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# Eco-Efficiency in two-stage structure: A DDF Approach

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**Abstract**

The importance of environment protection always raises a considerable among the researchers especially when pollutant and undesirable outputs are present. The non-parametric Data Envelopment Analysis (DEA) literature on network-structured performance analysis normally fail to express the undesirable intermediate production in the process. From the production efficiency point of view, symmetrically handling both of the desirable and the undesirable outputs in the assessment process in the two stage network structure is developed in this paper. The motivation of this study is the application of the directional distance function (DDF) to modeling the two-stage network structure with the undesirable intermediate measures upon employing Stackelberg theory. A real case on the thirteen poultry units in Iran has been illustrated to verify the applicability of the proposed approach.

**Keywords**:
Data Envelopment Analysis (DEA)
Directional Distance Function (DDF)
Two-stage network structure
Eco-efficiency
Undesirable outputs

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## INTRODUCTION

Data envelopment analysis DEA as a nonparametric technique pioneered by Charnes et al. (1978) under conditions of constant returns to scale (CRS) and varying returns to scale (VRS) by Banker et al. (1984) . Standard models of DEA measure the relative efficiencies of decision making units (DMUs) that use the same kind of inputs to produce the same kind of outputs in a specific time period. DEA has been widely applied to not only non-profit organizations but also for private sector firms. Apart from producing desirable outputs, the DMUs also generate undesirable ones chiefly when waste water or emissions must be controlled. Also the increment of world population and the resource scarcity arises the feeling of uncertainty of sustainability of environment. According to this problem, several studies have been published including the theory and application of DEA methodology to control undesirable production and eco-efficiency (for example: Sueyoshi et al. (2017), Kao (2014), Emrouznejad and Yang (2018) Chen and Delmas (2012), and Rashidi et al., (2015)).Upon the point of departure being the undesirable output interference, a directional distance function (DDF) approach stands as an important tool for measuring eco-efficiency. Based on the seminal work of Chambers et al. (1996) and Chung et al. (1997) this method has been widely applied in eco- efficiency. For example, refer to Liu (2018).

Interestingly, most of the above mentioned approaches considered the units as a block box and did not focus on internal structures of DMUs. In order to handle the situation, different methods such as Network DEA, game theory, standard DEA approach and efficiency decomposition were categorized. To do so, various perspectives were proposed in presence of undesirable factors in two-stage network structures.

For example, Lin et al. (2017) and Petridis et al. (2019) have applied directional distance function (DDF) to evaluate overall efficiency along with individual efficiency. Chen et al. (2010) evaluated the efficiency of two-stage processes based on DDF, where all the outputs from the first stage are intermediate measures and make the inputs for the second stage. Izadikhah and Saen (2016) proposed a two-stage model based on DDF that can deal with negative data. However they used the eigenvector method to determine the two weights at both stages then applied it to evaluate 29 Iranian supply chains of producing expendable medical devices. Wanke et al. (2018) designed a new combined model, that decreases inputs and increases outputs together, leading to a new way for assessing efficiency scores based on DDF with a two-stage structure under CRS. This model also approved the accuracy of the previous suggested models cannot deal with negative data. Tavana et al. (2018) designed a dynamic range directional measure based on a two-stage structure that allows for negative data. However, the existing can be applied to performance measurement systems characterized by positive and negative values. For the sake of completeness, they present a case study in to demonstrate the applicability in the banking industry and exhibit the efficacy of the RDM procedures to deal with negative data. Lin, R., and Liu, Q. (2021) extends the multiplier non-oriented series DEA model according to the DDF. The corresponding efficiency decomposition of this method is expressed as a weighted average of process DDF efficiency measures. Moreover, this approach can be applied under both CRS and VRS assumptions meantime, it can handle negative data well.

