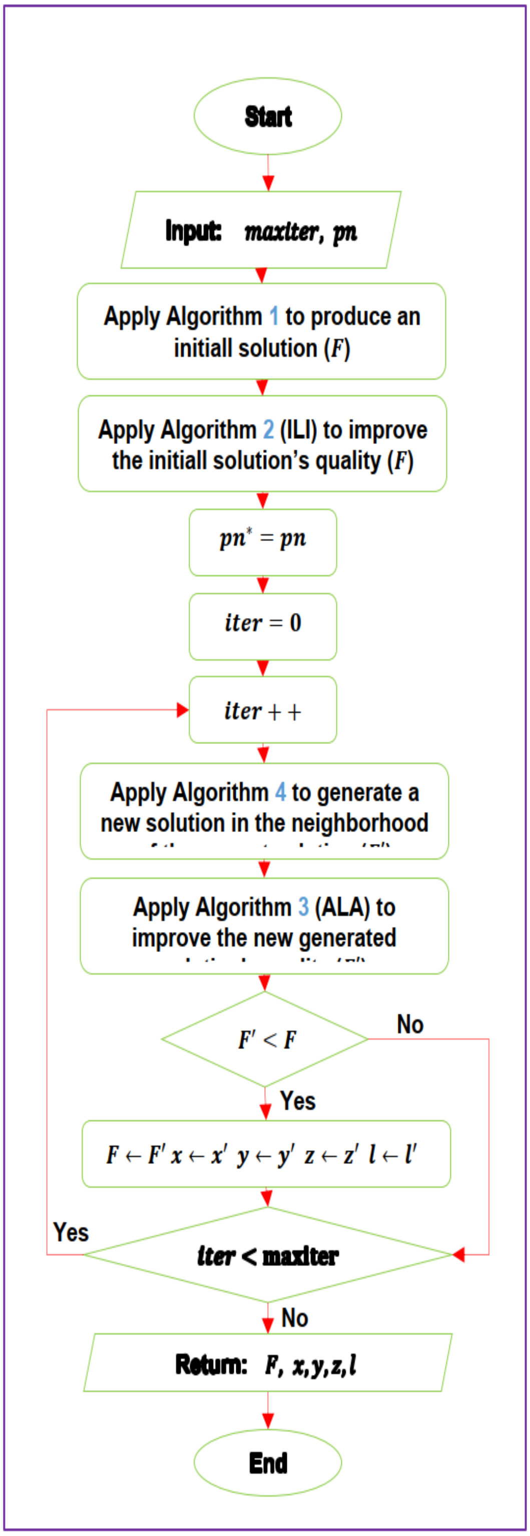
Start
Input: maxiter, pn
Apply Algorithm 1 to produce an initiall solution (F)
Apply Algorithm 2 (ILI) to improve the initiall solution's quality (F)
pn* = pn
iter = 0
iter ++
Apply Algorithm 4 to generate a new solution in the neighborhood
Apply Algorithm 3 (ALA) to improve the new generated
F' < F
No
Yes
F ← F' x ← x' y ← y' z ← z' l ← l'
Yes
iter < maxiter
No
Return: F, x,y,z,l
End

Fig. 1.

