Optimization
Iranian Journal of Optimization
Volume 11, Issue 2, 2019, 207-215
Research Paper
Online version is available on: www.ijo.iaurasht.ac.ir
Islamic Azad University
Rasht Branch
ISSN: 2588-5723
E-ISSN:2008-5427

Optimal Design of Open Channel Sections Using PSO Algorithm

Mohsen Monadi [1*], Mirali Mohammadi[2] and Hamed Taghizadeh[1]

[1] Department of Civil Engineering (Hydraulic Structures Engineering), Faculty of Engineering, Urmia University, Urmia, Iran
[2] Department of Civil Engineering, Faculty of Engineering, Hydraulic Structure and River Mechanics Group, Urmia University, Urmia, Iran

Received: 08 October 2018
Accepted: 24 June 2019

Abstract

This paper applies an evolutionary algorithm, the particle swarm optimization (PSO), to design the optimum open channel crass section. Depth, channel side slope and bottom width are considered as the variables for rectangular, triangular and trapezoidal channels, respectively. The objective function is minimizing the construction cost of the channel section. MATLAB software is used for programming and doing the optimization process. Manning's uniform flow formula has been used as a constraint for the optimization model. The cost function is included the cost of earthwork, the increment in the cost of earthwork with the depth below the ground surface and the cost of lining. Simple functions of unit cost terms have been used to express the optimum values of section variables. The optimum section variables are obtained for the case of minimum area or minimum wetted perimeter problems. The results of this study showed that the PSO is a robust algorithm to compute the optimum section variables in open channel design.

Keywords:
Optimization
Open Channel Design
PSO algorithm
Optimum Section Variables
MATLAB Software

* **Correspondence E_mail**: mohsen.monadi@gmail.com

INTRODUCTION

Conveyance of water from one place to another place for agricultural, municipal and power needs is a problem that human beings were encountered through the history. Although the techniques and materials used in the construction of conveyance lines have changed, open channels still keep their attractiveness in transportation of water. Todays, the most greatly used channel sections are rectangular, triangular, trapezoidal sections. These channels, in which the state of flows are called as open channel flows, are designed according to the laws of open channel flow.

Using these laws and combining them with the objective functions and constraints of the project, the section variables i.e., depth, side slope, bottom width of channels can be calculated. The section variables of the channel will vary according to the objective function and the constraints of every project to provide the channel section for conveying the required amount of water.

The objective of a project may be to convey the given flow rate with the least flow area or to convey the given discharge resulting in the minimum cost of construction. On the other side, the constraints of a project may be on the values of average flow velocity, top width, depth and also on the value of side slope. There are various studies on the values of section variables for different channel geometries considering the above objectives and constraints. Some of these are summarized below.

Chow, (1959) produced several characteristics of optimal sections. He expressed the relations between the section variables of the most hydraulically efficient sections for various channel types.

The relations obtained for the optimum section variables were adjusted by taking into account different parameters, including the effect of freeboard, Guo and Hughes, (1984) made a study on the optimum values of section variables which either minimizes frictional resistance or minimizes construction cost. They demonstrated their solutions for the trapezoidal channel sections.

Loganatha, (1991) gave three optimal conditions for the parabolic-channel design accounting for freeboard and limitations on velocity and channel dimensions.

Monadjemi, (1994) has carried out a detailed study on the relationships extracted for several channel types. He demonstrated that, considering the minimization of flow area as an objective will result in the same optimum values of section variables as of considering the minimization of wetted perimeter.

In addition to the relations between optimum section variables, Froehlich, (1994) suggested simple expressions for the optimum section variables of trapezoidal channel sections in terms of Q(discharge), n(Manning's roughness coefficient) and So(bed slope). He expanded his study for width and depth constrained trapezoidal channel sections and presented a graphical solution for the optimum values of section variables considering the width and depth constraints on the channel geometry.

Swamee, (1995) presented explicit equations of section variables for the minimum flow area considering resistance equation of uniform flow as a function of roughness height of channel bottom and kinematic viscosity of water.

