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Fermatean Fuzzy Type Statistical Concepts with Medical Decision-Making Application

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ABSTRACT

When a correlation between datasets is presented, it is clear from this statement that it quantifies how strongly these datasets are connected. Meanwhile, this coefficient is a well-known metric for assessing the link between two sets. The Fermatean fuzzy set is a significant extension of the extant intuitionistic and Pythagorean fuzzy sets, with the benefit of more comprehensively characterizing ambiguous data. In other words, Fermatean fuzzy sets are powerful and useful tools for representing imprecise information. The purpose of this work is to generate novel correlation coefficients using Fermatean fuzzy sets. These coefficients specify the degree and kind of correlation (positive or negative) between two Fermatean fuzzy sets. The new coefficient values will similarly be in the [-1,1] range. During formulation, pairs of membership and non-membership degrees were viewed as a vector representation containing the two elements. Furthermore, the novel approach was compared to existing methods. A medical diagnosis application and pattern recognition as a data mining application were used to exemplify the effectiveness of the proposed method.

1. Introduction

Several applications are used in the research in the literature to address decision-making (DM) issues. The correlation coefficients (C), which are utilized to assess the degree of dependence between two sets, are one of the techniques for determining the best choice. Any statistical association between two random variables or bivariate data, whether causal or not, is referred to in statistics as a "correlation." Calculating the covariance value, which depicts how these two variables have changed in relation to one another, is the easiest technique to determine whether two variables are connected. To further understand the covariance, it is helpful to consider the variance. The variance of a variable is a measure of how far the averaged data are from the mathematical mean. The change of two variables with regard to one another, or their relationship, may be demonstrated by computing the covariance value. The covariance is often negative when both variables are below the mean. In this instance, we may conclude that the two variables do not have a positive association. Nevertheless, employing covariance has a drawback since covariance is dependent on the unit of the variables. We will have trouble comprehending the covariance value if the two variables are measured in different units. It becomes challenging to define what being huge or tiny entails. We must normalize the covariance value in order to solve

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the unit problem in covariance. Each unit must be able to be converted to a common value in some way. We must make use of the standard deviation to do this. The correlation value is obtained by dividing the covariance formula by the standard deviation values.

Cs are used to gauge how strongly two variables are correlated. Because of this, there are several different areas of work, including engineering, physics, medicine, and economics. Current probabilistic methods have several benefits but some downsides as well. On a large scale, the complex system contains a lot of fuzzy ambiguity, making it difficult to understand the whole range of possible outcomes. Outcomes according to probability do not every time ensure helpful info to specialists due to the constraint of only being able to act on quantitative information. Also, there are times when there is not enough data to properly operate parameter statistics in day-to-day operations. Due to these limitations, the probabilistic approximation is insufficient to take into account the inherent uncertainties in the data, and the conclusions based on probability do not always offer experts valuable info. There are several approaches to overcoming these challenges. One of these choices' most successful outcomes for overcoming ambiguities and imprecision in DM is Fuzzy set ($\mathcal{F}$) theory-based methods.

Uncertainty is an important concept in DM problems. Unpredictable events are what define uncertainty. In unclear conditions, routine decisions may not be debated. Under vagueness situations, it is critical to weigh both the benefits and downsides of prospective results. At this point, it is critical to conduct a detailed examination of the environmental factors. Even when final judgments are not in dispute, utilizing past experiences and decisions is not every time beneficial, when there is uncertainty. Because of Zadeh's development of the $\mathcal{F}$ concept, language phrases that we inadvertently use on a daily basis are now calculable [42].

The classification system was able to transcend the restrictions of math, which had beforehand been limited to precision since fuzzy logic was used. This idea started a mode change that diffuse around the world as an outcome of its accomplished application in world circumstances. A distinct-function component is either an insider of a set in the conventional sense or it is not. Nevertheless, the F notion assesses whether or not a body is interested in a set by using a membership function(MF) that appoints an item a membership degree(MD) in [0, 1]. If the MD of an element of a set is π_A, its non-membership degree (ND) is $1-\pi_A$, according to $\mathcal{F}$-A.

Hence, the sum of the degrees of belonging and non-belonging is equal to one. Yet, this situation falls short of fully addressing the uncertainty in a number of areas. As a result, the intuitionistic fuzzy set ($\mathcal{IF}$) theory improved the generalization of the F theory [5].

Whereas $\mathcal{F}$ is intended to simply expose the MD specified in [0,1], IF defines the ND in addition to the MD. IF states that MD and ND are in [0, 1]. Yager [41] was the first to introduce Pythagorean fuzzy sets ($\mathcal{PF}$), which were occasionally created as extensions of $\mathcal{IF}$s since $\mathcal{IF}$s were insufficient for expressing uncertainty. Since MD and ND cannot be combined together to make a decision, PFs use the idea that $MD^2+ND^2\leq 1$. The literature has a number of studies on F and its several extensions ([10, 15, 23, 28, 30, 37]).

The Fermatean fuzzy set ($\mathcal{FFS}$) with $0\leq MD^3+ND^3\leq 1$ has been initiated by Senepati and Yager [32]. $\mathcal{FFS}$ is better at explaining uncertainties than $\mathcal{IF}$s and $\mathcal{PF}$. This work was continued by Senapati and Yager [33], who looked at a variety of new operations and arithmetic mean procedures over $\mathcal{FFS}$s. To solve MCDM difficulties, they also applied the $\mathcal{FF}$-weighted product model. $\mathcal{FFS}$-related new aggregation operators have been defined and [34] has examined the properties that go along with them. In a short time, many studies on $\mathcal{FFS}$ have entered the literature [1-3, 19, 24,-26].

In [9], a formula for C of Fs is guaranteed, while the correlation for fuzzy info based on traditional statistics is provided. According to the accepted concept of Cs, the C of fuzzy info has been investigated in [29]. IF and PF produced more thorough and precise results on the basis of the results produced by the F theory. Cs derived from IF and PF have been utilized in a wide variety of applications in the literature [4, 6, 8, 9, 27, 29, 36, 38, 39, 44].