On the other hand leader-follower DEA models are developed for measuring the efficiencies of the two stages. According to this method the first move is done by the leader, then the leader authorize the behavior of the follower. In confirmation of this statement, Zhou et.al (2018) explained that the leader-follower model is quite important and practical, in which the party with the manipulative power of the two stages controls the other party who becomes the follower in the relationship. As we can see, the study of leader-follower models is very meaningful. As a matter of fact, received little attention in the field of DEA. There exists a large number of variations of the two-stage network structure and an equally large number of models has been proposed for studying them (Zorriehhabib et.al(2019) ) Many studies have supported assessing the efficiencies of such two-stage systems. However, they do not envelope the DDF and leader-follower simultaneously. Besides, what needs to be implied is that the above two-stage and DDF models are developed under the multiplier form. In the existing research, there are very few studies on envelopment network DDF models. The motivation of this study is the application of DDF to modeling network DEA with undesirable intermediate measure in which the undesirable output of the first stage leaves the process (See figure 1). In this paper, we employ the directional distance function (DDF) to propose a new approach for measuring the efficiency in two-stage process. The main features and superiorities of the proposed model can be mention as bellow:

1: The proposed efficiency measurement method can be applied to both CRS and VRS assumptions.
2: The proposed two-stage DDF model can handle the problem with importance of stages to assess overall efficiency score. According to the purpose of the decision maker the main stage can be applied as the leader or the follower one.

The reminder of the paper is unfolded as follows: In the next section, a brief description on DDF will follow. Then, we proceed to DDF in a two-stage structure in Section 3. A practical example is given in Section 4. The conclusion section will summarize the findings and implications of the study. Finally, in Section 5 concluding remarks are presented.

## DIRECTIONAL DISTANCE FUNCTION(DDF)

The pioneer approach of Chambers et.al (1998) called as directional distance function (DDF) has been very common among researchers in presence of undesirable factors. This approach assumes that inputs and undesirable outputs are contracted and desirable outputs are expanded at the same rate. Suppose that there are $N$ DMUs. Each DMU is denoted by $DMU_j (j=1,...,N)$. Each unit has $m$ inputs $x_j=(x_{1j},...x_{mj})\geq 0$ to produce $S$ desirable outputs $v_j=(v_{1j},...v_{sj})\geq 0$ and $f$ undesirable outputs $w_j=(w_{1j},...,w_{fj})\geq 0$. Furthermore, assume that $x_j \neq 0$, $v_j \neq 0$, $w_j \neq 0$. So that, the production technology is defined as $S=\{(x,v,w) | x \text{ can produce } (v,w)\}$. From the perspective of axiomatic approach on efficiency measurement, the DDF is defined as:

$$D(x,v,w,-g_x,g_v,-g_w)=$$
$$\sup\{\beta | (x-\beta g_x, v+\beta g_v, w-\beta g_w)\in S\}.$$

The given direction vector $g=(-g_x^-, g_w^-, g_v^+)\neq 0_{m+r+k}$, $g^-\in R_+^m, g^-\in R_+^k, g^+\in R_+^r$ have exactly the same dimensions as any observation of the sample. Clearly, $\beta \geq 0$ denotes the scaling factor. From a managerial perspective, DDF approach measures the distance from a certain operation point (e.g. $DMU_o$) to the weakly efficient subset of the network production possibility set (PPS) along a given direction vector. It is, therefore, the largest step size that can be given along that direction from that operation point without abandoning the network production possibility set (PPS). Since the directional vector $g$ is the explicit directional vector in which the input–output combination will be scaled, mathematically, this vector may be constant, i.e., the directional vector is the same vector for all units, or may be variable, i.e. it is a specific vector for each unit. In the context of directional distance function, the DDF of under evaluated unit along the directional vector $g=(-g_x^-, g_w^-, g_v^+)$ can be computed by solving the following programming problem:

$$
\begin{aligned}
&Max \quad \beta_o \\
&s.t \quad \sum_{j=1}^{n}\lambda_j x_{ij} \leq x_{io}-\beta_o g_{io}, \quad i=1,...,m \qquad (1)\\
&\qquad \sum_{j=1}^{n}\lambda_j v_{rj} \geq v_{ro}+\beta_o g_{ro}, \quad r=1,..,R \qquad (2)\\
&\qquad \sum_{j=1}^{n}\lambda_j w_j = w_{ko}-\beta_o g_{ko}, \quad k=1,...,K \qquad (3)\\
&\qquad \sum_{j=1}^{n}\lambda_j = 1 \qquad (4)\\
&\qquad \lambda_j \geq 0, \; j=1,\dots,n
\end{aligned}
\qquad (1)
$$

