Optimization
Iranian Journal of Optimization
Volume 7, Issue 2, 2015, 857-864
Research Paper
Online version is available on: www.ijo.iaurasht.ac.ir

Islamic Azad University
Rasht Branch
E-ISSN:2008-5427

Sensitivity Analysis in Two-Stage DEA

Athena Forghani * and Esmaeil Najafi

Department of Industrial Engineering, Tehran Science and Research Branch, Islamic Azad University, Tehran, Iran

Received: 14 August 2015
Accepted: 25 October 2015

Keywords:
Two-stage DEA
Weighted sum model
Combined DEA model
Sensitivity analysis

Abstract

Data envelopment analysis (DEA) is a method for measuring the efficiency of peer decision making units (DMUs) which uses a set of inputs to produce a set of outputs. In some cases, DMUs have a two-stage structure, in which the first stage utilizes inputs to produce outputs used as the inputs of the second stage to produce final outputs. One important issue in two-stage DEA is the sensitivity of the results of an analysis to perturbations in the data. The current paper looks into combined model for two-stage DEA and applies the sensitivity analysis to DMUs on the entire frontier. In fact, necessary and sufficient conditions for preserving a DMU's efficiency classification are developed when various data changes are applied to all DMUs.

***Correspondence E-mail**: athenaforghani@yahoo.com

INTRODUCTION

Data envelopment analysis (DEA) is a methodology, developed by Charnes, Cooper, and Rhodes (1978), for measuring the relative efficiency of peer decision making units (DMUs) that have multiple inputs and outputs.

As noted in some cases, DMUs may have a two stage structure, in which the first stage uses inputs to generate outputs that become the inputs of the second stage and the second stage then utilizes these first stage outputs to produce its own outputs.

For the DMUs with a two-stage process, Seiford and Zhu (1999) applied a standard DEA model to measure their efficiencies in each stage separately, but the series relationship between the two stages has no way to be considered or reflected in this way. Kao and Hwang (2008) modified the standard DEA model by taking into account the series relationship of the two stages within the overall process and modeled the overall efficiency of two-stage process as the product of the efficiencies of two individual stages. Chen et al. (2009) introduced an additive approach for aggregating the efficiencies in two-stage process. Specifically, they modeled the overall efficiency of two-stage process as a weighted sum of the efficiencies of the two individual stages.

This paper applies the combined DEA model proposed by Joro et al. (1998) which minimizes the inputs and maximizes the outputs simultaneously, and proposes new models to evaluate the efficiency of two-stage process. Then, we focus on sensitivity analysis in two-stage problem.

During the recent years, the issue of sensitivity and stability of DEA results has been extensively studied. Some studies focus on the sensitivity of DEA results to the variable and model selection, e.g., Ahn and Seiford (1993) and Smith (1997). As in many other DEA sensitivity studies, the calculated frontiers of DEA models are stable if the frontier DMUs that determine the DEA frontier remain on the frontier after the particular data perturbations are made for all DMUs.

By updating the inverse of the basis matrix associated with a specific efficient DMU in a DEA linear programming problem, Charnes et al. (1985) study the sensitivity of DEA model to a single output change. This is followed by a series of sensitivity analysis articles by Charnes and Neralic in which sufficient conditions preserving efficiency are determined (Charnes & Neralic, 1990).

Another type of DEA sensitivity analysis is based on super-efficiency DEA approach in which a test DMU is not included in reference set (Andersen & Petersen, 1993; Seiford and Zhu, 1999). Charnes et al. (1992), Rousseau and Semple (1995) and Charnes et al. (1996) develop a super-efficiency DEA sensitivity analysis technique for the situation where simultaneous proportional change is assumed in all inputs and outputs for a specific DMU under consideration. This data variation condition is relaxed in Zhu (1996) and Seiford and Zhu (1998a) to a situation where inputs or outputs can be changed individually and the entire (largest) stability region which encompasses that of Charnes et al. (1992) is obtained. As a result, the condition for preserving efficiency of a test DMU is necessary and sufficient. Seiford and Zhu (1998b) generalize the technique in Zhu (1996) and Seiford and Zhu (1998a) to the worst-case scenario where the efficiency of the test DMU is deteriorating while the efficiencies of the other DMUs are improving. In their method, same maximum percentage data change of a test DMU and the remaining DMUs is assumed and sufficient conditions for preserving an extreme-efficient DMU's efficiency are determined.

