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Efficiency Evaluation in Presence of Undesirable and Negative Factors

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Abstract

Data envelopment analysis (DEA) has been proven as an excellent data-oriented efficiency analysis method for comparing decision making units (DMUs) with multiple inputs and multiple outputs. In conventional DEA models, it is assumed that the input or output variables are all non-negative and desirable. However, in some situations, a performance measure can take positive quantity for some DMUs and negative value for others. Also, undesirable (bad) inputs and outputs may be presented in the production process. Hence, the standard model cannot directly reflect the efficiency score. The paper proposes a modified model in which both undesirable and negative data are treated to improve the relative efficiency of the DMU under evaluation. The focus of this paper is on treating the negative data on the definition of the two non-negative variable and the decreasing of undesirable outputs. A real example of 20 bank branches shows applicability of the proposed approach.

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INTRODUCTION

Data envelopment analysis (DEA) is concerned with comparative assessment of efficiency of decision making units (DMUs). n the classical DEA models, the efficiency of a DMU is obtained by maximizing ratio of the weighted sum of its outputs to the weighted sum of its inputs, subject to the condition that this ratio does not exceed one for any DMU. Since the pioneering work of Charnes et al. (1978) and Banker et al. (1984), DEA has demonstrated to be an effective technique for measuring the relative efficiency of a set of homogeneous DMUs which utilize the same non- negative inputs to produce the same nonnegative outputs. In conventional DEA applications, given a set of available measures, it is assumed that the status of each measure is clearly stated as an input or an output variable in the production process prior to using DEA models (Amirteimoori et al., 2013; Esmaeilzadeh et al., 2015). What's more, in standard models, inputs need to be decreased and outputs need to be increased to improve the performance of a DMU. However, this principle is not applicable in real circumstances. A methodological contribution of DEA studies on efficiency assessment is the importance of an output separation into desirable (good) and undesirable (bad) outputs. Modeling undesirable outputs has received considerable attention not only for measuring efficiency but also for reducing pollution abatement factor. The approach has been critically debated in Fare et al. (1989), Chung et al. (1997), Seiford and Zhu (2002), Jahanshahloo et.al (2005), Pathomisiri et.al (2008), Tao et.al (2016) and Mehrabian et al. (1391).

As noted by Allen (1999) a symmetric case of undesirable inputs which should be maximized may also occur. For example, the aim of a recycling process is applying maximal quantity of the input waste (Eini et al., 2017)). The issue is still being explored. However, there are many occasions in which some inputs and/or outputs must take negative values. For example, if in a period of time the proportion of revenue to cost in business corporation comes out a low number, the profit actually meets a negative quantity. As another example, the temperature can be estimated as both positive and negative. In tackling with negative data Portela et al. (2004) introduced an alternative model called as RDM . Sharp et.al (2007) proposed one of most applicable model in examining negative data by the name of MSMB . Asmild and Pastor (2010) suggested MAE model in this regard. Kazemi-Matin and Salehi (1392) applied the bounded model by the name of BAM in presence of negative data. But in real occasions, undesirable inputs may generate negative outputs. As a specimen, if a bank cannot access its claim from the customers, definitely the bank meets loss. This damage can be considered as negative output. In some literature (Emrouznejad at al., 2010) and Tohidi and Matroud (2017)) there have been various approaches put forward for dealing with negative data. However, there is no standard model for dealing with such data and the issue is still being explored. This paper presents an approach to treat both undesirable and negative variables in a radial DEA model. The paper is unfolded as follows. Section 2 gives a brief explanation of the recent approaches that deal with negative data in DEA and are closest to our own approach in philosophy. Section 3 introduces a DEA-based approach for performance evaluation in presence of both undesirable and negative data. Section 4 illustrates the applicability and usefulness of the proposed approach in assessing two real example. Conclusion will end the paper.

PRELIMINARIES

To describe the DEA efficiency measurement, assume that there are DMUs and the performance of each DMU_j , $j \in J=\{1,\dots,n\}$ is characterized by a production process of m desirable inputs $x=(x_{1p},\dots,x_{mp})$ to yields s desirable outputs $y=(y_{1p},\dots,y_{sp})$.To assess the efficiency of DMU_o we have the following two linear programming problem , also known as the CCR model, as follows (Charnes et al., 1978):

$$
\begin{aligned}
& Min\ \theta_o \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o x_{io}\ ,\ i=1,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}\ ,\ r=1,\dots,s \\
& \lambda_j \ge 0 \quad j=1,\dots,n
\end{aligned}
\tag{1}
$$

This model is a constant return to scale (CRS) program and it assumes that all input/output variables are desirable and non-negative. The efficiency ratio ranges between zero and one, with DMU_o being considered relatively efficient if it receives a score of one. From a managerial perspective, this model delivers assessments and targets with an input minimization orientation. The extension of CCR model was called BCC model and has the following format:

$$\begin{aligned}
&Min\ \theta_o \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o x_{io}\ ,\ i=1,\ldots,m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}\ ,\ r=1,\ldots,s \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \geq 0\ j=1,\ldots,n
\end{aligned} \qquad (2)$$

Model (2) is formulated under variable return to scale (VRS) because it incorporates $\sum_{j=1}^{n} \lambda_j=1$.

