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Ranking Efficient DMUs Using the Infinity Norm and Virtual Inefficient DMU in DEA

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Abstract

In many applications, ranking of decision making units (DMUs) is a problematic technical task procedure to decision makers in data envelopment analysis (DEA), especially when there are extremely efficient DMUs. In such cases, many DEA models may usually get the same efficiency score for different DMUs. Hence, there is a growing interest in ranking techniques yet. The purpose of this paper is ranking extreme efficient DMUs in DEA based on exploiting the leave-one out and minimizing the maximum distance between DMU under evaluation and boundary efficient in input and output directions. The proposed method has been able to overcome the lacks of infeasibility and unboundedness in some DEA ranking methods.

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INTRODUCTION

In many cases ranking Decision Making Units (DMU) is important and essential process for the decision maker in DEA. The basic models of data envelopment analysis usually offer the same performance scores for some DMUs, and it occurs when the number of DMUs are less than the total number of input and output variables. In data envelopment analysis there are various methods for ranking DMUs, which some of the methods include Anderson and Peterson's method (AP method), Mehrabian, Alirezaei and Jahanshahloo's method (MAJ method), ℓ_1- norm and ℓ_∞-norm method. In order to rank every DMU, Ap method removes that DMU from the production possibility set, in that case efficiency boundary will change with the elimination of DMU, and then the maximum input reduction of DMU under evaluation is obtained according to efficiency boundary. Also the model related to AP method is a super efficiency model, but super efficiency models sometimes are infeasible. Mehrabian et al. (1999) proposed MAJ model to complete ranking of DMUs. The MAJ model was presented to solve the infeasibility problem of AP method but this method is infeasible in some cases. In order to overcome the defects of AP and MAJ, Jahanshahloo et al. (2004) proposed a method to rank efficient DMUs which their model was based on ℓ_1-norm. Wu and Yan (2010) using an effective transform changed the ℓ_1-norm into a linear model, which provides accurate optimized for every efficient DMU. Also Jahanshahloo et al. (2004) using gradient line ranked extreme efficient units. Rezaei Balf et al. (2012) using Chebyshev norm presented a model to rank efficient DMUs. Amirteymori et al. (2005) introduced a method based on distance to rank efficient DMUs. Hashimato (1997) presented a super efficiency model along with certain area in order to complete ranking of DMUs. Torgersen et al. (1996) proposed a method to rank efficient DMUs, which every efficient unit according to its importance was used as a pattern to inefficient DMU.Sexton et al. (1986) studied a method based on the cross-efficiency matrix to rank DMUs. Ranking method of cross-efficiency determines efficiency score of every DMU using a set of optimal weight, which these weights are obtained based on solving problem of DMU corresponding planning. Liu and Peng (2008) by determining Common Set of Weights ranked efficient DMUs. Bal et al. (2008) suggested a model which ranks DMUs based on definition of dispersion of input and output weights. Jahanshahloo and Firoozi Shahmirzadi (2013) modified the model which was proposed by Bal et al (2008). Khodabakhshi and Ariavash (2012) offered a method to rank DMUs, which according to that first, the minimum and maximum efficiency values of each DMU are computed under the assumption that the sum of efficiency values of all DMUs equals to unity. Then, the rank of each DMU is determined in proportion to convex combination of its minimum and maximum efficiency values. In this paper, we suggest a new method for ranking extreme efficient DMUs. Ziari and Raissi (2016) using minimizing distance ranked the efficient DMUs. Early, Ziari and Ziari (2016) proposed an approach for ranking efficient DMUs based on coeffitient of variation of input-output weights. The rest of the paper is organized as follows. In Section 2 , we review the concept of DEA framework. We review the some ranking methods in Section 3, Section 4 proposes the new model for ranking efficient units. Section 5 includes Some numerical examples. The last Section concludes the study.

DEA MODELS AND RAN KING MODELS REVIEW

DEA models review

DEA is a methodology for assessing the relative efficiency of decision making units (DMUs) where each DMU has multiple inputs used to secure multiple outputs.