Swamee et al. (2000) proposed explicit equations for section variables considering triangular, rectangular, trapezoidal and circular channel geometries. They considered the minimization of channel construction cost as the objective of the project.

Bhattacharjya and Satish, (2012) developed a new methodology to design a stable and optimal trapezoidal channel section using hybrid optimization techniques.

Ruben and Kamran, (2007) presented in detail the background and implementation of a particle swarm optimization algorithm suitable for constraint structural optimization tasks.

Turan and Yurdusev, (2011) obtained the optimized cross sections of different canal geometries using differential evolution algorithm and the findings of these exercises are compared with those of given in related literature.

Easa et al. (2011) determined an economical channel with trapezoidal cross-section considering the criterion for the side slope stability (soil conditions).

Bhattacharjya, (2007) presented a nonlinear optimization model for designing optimal channel section considering freeboard as an additional decision variable. And he solved the developed optimization model by sequential quadratic programing using MATLAB.

Kaveh et al. (2011) developed four different models for composite open channel and applied a new optimization algorithm called Charged System Search (CSS) to determine the optimal design of these models.

Kentli and Mercan, (2014) applied Genetic algorithm and sequential quadratic programming techniques to optimized Triangular, rectangular and trapezoidal cross-sections.

Tofiq and Guven, (2015) analyzed the effects of the aspect ratio (width to depth ratio, b/d) on the cost of a trapezoidal concrete-lined channel (either power channel or irrigation channel). They recommended b/d ratios and their limitations are discussed.

Niazkar and Afzali, (2015) utilized a novel optimization technique, invariably called the Modified Honey Bee Mating Optimization (MHBMO) algorithm, to optimized lined channel sections.

Liu Dong et al. (2016) designed the channel cross section with low water loss in irrigation areas using an improved cat swarm optimization (ICSO).

Kentli and Mercan, (2014) applied two different algorithms to optimal design of canal sections and results are compared with the one in literature. Genetic algorithm and sequential quadratic programming technique are used in the optimization. In this research Triangular, rectangular and trapezoidal cross-sections are optimized. It is seen that both algorithms are giving more accurate results than in literature.

Zhang et al. (2011) proposed a hybrid improved particle swarm optimization (IPSO) algorithm for the optimization of hydroelectric power scheduling in multi-reservoir systems.

Zhang et al. (2015) presented a comprehensive investigation of PSO. On one hand, we provided advances with PSO, including its modifications (including quantum-behaved PSO, bare-bones PSO, chaotic PSO, and fuzzy PSO), population topology (as fully connected, von Neumann, ring, star, random, etc.), hybridization (with genetic algorithm, simulated annealing, Tabu search, artificial immune system, ant colony algorithm, artificial bee colony, differential evolution, harmonic search, and biogeography-based optimization), extensions (to multiobjective, constrained, discrete, and binary optimization), theoretical analysis (parameter selection and tuning, and convergence analysis), and parallel implementation (in multicore, multiprocessor, GPU, and cloud computing forms).

In this study, optimum values of section variables for triangular, rectangular, trapezoidal channel sections are expressed in terms of known unit cost terms considering the minimization of channel construction cost as the objective and Manning's uniform flow equation as the constraint of the study. We used non-dimensional parameters so the model can be used for all sections with any shape. The novelty of this study is considering the cost per unit excavation per unit depth that in pervious researches did not consider in calculations. And also we used a robust optimization algorithm and by applying some improvements it can propose good results. So this optimization algorithm can be used for calculating the optimal open channel section considering all cost items of construction.

THEORY

Selection of the section variables such as channel side slope, bottom width and flow depth for open channel sections have been varied according to the objective of the designer. For different objectives, it is possible to have different values and relations between section variables.

Uniform flow

Uniform flow in an open channel can be achieved when a balance between the resisting forces and gravity forces acting on the body of water has been reached, Chow, (1959) The most widely used uniform flow formula in Iran is the Manning's formula and in international system of units (SI) it is given as;

$$Q=1/n\ AR^{2/3}\sqrt{S} \qquad (1)$$

where, Q= discharge in m^3/s, n= Manning's roughness coefficient, A= flow area in m^2, R= hydraulic reduce in m and S= Slope of the channel bottom. As it mentioned before Manning uniform flow formula is used to define the uniform flow and normal depth concept.