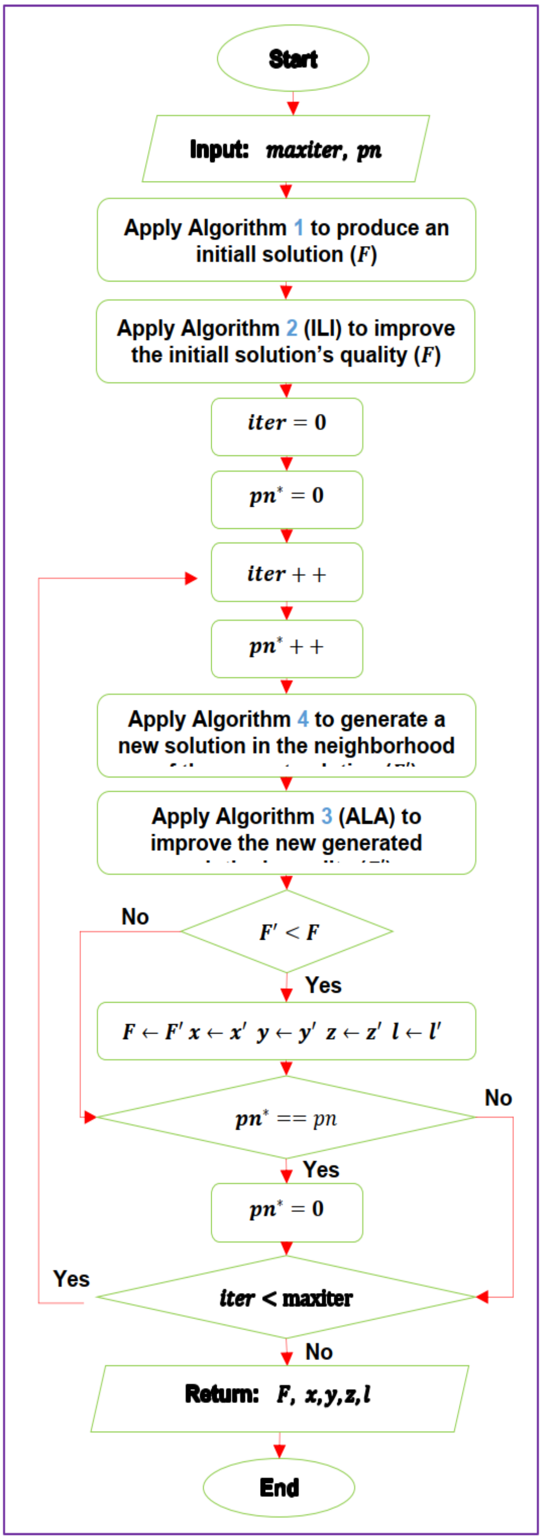
Start
Input: maxiter, pn
Apply Algorithm 1 to produce an initiall solution (F)
Apply Algorithm 2 (ILI) to improve the initiall solution's quality (F)
iter = 0
pn* = 0
iter ++
pn* ++
Apply Algorithm 4 to generate a new solution in the neighborhood
Apply Algorithm 3 (ALA) to improve the new generated
No
F' < F
Yes
F ← F' x ← x' y ← y' z ← z' l ← l'
No
pn* == pn
Yes
pn* = 0
Yes
iter < maxiter
No
Return: F, x,y,z,l
End

Fig. 2.

COMPUTATIONAL TRIALS

In this section, the results of computational trials on the proposed approach are presented. All algorithms were coded in C programing language and implemented on a Laptop with an intel core i5 2500 CPU @ 2GHz processor and 8 GB of RAM and IBM ILOG CPLEX 12.10 Concert Library was used to exact solving of the mathematical models. The presented methods were tested on a dataset which is available in the literature of this problem in which the number of customers are changed from 50 to 500 with a step length of 50. you can find this dataset in Irawan et al. (2020). The performance of the proposed methods is evaluated in terms of the quality of solutions. For this purpose, the quality of the solutions obtained by the proposed approach is compared with the best available solutions to this problem in the literature. We extracted these best solutions directly from Irawan et al. (2020). Two criteria, namely average solution and the number of best solutions, are used. The proposed algorithms are implemented ten times on each sample data, and the best solution, mean of solutions, the worst solution, and average running time are recorded for each test problem.

