FUZZY OPTIMIZATION AND MODELLING 4(3) (2023) 34-39

E-ISNN: 2676-7007

Contents lists available at FOMJ

Fuzzy Optimization and Modelling

Journal homepage: http://fomj.qaemiau.ac.ir/

Paper Type: Research Paper

Application of Grey Numbers and Neutrosophic Sets to Assessment Processes

Michael Voskoglou*

School of Engineering, University of Peloponnese, Patras, Greece

A R T I C L E I N F O

Article history:
Received 28 July 2023
Revised 10 October 2023
Accepted 10 October 2023
Available online 10 October 2023

Keywords:
Fuzzy Set
Neutrosophic Set
Grey System
Grey Number
Assessment under Fuzzy Conditions

A B S T R A C T

Zadeh extended in 1965 crisp sets to the concept of fuzzy set (FS) on the purpose of tackling mathematically the partial truths and the definitions with no clear boundaries. In a later stage FSs were extensively used for tackling the existing uncertainty in the real world. Neutrosophic sets (NSs) are extensions of FSs in which each element of the universal set is characterized, apart from Zadeh's membership degree, by the degrees of non-membership and indeterminacy. Grey numbers (GNs) are real numbers with known range, represented by a closed real interval, but with unknown exact value. GNs and NSs are used in this paper as tools for assessment processes under fuzzy conditions. The use of GNs enables the evaluation of the mean performance of a group of objects when qualitative grades (linguistic expressions) are used for the individual assessment of its members. The use of NSs, on the other hand, is useful for assessing the overall performance of a group, when one, due to incomplete assessment data, is not sure about the exactness of the grades assigned to its members. It is concluded that the suitable combination of two or more theories related to FSs give better assessment outcomes in general. Examples are also presented on student assessment illustrating our results.

1. Introduction

The assessment of a group's activities is a very important task, because it helps to correct mistakes and to improve the overall group's performance. Frequently, however, the assessment is performed not with numerical scores, but by using qualitative grades (linguistic expressions). This happens either because the existing data for the assessment are not exact, or for reasons of more elasticity (e.g. from teacher to students). In such cases the mean performance of the group cannot be assessed with the traditional method of calculating the mean value of the individual scores of its members. For this, fuzzy assessment methods have been introduced by the present author and by other researchers, e.g. see [1, 3, 5, 10-16].

In this work we present a method for estimating the mean performance of a group using *grey numbers*

* Correspondig author

- E-mail address: mvoskoglou@gmail.com(*Michael Voskoglou*)

DOI: 10.30495/fomj.2023.1992552.1104

(GNs) as tools [9] (section 6.2). Cases also appear, however, in which one is not sure about the exactness of the grades assigned to the members of a group under assessment, due to incomplete assessment data. To cover these cases, we also present here a method for evaluating the overall group's performance by using *neutrosophic sets (NSs)* as tools [17].

The rest of the paper is organized as follows: Section 2 contains the mathematical background from the theories of NSs and GNs needed for the understanding of this work. In sections 3 and 4 the assessment methods using GNs and NSs respectively are developed, illustrated by examples on student assessment. The article closes with the final conclusions and some hints for further research, contained in its last section 5.

2. Mathematical Background

2.1 Fuzzy and Neutrosophic Sets

Zadeh introduced in 1965 [20] the concept of *fuzzy set (FS)*, on the purpose of tackling mathematically the existing in everyday life partial truths and the definitions with no clear boundaries (like "high mountains", "young people", etc.). A FS is defined as follows:

Definition 1. Let U be the universe, then a FS F in U is of the form

$$F = \{(x, m(x)) : x \in U\} \tag{1}$$

In Equation (1) m: U $\rightarrow$ [0,1] is the *membership function* of F and m(x) is called the *membership degree* of x in F**.** The greater m(x), the more x satisfies the characteristic property of F. A crisp subset F of U is a FS in U with membership function such that $m(x) = 1$ if x belongs to F and 0 otherwise.