Cost structure

In this paper, minimization the cost of an open channel section, which is one of the most important goals in open channel construction, took into account as the objective function and some explicit relations have been derived for the section variables of rectangular, triangular and trapezoidal channel sections. Mostly, the cost per unit length (C) of a lined open channel section is defined as the addition of two terms; the cost of earthwork (C_e) and the cost of lining (C_l) in [Rial]/[L].

$$C = C_e + C_l \tag{2}$$

The cost of the earthwork per unit length, Ce, of a channel is the multiplication of unit cost of earthwork, αe ([Rial]/[L]3), by the related area of excavation or fill, A ([L]2).

$$C_e = \alpha_e A \tag{3}$$

Lining cost per unit length, Cl, of a channel can be computed by multiplying the unit cost of lining, αl ([Rial]/[L]2), which varies according to the type of material used, by the wetted perimeter, P ([L]), of the section.

$$C_l = \alpha_l P \tag{4}$$

By using Eq.1and Eq.4 we can rewrite Eq.2 as follow.

$$C = \alpha_e A + \alpha_l P \tag{5}$$

As it is clear for the same soil conditions, the cost of earthwork will increase with the increasing depth of excavation. So in this study in order to express the effect of this condition, an additional cost term ($\alpha_A ddA$, see Fig.1) for every unit area below the ground surface is added into the earthwork cost function Eq.3.

$$C_e = \alpha_e A + \alpha_A \int_0^{y_n} (y_n - y)\,dA \tag{6}$$

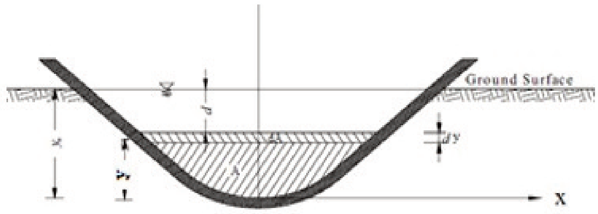

Fig. 1. Typical Channel Section

where, α_A = Additional cost of earthwork per unit volume of excavation for unit depth [Rial]/[L]^4

dA = Unit area of earthwork at height y or depth d [L]^2, d = Depth of area below the ground surface [L], A = The flow area at height y [L]^2, y and x represents the vertical and horizontal axes of channel geometry, respectively.

Keeping the lining cost unchanged, the total cost function of the channel section can be rewritten as follow.

$$C = \alpha_e A + \alpha_A \int_0^{y_n} A\,dA + \alpha_l P \tag{7}$$

It is noted that by applying some changes in the governing equations the algorithm can be used for all open channel sections such as circular, parabolic and etc. And also the cost functions can be calculated for all types of soil and lining material.

Optimization algorithm

An optimization model mostly consists of an objective function and different constraint functions that control the value of the objective function. In this study particle swarm optimization (PSO) algorithm has been used to optimize the open channel sections. Swarm Intelligence (SI) is mainly defined as the behavior of natural or artificial self-organized of decentralized systems. Swarms interact locally with each other or with external agents, i.e. environment, and can be in the form of bird flocks, ants, bees etc. introduced by Srinivas and Deb, (1994) for optimizing continuous nonlinear functions. Particle Swarm Optimization (PSO) defined a new era in SI. PSO is a population based method for optimization. The population of the potential solution is called as swarm and each individual in the swarm is defined as particle. The particles in the swarm search their best solution based on own experience and the other particles of the same swarm. The position x of a particle i at iteration k + 1 is updated as shown in Eq. 8.

$$x_{k+1}^{i} = x_{k}^{i} + v_{k+1}^{i}\,\Delta t \tag{8}$$

Where x_k^i is the current position of a particle in iteration k, x_{k+1}^i is the updated position of a particle in iteration k+1, v_{k+1}^i is the corresponding updated velocity vector, and Δt is the time step value proposed by Shi and Eberhart (1998). Throughout the present work a unit time step is used. The velocity vector of each particle is calculated as shown in Eq. 9.