It is proved that the above model is always feasible and in optimality we have $\theta^*_0 \leq 1$. Obviously, all variables in CCR and BCC models are non-negative. In order to tackle with negative data, a semi-oriented radial measure (SORM) model was introduced by Emrouznejad et al. (2010). On the face of what the authors have done was to create two variables from a single variable that takes positive values for some and negative for other DMUs. This enables treating the negative output values as inputs in that the model seeks improved solutions which reduce the absolute value of the negative output. Note that this happens only for DMUs that have a negative value on the output concerned while the same variable is treated as a normal output for those DMUs that have a positive level on that variable. Similarly, the negative input values can be treated as outputs which increases the absolute value of the negative output. In DEA, each observed unit is characterized by a pair of input and output vector (X_j, Y_j), $j \in J=\{1,\ldots,n\}$. Following Emrouznejad et al. (2010), set of input variables $I=\{1,\ldots,m\}$, are partitioned as $I' \cup I''=I$ and the set of output variables $R=\{1,\ldots,s\}$, are divided to $R' \cup R''=R$ Subsets and as well as and are assumed to be mutually disjoint, that is to say $I' \cap I''=\phi$ and $R' \cap R''=\phi$ Subsets I' and R' are subject to positivity condition and subsets I'' and R'' are negative variable. The sets have the following format:

$$\begin{aligned}
&Min\ \theta \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}\ \ i \in I'
\end{aligned}$$

n

Based on the preceding notations, every negative input activity can be written as:

$$\forall i \in I''\ \ x_{ij} = x_{ij}^1 - x_{ij}^2$$

$$x_{ij}^1 = \begin{cases} x_{ij} & x_{ij} \geq 0 \\ 0 & x_{ij} < 0 \end{cases}$$

$$x_{ij}^2 = \begin{cases} 0 & x_{ij} \geq 0 \\ -x_{ij} & x_{ij} < 0 \end{cases}$$

Similarly, negative output can be transformed to:

$$\forall r \in R''\ \ y_{rj} = y_{rj}^1 - y_{rj}^2$$

$$y_{rj}^1 = \begin{cases} y_{rj}\ , & y_{rj} \geq 0 \\ 0\ , & y_{rj} < 0 \end{cases}$$

$$y_{rj}^2 = \begin{cases} 0\ , & y_{rj} \geq 0 \\ -y_{rj}\ , & y_{rj} < 0 \end{cases}$$

To measure efficiency improvement potential in negative variables a modified input efficiency measure is needed. The modified input oriented efficiency scores can be computed by solving the following model:

$$\begin{aligned}
&Min\ \theta \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}\ \ i \in I' \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq \theta x_{io}^1\ i \in I'' \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^2 \geq \theta x_{io}^2\ \ i \in I'' \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}\ \ r \in R' \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^1 \geq y_{ro}^1\ \ r \in R''' \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^2 \leq y_{ro}^2\ \ r \in R'' \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \geq 0\ j=1,\ldots,n
\end{aligned} \qquad (3)$$

Model (3) represents the general case for an input oriented variable return to scale (VRS) DEA model which has both inputs and outputs which take positive values for some DMUs and negative for others. The efficiency measure of model (3) will reflect radial contraction only of absolute input values and then only when there is no slack in either one of the constraints in model (3) which relate to the two auxiliary variables created from the original variable. For this reason, the efficiency measure θ in model (3) is refereed as input reduction semi-oriented radial measure (SORM). A negative input level (e.g. contributory rather than competing sales outlets where competing establishments are a positive input) is a good thing and targets which suggest replacing contributory with competing sales outlets would not be seen as sensible.