Setting the value of parameters

The value of parameters was determined based on a number of preliminary trials, as follows:

t_1	=	$2 \times m$	t_2	=	m
cn	=	8	pn	=	5
t_3	=	$\frac{m}{2}$	ϵ_1	=	0.00001
sn	=	2	ϵ_2	=	0.1

Numerical results

The numerical results of computational trials on sample data are presented in **Error! Reference source not found.**. The best value for each test problem is shown in bold. According to **Error! Reference source not found.**, the lowest average is associated with the IRS1 method. In addition, the average of the “Best” and “Average” columns for each of the two proposed methods is lower than the average of the “best” column for the previous best solutions from the literature. Furthermore, both the two proposed methods have obtained the best solution in eight cases, while the value of this criterion for the previous best solutions is 4. Therefore, the quality of the solutions obtained by the proposed methods is better than the previous best solutions in terms of both the average value of solutions and the number of the best solutions. The best solutions among the results of the two proposed methods are extracted from **Error! Reference source not found.** and presented in **Error! Reference source not found.** in the ”New best” column to conduct a better comparison. According to **Error! Reference source not found.**, for ten test problems, the number of best solutions found by the proposed methods is 10, while this number for the existing methods in the literature is 4. In addition, the average value of the best solutions obtained by the proposed methods is less than the value of this criterion for the best solutions in the literature. Moreover, the proposed methods have improved the best solutions in six of the ten test problems. Therefore, the best solutions of the proposed methods can be used as the best existing solutions to this problem for comparison in future researches.

Table 1: Computational results

IRS2					Previous	IRS1				
Average	Worst	time(s)	m	n	Best	Best	Average	Worst	time(s)	Best
4145.87	**4145.87**	160.7	5	50	**4145.87**	**4145.87**	**4145.87**	**4145.87**	178.5	**4145.87**

11008.13	11009.68	177.3	10	100	**11005.81**	**11005.81**	11006.58	11009.68	217.9	**11005.81**
18123.85	18322.45	478.2	15	150	**18087.01**	**18087.01**	18186.17	18322.45	593.3	**18087.01**
27549.5	27555.45	884.2	20	200	**27533.93**	**27533.93**	27544.73	27559.67	922	27533.93
39588.46	39632.6	1437.5	25	250	39655.11	**39145.89**	39467.55	39718.08	1551.5	39562.75
53189.32	53231.07	2019	30	300	53193.37	**53180.58**	53243	53345.41	2014.4	**53180.77**
70875.46	71042.99	2585.6	35	350	70797.9	**70780.58**	70909.91	71187.29	2496.4	70780.58
78080.15	78141.03	2931	40	400	78159.08	**77998.76**	78109.6	78292	2794.3	**78040.86**
103285.4	103630.4	3610.9	45	450	103746.5	103091.1	103439.3	103819.1	3607.6	**103067**
119907.4	120228.6	3853.5	50	500	120199.5	119899.5	120108.5	120308.2	3735	**119735.3**
52575.36	52694.01	1813.79	Average		52652.4	**52486.9**	52616.12	52770.77	1811.09	52513.99
			#Best		**4**	**8**				**8**

Table 2: Comparison between the best solutions obtained by the proposed methods and the previous best solutions

n	m	Previous best	New best
50	5	**4145.87**	**4145.87**
100	10	**11005.81**	**11005.81**
150	15	**18087.01**	**18087.01**
200	20	**27533.93**	**27533.93**
250	25	39655.11	**39145.89**
300	30	53193.37	**53180.58**
350	35	70797.90	**70780.58**
400	40	78159.08	**77998.76**
450	45	103746.48	**103066.98**
500	50	120199.45	**119735.31**
Average		52652.40	**52468.07**
#Best		4	**10**

CONCLUSION

SSCMFWP-SC has been the topic studied here. Finding location of facility construction and simultaneously allocating customers and machineries to the facilities has been aimed at by the problem. That is, total transportation cost between customers and facilities and their setup costs would be minimized. To solve the problem, a heuristic method named iterative random search have been provided in two versions. Recommended methods have been tested on a dataset available in literature and the solutions resulted have been compared to the best available ones in literature. Considering the results, it was observed that in all sample problems, recommended methods have had better performance both in terms of average solution and numbers of best solutions criteria. Moreover, recommended methods in all of the 10 sample problems, have obtained best solutions and in 6 problems from among 10 sample problems, best available solutions in literature have been improved. So, best solutions resulted from recommended methods can be used in future studies as best solutions to the problem as a source of comparison.

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