$$v_{k+1}^{i} = wv_{k}^{i} + c_1 r_1 \frac{(p_k^i - x_k^i)}{\Delta t} + c_2 r_2 \frac{(p_k^g - x_k^i)}{\Delta t} \tag{9}$$

where w is the inertia weight coefficient, v_k^i is the velocity vector at iteration k, r1 and r2 represent random numbers between 0 and 1, c1 and c2 represent personal and global learning coefficient respectively, p_k^i represents the best ever particle position of particle i, and p_k^g corresponds to the global best position in the swarm up to iteration k. All of the adjustment parameters in Eq. 8 and Eq. 9 are defined according to the previous studies and try and error as listed in Table 2. The framework of PSO is shown in Fig. 2.

The objective function of this study is taken as the minimization of cost and the uniform flow equation is treated as a constraint and put into the optimization model as below.

Objective Function:

$$\text{Minimize } C = \alpha_e A + \alpha_A \int_0^{y_n} A\,dA + \alpha_l P \tag{10}$$

$$\text{Subjected to: } Q - \frac{1}{n} A R^{\frac{2}{3}} \sqrt{S} = 0 \tag{11}$$

The all terms of this model are in dimensional forms and the user confront with some difficulties when he/she wants apply it for different channel sections. So that in this study the above equations are put into account in non-dimensional forms. This conversion is done by defining a length scale, γ as follows.

$$\gamma = \left(\frac{Qn}{\sqrt{S}}\right)^{\frac{3}{8}} \tag{12}$$

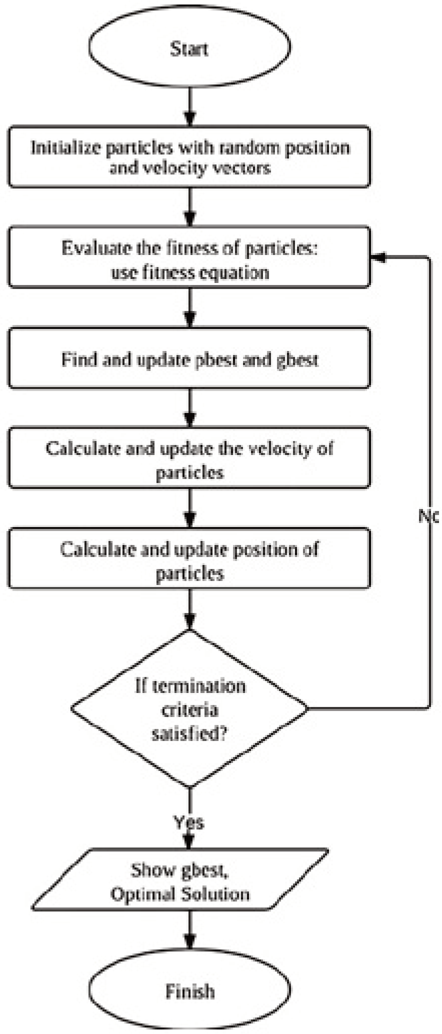

Fig. 2. The framework of particle swarm optimization algorithm.

The non-dimensional forms of the used parameters

The non-dimensional forms of total cost, unit cost of lining, additional unit cost of earthwork, flow area, wetted perimeter, bottom width of the channel, normal depth of the flow are listed below, respectively.

$$C_* = \frac{C}{\alpha_e \gamma^2} \tag{13}$$

$$\alpha_{l*} = \frac{\alpha_l}{\alpha_e \gamma} \quad (14)$$

$$\alpha_{A*} = \frac{\alpha_A \gamma}{\alpha_e} \quad (15)$$

$$A_* = \frac{A}{\gamma^2} \quad (16)$$

$$P_* = \frac{P}{\gamma} \quad (17)$$

$$b_* = \frac{b}{\gamma} \quad (18)$$

$$y_{n*} = \frac{y_n}{\gamma} \quad (19)$$

The non-dimensional objective functions and constraints

The final non-dimensional equations for the optimization model of the sections can be written as follow.