Constraints (1) and (2) respectively impose the corresponding exogenous input reductions and final outputs increases. The corresponding right hand size relates the target total input consumption to the current input consumption thus bounding from below the maximum step size $\beta_o$ that can be achieved along the direction given by vector $g=(-g_x^-, g_w^-, g_v^+)$. Similarly, the right hand side of constraint (3) imposes, for each undesirable output, the possible reduction that can be obtained using that reduction, as before, to bound the maximum step size along direction $g=(g_x, g_v, g_w)$. Obviously, if $\beta_o=0$ it means that

The unit evaluated is located at the frontier of best practice and is therefore efficient in the direction $g=(-g_x^-, g_w^-, g_v^+)$. Otherwise, the unit will be evaluated as an inefficient.

## DDF APPROACH IN TWO-STAGE NETWORK PROCESS

In this section a two-stage decision process within which the intermediate measures consist of desirable and undesirable outputs, is introduced. Consider a two-stage production process as shown in Fig. 1.

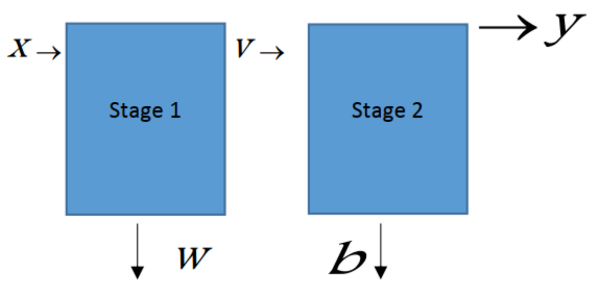


Fig. 1. A general two-stage model considering undesirable outputs

Again suppose that there are $N$ DMUs and for the first stage of $DMU_j (j=1,...,N)$, the observe data on the vectors of inputs, desirable outputs and undesirable outputs are $x_j=(x_{1j},...x_{mj})\geq 0$ ; $v_j=(v_{1j},...v_{sj})\geq 0$, $w_j=(w_{1j},...,w_{fj})\geq 0$ respectively; The outputs $(v_j, w_j)$ are used as the inputs for the second stage. The second stage fed up with $(v_j, w_j)$ and external input $z=(z_{1j},...,z_{Tj})$. The final output of $DMU_j (j=1,...,N)$ is presented by $y=(y_{1j},...,y_{sj})$. Referring to the concept of DDF, the first stage will determine the expansion of desirable output and the contraction of undesirable outputs at the same time. Mathematically, the value of efficiency for the first stage can be obtained by solving the following DEA type model:

$$
\begin{aligned}
& e_1 = Max \quad \beta_o \\
s.t \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \leq x_{io} - \beta_o g_{io}, \quad i=1,\ldots,m & (1)\\
& \sum_{j=1}^{n}\lambda_j v_{rj} \geq v_{ro} + \beta_o g_{ro}, \quad r=1,..,R & (2)\\
& \sum_{j=1}^{n}\lambda_j w_{kj} = w_{ko} - \beta_o g_{ko} \quad , \quad k=1,\ldots,K & (3)\\
& \sum_{j=1}^{n}\lambda_j = 1 & (4)\\
& \sum_{i=1}^{m} g_{io} + \sum_{r=1}^{R} g_{ro} + \sum_{k=1}^{K} g_{ko} = 1 & (5)\\
& \lambda_j \geq 0, \; j=1,\ldots,n
\end{aligned}
$$

(2)