In real-world implementations, $\mathcal{FFN}$s have a very large capacity to duplicate uncertain and imprecise information; hence, our study improves the $\mathcal{FFN}$-based C to solve MCGDM challenges. By taking into account MD and ND, the new Cs between the two $\mathcal{FFS}$s have been determined. The $\mathcal{FF}$ environment served as the first definition of new informational energy ($\mathcal{IE}$) in this work. $\mathcal{IE}$ measures a random variable's level of uncertainty and increases as randomness drops. IE is precisely convex at all times. As the novel *C*s and other sets have different weights in reality, weighted *C*s have been developed according to $\mathcal{IE}$. Many methods have been developed to find the *C*s between $\mathcal{F}$s. Nevertheless, these tactics fail when certain values are both MDs and NDs of the same element. As it may show the link between the $\mathcal{FFS}$s, this study is interested in identifying the correlation between $\mathcal{FFE}$. Our results fall inside the [0, 1] interval since we have left out the equation's negative part, commonly known as the reverse correlation because we are working in a fuzzy environment. The hypothesis that C belongs to $\mathcal{FFS}$s was developed and put out in light of the fact that $\mathcal{FFS}$s are useful tools for figuring out relationships between pieces of confusing information. To demonstrate the success of the new strategies, two applications were used: Medical diagnosis and pattern recognition. The old and new *C*s were contrasted.

Originality: The traditional *C*s have undergone a number of extensions, including $\mathcal{F}$, $\mathcal{IF}$, and $\mathcal{PF}$ *C*s. The *C*s are now performing better thanks to these extensions. Considering $\mathcal{IF}$s, $\mathcal{PF}$ and $\mathcal{FFS}$s shows that $\mathcal{FFS}$s are better able to deal with uncertainty and missing information issues. In this work, the $\mathcal{IF}$ *C*s and $\mathcal{PF}$ *C*s investigations were taken into consideration for developing the $\mathcal{FF}$ *C*s. $\mathcal{FFS}$s have the potential to encompass more components than $\mathcal{IF}$s and $\mathcal{PF}$s since they satisfy the $MD^3 + ND^3 \leq 1$ criteria for an object. There is a medical application for the new *C*s. We compared newly suggested *C*s with *C*s based on $\mathcal{IF}$ and $\mathcal{PF}$ from prior trials.

The contributions of this study can be given as follows:

I. Most correlation coefficients that are now used in fuzzy or nonstandard fuzzy theory have values between *0* and *1*, which merely shows the strength of the connection. Uncertain concepts like ugly vs. beautiful (negatively correlated), heavy vs. big (positively correlated), etc. can be easily correlated using a correlation coefficient with a value in [1, 1], but the correlation coefficient with a value in [0, 1] is insufficient to describe the correlation between ugly and beautiful.

II. The bulk of studies on fuzzy and non-standard fuzzy sets use fabricated data to support their comparative measures. The current $\mathcal{IF}$ and $\mathcal{PF}$-based correlation coefficients do not satisfy all or some of these requirements. In this study, we thus propose some new correlation coefficients for $\mathcal{FFS}$s that exceed the existing correlation coefficients while accounting for these aspects.

III. For $\mathcal{FFS}$s with values in [1, 1], we suggest four new correlation coefficients, and we also go over some of their attractive characteristics.

IV. With the help of our proposed $\mathcal{FF}$-correlation coefficients, medical diagnostics, and data mining applications are studied. Further, the proposed $\mathcal{FF}$ *C*s are compared with various measures of compatibility already existing under $\mathcal{FF}$ conditions.

2. Preliminaries

In this section, we introduce some basic concepts related to $\mathcal{FFS}$s.

Definition 1. The set $M = \{(u,\ f_M(u),\ g_M(u)) : u\ \in U\}$ is called $\mathcal{FFS}$ with $f_M, g_M \in [0,1]$, $0\ \leq\ f_M^3 + g_M^3 \leq 1$. The hesitation degree is represented as $h_M = (1 - f_M^3 - g_M^3)^{1/3}$.

Fermatean fuzzy number ($\mathcal{FFN}$) and the set of all $\mathcal{FFS}$ s in U are denoted by $\left(f_M(u), g_M(u)\right)$, and $\Gamma(U)$, respectively.

For two $\mathcal{FFN}$s $M = (f_M, g_M)$ and $N = (f_N, g_N)$,

a. $\overline{M} = (g_M, f_M)$,

b. $M \boxtimes N = \left(f_M\, f_N\ , (g_M^3 + g_N^3 - g_M^3 . g_N^3)^{\frac{1}{3}}\right)$

c. $M \boxplus N = \left((f_M^3 + f_N^3 - f_M^3 . f_N^3)^{1/3},\ g_M^3 g_N^3\right)$

d. $z.M = \left((1 - (1 - f_M^3)^z)^{1/3},\ g_M^z\right)$

e. $M^z = \left(f_M^z,\ (1-(1-g_M^3)^z)^{1/3}\right)$

Definition 2. For two $\mathcal{FFN}$s $M = (f_M, g_M)$ and $N = (f_N, g_N)$,

a. $M \cup N = (max\{f_M, f_N\},\ min\{g_M, g_N\})$
b. $M \cap N = (min\{f_M, f_N\},\ max\{g_M, g_N\})$
c. $M^c = (g_M, f_M)$
d. $M \preccurlyeq N$ if and only if $f_M \le f_N$, $g_M \le g_N$.