It is assumed that in DEA there are DMUs and for each DMU_j (j=1,..., n) is considered a column vector of inputs $(X_{1j}, X_{2j},\ldots, X_{mj})^T$ in order to produce a column vector of outputs $(Y_{1j}, Y_{2j},\ldots, Y_{sj})^T$. Here, the superscript (T) indicates a vector transpose. The production possibility set with constant returns to scale T_c is defined as:

$$T_c=\left\{(x_1,x_2,\ldots,x_m,y_1,y_2,\ldots,y_s)\,|\,x_i\ge\sum_{j=1}^{n}\lambda_j x_{ij},\right.$$

$$\left.i=1,2,\ldots,m,\, y_r\le\sum_{j=1}^{n}\lambda_j y_{rj},\, r=1,2,\ldots,s\right\}$$

According to the above definition, the following input-oriented CCR model (see Charnes et al., se1989) in the envelopment form with constant Returns to Scale measures the level of DEA efficiency (θ) of the th DMU:

$$\begin{aligned} \theta^* &= \min \theta \\ s.t. \ & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{ik}, \quad i=1,..,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk}, \quad r=1,..,s \\ & \lambda_j \geq 0, \quad j=1,..,n \end{aligned} \tag{1}$$

Here, $\lambda=(\lambda_1,\dots \lambda_n)^T$ is a column vector of unknown variables used for components of the input and output vectors by a combination. θ^* represents the efficiency score of DMU_k in (1), where the superscript (*) indicates optimality.

DMU_k is relative efficient if and only if on optimality, the objective of (1) equals to one and all the slacks are zero.Similarly, the output-oriented CCR model, corresponding to (1), is formulated as follows:

$$\begin{aligned} \phi^* &= \max \phi \\ s.t. \ & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik}, \quad i=1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \phi y_{rk}, \quad r=1,\dots,s \\ & \lambda_j \geq 0, \quad j=1,\dots,n \end{aligned} \tag{2}$$

Here, $1/\phi^*$ intends the DEA efficiency score in the output-oriented model. Also, the following input-oriented BCC model (see Banker et al., 1984) in the envelopment form with variable Returns to Scale measures the level of DEA efficiency (θ) of the k th DMU (X_k, y_k):

$$\begin{aligned} \theta^* &= \min \theta \\ s.t. \ & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{ik}, \quad i=1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk}, \quad r=1,\dots,s \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, \quad j=1,\dots,n \end{aligned} \tag{3}$$

DMU_k is relative efficient if and only if on optimality, the objective of (3) equals to one and all the slacks are zero.

$$\sum_{j=1}^{n} \lambda_j = 1$$

Similarly, the output-oriented BCC model, corresponding to (3) which obtains from (2) by adding constraint,

Moreover, the following additive model is based on input and output slacks which accounts the possible input decreases as well as output increases simultaneously.

$$\begin{aligned} \max \ & \sum_{j=1}^{n} s_i^- + \sum_{j=1}^{n} s_r^+ \\ s.t. \ & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- \leq x_{ik}, \quad i=1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ \geq \phi y_{rk}, \quad r=1,\dots,s \\ & \lambda_j, s_i^-, s_r^+, \geq 0, \end{aligned} \tag{4}$$

which λj, j=1, 2,…, n are wheights of DMUs, s_i^-, i=1,2,...m and s_r^+, r=1,2,...s are slacks or surplus variables. DMU is relative efficient if and only if on optimality, the objective of (4) equals to zero.

Review of some ranking models

In this subsection some ranking models are reviewed in data envelopment analysis. The first ranking model proposed by Anderson and Peterson (1993) which is the supper efficiency model. In the AP model DMU under evaluation is excluded from reference set and by using other units, the rank of given DMU is obtained.

The AP model using the CRS super-efficiency model is as follows:

$$\begin{aligned} AP: \quad & \min \ \theta \\ s.t. \ & \sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} \leq \theta x_{ik}, \quad i=1,\dots,m \\ & \sum_{j=1, j\neq k}^{n} \lambda_j y_{rj} \geq y_{rk}, \quad r=1,\dots,s \\ & \lambda_j \geq 0, \quad j=1,\dots,n, \ j\neq k \end{aligned} \tag{5}$$

The main drawbacks of this model are infeasibility and instability for some DMUs. It is said that a model is stable if a DMU under evaluation is efficient, it is remains efficient after perturbation on data. The second ranking model under investigation proposed by Mehrabian et al. (1999) in order to solve infeasibility of AP models in some cases. The following model is MAJ model:

$$
\begin{aligned}
MAJ: \quad & min\ 1+w \\
& s.t. \sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} \leq x_{ik}+w, & i=1,\dots,m \\
& \sum_{j=1, j\neq k}^{n} \lambda_j y_{rj} \geq y_{rk}, & r=1,\dots,s \\
& \lambda_j \geq 0, & j=1,\dots,n,\ j\neq k
\end{aligned} \tag{6}
$$