Zhu (2001) focuses on the DEA sensitivity analysis methods based on super-efficiency DEA models. For the DEA sensitivity analysis based on the inverse of basis matrix, the reader is referred to Neralic (1994). It is well known that certain super-efficiency DEA models may be infeasible for some extreme-efficient DMUs. Seiford and Zhu (1999) develop the necessary and sufficient conditions for infeasibility of various super-efficiency DEA models. Seiford and Zhu (1998a) discover the relationship between infeasibility and stability of efficiency classification. That is, infeasibility means that the CCR efficiency of the test DMU remains stable to data changes in the test DMU. Furthermore, Seiford and Zhu (1998b) show that this relationship is also true for the simultaneous data change case and other DEA models, such as BCC model of Banker et al. (1984) and additive model of Charnes et al. (1985).

In fact, Zhu (2001) generalizes the results in Seiford and Zhu (1998b) to a situation where variable percentage data changes are assumed for a test DMU and for the remaining DMUs. He considers the same worst-case analysis as in Seiford and Zhu (1998b). It is shown that a particular super-efficiency score can be decomposed into two data perturbation components of a particular test DMU and the remaining DMUs. Also, necessary and sufficient conditions for preserving a DMU's efficiency classification are developed when various data changes are applied to all DMUs. As a result, DEA sensitivity analysis can be easily applied as for various super-efficiency DEA models including combined DEA model. In this paper, we use the proposed combined model by Zhu (2001) for two-stage process to a situation where variable percentage data changes are assumed for a test two-stage DMU and for the remaining two-stage DMUs.

The rest of the paper is organized as follows: Section 2 briefly reviews the combined DEA model; Section 3 briefly reviews the two-stage process; Section 4 presents a combined DEA model for two stage process; Section 5 and 6 briefly define the data variations in a test frontier DMU and the remaining DMUs and review sensitivity analysis model using combined DEA model; Section 7 presents new models to analyze the sensitivity in two-stage process and Section 8 gives a numerical example to show applicability of the two-stage proposed model; Section 9 concludes the paper.

COMBINED DEA MODELS

Consider the usual single stage process. Suppose there are n DMUs to be evaluated and each DMU_j $(j = 1,\dots,n)$ has m inputs x_{ij} $(i = 1,\dots,m)$ and s outputs y_{rj} $(r = 1,\dots,s)$. Here we consider the Combined CCR model proposed by Joro et al. (1998) as

$$
\begin{aligned}
\min \quad & \theta \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le (1+\theta) x_{io}, \qquad i = 1,\dots,m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge (1-\theta) y_{ro}, \qquad r = 1,\dots,s, \\
& \lambda_j \ge 0, \qquad j = 1,\dots,n.
\end{aligned} \tag{1}
$$

This model minimizes the inputs and maximizes the outputs, simultaneously. In addition, the optimal value of objective function is $0 \le \theta_o^* < 1$ and DMU_o is efficient if and only if $\theta_o^* = 0$.

The model (1) is equivalent to the following fractional programming problem:

$$
\begin{aligned}
\max \quad & \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{r=1}^{s} u_r y_{ro} + \sum_{i=1}^{m} v_i x_{io}} \\
s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1,\dots,n, \\
& v_i, u_r \ge 0, \qquad i = 1,\dots,m, \\
& \qquad\qquad r = 1,\dots,s.
\end{aligned} \tag{2}
$$

TWO STAGE PROCESS

Consider the two-stage process shown in Fig. 1. Suppose there are n DMUs to be evaluated and each DMU_j $(j = 1,\dots,n)$ has m inputs x^{ij} $(i = 1,\dots,m)$ and D outputs z_{dj} $(d = 1,\dots,D)$ in the first stage. The D outputs then become the inputs of the second stage and are referred to as intermediate measures. The outputs from the second stage are denoted as y_{rj} $(r = 1,\dots,s)$. The weights v_i $(i = 1,\dots,m)$, η_d $(d = 1,\dots,D)$ and u_r $(r = 1,\dots,s)$ are considered as for the inputs of the first stage, the intermediate measures and the output weights in the second stage, respectively.