Definition 3. For two $\mathcal{FFN}$s $M = (f_M, g_M)$ and $N = (f_N, g_N)$, the score functions, and the accuracy functions are:

$$SC(M) = f_M^3 - g_M^3, \qquad SC(N) = f_N^3 - g_N^3,$$
$$AC(M) = f_M^3 + g_M^3, \qquad AC(N) = f_N^3 + g_N^3.$$

Lemma 1. For two $\mathcal{FF}$ s $M = (f_M, g_M)$ and $N = (f_N, g_N)$,

a. If $SC(M) < SC(N)$, then $M < N$,
b. If $SC(M) = SC(N)$, $AC(M) < AC(N)$, then $M < N$,
c. If $SC(M) = SC(N)$, $AC(M) = AC(N)$, then $M = N$.

3. Novel Statistical Concepts

In this section, a few brand-new correlation coefficients for $\mathcal{FFS}$s that, in addition to the degree of connection, also indicate whether the two $\mathcal{FFS}$s are positively or negatively associated have been offered. Throughout this paper, let $\Gamma(U)$ denote the set of all $\mathcal{FFS}$s in the universe of discourse $U = \{t_1, t_2, \dots, t_l\}$.

First, $\mathcal{FF}$-type correlation coefficients based on variance and covariance have been offered. In order to define the correlation coefficients for $\mathcal{FFS}$s, some terms like average, variance, and covariance of FFSs have been defined.

Definition 4. For any $M, N \in \Gamma(U)$,

a. $Ort(M)$ is said to be the average of M, where

$$Ort(M) = \left(\overline{f_M}, \overline{g_M}\right) = \left(\frac{1}{n}\sum_{k=1}^{n} f_M(t_k), \qquad \frac{1}{n}\sum_{k=1}^{n} g_M(t_k),\right) \tag{1}$$

b. $VRNC(M)$ is said to be the variance of M, where

$$VRNC(M) = \frac{1}{n-1}\sum_{k=1}^{n}\left(\left[f_M^3(t_k) - \overline{f_M^3}\right]^2 + \left[g_M^3(t_k) - \overline{g_M^3}\right]^2\right) \tag{2}$$

c. $K(M, N)$ is called the covariance of M, for

$$K(M,N) = \frac{1}{n-1}\sum_{k=1}^{n}\left(\left\{\left[f_M^3(t_k) - \overline{f_M^3}\right] \times \left[f_N^3(t_k) - \overline{f_N^3}\right]\right\} + \left\{\left[g_M^3(t_k) - \overline{g_M^3}\right] \times \left[g_N^3(t_k) - \overline{g_N^3}\right]\right\}\right) \tag{3}$$

Proposition 1. For any $M, N \in \Gamma(U)$,

a. $K(M,N) = K(N,M)$,
b. $K(M,M) = VRNC(M)$,
c. $|VRNC(M)| \le \sqrt{VRNC(M)VRNC(N)}$.

Proof: a.

$$K(M,N) = \frac{1}{n-1}\sum_{k=1}^{n}\left(\left\{\left[f_M^3(t_k) - \overline{f_M^3}\right] \times \left[f_N^3(t_k) - \overline{f_N^3}\right]\right\} + \left\{\left[g_M^3(t_k) - \overline{g_M^3}\right] \times \left[g_N^3(t_k) - \overline{g_N^3}\right]\right\}\right)$$

$$= \frac{1}{n-1}\sum_{k=1}^{n}\left(\left\{\left[f_N^3(t_k) - \overline{f_N^3}\right] \times \left[f_M^3(t_k) - \overline{f_M^3}\right]\right\} + \left\{\left[g_N^3(t_k) - \overline{g_N^3}\right] \times \left[g_M^3(t_k) - \overline{g_M^3}\right]\right\}\right) = K(N,M).$$

b. $K(M,M)=\frac{1}{n-1}\sum_{k=1}^{n}(\{[f_M^3(t_k)-\overline{f_M^3}]\times[f_M^3(t_k)-\overline{f_M^3}]\}+\{[g_M^3(t_k)-\overline{g_M^3}]\times[g_M^3(t_k)-\overline{g_M^3}]\})$

$$=\frac{1}{n-1}\sum_{k=1}^{n}\left(\left[f_M^3(t_k)-\overline{f_M^3}\right]^2+\left[g_M^3(t_k)-\overline{g_M^3}\right]^2\right)=VRNC(M).$$

c. $K(M,N)^2=\left(\frac{1}{n-1}\sum_{k=1}^{n}(\{[f_M^3(t_k)-\overline{f_M^3}]\times[f_N^3(t_k)-\overline{f_N^3}]\}+\{[g_M^3(t_k)-\overline{g_M^3}]\times[g_N^3(t_k)-\overline{g_N^3}]\})\right)^2$

$$\leq\left(\frac{1}{n-1}\sum_{k=1}^{n}\{[f_M^3(t_k)-\overline{f_M^3}]+[g_M^3(t_k)-\overline{g_M^3}]\}\right)\times\left(\frac{1}{n-1}\sum_{k=1}^{n}\{[f_N^3(t_k)-\overline{f_N^3}]+[g_N^3(t_k)-\overline{g_N^3}]\}\right)$$

$=VRNC(M)VRNC(N)$.

Therefore, $|VRNC(M)|\leq\sqrt{VRNC(M)VRNC(N)}$.

Definition 5. For any $M,N\in\Gamma(U)$,

$$C(M,N)=\frac{K(M,N)}{\sqrt{VRNC(M)VRNC(N)}} \tag{4}$$

is called the *C* of *M,N* where *K*(*M, N*) and *VRNC*(*M*) utilized as defined by (2) and (3), respectively.

The new *C* will be more universal if the FFS hesitation degree *h* is also employed for it (3.4). This definition of *C* will make it much easier to categorize the sample and will greatly improve its ability to address practical situations. We now provide a revised definition of *C*.