Uncertainty is defined as the shortage of precise knowledge or complete information on the data that describe the state of a corresponding situation. It was only in a second stage that FSs were used to embrace uncertainty modelling. This happened when membership functions were reinterpreted as possibility distributions. Zadeh [21] articulated the relationship between possibility and probability, noticing that what is probable must preliminarily be possible.

Probability theory used to be for a long period the unique tool in hands of the specialists for dealing with problems connected to uncertainty. Probability, however, was proved to be suitable only for tackling the cases of uncertainty which are due to *randomnes*. Randomness characterizes events with known outcomes which, however, cannot be predicted in advance, like the games of chance. FSs, apart from randomness, tackle also successfully the uncertainty due to *vagueness*, which is created when one is unable to distinguish between two properties, such as "a good player" and "a mediocre player".

Following the introduction of FSs, a series of extensions and relative theories have been proposed for managing more effectively all the forms of the existing uncertainty. Among them, Atanassov in 1986 [2] added to Zadeh's membership degree the degree of *non-membership* in [0, 1] and extended the concept of FS to the notion of *intuitionistic FS (IFS)*.

Smarandache in 1995 [8], motivated by the various neutral situations appearing in real life - like <positive, zero, negative>, <small, medium, high>, <win, draw, defeat>, etc. – added to the degrees of membership and non-membership the degree of *indeterminacy*, also in [0, 1], and extended the concept of IFS to the concept of NS. In this work we will make use only of the simplest form of a NS, which is known as *single valued NS (SVNS)* [19]**.**

Definition 2. A SVNS A in the universal set U is defined as the set of the ordered tuples

$$A = \{(x, T(x), I(x), F(x)) : x \in U, T(x), I(x), F(x) \in [0,1], \quad 0 \leq T(x) + I(x) + F(x). \tag{2}$$

In (2) T(x), I(x), F(x) are the degrees of *truth* (or membership), of *indeterminacy* (or neutrality) and of *falsity* (or non-membership) of x in A respectively, called the *neutrosophic components* of x. For simplicity, we represent A by $A < T, I, F >$ and the elements of A in the form of *neutrosophic triplets (NTs)* (t, i, f), with t, i, f in [0, 1].

Example 1. Let U be the set of the players of a football team and let A be the SVNS of the good players of U. Then each player x of U is characterized by a NT (t, i, f), with t, i, f in $[0, 1]$. For instance, $x(0.7, 0.1, 0.4) \in A$ means that there is a 70% belief that x is a good player, a 10% doubt about it and at the same time a 40% belief that x may not be a good player. In particular, $x(0,1,0) \in A$ means that we do not know absolutely nothing about x's affiliation with A.

When $T(x) + I(x) + F(x) < 1T(x) + I(x) + F(x) < 1$, it leaves room for *incomplete* information, when $T(x)+I(x)+F(x)=1$ for *complete* information, and when $T(x) + I(x) + F(x) > 1$ for *inconsistent* information about x in A. A SVNS may contain simultaneously elements corresponding to all kinds of the previous information. All notions and operations defined on FSs are extended in a natural way to SVNSs [9]

Since the NTs of a SVNS A are ordered triplets, one may define addition among them and scalar multiplication of a positive number with a NT in the usual way, as follows:

Definition 3. Let (t_1, i_1, f_1), (t_2, i_2, f_2) be NTs in A and let r be a positive number. Then:

- The *sum* $(t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1+ t_2, i_1+ i_2, f_1+ f_2)$ (3)
- The *scalar product* $r(t_1, i_1, f_1) = (rt_1, r i_1, f_1)$ (4)

The sum and the scalar product of the NTs of a SVNS A with respect to the previous definition need not be a NT of A, since it may happen that $(t_1+ t_2)+(i_1+ i_2)+(f_1+ f_2)>3$ or $rt_1+ri_1+ rf_1>3$. With the help of Definition 2, however, one can define the *mean value* of a finite number of NTs of A, which is always a NT in A, as follows:

Definition 4. Let A be a SVNS and let (t_1, i_1, f_1), (t_2, i_2, f_2), …., (t_k, i_k, f_k) be a finite number of NTs of A. Assume that (t_i, i_i, f_i) appears n_i times in an application, i = 1,2,…., k. Set $n = n1 + n2 + \cdots. +nk$. Then the *mean value*
of all these elements of A is defined to be the NT

$$(tm, im, fm) = \frac{1}{n}[n1(t_1, i_1, f_1) + n2(t_2, i_2, f_2) + \cdots. +nk(t_k, i_k, f_k)] \quad (5)$$

2.2 Grey Numbers

Approximate data are frequently used nowadays in many problems of everyday life, science and engineering, because many continuously changing factors are usually involved in large and complex systems. Deng in 1982 [4] introduced the *grey system (GS)* theory as an alternative to the theory of FSs for tackling such kinds of data. A GS is understood to be a system that lacks information such as structure message, operation mechanism and/or behaviour document. The theory of GSs was developed mainly in China and has found many and important applications to everyday life, science and engineering, including medicine diagnostics, psychology, sociology, control systems, economics, agriculture, opinion polls, etc. , where the data cannot be easily determined and estimates of them are used in practice. For general facts on GSs we refer to the classical on the subject book of Liu & Lin [6].

The main tool for handling the approximate data of a GS is the use of GNs. A GN T is understood to be a real number with known range, given by a closed real interval of the form [a, b], but with unknown exact value The GN T, however, may differ from the interval [a, b] with respect to the presence of a *whitenization function* f: [a, b] → [0, 1], such that the closer is f(t) to 1, the better t ∈ [a, b] approximates the unknown value of T. When no such function exists, it is logical to consider as the crisp representative of T the real number

$$V(T) = \frac{a+b}{2} \quad (6)$$

The known arithmetic of the real intervals [4] is used to perform the basic arithmetic operations between GNS. Let $T_1 \in [a_1, b_1]$ and $T_2 \in [a_2, b_2]$ be given GNs and let r be a positive number. In this paper we will make use only of the *addition* and of the *scalar product* of GNs, which are defined respectively by the relations

$$G_1 + G_2 \in [x_1 + y_1, x_2 + y_2] \text{and } rG_1 \in [rx_1, ryx_1]. \quad (7)$$

3. Grey Assessment

A scale of qualitative grades, which is frequently used in assessment cases, is the following: A = excellent, B = very good, C = good, D = fair and F= fail. In certain cases the grade E is also inserted between D and F, or intermediate grades like A_-, B_+, B_-, etc. are used, but this does not affect the generality of our method.

In order to evaluate the mean performance of a group with respect to the previous grades, we assign the numerical scale 1-100 to these grades as follows: A → [85, 100], B → [75, 84], C → [60, 74], D → [50, 59], F → [0, 49]. This assignment, although it is compatible to the common sense, it is not unique. For a more strict assessment, for example, one could consider, instead of the previous one, the assignment A → [90, 100], B → [80, 89], C → [70, 79], D → [60, 69], F → [0, 59], etc. Neither this fact, however, affects the generality of our method.

We introduce now the following GNs, denoted for simplicity with the same letters: A ∈ [85, 100], B ∈ [75, 84], C ∈ [60, 74], D ∈ [50, 59], F ∈ [0, 49].

Let us consider a group G of n objects under assessment. Assume that the performance of n_A of these objects was evaluated by A, of n_B by B, of n_C by C, of n_D by D and of n_F objects by F, so that

$$nA + nB + nC + nDD + nF = n$$

With the help of equations (6) and (7) we define the *mean value* of the corresponding GNs to be the GN

$$M = \frac{1}{n}[n_A A + n_B B + n_C C + n_D D + n_F F] \quad (8)$$

Then the *mean performance* of the group G can be estimated, with the help of equation (5), by the real value V(M).