In the above model, the set of constraints guarantees decreasing of inputs. Furthermore, the inequality constraints on the desirable outputs impose that theses desirable outputs are increasing. Together with the strict equality constraints on the undesirable outputs side in (3) and the parameter $\beta_o$ on the right hand side of restrictions (1) - (3), both desirable outputs and undesirable outputs are weakly disposable when variable returns to scale are imposed. Finally, the model also satisfies null-joint ness property. The last constraint (5) deals with directional vector to avoid the unbounded solution. In terms of the abatement potential in undesirable outputs of the first stage, the efficiency of the second stage can be evaluated. Having obtained the efficiency of the first stage - applying the leader-follower theory – and preserving the efficiency statues of the first stage, the optimal value of the second stage is obtained through following model:

$$e_2 = Max \quad \gamma_o$$

$$s.t \quad \sum_{j=1}^{n} \lambda'_j v^*_{rj} \le v^*_{ro} - \gamma_o g'_{ro}, \quad r = 1,\dots,R \qquad (1)$$

$$\sum_{j=1}^{n} \lambda'_j y_{cj} \ge y_{co} + \gamma_o g'_{co}, \quad c = 1,..,C \qquad (2)$$

$$\sum_{j=1}^{n} \lambda'_j b_{fj} = b_{fo} - \gamma_o g'_{fo} \quad ,f = 1,\dots,F \qquad (3)$$

$$\sum_{j=1}^{n} \lambda'_j = 1 \qquad (4)$$

$$\sum_{c=1}^{C} g'_{co} + \sum_{r=1}^{R} g'_{ro} + \sum_{f=1}^{F} g'_{fo} = 1 \qquad (5)$$

$$\lambda'_j \ge 0, \; j = 1,\dots,n$$

(3)

Based upon leader-follower game theory for two-stage process, the second stage only considers optimal solutions that maintain the first stage's efficiency statues. To address this issue, the second stage treats the triple $(v^*, y \;, b)$ subject to the restriction that efficiency score of the first stage remains at optimality. By applying leader-follower theory we assume that the first stage is more important. According to the importance of the first stage then it is labeled as the leader stage. As constraint (1) of model (3) admits the optimal values of the first stage are treated in the second stage. Constraint (2) is imposing the desirable final output expansion at the same time employing the directional vector for second stage. As before, $\gamma_o$ denotes the largest step size that can be given along that direction from that operation point without abandoning the network production possibility set. It should be pointed out that a system is efficient if and only if the two component processes are efficient.

## EMPIRICAL EXAMPLE

The applicability of the proposed approach is illustrated by an empirical data set consisting of thirteen DMUs. The data set was driven from Amirteimoori et.al. (2014). In the first stage, there are three inputs characterized by $x_1^1$, $x_2^1$, $x_3^1$.Also the desirable intermediate measure are reported as $v_1^1, v_2^1$ . Finally the undesirable measure which leaves the system in the first stage is characterized as $w_1^1$ . The final desirable output of the second stage are reported by $y_1^2$, $y_2^2$ and the undesirable output is recorded by $w_1^2$ .

With reference to Amirteimoori et.al (2014), the process has been divided into two sub-process. The first sub section one is related to the first 21 days of the broiler production and the second stage is the remaining time of the production period. Fig.2. illustrates the process as a two-stage structure.

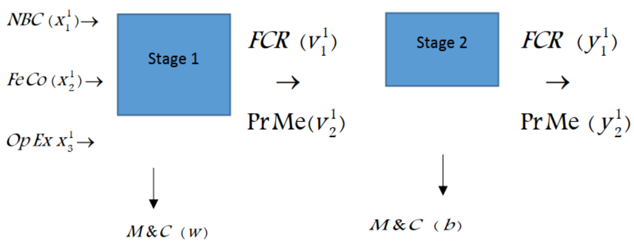


Fig. 2. Poultry farm as a two-stage system

As fig. 2. records, the first stage applies three input $x_1^1$, $x_2^1$, $x_3^1$ to produce two desirable outputs $v_1^1, v_2^1$ and only one undesirable output $w_1^1$ . The undesirable output of the first stage leaves the system, but second stage fed up with intermediate measure. The final outputs of the system are denoted by $y_1^2$, $y_2^2$ as final desirable output and $w_1^2$ as final undesirable output. Table 1 lists the data set and definitions and units of measurement of the different variables.