Definition 6. For any $M,N\in\Gamma(U)$, then the *C* between *M, N*,

$$C_H(M,N)=\frac{K_H(M,N)}{\sqrt{VRNC_H(M)VRNC_H(N)}} \tag{5}$$

where

$$VRNC_H(M)=\frac{1}{n-1}\sum_{k=1}^{n}\left(\left[f_M^3(t_k)-\overline{f_M^3}\right]^2+\left[g_M^3(t_k)-\overline{g_M^3}\right]^2+\left[h_M^3(t_k)-\overline{h_M^3}\right]^2\right) \tag{6}$$

and

$$K_H(M,N)=\frac{1}{n-1}\sum_{k=1}^{n}(\{[f_M^3(t_k)-\overline{f_M^3}]\times[f_N^3(t_k)-\overline{f_N^3}]\}+\{[g_M^3(t_k)-\overline{g_M^3}]\times[g_N^3(t_k)-\overline{g_N^3}]\}$$
$$+\{[h_M^3(t_k)-\overline{h_M^3}]\times[h_N^3(t_k)-\overline{h_N^3}]\}) \tag{7}$$

and also

$$\overline{h_M}=\frac{1}{n}\sum_{k=1}^{n}h_M(t_k),\qquad \overline{h_N}=\frac{1}{n}\sum_{k=1}^{n}h_N(t_k).$$

Theorem 1. For any $M,N\in\Gamma(U)$, the following conditions are held:

a. $K(M,N)=K(N,M)$,
b. $-1\leq K(M,N)\leq 1$,
c. If $M=\alpha N$ for some α, then

$$C(M,N)=\begin{cases}1, & \alpha>0\\ -1, & \alpha<0.\end{cases}$$

4. Novel Pearson type Correlation Coefficients Formula

Karl Pearson's Coefficient of Correlation is also known as the Product Moment Correlation Coefficient and was developed by Karl Pearson. Along with the scatter diagram and Spearman's rank correlation, it is one of the three most effective and widely used techniques for determining the degree of correlation. The quantitative Karl Pearson correlation coefficient method provides a number to determine the strength of the linear relationship between *X* and *Y*. In this section, new Pearson type-correlation coefficients pertaining to $\mathcal{FFS}$s have been presented.

Let's start by outlining a few of the equations that will be utilized in this section:

$$\lambda_1 = \frac{\sum_{k=1}^{n}\{[f_M^3(t_k) - \overline{f_M^3}] \times [f_N^3(t_k) - \overline{f_N^3}]\}}{\sqrt{[f_M^3(t_k) - \overline{f_M^3}]^2}\sqrt{[f_N^3(t_k) - \overline{f_N^3}]^2}} \quad (8)$$

$$\lambda_2 = \frac{\sum_{k=1}^{n}\{[g_M^3(t_k) - \overline{g_M^3}] \times [g_N^3(t_k) - \overline{g_N^3}]\}}{\sqrt{[g_M^3(t_k) - \overline{g_M^3}]^2}\sqrt{[g_N^3(t_k) - \overline{g_N^3}]^2}} \quad (9)$$

$$\lambda_3 = \frac{\sum_{k=1}^{n}\{[h_M^3(t_k) - \overline{h_M^3}] \times [h_N^3(t_k) - \overline{h_N^3}]\}}{\sqrt{[h_M^3(t_k) - \overline{h_M^3}]^2}\sqrt{[h_N^3(t_k) - \overline{h_N^3}]^2}} \quad (10)$$

Definition 7. Take $M, N \in \Gamma(U)$. Using Equations (8) and (9),

$$C_P(M,N) = \frac{1}{2}(\lambda_1 + \lambda_2) \quad (11)$$

is obtained. The Equation (11) is said to be C.

Definition 8. Take $M, N \in \Gamma(U)$. Using Equations (8), (9), and (10),

$$C_{PH}(M,N) = \frac{1}{2}(\lambda_1 + \lambda_2 + \lambda_3) \quad (12)$$

is obtained. The Equation (12) is said to be C.

Theorem 2. Take $M, N \in \Gamma(U)$.

a. $C_P(M,N) = C_P(N,M)$,

b. $-1 \le C_P(M,N) \le 1$,

c. If $M = \alpha N$, for some α, then

$$C_P(M,N) = \begin{cases} 1, & \alpha > 0 \\ -1, & \alpha < 0. \end{cases}$$

Proof: The proof is item a is clear.

b. Let's utilize the Cauchy-Schwarz inequality:

$$(\lambda_1)^2 = \left(\frac{\sum_{k=1}^{n}\{[f_M^3(t_k) - \overline{f_M^3}] \times [f_N^3(t_k) - \overline{f_N^3}]\}}{\sqrt{[f_M^3(t_k) - \overline{f_M^3}]^2}\sqrt{[f_N^3(t_k) - \overline{f_N^3}]^2}}\right)^2$$

$$= \frac{\left(\sum_{k=1}^{n}\{[f_M^3(t_k) - \overline{f_M^3}] \times [f_N^3(t_k) - \overline{f_N^3}]\}\right)^2}{\left(\sqrt{[f_M^3(t_k) - \overline{f_M^3}]^2}\sqrt{[f_N^3(t_k) - \overline{f_N^3}]^2}\right)^2}$$

$$\le \frac{\left[\sum_{k=1}^{n}[f_M^3(t_k) - \overline{f_M^3}]^2 \times \sum_{k=1}^{n}[f_N^3(t_k) - \overline{f_N^3}]^2\right]}{\left[\sum_{k=1}^{n}[f_M^3(t_k) - \overline{f_M^3}]^2 \times \sum_{k=1}^{n}[f_N^3(t_k) - \overline{f_N^3}]^2\right]} = 1.$$

Then, $|(\lambda_1)^2| \le 1$. Similarly, $|(\lambda_2)^2| \le 1$. Therefore

$$|C_P(M,N)| = \left|\frac{1}{2}(\lambda_1 + \lambda_2)\right| \le \frac{1}{2}(|\lambda_1| + |\lambda_2|) \le \frac{1}{2}(1+1) = 1.$$