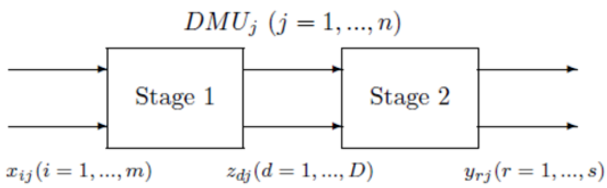

Fig. 1. Two-stage process

COMBINED MODEL FOR TWO-STAGE PROCESS

According to model (2), we define the CRS efficiencies of DMUj in the first and the second stage as

$$\theta_j^1 = \frac{\sum_{d=1}^{D} \eta_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij}}{\sum_{d=1}^{D} \eta_d z_{dj} + \sum_{i=1}^{m} v_i x_{ij}},$$

and

$$\theta_j^2 = \frac{\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} \eta_d z_{dj}}{\sum_{r=1}^{s} u_r y_{rj} + \sum_{d=1}^{D} \eta_d z_{dj}}.$$

Like Chen et al. (2009), we define the overall efficiency of DMU_j as the weighted sum of the two individual efficiencies, namely $\theta_j = w_1\theta_j^1 + w_2\theta_j^2$, where w_1, w_2 are the weights satisfying $w_1+w_2=1$. It's also assumed that $\eta^1_d = \eta^2_d = \eta_d$ for $d = 1,...,D$. We thus propose deriving the overall efficiency of the process by solving the following problem:

$$\theta_o^* = \max \quad \omega_1 . \frac{\sum_{d=1}^{D} \eta_d z_{do} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{d=1}^{D} \eta_d z_{do} + \sum_{i=1}^{m} v_i x_{io}} + \omega_2 . \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{d=1}^{D} \eta_d z_{do}}{\sum_{r=1}^{s} u_r y_{ro} + \sum_{d=1}^{D} \eta_d z_{do}}$$

$$\begin{aligned} s.t. \quad & \frac{\sum_{d=1}^{D} \eta_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1, \dots, n, \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} \eta_d z_{dj}} \le 1, \qquad j = 1, \dots, n, \\ & v_i, \eta_d, u_r \ge 0, \qquad i = 1, \dots, m, \quad r = 1, \dots, s. \\ & d = 1, \dots, D, \end{aligned} \tag{3}$$

The model (3) can be converted into a linear form by choosing the weights

$$\omega_1 = \frac{\sum_{i=1}^{m} v_i x_{io} + \sum_{d=1}^{D} \eta_d z_{do}}{\sum_{i=1}^{m} v_i x_{io} + 2\sum_{d=1}^{D} \eta_d z_{do} + \sum_{r=1}^{s} u_r y_{ro}},$$

$$\omega_2 = \frac{\sum_{d=1}^{D} \eta_d z_{do} + \sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io} + 2\sum_{d=1}^{D} \eta_d z_{do} + \sum_{r=1}^{s} u_r y_{ro}}. \tag{4}$$

Note that w_1 and w_2 are intended to represent the relative importance or contribution of the performances of stages 1 and 2, respectively, to the overall performance of the DMU. By setting w_1 and w_2 in the objective function of the model (3), it can be written as

$$\theta_o^* = \max \quad \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{i=1}^{m} v_i x_{io} + 2\sum_{d=1}^{D} \eta_d z_{do} + \sum_{r=1}^{s} u_r y_{ro}}$$

$$\begin{aligned} s.t. \quad & \frac{\sum_{d=1}^{D} \eta_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1, \dots, n, \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} \eta_d z_{dj}} \le 1, \qquad j = 1, \dots, n, \\ & v_i, \eta_d, u_r \ge 0, \qquad i = 1, \dots, m, \quad d = 1, \dots, D, \\ & r = 1, \dots, s. \end{aligned} \tag{5}$$

The optimal value of this model is $0<\theta_o^* \le 1$ and DMUo is efficient if and only if $\theta_o^*=1$. We use the Stackelberg game to calculate the efficiency scores of the sub-DMUs. We assume that the first sub-DMU is the leader and calculate its efficiency scores. We then calculate the efficiency scores of the second sub-DMU (i.e., the follower), based on the achieved efficiency scores for the first the leader.