PROPOSSED APPROACH

Suppose we have nDMUs, and each DMU_j, $j \in J=\{1,...,n\}$ uses m inputs x_{ij} $(i=1,...,m)$ to generate outputs y_{rj} $(r=1,...,s)$. Following Emrouznejad et al. (2010) the set of input variables are divided into two categories as $I=I^g \cup I^b$ and the set of output variables as $R=R^g \cup R^b$, where subsets I^g and R^g are subject to desirable input and output and subsets I^b and R^b indicate the undesirable variables. Without loss of generality, it is assumed that the subsets are mutually disjoint also $|I^g|=p \le m$ and $|R^g|=q \le s$. As Koopmans (1951) stated, in case of inefficiency the undesirable outputs should be reduced to improve the efficiency. The same assumption is for undesirable inputs; they need to increase for efficiency improvement. Additionally, it may be different with before mentioned models, i.e., model (1) and model (2). To be more precise, to improve the performance of a DMU desirable inputs need to be decreased and undesirable inputs need to be increased. What's more, the increasing of desirable outputs and decreasing of the undesirable outputs is preferred in production process. As Thanassoulis et al. (2008) pointed out one of the possibilities for dealing with negative data is employing the combination of output variables such as profit or loss accompanying with the air pollution which reflected as negative data. Assume that we are operating in an environment in which negative data prevails. Equipped with SORM model discussed in previous section, the subsets are redefined as follows:

$$I' = \{i \in I : \forall j \in J, x_{ij} \ge 0\}$$
$$I'' = \{i \in I : \exists j \in J, x_{ij} < 0\}$$
$$R' = \{r \in R : \forall j \in J, y_{rj} \ge 0\}$$
$$R'' = \{r \in R : \exists j \in J, y_{rj} < 0\}$$

Based on the preceding notations, the set of input and output variables are defined as $I=I^g \cup I^b \cup I' \cup I''$ and $R=R^g \cup R^b \cup R' \cup R''$ respectively. Every feasible activity which is characterized as negative data can be stated as the difference of two non-negative variables. Hence we have:

$$\forall i \in I'' \quad x_{ij} = x_{ij}^1 - x_{ij}^2$$
$$x_{ij}^1 = \begin{cases} x_{ij} & x_{ij} \ge 0 \\ 0 & x_{ij} < 0 \end{cases}$$
$$x_{ij}^2 = \begin{cases} 0 & x_{ij} \ge 0 \\ -x_{ij} & x_{ij} < 0 \end{cases}$$

And

$$\forall r \in R'' \quad y_{rj} = y_{rj}^1 - y_{rj}^2$$
$$y_{rj}^1 = \begin{cases} y_{rj} \;, & y_{rj} \ge 0 \\ 0 \;, & y_{rj} < 0 \end{cases}$$
$$y_{rj}^2 = \begin{cases} 0 \;, & y_{rj} \ge 0 \\ -y_{rj} \;, & y_{rj} < 0 \end{cases}$$

The modified input efficiency scores in simultaneous presence of undesirable and negative factors can be computed by solving the following radial DEA-based programming, which is computable by standard algorithms and solver software.

$$\begin{aligned} & Min\ \theta \\ & s.t. \\ & \sum_{j=1}^{n} \lambda_j x_{ij}^g \le \theta x_{io}^g \quad i \in I^g \\ & \sum_{j=1}^{n} \lambda_j x_{ij}^b \ge x_{io}^b \quad i \in I^b \\ & \sum_{j=1}^{n} \lambda_j x_{ij}^1 \le \theta x_{io}^1 \quad i \in I'' \\ & \sum_{j=1}^{n} \lambda_j x_{ij}^2 \ge \theta x_{io}^2 \quad i \in I'' \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^g \ge y_{ro}^g \quad r \in R^g \end{aligned}$$

$$
\begin{aligned}
&\sum_{j=1}^{n} \lambda_j y_{rj}^{b} \le \theta y_{ro}^{b} \quad r \in R^{b} \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{1} \ge y_{ro}^{1} \quad r \in R^{m} \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{2} \le y_{ro}^{2} \quad r \in R^{n} \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0 \; j = 1,\ldots,n
\end{aligned} \tag{4}
$$

It is worth to note that the feasible space of model (4) is the subset of feasible space of model (1). As the model(4) presents, the abatement factor θ incorporates with desirable input, undesirable output and negative input variable. To improve the performance of under evaluation units, desirable inputs imposed by the constraint $\sum_{j=1}^{n} \lambda_j x^{g}_{ij} \le \theta x_{io}^{g}$, $i \in I^g$ have to be decreased and undesirable outputs $\sum_{j=1}^{n} \lambda_j y^{b}_{rj} \le \theta y^{b}_{ro}$, $r \in R^b$ have to be decreased. The model also yields a measure of efficiency for *DMUo*, which is the optimal value of θ.This measure reflects the radial contraction of the negative valued inputs. However, for each input that takes positive and negative values the model creates two variables, one for negative values and one for positive values. It is noteworthy that when *DMUo* receives a score of one in model(4) it is relatively efficient, otherwise it is inefficient.

Theorem1: Model (4) is always feasible and bounded.

Proof: refer to Emrouznejad et al. (2010).