The following example illustrates the previous assessment method.

Example 2: The teacher of a student class assessed the performance of his students as follows: Students s_1 - s_3 by A, s_4 - s_7 by B, s_8 - s_{10} by C, s_{11} – s_{16} by D, and the remaining four students by F. Evaluate the mean performance of the class.

Solution: With the help of equation (8) one finds that the mean value of the grades obtained by the students of the class is equal to

$$M = \frac{1}{20}\{3[85, 100]+ 4[75, 84]+ 3[60, 74]+ 6[50, 59]+ 4[0,49]\}.$$

Therefore, with the help of equations (7), it turns out that

$$M = \frac{1}{20}[1035, 1408] = [51.75, 70.4].$$

Thus, by equation (6), one finds that $V(M) = \frac{51.75+70.4}{2} = 61.075$, which shows that the student class demonstrated a good (C) mean performance.

4. Neutrosophic Assessment

When one is not sure about the exactness of the grades assigned to the members of a group under assessment, then the use of a suitable NS can help to evaluate satisfactorily the overall group's performance. This is illustrated by the following example.

Example 3. The new teacher of a student class is not sure yet about the quality of each of the students. In order to assess the overall quality of the class, therefore, he decided to define the NS of the very good students of the class, in which he characterized each student by a NT as follows: s_1(1, 0, 0), s_2(0.9, 0.1, 0.1), s_3(0.8, 0.2, 0.1), s_4(0.4, 0.5, 0.8), s_5(0.4, 0.5, 0.8), s_6(0.3, 0.7, 0.8), s_7(0.3, 0.7, 0.8), s_8(0.2, 0.8, 0.9), s_9(0.1, 0.9, 0.9), s_{10}(0.1, 0.9, 0.9} and the remaining 10 students of the class by (0, 0, 1). This means that the teacher is absolutely sure that s_1 is a very good student, 90% sure that s_2 is a very good student too, but at the same time he has a 10% doubt about it and a 10% belief that s_2 may not be a very good student, etc. For the last 10 students the teacher is absolutely sure that they are not very good students. How he evaluated then the overall quality of the student class?

Solution: The overall quality of the student class can be evaluated by calculating the mean value of the formed by the teacher NTs, which, according to equation (4), is equal to M = $\frac{1}{20}$ [(1, 0, 0)+(0.9, 0.1, 0.1)+(0.8, 0.2, 0.1)+2(0.4, 0.5, 0.8)+2(0.3, 0.7, 0.8)+(0.2, 0.8, 0.9)+2(0.1, 0.9, 0.9)+10(0, 0, 1)]. Thus, by equations (3) and (4) one obtains that M = $\frac{1}{20}$ (4.5, 5.3, 16.3) = (0.225, 0.265, 0.815). This means that a random student of the class has a 22.5 % probability to be a very good student, however, there exists also a 26.5% doubt about it and an 81.5% probability to be not a very good student.

Remark. The choice of the NS of the very good students of the class was not the unique option for the teacher, who could choose the NS of the excellent (or good, etc.) students instead and obtain analogous results with the same method.

5. Conclusions and Future Research Directions

In this work we presented two methods for assessment under fuzzy conditions. The grey assessment method can be used for evaluating the mean performance of a group, when the individual performance of its members is assessed with qualitative grades (linguistic expressions). The neutrosophic method on the other hand, is suitable to be used for evaluating the overall performance of a group, when the assessment data are not complete, and therefore, one is not sure about the exactness of the grades assigned to the group's members. The outcomes of this method, however, depending on the choice of the corresponding NS, give only an approximate estimation of the overall performance of a group of objects with respect of a certain activity and they cannot assess the group's mean performance.