The efficiency of the first stage θ_o^{1*} can be computed by solving of the following model:

$$\theta_o^{1*} = \max \quad \frac{\sum_{d=1}^{D} \eta_d z_{do} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{d=1}^{D} \eta_d z_{do} + \sum_{i=1}^{m} v_i x_{io}}$$

$$\begin{aligned} s.t. \quad & \theta_o^* = \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{i=1}^{m} v_i x_{io} + 2\sum_{d=1}^{D} \eta_d z_{do} + \sum_{r=1}^{s} u_r y_{ro}} \\ & \frac{\sum_{d=1}^{D} \eta_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1, \dots, n, \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} \eta_d z_{dj}} \le 1, \qquad j = 1, \dots, n, \\ & v_i, \eta_d, u_r \ge 0, \qquad i = 1, \dots, m, \quad d = 1, \dots, D, \\ & r = 1, \dots, s. \end{aligned} \tag{6}$$

The efficiency for the second stage is then calculated as $\theta_o^{2*}=(\theta_o^*-w_1^*\theta_o^{1*})/w_2^*$, where w^{1*}, w^{2*} represent optimal weights obtained from model (5) by way of (4). We can also compute the efficiency of the second stage θ_o^{2*} by solving the following model and then calculate the efficiency second stage as $\theta_o^{1*}=(\theta_o^*-w_2^*\theta_o^{2*})/w_1^*$:

$$\theta_o^{2*} = \max \quad \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{d=1}^{D} \eta_d z_{do}}{\sum_{r=1}^{s} u_r y_{ro} + \sum_{d=1}^{D} \eta_d z_{do}}$$

$$\begin{aligned} s.t. \quad & \theta_o^* = \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{i=1}^{m} v_i x_{io} + 2\sum_{d=1}^{D} \eta_d z_{do} + \sum_{r=1}^{s} u_r y_{ro}} \\ & \frac{\sum_{d=1}^{D} \eta_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1, \dots, n, \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} \eta_d z_{dj}} \le 1, \qquad j = 1, \dots, n, \\ & v_i, \eta_d, u_r \ge 0, \qquad i = 1, \dots, m, \quad d = 1, \dots, D, \end{aligned} \tag{7}$$

Note: Using the Charnes and Cooper's transformation, all three above models can be converted into the linear programs (LP) for solution.

SENSITIVITY ANALYSIS

According to Zhu (2001), the frontier points in DEA are of primary importance as they define the DEA frontier. The current section will discuss the stability of efficiency classification for such *DMUs* whether DMU_o will still be a frontier point after data perturbations in all the *DMUs*. Since an increase of any output or a decrease of any input cannot worsen the efficiency of DMU_o, the attention is restricted to decrease in outputs and increase in inputs for DMU_o. In order to simultaneously consider the data changes for other *DMUs*, as in Seiford and Zhu (1998), increased output and decreased input is suppsed for all other *DMUs*. The discussion is based on a worst-case scenario in which efficiency of DMU_o declines and the efficiencies of all other DMU_j $(j \neq o)$ improve.

Let I and O denote, respectively, the input and output subsets in which the data changes. Then the simultaneous data perturbations in input/output of all DMU_j $(j \neq o)$ and DMUo can be written as percentage data perturbation (variation):

for DMU_o,

$$\begin{cases} \hat{x}_{io} = \delta_i x_{io}, & \delta_i \geq 1, i \in I, \\ \hat{x}_{io} = x_{io}, & i \notin I, \end{cases}$$

and

$$\begin{cases} \hat{y}_{ro} = \tau_r y_{ro}, & 0 < \tau_r \leq 1, r \in O, \\ \hat{y}_{ro} = y_{ro}, & r \notin O, \end{cases}$$

for DMU_j $(j \neq o)$,

$$\begin{cases} \hat{x}_{io} = x_{io}/\tilde{\delta}_i, & \tilde{\delta}_i \geq 1, i \in I, \\ \hat{x}_{io} = x_{io}, & i \notin I, \end{cases}$$

and

$$\begin{cases} \hat{y}_{ro} = y_{ro}/\tilde{\tau}_r, & 0 < \tilde{\tau}_r \leq 1, r \in O, \\ \hat{y}_{ro} = y_{ro}, & r \notin O. \end{cases}$$

Where ˆ represents adjusted data.