Table 1: Observed data for thirteen poultry farms

| ***DMU*** | ***Stage 1(golden time)*** | | | | | | ***Stage 2(chicken to broiler)*** | | |
|---|---|---|---|---|---|---|---|---|---|
| | $x_1^1$ | $x_2^1$ | $x_3^1$ | w | $v_{1j}^1$ | $v_{2j}^1$ | $v_{1j}^2$ | $v_{2j}^2$ | $w_{1j}^2$ |
| | *NBC* | *FeCo* | *OpEx* | *M&C* | *FCR* | *Pr Me* | *FCR* | *Pr Me* | *M&C* |
| 1 | 12700 | 148500 | 57370 | 467 | 1.69 | 6691.5 | 1.98 | 28582.2 | 173 |
| 2 | 14670 | 171740 | 63900 | 513 | 1.65 | 7871.3 | 1.93 | 32387.2 | 197 |
| 3 | 13300 | 154930 | 63220 | 1263 | 1.72 | 6921.3 | 2.00 | 28506.3 | 306 |
| 4 | 15000 | 182880 | 60590 | 421 | 1.71 | 8280.9 | 1.95 | 34075.0 | 79 |
| 5 | 12000 | 147490 | 57030 | 758 | 1.68 | 6340.5 | 1.98 | 26256.5 | 256 |
| 6 | 14000 | 105080 | 63640 | 1098 | 1.70 | 7134.8 | 1.97 | 29828.0 | 263 |
| 7 | 13000 | 168930 | 62020 | 646 | 1.75 | 7202.4 | 2.03 | 30158.7 | 144 |
| 8 | 14900 | 175430 | 71680 | 821 | 1.62 | 7475.9 | 2.04 | 33414.6 | 214 |
| 9 | 13500 | 169520 | 62300 | 518 | 1.71 | 7399.7 | 1.94 | 30439.0 | 246 |
| 10 | 12800 | 144130 | 60930 | 623 | 1.63 | 6356.4 | 2.03 | 28223.5 | 167 |
| 11 | 19800 | 235970 | 80960 | 1042 | 1.67 | 10373.2 | 2.01 | 44581.2 | 336 |
| 12 | 11000 | 133540 | 51340 | 385 | 1.68 | 5933.8 | 2.00 | 25683.4 | 89 |
| 13 | 12600 | 118870 | 57210 | 479 | 1.63 | 5933.8 | 1.88 | 28405.3 | 186 |

In order to evaluate the system performance, the proposed DDF approach, in Section3 has been employed to the data set. First, the optimal scores of the first stage are recorded and the results are shown in Table (3). Upon obtaining the optimal intermediate outputs of the first stage, the second stage is evaluated while the efficiency of the first stage remains unchanged. The results are shown in the third column of Table 3.

Table 3: Total efficiency along with stages scores

| **DMU** | $e_1$ **stage 1** | $e_2$ **stage 2** (overall inefficiency) |
|---|---|---|
| **DMU01** | 231.20 | 507.47 |
| **DMU02** | 202.96 | 387.12 |
| **DMU03** | 1131.87 | 517.75 |
| **DMU04** | 0.00 | 0.00 |
| **DMU05** | 544.08 | 357.08 |
| **DMU06** | 0.00 | 194.00 |
| **DMU07** | 0.00 | 60.66 |
| **DMU08** | 1029.66 | 1028.14 |
| **DMU09** | 111.53 | 168.04 |
| **DMU10** | 709.92 | 603.07 |

| | | |
|---|---|---|
| **DMU11** | 0.00 | 2349.30 |
| **DMU12** | 0.00 | 0.00 |
| **DMU13** | 0.00 | 0.00 |

As Table 3 shows, there are six efficient units in the first stage $(\beta_o^* = 0)$ Efficient poultry farms are DMU4, DMU6, DMU7, DMU11, DMU12 DMU13 Keeping these efficient units in the second stage, there are three efficient units in aggregate sense.