Hence $C_P(M,N) \in [-1,1]$.

c. If $M = \alpha N$, then $f_M(t_k) = \alpha f_N(t_k)$, $g_M(t_k) = \alpha g_N(t_k)$. For $\alpha > 0$,

$$\lambda_1 = \frac{\sum_{k=1}^{n}\{[f_M^3(t_k) - \overline{f_M^3}] \times [f_N^3(t_k) - \overline{f_N^3}]\}}{\sqrt{[f_M^3(t_k) - \overline{f_M^3}]^2}\sqrt{[f_N^3(t_k) - \overline{f_N^3}]^2}} = \frac{\sum_{k=1}^{n}\{[\alpha f_M^3(t_k) - \alpha\overline{f_M^3}] \times [f_N^3(t_k) - \overline{f_N^3}]\}}{\sqrt{[\alpha f_M^3(t_k) - \alpha\overline{f_M^3}]^2}\sqrt{[f_N^3(t_k) - \overline{f_N^3}]^2}}$$

$$= \frac{\alpha^3 \sum_{k=1}^{n}\{[f_M^3(t_k) - \overline{f_M^3}] \times [f_N^3(t_k) - \overline{f_N^3}]\}}{\alpha^3 \sqrt{[f_M^3(t_k) - \overline{f_M^3}]^2} \sqrt{[f_N^3(t_k) - \overline{f_N^3}]^2}} = 1.$$

Similarly, $\lambda_2 = 1$. Hence, $C_P(M, N) = \frac{1}{2}(\lambda_1 + \lambda_2) = 1$.
Take $\alpha > 0$. Choose $\alpha = -a$ for $\alpha > 0$. Therefore,

$$\lambda_1 = \frac{(-1)a^3 \sum_{k=1}^{n}\{[f_M^3(t_k) - \overline{f_M^3}] \times [f_N^3(t_k) - \overline{f_N^3}]\}}{a^3 \sqrt{[f_M^3(t_k) - \overline{f_M^3}]^2} \sqrt{[f_N^3(t_k) - \overline{f_N^3}]^2}} = (-1)^3 = -1.$$

Likewise, $\lambda_2 = 1$, and the proof is complete.

Theorem 3. Take $M, N \in \Gamma(U)$. Then,

d. $C_{PH}(M, N) = C_{PH}(N, M)$,
e. $-1 \leq C_{PH}(M, N) \leq 1$,
f. If $M = \alpha N$, for some α, then

$$C_{PH}(M, N) = \begin{cases} 1, & \alpha > 0 \\ -1, & \alpha < 0. \end{cases}$$

5. Applications

5.1. Medical Application

For this study, Kirisci and Şimşek [28] example of infectious illness was was modified to illustrate how the recommended strategy in MCDM may be applied. The illness state of the patients will be determined by taking into account their symptoms and using the distance, similarity, and correlation criteria. The ailment from which the patient suffers the most will be identified based on the findings.

Let $H = \{H_1, H_2, H_3, H_4\}$ and

$$U = \{Hepatitus\ C, Crimean\ Hemorrhagic\ Fever(CCHF), influenza(H1N1), sandly\ fever, norovirus\} = \{U_1, U_2, U_3, U_4, U_5\}$$

be the set of patients and the set of five infectious diseases, respectively. Further, let $S = \{chest\ pain, cough, stomach\ pain, headache, temprature\} = \{s_1, s_2, s_3, s_4, s_5\}$ be the set of symptoms.

The links between disease-symptoms (DS) and patient-symptoms (PS) will be analyzed using the maximization of correlation, minimization of distance, and maximization of similarity.

Table 1. Disease-symptoms

	s_1	s_2	s_3	s_4	s_5
U_1	(0.1, 0.9)	(0.2, 0.9)	(0.8, 0.5)	(0.4, 0.5)	(0.9, 0.2)
U_2	(0.2, 0.7)	(0.7, 0.6)	(0.7, 0.4)	(0.8, 0.4)	(0.9, 0.1)
U_3	(0.4, 0.6)	(0.9, 0.2)	(0.1, 0.7)	(0.7, 0.5)	(0.8, 0.4)
U_4	(0.5, 0.7)	(0.2, 0.7)	(0.6, 0.6)	(0.8, 0.3)	(0.9, 0.1)
U_5	(0.3, 0.7)	(0.2, 0.8)	(0.8, 0.5)	(0.9, 0.1)	(0.4, 0.6)

Table 2. Patient-symptoms

	s_1	s_2	s_3	s_4	s_5
H_1	(0.0, 0.6)	(0.6, 0.3)	(0.8, 0.1)	(0.2, 0.6)	(0.8, 0.3)
H_2	(0.1, 0.4)	(0.4, 0.5)	(0.6, 0.3)	(0.7, 0.4)	(0.8, 0.1)
H_3	(0.1, 0.5)	(0.8, 0.1)	(0.3, 0.7)	(0.5, 0.7)	(0.8, 0.2)
H_4	(0.3, 0.5)	(0.0, 0.8)	(0.2, 0.6)	(0.6, 0.5)	(0.9, 0.1)

Table 3. Values of C

	U_1	U_2	U_3	U_4	U_5
H_1	0.6347	-0.1614	**0.7791**	-0.0138	0.5217
H_2	0.1290	-0.3876	**0.7958**	-0.2451	0.4592
H_3	0.2193	0.2682	**0.5135**	-0.1405	0.1470
H_4	0.5315	0.3758	0.5016	0.4816	**0.6973**

Table 4. Values of C_H

	U_1	U_2	U_3	U_4	U_5
H_1	0.4233	-0.5857	**0.7225**	-0.0079	0.2851
H_2	0.0894	-0.3165	**0.7143**	-0.2017	0.1928
H_3	0.0065	0.1436	**0.2619**	-0.0291	0.0790
H_4	0.5315	0.3758	0.5016	0.4816	**0.6184**