SENSITIVITY ANALYSIS MODEL

We are only interested in whether a DMU remains on the frontier, rather than in its original efficiency classification. Therefore, we consider the following modified DEA measure for simultaneous variations of inputs and outputs proposed by Zhu (2001) as

$$\begin{aligned} \min \quad & \theta \\ s.t. \quad & \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} \leq (1+\theta) x_{io}, && i \in I, \\ & \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} \leq x_{io}, && i \notin I, \\ & \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj} \geq (1-\theta) y_{ro}, && r \in O, \\ & \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj} \geq y_{ro}, && r \notin O, \end{aligned} \tag{8}$$

$\lambda_j \geq 0$, $j = 1, \ldots, n$, $j \neq 0$

If $I = 1,2,\ldots,m$ and $O = 1,2,\ldots,s$, then (8) is identical to the model of Charnes et al. (1996) when variations in the data are only applied to DMU_o. Note that if DMU_o is a frontier point, then $\lambda \geq 0$.

Theorem 1. Suppose DMUo is a frontier point.

If $1 \leq \delta_i \, \tilde{\delta}_i \leq \sqrt{(1+\lambda^*)}$ and $\sqrt{(1-\lambda^*)} \leq \tau_r \, \tilde{\tau}_r \leq 1$,

then DMU_o remains as a frontier point, where λ^* is the optimal value to (8).

The dual of the model (8) in fractional form is:

$$\begin{aligned} \max \quad & \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{r \in O} u_r y_{ro} + \sum_{i \in I} v_i x_{io}} \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, \ldots, n, \\ & v_i, u_r \geq 0, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s. \end{aligned} \tag{9}$$

SENSITIVITY ANALYSIS IN TWO –STAGE PROCESS

Let I, F and O denote, respectively, the first-stage input, intermediate measure and second-stage output subsets in which the data changes. According to the introduced models in the previous section, we propose the following models as for two-stage process:

The overall efficiency of the whole process:

$$\theta_o^* = \max \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{i\in I} v_i x_{io} + 2\sum_{d\in F} \eta_d z_{do} + \sum_{r\in O} u_r y_{ro}}$$

$$\begin{aligned} s.t.\quad & \frac{\sum_{d=1}^{D} \eta_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1,\dots,n, \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} \eta_d z_{dj}} \le 1, \qquad j = 1,\dots,n, \\ & v_i, \eta_d, u_r \ge 0, \qquad i = 1,\dots,m, \\ & d = 1,\dots,D, \qquad r = 1,\dots,s. \end{aligned} \tag{10}$$

The efficiency of the first stage:

$$\theta_o^{1*} = \max \frac{\sum_{d=1}^{D} \eta_d z_{do} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{d\in F} \eta_d z_{do} + \sum_{i\in I} v_i x_{io}}$$

$$\begin{aligned} s.t.\quad & \theta_o^* = \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{i\in I} v_i x_{io} + 2\sum_{d\in F} \eta_d z_{do} + \sum_{r\in O} u_r y_{ro}} \\ & \frac{\sum_{d=1}^{D} \eta_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1,\dots,n, \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} \eta_d z_{dj}} \le 1, \qquad j = 1,\dots,n, \\ & v_i, \eta_d, u_r \ge 0, \qquad i = 1,\dots,m, \\ & d = 1,\dots,D, \qquad r = 1,\dots,s. \end{aligned} \tag{11}$$

The efficiency of the second stage:

$$\theta_o^{2*} = \max \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{d=1}^{D} \eta_d z_{do}}{\sum_{r\in O} u_r y_{ro} + \sum_{d\in F} \eta_d z_{do}}$$

$$\begin{aligned} s.t.\quad & \theta_o^* = \frac{\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}}{\sum_{i\in I} v_i x_{io} + 2\sum_{d\in F} \eta_d z_{do} + \sum_{r\in O} u_r y_{ro}} \\ & \frac{\sum_{d=1}^{D} \eta_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1,\dots,n, \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} \eta_d z_{dj}} \le 1, \qquad j = 1,\dots,n, \\ & v_i, \eta_d, u_r \ge 0, \qquad i = 1,\dots,m, \\ & d = 1,\dots,D, \qquad r = 1,\dots,s. \end{aligned} \tag{12}$$

Using the Charnes and Cooper's transformation, all three above models can easily be converted into the linear program (LP) for solution.

By the additional constraint, $\sum_{j=1, j\neq o}^{n} \lambda_j = 1$ the approach can easily be modified to study the sensitivity of other DEA models that satisfy different returns to scale (Seiford and Thrall, 1990; Charnes et al., 1994; F¨are et al., 1994).