For Rectangular Section:

$$\text{Minimize } C_* = b_* y_{n*} + \frac{\alpha_{A*} b_* y_{n*}^2}{2} \cdot + \alpha_{l*}(2y_{n*} + b_*) \quad (20)$$

$$\text{Subjected to: } 1 - \frac{(b_* y_{n*})^{\frac{5}{3}}}{(2y_{n*} + b_*)^{\frac{2}{3}}} = 0 \quad (21)$$

For Triangular Section:

$$\text{Minimize } C_* = m{y_{n*}}^2 + \frac{\alpha_{A*} m y_{n*}^3}{3} + 2\alpha_{l*} y_{n*}\sqrt{1+m^2} \quad (22)$$

$$\text{Subjected to: } 1 - \frac{(m{y_{n*}}^2)^{\frac{5}{3}}}{(2y_{n*}\sqrt{1+m^2})^{\frac{2}{3}}} = 0 \quad (23)$$

For Trapezoidal Section:

$$\text{Minimize } C_* = (b_* y_{n*} + m{y_{n*}}^2) + \alpha_{A*}\left(\frac{b_* y_{n*}^2}{2} + \frac{m y_{n*}^3}{3}\right) + \alpha_{l*}(b_* + 2y_{n*}\sqrt{1+m^2}) \quad (24)$$

$$\text{Subjected to: } 1 - \frac{(b_* y_{n*} + m{y_{n*}}^2)^{\frac{5}{3}}}{(b_* + 2y_{n*}\sqrt{1+m^2})^{\frac{2}{3}}} = 0 \quad (25)$$

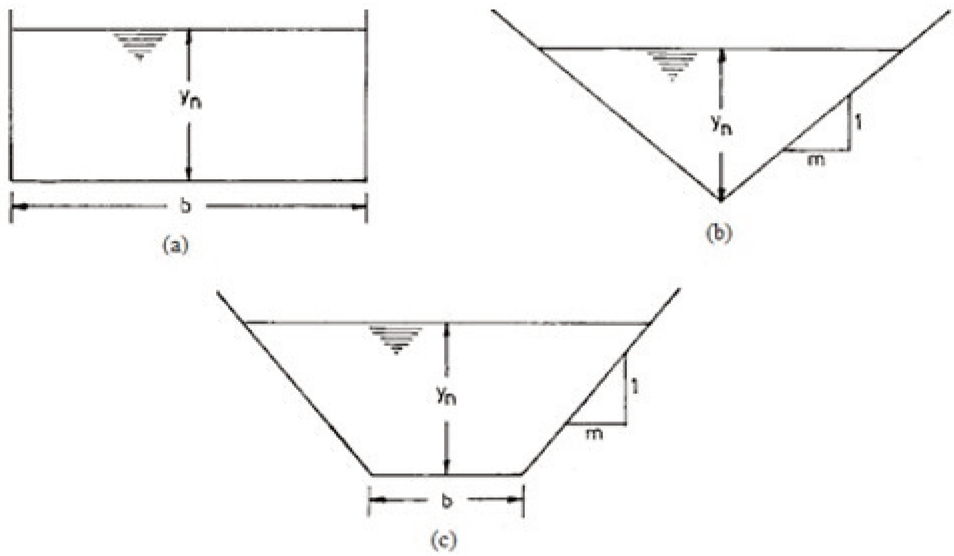

Fig. 3. Channel sections: (a) rectangular section, (b) triangular section, (c) trapezoidal section. (Note: m is the side slope of channel)

RESULT AND DISCUSSION

As it is mentioned before, in this study particle swarm optimization (PSO) algorithm has been used to optimize the open channel sections. MATLAB software has been used for programming and all parameters are expressed in non-dimensional form using dimensional analysis. A practical concrete channel is used to demonstrate the application of proposed optimization algorithm. All design variables are listed in Table 1.

Table 1: The design variables of the proposed example

Design Variables	Value
Flow Rate (m^3/s)	5
Longitudinal Slope (%)	0.01
Manning's roughnesscoefficient	0.012
α_e (Rial/m^3)	29100
α_l (Rial/m^2)	940600
α_A (Rial/m^4)	10800
γ	1.95

Inertia weight coefficient

We used a dynamic update for inertia weight coefficient as described by Eq. 26 with an inertia weight damping ration w_{damp} = 0.99 and inertia weight w = 1.

$$w_{k+1} = w_{damp}\, w_k \tag{26}$$

where w_k and w_{k+1} are the inertia weight at iteration k and k+1, respectively.