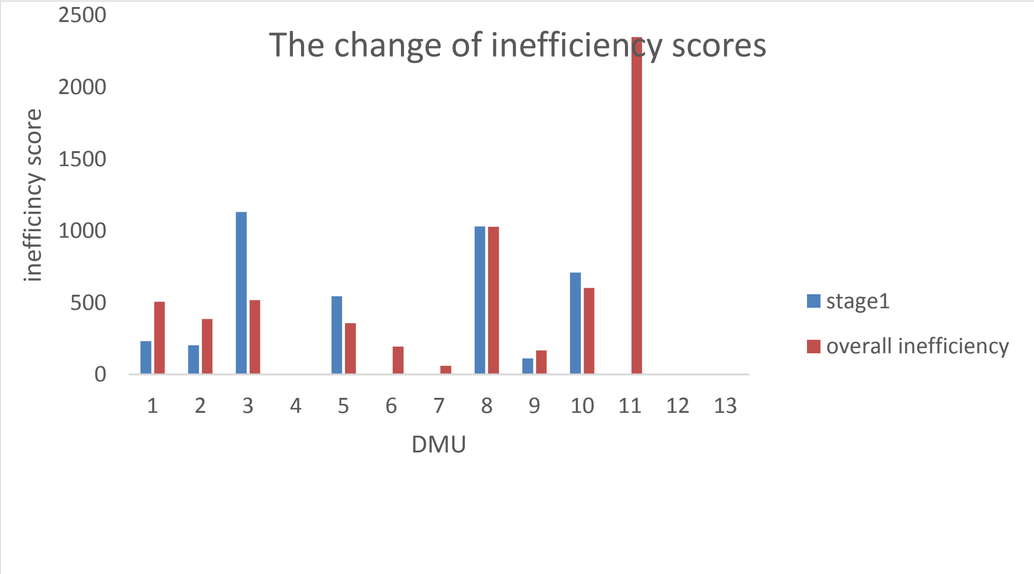


Fig. 3. Inefficiency scores of stage 1 and the whole system

Referring to the both columns of Table 3, we see that the reduction of undesirable factors for unit 4 is zero. This means that for this unit the DDF approach will reduce amount is zero. We come to conclusion that DMU4 is overall efficient. In other words productive efficiency score of the DMU4 is one. According to Table 3, three DMUs are efficient. DMU4, DMU12 and DMU13 are efficient and their inefficiency scores are zero. (See Fig.3)

## CONCLUSION

The growing interest in eco-efficiency performance and environmental protection is a major matter of concern for regulators, firms' managers and academics in the field of environmental Economy. Besides, the growing recognition of pollutant reduction and environment protection, aimed at achieving predetermined standards of environmental quality. This setting calls for innovative methods to assess firms' environmental performance in presence of undesirable factors. In this paper, a directional distance (DDF) approach considering that intermediate undesirable factors are generated in a two stage network structure has been proposed. When the intermediate measures in two-stage processes consist of desirable and undesirable, the existing approaches in network DEA cannot provide a good estimation of the efficiency. The paper introduces a DDF approach which leads to fair assessment. Using a dataset of 13 poultry units in Iran the corresponding DDF efficiency scores have been computed and leads to valid results in terms of undesirable output's reduction. This denotes that the DDF approach is able to compute much larger improvements for undesirable outputs.

## CONCLUDING REMARKS

In this paper, we developed our method based on two-stage and DDF model for nonnegative data. In real world, in some cases, the inputs, intermediate measures, and outputs may have negative values. Therefore, for future works one can apply our proposed method for handling negative data. Similar works can be expanded in the presence of stochastic and fuzzy data. Moreover, one can apply our proposed model for evaluating sustainability of supply chains. For example, industries which have network structure can be a candidate.

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