Table 1: Data set for 24 Taiwanese non-life insurance companies

DMU		X_1	X_2	Z_1	Z_2	Y_1	Y_2
1	Taiwan Fire	1,178,744	673,512	7,451,757	856,735	984,143	681,687
2	Chung Kuo	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3	Tai Ping	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4	China Mariners	601,320	594,259	3,174,851	371,863	248,709	177,331
5	Fubon	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6	Zurich	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
7	Taian	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
8	Ming Tai	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
9	Central	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
10	The First	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
11	Kuo Hua	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12	Union	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
13	Shingkong	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
14	South China	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
15	Cathay Century	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
16	Allianz President	1,211,716	415,071	5,606,013	402,881	854,054	197,947
17	Newa	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
18	AIU	757,515	547,997	3,631,484	995,620	692,731	163,927
19	North America	159,422	182,338	1,141,951	483,291	519,121	46,857
20	Federal	145,442	53,518	316,829	131,920	355,624	26,537
21	Royal & Sunalliance	84,171	26,224	225,888	40,542	51,950	6491
22	Aisa	15,993	10,502	52,063	14,574	82,141	4181
23	AXA	54,693	28,408	245,910	49,864	0.1	18,980
24	Mitsui Sumitomo	163,297	235,094	476,419	644,816	142,370	16,976

NUMERICAL EXAMPLE FOR NEW – STAGE MODEL

As a numerical illustration, we show in Table 1 the data set for 24 Taiwanese nonlife insurance companies (DMUs) taken from Kao and Hwang (2008), where Operation expenses (X_1) and Insurance expenses (X_2) are two inputs to the first stage, Direct written premiums (Z_1) and Reinsurance premiums (Z_2) are two intermediate measures, i.e. the outputs of the first stage and the inputs to the second stage, and Underwriting profit (Y_1) and Investment profit (Y_2) are two outputs to the second stage.For such a two-stage structure and the data set, Table 2 shows the efficiency decompositions of the 24 DMUs under CRS assumption. The ranking of our model is almost the same as Chen et al's (2009) model. However, the efficiency scores which are resulted from the proposed model in this paper are higher than the results of the others, which shows the potential applications of the proposed model.

CONCLUSION

It has been realized that DMUs may have a two-stage structure in some applications, where the first stage utilizes inputs to generate outputs that become the inputs of the second stage and the second stage then employs the first stage outputs to produce its own outputs. As it's mentioned in Chen et al. (2009), Kao and Hwang's model just reduces the inputs of the first stage, while Chen et al.'s approach describes (in the input oriented case) the extend to which the aggregate of all component inputs can be reduced while producing the same level of aggregate outputs. In this paper, we have used combined DEA model, which minimizes inputs and maximizes outputs simultaneously. We proposed the combined DEA model for two stage process that can be applied under both CRS and VRS assumptions. Similarly, these models can be used for the process with more than two stages. Furthermore, numerical illustrations have been provided wherever necessary to show the potential applications of the proposed new DEA models.In addition, the

Table 2: The results of combined CRS models for two-stage process

DMU		θ_o^*	w_1	w_2	θ_o^{1*}	θ_o^{2*}
1	Taiwan Fire	0.918	0.541	0.459	0.996	0.827
2	Chung Kuo	0.896	0.552	0.448	0.999	0.770
3	Tai Ping	0.899	0.550	0.450	0.817	1.000
4	China Mariners	0.747	0.626	0.374	0.840	0.592
5	Fubon	0.932	0.534	0.466	0.908	0.960
6	Zurich	0.816	0.592	0.408	0.980	0.577
7	Taian	0.735	0.633	0.367	0.859	0.521
8	Ming Tai	0.734	0.633	0.367	0.841	0.549
9	Central	0.759	0.620	0.380	1.000	0.365
10	The First	0.833	0.584	0.416	0.926	0.702
11	Kuo Hua	0.674	0.663	0.337	0.843	0.342
12	Union	0.936	0.532	0.468	1.000	0.863
13	Shingkong	0.715	0.642	0.358	0.895	0.391
14	South China	0.732	0.634	0.366	0.840	0.544
15	Cathay Century	0.893	0.553	0.447	1.000	0.761
16	Allianz President	0.780	0.610	0.390	0.939	0.531
17	Newa	0.760	0.620	0.380	0.839	0.630
18	AIU	0.740	0.630	0.370	0.885	0.492
19	North America	0.827	0.586	0.414	1.000	0.583
20	Federal	0.867	0.566	0.434	0.965	0.739
21	Royal & Sunalliance	0.702	0.649	0.351	0.857	0.416
22	Aisa	0.852	0.574	0.426	0.742	1.000
23	AXA	0.813	0.593	0.407	0.915	0.666
24	Mitsui Sumitomo	0.704	0.648	0.352	1.000	0.160
Mean		0.803			0.912	0.624