The third ranking model proposed by Jahanshahloo et al. (2004). Their proposed method to rank the extremely efficient DMUs in DEA models with constant and variable Returns to Scale by using the omitted DMU under evaluation from production possibility set and applying ℓ_1-norm. It is shown that the proposed method is able to overcome the existing difficulties in The AP (1993) and MAJ (1999) models. On the other hand, the proposed model is the form of nonlinear programming which is difficult to be solved. The model of Jahanshahloo et al. (2004) is presented as follows:

$$
\begin{aligned}
\ell_1-norm: \quad & min \sum_{i=1}^{m} |x_i - x_{ik}| + \sum_{r=1}^{s} |y_r - y_{rk}| \\
& s.t. \sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} \leq x_i, & i=1,\dots,m \\
& \sum_{j=1, j\neq k}^{n} \lambda_j y_{rj} \geq y_r, & r=1,\dots,s \\
& x_i \geq 0,\ y_r \geq 0 & i=1,\dots,m,\ r=1,\dots,s \\
& \lambda_j \geq 0, & j=1,\dots,n,\ j\neq k
\end{aligned} \tag{7}
$$

The fourth ranking model proposed by Rezai Balf et al. (2012) which applies for ranking extreme efficient units using the leave-one-out idea and ℓ_∞-norm. The proposed model is always feasible and so, it is able to remove the existing difficulties in some methods, such as Andersen and Petersen (1993). The model of Rezai Balf et al. (2012) is formulated as follows:

$$
\begin{aligned}
\ell_\infty-norm: \quad & min\ v_k \\
& s.t.\ v_k \geq \sum_{j=1 j\neq k}^{n} \lambda_j x_{ij} - x_{ik}, & i=1,\dots,m \\
& v_k \geq y_{rk} - \sum_{j=1 j\neq k}^{n} \lambda_j y_{rj}, & r=1,\dots,s \\
& \lambda_j \geq 0, & j=1,\dots,n, j\neq k \\
& v_k \geq 0
\end{aligned} \tag{8}
$$

The fifth ranking model presented by M. Ziari and S. Ziari (2016) which uses for ranking extreme efficient units based on leave-one-out idea and minimizing distance between DMU under evaluation and transformed efficiency boundary. The proposed linear model is always feasible and so, it is able to remove the existing difficulties in some methods, such as Andersen and Petersen (1993) and nonlinear ℓ_1-norm model. This model is formulated as follows:

$$
\begin{aligned}
& min \sum_{i=1}^{m} \alpha_i + \sum_{r=1}^{s} \beta_r \\
& st. \sum_{j=1 j\neq k}^{n} \lambda_j x_{ij} \leq x_{ik} - \alpha_i, & i=1,\dots,m \\
& \sum_{j=1 j\neq k}^{n} \lambda_j y_{rj} \geq y_{rk} + \beta_r, & r=1,\dots,s \\
& \lambda_j \geq 0, & j=1,\dots,n, j\neq k, \\
& \alpha_i \geq 0,\ \beta_r \geq 0 & i=1,\dots,m,\ r=1,\dots,s,
\end{aligned} \tag{9}
$$

which $\alpha=(\alpha_1,\dots,\alpha_m)$, $\beta=(\beta_1,\dots,\beta_s)$ and $\lambda=(\lambda_1,\dots \lambda_{k-1},\dots,\lambda_n)$, are the variables of the model (9).

The proposed ranking model for efficient DMUs

In this section, by considering the CCR production possibility set T_c and by assuming the DMU_k be extremely efficient, the production possibility set T'_c is obtained by removing (X_k, Y_k) from T_c:

$$
T'_c = \left\{ (x_1, x_2, \dots, x_m, y_1, y_2, \dots, y_s) \mid x_i \geq \sum_{j=1 j\neq k} \lambda_j x_{ij},\ i=1,2,\dots,m, y_r \leq \sum_{j=1 j\neq k}^{n} \lambda_j y_{rj}, r=1,2,\dots,s \right\}
$$

In order to attain DMU_k ranking the following model is suggested according to model (10). This model is based on elimination idea of DMU_k from reference set and minimizing the maximum distance between DMU under evaluation in input and output directions and boundary of T'_c. The proposed model is as follows:

$$
\begin{aligned}
& \min\ z = \max\{\delta_1, \delta_2\} \\
& s.t. \sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} \le x_{ik} + \delta_1, \qquad i = 1,\dots,m \\
& \sum_{j=1, j\neq k}^{n} \lambda_j y_{rj} \ge y_{rk} - \delta_2, \qquad r = 1,\dots,s \\
& \lambda_j \ge 0, \qquad j = 1,\dots,n, j \neq k, \\
& \delta_1, \delta_2 \ge 0,
\end{aligned} \tag{10}
$$

where in above model $\lambda=(\lambda_1,\dots, \lambda_{k-1}, \lambda_{k+1},\dots, \lambda_n)$, δ_1 and δ_2 are variables of model.