The suitable combination of two or more theories relative to FSs seems to give better results in general, not only for assessment, but also for decision making [18] and probably for other cases related to the management of the existing in everyday life uncertainty. This is, therefore, a promising area for further research.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper

References

1. Antony Crispin Sweety, C.. Jansi, R..(2021), Fermatean Neutrosophic Sets, *International Journal of Advanced Research in Computer and Communication Engineering*, 10(6), 24-27.
2. Atanassov, K.T. (1986), Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*, 20(1), 87-96.
3. Broumi, S., et al. (2023), Faculty Performance Evaluation Through Multicriteria Decision Analysis Using Interval -Valued Fermatean Neutrosophic Sets, *Mathematics*, 11, 3817.
4. Deng, J. (1982), Control Problems of Grey Systems, *Systems and Control Letters*, 288.
5. Jeevaraj S. (2021), Ordering of interval valued Fermatean fuzzy sets and its applications, *Expert Systems with Applications*, 185,115613.
6. Liu, S. F. & Lin, Y. (Eds.) (2010), *Advances in Grey System Research*, Springer, Berlin – Heidelberg, Germany.
7. Moore, R.A., Kearfort, R. B. & Clood, M.J. (1995), *Introduction to Interval Analysis*, 2nd Printing, SIAM, Philadelphia, USA.
8. Smarandache, F. (1998), Neutrosophy/ Neutrosophic probability, set, and logic, Proquest, Michigan, USA.
9. Voskoglou, M.Gr. (2019), Assessing Human-Machine Performance under Fuzzy Conditions, *Mathematics*, 7, article 230.
10. Voskoglou, M.Gr. (2022), Soft sets as tools for assessing human-machine performance, *Egyptian Computer Science Journal*, 46(1), 1-6.
11. Voskoglou, M.Gr. (2022), Use of Grey Numbers and Soft Sets as Assessment Tools, *Asian Journal of Pure and Applied Mathematics*, 4(3), 171-177.
12. Voskoglou, M.Gr. (2022), Use of Soft Sets and the Bloom's Taxonomy for Assessing Learning skills, *Transactions on Fuzzy Sets and Systems*, 1(1), 106-113.
13. Voskoglou, M.Gr., Broumi, S. (2022), A Hybrid Method for the Assessment of Analogical Reasoning skills, *Journal of Fuzzy Extension and Applications*, 3(2), 152-157.

14. Voskoglou, M.Gr., Broumi, S., Smarandache, F. (2022), A Combined Use of Soft and Neutrosophic Sets for Student Assessment with Qualitative Grades, *Journal of Neutrosophic and Fuzzy Systems*, 4(1), 15-20.
15. Voskoglou, M.Gr. (2022), A Hybrid Method for Assessment with Linguistic Grades, *Oriental Journal of Physical Sciences*, 7(1), 26-29.
16. Voskoglou, M.Gr. (2023), Assessing the Effectiveness of Flipped Learning for Teaching Mathematics to Management Students, *American Journal of Applie Mathematics and Statistics*, 11(1), 30-34.
17. Voskoglou, M.Gr. (2023), Neutrosophic Assessment of Student Mathematical Skills, *Physical and Mathematical Education*, 38(2), 22-26.
18. Voskoglou, M.Gr. (2023), An Application of Neutrosophic Sets to Decision Making, *Neutrosophic Sets and Systems*, 53, 1-9.
19. Wang, H., Smarandanche, F., Zhang, Y. and Sunderraman, R. (2010), Single Valued Neutrosophic Sets, *Review of the Air Force Academy (Brasov)*, 1(16), 2010, 10-14.
20. Zadeh, L.A. (1965), Fuzzy Sets, *Information and Control*, 8, 338-353.
21. Zadeh, L.A. (1978), Fuzzy Sets as a basis for a theory of possibility, *Fuzzy Sets and Systems*, 1, 3-28.

Voskoglou, M. (2023). Application of Grey Numbers and Neutrosophic Sets to Assessment Processes. *Fuzzy Optimization and Modeling Journal*, *4*(3), 34-39.

https://doi.org/10.30495/fomj.2023.1992552.1104

Received: 28 July 2023 Revised: 10 October 2023 Accepted: 10 October 2023