Table 5. Values of C_P

	U_1	U_2	U_3	U_4	U_5
H_1	**0.6198**	0.5983	0.3704	-0.2177	0.1912
H_2	**0.7354**	-0.1508	0.1005	0.1606	-0.0810
H_3	0.2268	0.1325	**0.4913**	-0.0190	-0.0125
H_4	0.1798	0.6784	0.0672	0.1609	**0.7164**

Table 6. Values of C_{PH}

	U_1	U_2	U_3	U_4	U_5
H_1	**0.6829**	0.4136	0.1184	-0.0189	0.0463
H_2	0.3381	**0.6932**	0.1057	0.2460	-0.1340
H_3	0.0136	0.1695	**0.2656**	-0.1298	-0.0450
H_4	0.1073	0.3407	0.0506	-0.2741	**0.4713**

5.2. Data Mining Application: Pattern Recognition

Pattern recognition on the other hand is an engineering application of data mining and machine learning, it is a process of recognizing patterns such as images or speech. Once a neural net is trained using machine learning algorithms it can be used for pattern recognition. Other methods, even ones not related to machine learning and data mining can be used for pattern recognition such as a fully handcrafted pattern recognition system.

Pattern recognition can be defined as the classification of data based on knowledge already gained or on statistical information extracted from patterns and/or their representation. One of the important aspects of pattern recognition is its application potential. In this context, pattern recognition refers to the classification of an unknown pattern into some known patterns with the help of compatibility measures. Here, use of our proposed correlation coefficients in pattern recognition has been made.

Now, we take some known patterns $M_i = \{(u_k, f_M(u_k), g_M(u_k)) : u_k \in U, \ k = 1,2,\ldots,l\}$ for $i = 1,2,\ldots,m$. Further, we chooses an unknown pattern $N = \{(u_k, f_N(u_k), g_N(u_k)) : u_k \in U, \ k = 1,2,\ldots,l\}$.

In this application, our aim is to classify the unknown model N to one of the known models M_i. In order to achieve this goal, the unknown N model will be assigned to one of the known M_i $(i = 1,2,\ldots,m)$ models according to the following methods:

Firstly, we will use the distance method. Let $D(M_i, N)$ denote the distance between the known pattern. M_i and the unknown pattern N. According to this method, N is assigned to M_{i^*}, where $i^* = argmin_i D(M_i, N)$ for $i = 1,2,\ldots,m$.

Secondly, we will use the correlation method. Let $C(M_i, N)$ denote the correlation between the known pattern. M_i and the unknown pattern N. According to this method, N is assigned to M_{i^*}, where $i^* =$

$argmax_i C(M_i, N)$ for $i = 1,2, \ldots, m$.

Thirdly, we will use the similarity method. Let $S(M_i, N)$ denote the similarity between the known pattern. M_i and the unknown pattern N. According to this method, N is assigned to M_{i^*}, where $i^* = argmax_i S(M_i, N)$ for $i = 1,2, \ldots, m$.

Shahzaib et al's [35] distance measures:

$$D_{SHAH1}(M,N) = \frac{1}{2|U|} \sum_{u_i \in U} \left(|f_M^3 - f_N^3| + |g_M^3 - g_N^3| + |h_M^3 - h_N^3| \right)$$

$$D_{SHAH2}(M,N) = \frac{1}{4|U|} \left[\sum_{u_i \in U} \left(|f_M^3 - f_N^3| + |g_M^3 - g_N^3| + |h_M^3 - h_N^3| \right) + \sum_{u_i \in U} \left(|f_M^3 - f_N^3| + |g_M^3 - g_N^3| \right) \right]$$

Kirisci's [24] Euclidean distance measures:

$$D_{KIRISCI1}(M,N) = \frac{1}{2n} \left(\sum_{u_i \in U} \left(|f_M^3 - f_N^3| + |g_M^3 - g_N^3| + |h_M^3 - h_N^3| \right) \right)^2$$

$$D_{KIRISCI1}(M,N) = \frac{1}{2n} \left(\sum_{u_i \in U} \omega_i \left(|f_M^3 - f_N^3| + |g_M^3 - g_N^3| + |h_M^3 - h_N^3| \right) \right)^2$$

Shahzaib et al's [35] similarity measures:

$$S_{SHAH1}(M,N) = 1 - \frac{1}{2|U|} \sum_{u_i \in U} \left(|f_M^3 - f_N^3| + |g_M^3 - g_N^3| + |h_M^3 - h_N^3| \right)$$

$$S_{SHAH2}(M,N) = 1 - \frac{1}{4|U|} \left[\sum_{u_i \in U} \left(|f_M^3 - f_N^3| + |g_M^3 - g_N^3| + |h_M^3 - h_N^3| \right) + \sum_{u_i \in U} \left(|f_M^3 - f_N^3| + |g_M^3 - g_N^3| \right) \right]$$

Kirisci's [24] cosine similarity measures:

$$S_{KIRISCI1}(M,N) = \frac{1}{n} \sum_{k=1}^{n} \frac{f_M^3(u_i) f_N^3(u_i) + g_M^3(u_i) g_N^3(u_i) + h_M^3(u_i) h_N^3(u_i)}{(f_M^6(u_i) + g_M^6(u_i) + h_M^6(u_i))^{1/3} (f_N^6(u_i) + g_N^6(u_i) + h_N^6(u_i))^{1/3}}$$

$$S_{KIRISCI2}(M,N) = \frac{1}{n} \sum_{k=1}^{n} \omega_i \frac{f_M^3(u_i) f_N^3(u_i) + g_M^3(u_i) g_N^3(u_i) + h_M^3(u_i) h_N^3(u_i)}{(f_M^6(u_i) + g_M^6(u_i) + h_M^6(u_i))^{1/3} (f_N^6(u_i) + g_N^6(u_i) + h_N^6(u_i))^{1/3}}$$