Remark. In the model (9), if $\delta_1 = \delta_2$ then we obtain the ℓ_∞-norm model.

It can easily be converted the nonlinear above model into the following linear form:

$$
\begin{aligned}
& \min\ z = \delta \\
& s.t. \sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} \le x_{ik} + \delta_1, \qquad i = 1,\dots,m \\
& \sum_{j=1, j\neq k}^{n} \lambda_j y_{rj} \ge y_{rk} - \delta_2, \qquad r = 1,\dots,s \\
& \delta_1 \le \delta, \\
& \delta_2 \le \delta, \\
& \lambda_j \ge 0, \qquad j = 1,\dots,n, j \neq k, \\
& \delta, \delta_1, \delta_2 \ge 0,
\end{aligned} \tag{11}
$$

where in above model $\lambda=(\lambda_1,\dots, \lambda_{k-1}, \lambda_{k+1},\dots, \lambda_n)$, δ_1, δ_2, and δ are variables of model.

Notice that in order to gain rank of every efficient decision making unit like DMU_k use the model (11).

Theorem 1. The model (11) is feasible and bounded.

Proof. Let $\lambda_t=1$ for $t\neq k$ and $\lambda_j=0$ for $j=1,\dots, n, j\neq k,t$.

Also we put $\delta_1=max\ \{xit-xik,\ i=1,2,\dots,m\}$, $\delta_2=min\ \{yik-yit,\ r=1,2,\dots,s\}$ and $\delta=max2\{\delta_1, \delta_2\}$.

Obviously, it can be seen that $(\lambda, \delta_1, \delta_2, \delta)$ according to above selection is a feasible solution of the model (11). Moreover, the objective function of model (11) is bounded below zero, because the variables of model are non-negative Ω.

EXTENSION TO VARIABLE RETURNS TO SCALE

In this section, the proposed model (model (10)) is extended to variable Returns to Scale model. For this purpose, the model (10) is reformulated by adjoining the following convexity constraint to the model:

$$
\sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j = 1,\ \lambda_j \ge 0
$$

So, in order to get the ranking score under variable returns to Scale assumption is solved the following model:

$$
\begin{aligned}
& \min\ z = \delta \\
& s.t. \sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} \le x_{ik} + \delta_1, \qquad i = 1,\dots,m \\
& \sum_{j=1, j\neq k}^{n} \lambda_j y_{rj} \ge y_{rk} - \delta_2, \qquad r = 1,\dots,s \\
& \delta_1 \le \delta, \\
& \delta_2 \le \delta, \\
& \sum_{j=1, j\neq k}^{n} \lambda_j = 1, \\
& \lambda_j \ge 0, \qquad j = 1,\dots,n, j \neq k, \\
& \delta, \delta_1, \delta_2 \ge 0,
\end{aligned} \tag{12}
$$

where in above model, $\lambda=(\lambda_1,\dots, \lambda_{k-1}, \lambda_{k+1},\dots, \lambda_n)$, δ_1, δ_2, and δ are variables of model.

Theorem 2. The model (12) is feasible and bounded.

Proof. The proof of this theorem is similar to the proof of Theorem 1.

Example 2 show that the application of model (12).

ILLUSTRATED EXAMPLE

In this section, we employ the above DEA models (11) and (12) on the two data sets which they are introduced here, with the assumption of Constant and Variable Returns to Scale.