Model (4) can be readily modified to assess *DMUo* in the output orientation. This is done in model (5) which yields an output augmentation semi-oriented radial measure (SORM) of efficiency, *whereh** is the optimal value in model (5). The model (5) has the following format:

$$
\begin{aligned}
&Max \; h \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^{g} \le x_{io}^{g} \quad i \in I^{g} \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^{b} \ge h x_{io}^{b} \quad i \in I^{b} \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^{1} \le x_{io}^{1} \quad i \in I^{n} \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^{2} \ge x_{io}^{2} \quad i \in I^{n} \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{g} \ge h y_{ro}^{g} \quad r \in R^{g} \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{b} \le y_{ro}^{b} \quad r \in R^{b} \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{1} \ge h y_{ro}^{1} \quad r \in R^{m} \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{2} \le h y_{ro}^{2} \quad r \in R^{n} \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0 \; j = 1,\ldots,n
\end{aligned} \tag{5}
$$

Theorem2: Model (5) is feasible and bounded.

Proof: It can easily be seen that ($h=1$, $\lambda_o=1$, $\lambda_j \neq 0$, $j \in J=\{1,\ldots,n\}$, $j \neq 0$ is a feasible solution in Model (5). Since, the feasible space of model (5) is the subset of feasible space of model (1), model (5) cannot yield an efficiency rating lower than yielded by model (1). So, the model (5) is bounded. The proof is complete.

The reasoning expounded in respect of model (4) can be readily transferred to model (5) to show that the feasible solutions to model (5) are a subset of those of model (1). Output augmentation h was employed on negative output variables to improve the performance of *DMUo*. Interestingly enough, as model (5) claims the desirable inputs are decreased and undesirable inputs have to be increased. It is proved that the model (5) is always feasible and bounded.

EMPRICAL EXAMPLE

We next illustrate the proposed methodology by applying it to the real-world data of 20 branches of a private bank in Iran. Input variables are personnel and administrative costs (desirable input x_1), long-term deposit (undesirable input x_2), and the profit and loss from the deposit (including both positive and negative quantity). Output variables are facilities (desirable output y_1) and deferred claim(undesirable output y_2).

Note that all variables have non- negative structure but the third input, which take positive values for some DMUs and negative for others. We have also applied to the data in Table 1 the input-oriented model (4) and the output-oriented model (5). Furthermore, GAMS software were used for computation. The obtained radial efficiency scores are presented in Table 2.

It is worth to mention that the input-oriented Model (4) achieves the efficiency score lower than one. In general, the efficiency scores of output-oriented Model (5) must be always larger than one. In all cases an efficiency score of 1 (100%) means that *DMUo* is efficient in the sense that at least one negative component of input cannot improve further. both models can estimate suitable improved targets for variables in presence of both undesirable and negative values. As Table 2 records nine out of twenty units are efficient in both estimations. the last unit (unit#20) has the minimum efficiency score in both models. The results of these models can be used for efficiency improvement and identifying the inefficiency sources.

CONCLUSION

The standard DEA model cannot be used for efficiency assessment of decision making units with negative data. Also, all variables assumed as desirable. The current paper modified a model to deal with undesirable factors and negative data simultaneously in radial DEA model. The modified radial model not only achieve an admissible efficiency score but also preserves the features of the original model. Finally, an application in bank branches used to show the usefulness of the model. Further research can be done to transform other DEA models, e.g. slack-based model, using similar concept.

Table1: Data Set for 20 DMUs

DMU	Personnel and administrative costs	Long-term deposit	Profit and loss from the deposit	Facilities	Deferred claim
1	6190	61918	-20	101870	8
2	6936	12222	5	55104	6
3	5556	20457	53	60709	10
4	5843	24520	20	60165	5
5	8951	6976	-5	72245	4
6	14403	63578	25	31614	5
7	7754	34218	-40	48722	8
8	7528	21440	73	80376	6
9	7999	12026	-62	30833	5
10	2839	7379	8	44741	3
11	4219	10168	-15	65552	6
12	5186	15279	-10	39927	4
13	7075	43649	23	43477	7
14	4873	15054	-30	77284	6
15	6495	17960	-26	43009	5
16	10274	57137	60	55867	8
17	9739	47739	50	107056	10
18	9248	47214	10	67709	6
19	5744	31623	20	41278	5
20	8128	28150	35	62499	9

Table2: Efficiency Scores

DMU	Efficiency Score Model (4)	Efficiency Score Model (5)
1	1	1
2	6768/0	5459/1
3	6796/0	5000/1
4	8881/0	1544/1
5	1	1
6	1	1
7	8050/0	1152/1
8	9169/0	0816/1
9	1	1
10	1	1
11	1	1
12	1	1
13	8543/0	2797/1
14	1	1
15	9156/0	2041/1
16	8216/0	0981/1
17	1	1
18	9666/0	0401/1
19	9547/0	0896/1
20	5349/0	6714/1

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