Example 1. Let M_1, M_2 and M_3 be three known patterns and $U = \{u_1, u_2, u_3, u_4\}$. Then, the known patterns M_i is given as

$$M_1 = \{(u_1, 0.46, 0.68), (u_2, 0.55, 0.73), (u_3, 0.38, 0.84), (u_4, 0.70, 0.25)\}$$
$$M_2 = \{(u_1, 0.81, 0.34), (u_2, 0.24, 0.91), (u_3, 0.27, 0.79), (u_4, 0.35, 0.65)\}$$
$$M_3 = \{(u_1, 0.18, 0.85), (u_2, 0.43, 0.67), (u_3, 0.54, 0.39), (u_4, 0.70, 0.25)\}$$

An unknown pattern N is shown as

$$N = \{(u_1, 0.67, 0.33), (u_2, 0.39, 0.44), (u_3, 0.51, 0.55), (u_4, 0.42, 0.29)\}$$

Our aim is to find out the pattern to which N belongs. For this, we calculate the correlation/distance/similarity between N and M_i (i = 1, 2, 3), and the results are listed in Table 7. Weights are $\omega = \{0{,}36,\ 0.14,\ 0.22, 0.28\}$.

Table 7. Calculated values of various measures

	(M_1, N)	(M_2, N)	(M_3, N)	Outcomes
$C(M,N)$	0.403	0.485	0.332	M_1
$C_H(M,N)$	0.462	0.617	0.378	M_2
$C_P(M,N)$	0.681	0.734	0.543	M_2
$C_{PH}(M,N)$	0.605	0.698	0.499	M_2
$D_{SHAH1}(M,N)$	0,378	0.500	0,434	M_2
$D_{SHAH2}(M,N)$	0.279	0.386	0,312	M_2
$D_{KIRISCI1}(M,N)$	0.422	0.422	0.4221	**Not Classified**
$D_{KIRISCI2}(M,N)$	0.026	0.022	0.035	M_3
$S_{SHAH1}(M,N)$	0.622	0.500	0.566	M_1
$S_{SHAH2}(M,N)$	0.721	0.614	0.688	M_1
$S_{KIRISCI1}(M,N)$	0.031	0.038	0.025	M_2
$S_{KIRISCI2}(M,N)$	0.007	0.007	0.007	**Not Classified**

5.3. Comparative Analysis

The Cs suggested in this work will be contrasted with previously with previously investigated measuring methods.

First method [7]:

$$C_{CHENG_{CHANG}}(M,N) = 1 - \frac{1}{n}\sum_{k=1}^{n}\left([f_M(t_k) - f_N(t_k)] \times \left[1 - \frac{h_M(t_k) + h_N(t_k)}{2}\right]\right) + \left[\int_0^1 |f_M(t_k) - f_N(t_k)| dx\right] \times \left[\frac{h_M(t_k) + h_N(t_k)}{2}\right] \quad (13)$$

where

$$f_{M_{tk}}(x) = \begin{cases} 1, & if\ x = f_M(t_k) = 1 - g_M(t_k), \\ \dfrac{1 - g_M(t_k) - x}{1 - f_M(t_k) - g_M(t_k)}, & if\ x \in [f_M(t_k), 1 - g_M(t_k)], \\ 0, & otherwise. \end{cases}$$

Second method [31]:

$$C_{PENG1}(M,N) = \frac{1}{n}\sum_{k=1}^{n}\left[\frac{(f_M(t_k))^2 \wedge (f_N(t_k))^2 + \left((g_M(t_k))^2\right) \wedge \left((g_N(t_k))^2\right)}{(f_M(t_k))^2 \vee (f_N(t_k))^2 + \left((g_M(t_k))^2\right) \vee \left((g_N(t_k))^2\right)}\right] \quad (14)$$

$$C_{PENG1}(M,N) = \frac{1}{n}\sum_{k=1}^{n}\left[\frac{(f_M(t_k))^2 \wedge (f_N(t_k))^2 + \left(1 - (g_M(t_k))^2\right) \wedge \left(1 - (g_N(t_k))^2\right)}{(f_M(t_k))^2 \vee (f_N(t_k))^2 + \left(1 - (g_M(t_k))^2\right) \vee \left(1 - (g_N(t_k))^2\right)}\right] \quad (15)$$

Third method [40]:

$$C_{WEI}(M,N)=\frac{1}{n}\sum_{k=1}^{n}\left[\frac{\left(f_M(t_k)\right)^2\left(f_N(t_k)\right)^2+\left(\left(g_M(t_k)\right)^2\right)\left(\left(g_N(t_k)\right)^2\right)}{\sqrt{\left(f_M(t_k)\right)^4+\left(g_M(t_k)\right)^4}+\sqrt{\left(f_N(t_k)\right)^4+\left(g_N(t_k)\right)^4}}\right] \quad (16)$$

Fourth method [43]:

$$C_{ZHANG}(M,N)=\frac{1}{n}\sum_{k=1}^{n}\frac{\left[|f_M(t_k)^2-g_N(t_k)^2|+|g_M(t_k)^2-f_N(t_k)^2|+|h_M(t_k)^2-h_N(t_k)^2|\right]}{\left[\begin{array}{c}|f_M(t_k)^2-f_N(t_k)^2|+|g_M(t_k)^2-g_N(t_k)^2|+|h_M(t_k)^2-h_N(t_k)^2|+|f_M(t_k)^2-g_N(t_k)^2|\\+|g_M(t_k)^2-f_N(t_k)^2|+|h_M(t_k)^2-h_N(t_k)^2|\end{array}\right]} \quad (17)$$

Table 8 shows that all tests produced the same results, proving that patients H_3 and H_4 had the norovirus and influenza A (H1N1) viruses, respectively. Six measurements for the H_1 patient revealed that he had hepatitis C, two measurements revealed that he had influenza A (H1N1) disease, and one measurement revealed that he had sandfly fever disease when the findings of all techniques were combined. Similarly, five tests revealed Hepatitis C, three revealed influenza A(H1N1), and one revealed Crimean-Congo Haemorrhagic Fever (CCHF) in patient H_2. The findings obtained using the Cs indicated in this investigation were discovered to be identical to the measurements of (13) − (17).