Example 1. As it can be seen from the Table 1, the data set consists of 19 DMUs with 2 inputs and 2 outputs. The data originally is used by Rezai Balf et al. (2012). Table 2 reports the results of ranking for 6 extremely efficient DMUs

Table 1: Input and output data for Example 1

DMU	Input 1	Input 2	Output 1	Output 2
1	81	87.6	5191	205
2	85	12.8	3629	0
3	56.7	55.2	3302	0
4	91	78.8	3379	8
5	216	72	5368	639
6	58	25.6	1674	0
7	112.2	8.8	2350	0
8	293.2	52	6315	414
9	186.6	0	2865	0
10	143.4	105.2	7689	66
11	108.7	127	2165	266
12	105.7	134.4	3963	315
13	235	236.8	6643	236
14	146.3	124	4611	128
15	57	203	4869	540
16	118.7	48.2	3313	16
17	58	47.4	1853	230
18	14	650.8	4578	217
19	0	91.3	0	508

(D_1, D_2, D_5, D_9, D_{15}, D_{19}) in model (11) with constant Returns to Scale and the proposed method are compared with Ap, MAJ, ℓ_1 and ℓ_∞. The results imply that the model proposed in this paper provides a easy tool for ranking extremely efficient DMUs. The value of inputs and outputs.

Example 2 (Empirical example). we employ the DEA model (12) on the empirical example used in(Zhu,1998), with the assumption of variable Returns to Scale. The data set in Table 5 provides 13 open coastal Chinese cities and five Chinese special economic zones in 1989. Two inputs and three outputs were chosen to characterize the technology of those cities/zones. Two inputs include Investment in fixed assets by state-owned enter-prises, Foreign funds actually used. Three outputs include Total industrial output value, Total value of retail sales and Handling capacity of coastal ports. Table 6 reports the results

Table 2: Results of ranking by different models

DMU	1	2	5	9	15	19
AP ranking results	4	1	3	-	2	-
MAJ ranking results	5	3	2	6	4	1
ℓ_1 -norm ranking results	4	3	2	6	5	1
ℓ_∞ -norm ranking results	5	2	3	6	4	1
Method in [25]	5	3	4	2	6	1
Value of obj. function model (11)	0.029	0.063	0.070	0.038	0.046	0181
Proposed model ranking results	6	3	2	5	4	1

Table 3: The value of inputs and outputs

DMU # Cities/Zones	Input 1	Input 2	Output 1	Output 2	Output 3
Dalian	2874.8	16,738	160.89	80,800	5092
Qinhuangdao	946.3	691	21.14	18,172	6563
Tianjin	6854.0	43,024	375.25	44,530	2437
Qingdao	2305.1	10,815	176.68	70,318	3145
Yantai	1010.3	2099	102.12	55,419	1225
Weihai	282.3	757	59.17	27,422	246
Shanghai	17,478.6	116,900	1029.09	351,390	14,604
Lianyungang	661.8	2024	30.07	23,550	1126
Ningbo	1544.2	3218	160.58	59,406	2230
Wenzhou	428.4	574	53.69	47,504	430
Guangzhou	6228.1	29,842	258.09	151,356	4649
Zhanjiang	697.7	3394	38.02	45,336	1555
Beihai	106.4	367	7.07	8236	121
Shenzhen	4539.3	45,809	116.46	56,135	956
Zhuhai	957.8	16,947	29.20	17,554	231
Shantou	1209.2	15,741	65.36	62,341	618
Xiamen	972.4	23,822	54.52	25,203	513
Hainan	2192.0	10,943	25.24	40,267	895

Table 4: Results for several models ranking

DMU	1	2	5	6	7	9	10	11	13	16
AP ranking results	9	1	8	4	6	3	2	7	5	10
MAJ ranking results	1	8	3	4	9	7	10	6	2	5
ℓ_1 -norm ranking results	4	8	3	6	9	1	10	7	2	5
ℓ_∞ -norm ranking results	3	8	4	6	9	1	10	7	2	5
Method in (Ziari, M., & Ziari, S, 2016)	1	7	3	2	8	9	10	5	6	4
Value of obj. fun. model (12)	0.009	0.124	0.002	0.007	0.720	0.024	0.013	0.036	0.010	0.001
Proposed model (12)	7	2	9	8	1	4	5	3	6	10

of ranking for 10 extremely efficient DMUs in model (12) with variable returns to scale and the proposed method are compared with others methods.

We note that the results of proposed model closer to the results of AP model.

CONCLUSION

Many researchers have offered numerous methods to rank DMUs in data envelopment analysis, but most of them have defects as inefficiency.

In this study, a new method using the idea of super efficiency model, minimizing the maximum distance between DMU under evaluation and boundary efficient in input and output directions for ranking efficient units is proposed. The suggested model is able to rank all the extreme efficient units under the assumption of returns to constant and variable scale. Also the presented model is always feasible and bounded and therefore eliminates some defects of ranking methods of extreme DMUs. In this study ranking results of proposed model indicates that the proposed model performed well.

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