Personal and global learning coefficients

The coefficients were varied according to the formula $c_2 = 4 - c_1$, where c1 was varied in the interval [4, 0]. It can be seen clearly that when only personal ($c_1 = 4$, $c_2 = 0$) or only global values ($c_1 = 0$, $c_2 = 4$) are used the resulting history converges very fast. But when the results do not improve after the initial convergence indicating that the algorithm reaches a local suboptimal and it is not able to escape from it, due to the lack of information exchange either from global or personal sources respectively. When higher emphasis is placed on the social exchange of information ($c_1 = 1$, $c_2 = 3$) the algorithm is able to gradually converge to better regions of the design space, but again if a local optimum is found it cannot escape from it. When the personal and global coefficients are equal ($c_1 = 2$, $c_2 = 2$ that it results $w = 1$) solutions converge near the global optimum solution.

The number of individual and iteration

After 20 trials the number of individuals and iterations has been considered equal to 30 and 500, respectively. It observed that with the number of individual and iteration more than 30 and 500, respectively, the results are not improve and with the numbers less than these amounts the results are not accurate enough and the proposed algorithm do not work well.

In this study, using the results of the previous studies and after 20 trials the adjustment parameters of the optimization algorithm are recommended as Table 2.

It is note that the number of optimal variables for each channel section is different and they are equal to 2, 1 and 3 for rectangular, triangular and trapezoidal channel sections, respectively. To validate the proposed optimization algorithm the problem defined by Swamee et al. (2000) has been used and the results show that the proposed algorithm is so useful and appropriate for the problem. The optimal channel sections have been calculated using the proposed optimization algorithm for each channel section and the results are listed in Table 3.

Table 2: The adjustment parameters of the proposed optimization algorithm

Adjustment Parameter	Value
ω	1
w_{damp}	0.99
Number of individual	30
Maximum Iteration	500
Δt	1
r_1	Random number between 0 and 1
r_2	Random number between 0 and 1
c_1	2
c_2	2
Lower bound	0
Upper bound	10
Maximum Velocity	0.1(upper bound – lower bound)
Minimum Velocity	-Maximum Velocity

Table 3: The obtained optimal channel section variables.

	Optimal Parameter	Value
Rectangular	*Normal Depth (m)*	1.78
	Width (m)	3.60
Triangular	*Normal Depth (m)*	2.53
	Side Slope	1V:1H
Trapezoidal	*Normal Depth (m)*	2.10
	Bottom Width (m)	1.63
	Side Slope	1V:0.65H

CONCLUSIONS

The optimal open channel cross section design has been revisited using PSO algorithm. In this study, three different channel shapes, including rectangular, triangular and trapezoidal, are considered. The objective function is minimizing the cost of channel construction considering depth, side slope and bottom width as the variables. Manning's uniform flow equation is considered as the constraint of the problem. The cost function is arranged to include the cost of lining, cost of earthwork and the increment in the cost of earthwork with the depth below the ground surface. The best results for the problem have been obtained using equal selection of the personal and global pressure parameters ($c_1 = c_2 = 2$), and a dynamic inertia weight update. The obtained results, from the three practical cases using an improved PSO algorithm, are illustrated the robustness of the algorithm to find the optimal channel section. In this study, the latest channel cost record was collected and analyzed to provide the cost function directly related to the channel cross-sectional geometry. Unique values of optimum section variables are obtained for the case of minimum area or minimum wetted perimeter problems as listed in Table 3. Also in addition to the traditional channel cost functions which consist of the cost of earthwork and the cost of lining, an additional cost per unit excavation per unit depth is also considered in the channel cost function. According to the results the proposed optimization algorithm is robust and appropriate for optimizing construction cost of open channel sections and can be used for the wide range of practical optimization channel section design with each shape and material.

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