current paper develops a new approach for the sensitivity analysis of DEA models according to the proposed combined model by Zhu (2001) for two-stage process. The new sensitivity analysis approach simultaneously evaluates whether a DMU remains on the frontier, rather than in its original efficiency classification. We also develop necessary and sufficient conditions for preserving efficiency when data changes are made for all DMUs. It is obvious that larger optimal values to DEA models correspond to greater stability of the test DMU in preserving efficiency.It is hoped that the research conducted in this study can enrich the theory of DEA and provide more alternative ways for measuring the performance and analyzing the sensitivity of two-stage process.

REFERENCE

Ahn, T., & Seiford, L. M. (1993). Sensitivity of DEA to models and variable sets in a hypothesis test setting: The efficiency of university operations. *Creative and innovative approaches to the science of management*, 191-208.

Andersen, P., & Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management science, 39*(10), 1261-1264.

Charnes, A., Cooper, W.W., Golany, B., Seiford, L., & Stutz, J. (1985). Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions. *Journal of econometrics, 30*(1-2), 91-107.

Charnes, A., Cooper, W.W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European journal of operational research, 2*(6), 429-444.

Charnes, A., Haag, S., Jaska, P., & Semple, J. (1992). Sensitivity of efficiency classifications in the additive model of data envelopment analysis. *International Journal of Systems Science, 23*(5), 789-798.

Charnes, A., Rousseau, J. J., & Semple, J. H. (1996). Sensitivity and stability of efficiency classifications in data envelopment analysis. *Journal of Productivity Analysis, 7*(1), 5-18.

Charnes, A., & Neralić, L. (1990). Sensitivity analysis of the additive model in data envelopment analysis. *European Journal of Operational Research, 48*(3), 332-341.

Chen, Y., Cook, W. D., Li, N., & Zhu, J. (2009). Additive efficiency decomposition in two-stage DEA. *European Journal of Operational Research, 196*(3), 1170-1176.

Fare, R., Grosskopf, S., & Lovell, C. K. (1994). *Production frontiers*. Cambridge University Press.

Kao, C., & Hwang, S. N. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European journal of operational research, 185*(1), 418-429.

Neralic, L. (1994). Sensitivity analysis in data envelopment analysis: A review. In *Proceedings of the 2nd Slovenian Symposium on Operations Research SOR* , 94, 29-42.

Rousseau, J. J., & Semple, J. H. (1995). Radii of classification preservation in data envelopment analysis: A case study of 'Program Follow-Through'. *Journal of the Operational Research Society, 46*(8), 943-957.

Seiford, L. M., & Thrall, R. M. (1990). Recent developments in DEA: the mathematical programming approach to frontier analysis. *Journal of econometrics, 46*(1-2), 7-38.

Seiford, L. M., & Zhu, J. (1998a). Stability regions for maintaining efficiency in data envelopment analysis. *European Journal of Operational Research, 108*(1), 127-139.

Seiford, L. M., & Zhu, J. (1998b). Sensitivity analysis of DEA models for simultaneous changes in all the data. *Journal of the Operational Research Society, 49*(10), 1060-1071.

Seiford, L. M., & Zhu, J. (1999). Infeasibility of super-efficiency data envelopment analysis models. INFOR: *Information Systems and Operational Research, 37*(2), 174-187.

Smith, P. (1997). Model misspecification in data envelopment analysis. *Annals of Operations Research, 73*, 233-252.

Joro, T., Korhonen, P., & Wallenius, J. (1998). Structural comparison of data envelopment analysis and multiple objective linear programming. *Management science, 44*(7), 962-970.

Zhu, J. (1996). Robustness of the efficient DMUs in data envelopment analysis. *European Journal of operational research, 90*(3), 451-460.

Zhu, J. (2001). Super-efficiency and DEA sensitivity analysis. *European Journal of operational research, 129*(2), 443-455.