Table 8. Comparison results

	H_1	H_2	H_3	H_4
C	U_3	U_3	U_3	U_5
C_H	U_3	U_3	U_3	U_5
C_P	U_1	U_1	U_3	U_5
C_{PH}	U_1	U_2	U_3	U_5
$C_{CHENG\ CHENG}$	U_1	U_1	U_3	U_5
C_{PENG1}	U_1	U_1	U_3	U_5
C_{PENG2}	U_1	U_1	U_3	U_5
C_{WEI}	U_4	U_3	U_3	U_5
C_{ZHANG}	U_1	U_1	U_3	U_5

6. Discussion

There are several applications for fuzzy and non-standard fuzzy C implementations, including image segmentation, cluster analysis, pattern classification, etc. While appearing to have the same effects, these acts in this scenario lead to different results. Various findings are achieved using various similarity, distance, and correlation measurements. This discrepancy results from these metrics' inability to assess the full degree of uncertainty or needed accuracy in such sets.

The benefits of the suggested techniques can be explained as follows: A beneficial, useful, and greatly generalized model of $\mathcal{IF}$s and $\mathcal{PF}$s is the $\mathcal{FFS}$ method. In this situation, professionals offer their opinions concerning the degree of membership with more independence. If uncertain data are forced to adopt the constrained form of $\mathcal{IFN}$s and $\mathcal{PFN}$s, then the ability to choice the optimal alternative from a collection of alternatives is hindered. Data mutilation would result from the aforementioned situations. To offer effective answers in such pressing situations, a more comprehensive version is required. Given that they are a useful extension of $\mathcal{IF}$s and $\mathcal{PF}$s, $\mathcal{FFS}$s produce more accurate and precise results when applied to real-world MCGDM situations that contain $\mathcal{FF}$ information.

The first differentiating aspect of $\mathcal{FF}$-based Cs is that they are more comparable to human judgment due to the presence of additional cognitive ambiguity features such as MD, ND, and neutrality. Once they get their values in the range [-1, 1], the new *C*s are able to appreciate two negatively associated qualities. Given these advantages, the proposed *C*s appear to be useful for estimating real-world knowledge in data challenges.

Notwithstanding all of the benefits, the new *C*s have several restrictions. The *C*s for $\mathcal{FFS}$s are difficult to implement to the clean data present in storages and other places. This may be achieved via transformation equations or by establishing a multi-dimensional language database.

Weakness of the new approach: The proposed extension of the *C* method uses variance and covariance. When *C* is calculated according to the values obtained in these concepts, the formation of equal values will complicate the decision-making process. In this case, the variance and covariance values need to be reconsidered.

7. Conclusion

First, let us go through the benefits of the provided strategy and how it differs from others. $\mathcal{FFS}$s, as is well known, can examine situations with ambiguity and insufficient info more effectively than $\mathcal{IF}$s. Given that the sets of Pythagorean and intuitionistic MDs are not as large as the sets of Fermatean FMs [32], it is obvious that $\mathcal{FFS}$s will have far more comprehensive options for finding and resolving uncertainty than $\mathcal{IF}$ and $\mathcal{PF}$.

As defined by MD + ND ≤ 1, $\mathcal{IF}$ is a accomplished extension of $\mathcal{F}$ for dealing with ambiguity. In some circumstances, it will be MD+ND>1. The $\mathcal{IF}$ approach will be inadequate to fix this problem in these circumstances. $\mathcal{PF}$, which Yager pioneered, has evolved to address this shortcoming. $\mathcal{PF}$ is a natural expansion of $\mathcal{F}$ theory that yields positive outcomes. However, there are some cases where $MD^2 + ND^2 > 1$ becomes. In this case $\mathcal{PF}$ will not be an effective solution technique.

In the literature, there exist *C*s derived using $\mathcal{IF}$s and $\mathcal{PF}$s, as well as methods established employing these *C*s. As previously stated, some instances may not be represented by $\mathcal{IF}$s and $\mathcal{PF}$s, and therefore suitable outcomes from their respective algorithms may not be produced. The *C*s acquired with $\mathcal{IF}$s and $\mathcal{PF}$s are a subset of the *C*s obtained with $\mathcal{FFS}$s. The proposed *C* is hence more generic than current ones and better suited to handling real-world situations.

Correlation is a statistical method that reveals the relationship between two components. A *C* is the primary consequence of a correlation. This research is devoted to characterizing a *C* for $\mathcal{FFS}$. In this paper, the restriction criteria MD+ND ≤ 1 for $\mathcal{IF}$ and $MD^2 + ND^2 \leq 1$ for $\mathcal{PF}$ were extended to the $\mathcal{FFS}$ *C*. The numerical example that demonstrates supplied *C* may readily operate in situations where the existing *C*s in the $\mathcal{IF}$ and $\mathcal{PF}$ structures fail. The essential properties of $\mathcal{FFS}$s are that each $\mathcal{FFS}$ has an MF and an NF based on the items in the sample space. As a result, the association between $\mathcal{FFS}$s has distinct properties. The *C*s of $\mathcal{FFS}$s according to both MFs and NFs are examined, as have the results acquired by numerous researchers in prior studies. The example cases demonstrate how the provided *C* in the $\mathcal{FFS}$ structure may easily operate the real-life problem with their goals. The benefits of *C*s characterized in the $\mathcal{FFS}$ are demonstrated by computed outcomes as follows: The findings obtained with the proposed *C*s are more sensitive. As a consequence, computational overheads are decreased, and the outcomes are more applicable to real-world settings.

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