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Anti-Fuzzy Bi-Ideals in Semirings Under S-norms

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A R T I C L E I N F O

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ABSTRACT

In this paper, by using S-norms, the notion of anti fuzzy bi-ideals in semirings will be defined and investigated some properties of them. Next prime, strongly prime, semiprime, irreducible, strongly irreducible of them will be introduced and obtained some results about them. Latter, they will be investigated under regular and intra-regular semirings. Finally, they will be characterized under totally ordered by inclusion.

1. Introduction

In algebra, ring theory is the study of rings algebraic structures in which addition and multiplication are defined and have similar properties to those operations defined for the integers. Ring theory studies the structure of rings, their representations, or, in different language, modules, special classes of rings (group rings, division rings, universal enveloping algebras), as well as an array of properties that proved to be of interest both within the theory itself and for its applications, such as homological properties and polynomial identities. In abstract algebra, a semiring is an algebraic structure similar to a ring, but without the requirement that each element must have an additive inverse. The term rig is also used occasionally this originated as a joke, suggesting that rigs are rings without negative elements, similar to using rng to mean a ring without a multiplicative identity. Von Neumann regular rings were introduced by von Neumann (1936) under the name of "regular rings", during his study of von Neumann algebras and continuous geometry. Since its inception in 1965, the theory of fuzzy sets has advanced in a variety of ways and in many disciplines. Applications of this theory can be found, for example, in artificial intelligence, computer science, medicine, control engineering, decision theory, expert systems, logic, management science, operations research, pattern recognition, and robotics. Mathematical developments have advanced to a very high standard and are still forthcoming to day. The theory of fuzzy sets

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was first inspired by Zadeh [26]. Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among mathematicians working in different fields of mathematics. Fuzzy ideals in rings were introduced by Liu [5] and it has been studied by several authors. Mukhrejee and Sen [6, 7] defined fuzzy ideal of of semirings. Jun [3] and Kim and Park [4] have also studied fuzzy ideals in semirings. Lattice theory is the study of sets of objects known as lattices. It is an outgrowth of the study of Boolean algebras, and provides a framework for unifying the study of classes or ordered sets in mathematics. The study of lattice theory was given a great boost by a series of papers and subsequent textbook written by Birkhoff (1967). Shabir, Jun and Bano [22] introduced the notions of prime, strongly prime, semiprime and irreducible fuzzy bi-ideals of a semigroup.The author by using norms, investigated some properties of fuzzy algebraic structures[12-21]. In this paper, Firstly, we define anti fuzzy bi-ideal of semiring R under s-norm S as $AFBIS(R)$. Secondly, we define symbol $\vee$ and sum and product of two $\mu, \vartheta \in AFBIS(R)$ and we investigate some properties about them. Thirdly, we define prime, strongly prime, idempotent, semiprime, irreducible and strongly irreducible of $\mu \in AFBIS(R)$ and prove some results about it. Fourthly, we prove that if R be a regular and intra-regular semiring and $\mu \in AFBIS(R)$, then μ is a strongly prime if and only if μ be a strongly irreducible. Also if the set $A = \{\mu : \mu \in AFBIS(R)\}$ is totally ordered by inclusion $(\leq)$, then each $\mu \in A$ is a prime. Finally, we prove that the following assertions are equivalent:

(1) Set of $A = \{\mu : \mu \in AFBIS(R)\}$ is totally ordered by inclusion $(\leq)$.
(2) Each $\mu \in A$ is strongly irreducible.
(3) Each $\mu \in A$ is an irreducible.

2. Preliminaries

This section contains some basic definitions and preliminary results which will be needed in the sequel.

Definition 1. (See [24]) A semiring is a set R equipped with two binary operations $+$ and ., called addition and multiplication, such that:

(1) $(R, +)$ is a commutative monoid with identity element 0:

$$(a + b) + c = a + (b + c)$$
$$0 + a = a + 0 = a$$
$$a + b = b + a.$$

(2) $(R, .)$ is a monoid with identity element 1:

$$(a.b).c = a.(b.c)$$
$$1.a = a.1 = a.$$

(3) Multiplication left and right distributes over addition:

$$a.(b + c) = a.b + a.c$$
$$(a + b).c = a.c + b.c$$

(4) Multiplication by 0 annihilates R:

$$0.a = a.0 = 0.$$

The symbol . is usually omitted from the notation; that is, $a.b$ is just written ab. Similarly, an order of operations is accepted, according to which . is applied before +; that is, $a+bc$ is $a+(bc)$. A commutative semiring is one whose multiplication is commutative. Throughout this paper, R stands for the semiring $(R,+,.)$ with an identity element 1_R and zero element 0_R.

Example 1. By definition, any ring is also a semiring. A motivating example of a semiring is the set of integer numbers Z under ordinary addition and multiplicationand it is commutative semiring.

Definition 2. (See [8]) A non-empty subset B of a semiring R is called a bi-ideal of R if
(1) $a+b\in B$,
(2) $ab\in B$,
(3) $arb \in B$,
for all $a,b \in B$ and $r\in R$.

Definition 3. (See [25]) (1) A semiring R is called von Neumann regular or simply regular if for each $a\in R$ there exists $x\in R$ such that $a=axa$.
(2) A semiring R is called an intra-regular semiring if for each $a\in R$ there exists $x_i,y_i\in R$ such that $a = \sum_{i=1}^{n} x_i a^2 y_i$.

Definition 4. (See [2]) A (non-strict) partial order is a binary relation ≤ over a set P satisfying particular axioms which are discussed below. When $a\leq b$, we say that a is related to b. (This does not imply that b is also related to a, because the relation need not be symmetric.) The axioms for a non-strict partial order state that the relation ≤ is reflexive, antisymmetric, and transitive. That is, for all $a,b,c\in P$, it must satisfy:
(1) $a\leq a$ (reflexivity: every element is related to itself).
(2) if $a\leq b$ and $b\leq a$, then $a= b$ (antisymmetry: two distinct elements cannot be related in both directions).
(3) if $a\leq b$ and $b\leq c$, then $a\leq c$ (transitivity: if a first element is related to a second element, and, in turn, that element is related to a third element, then the first element is related to the third element).

In other words, a partial order is an antisymmetric preorder. A set with a partial order is called a partially ordered set (also called a poset). The term ordered set is sometimes also used, as long as it is clear from the context that no other kind of order is meant. In particular, totally ordered sets can also be referred to as "ordered sets", especially in areas where these structures are more common than posets. For a,b, elements of a partially ordered set P, if $a\leq b$ or $b\leq a$, then a and b are comparable. Otherwise they are incomparable. For example $\{x\}$ and $\{x,y,z\}$ are comparable, while $\{x\}$ and $\{y\}$ are not. A partial order under which every pair of elements is comparable is called a total order or linear order; a totally ordered set is also called a chain (e.g., the natural numbers with their standard order). A subset of a poset in which no two distinct elements are comparable is called an antichain. For example set of singletons $\{\{x\},\{y\},\{z\}\}$.

Lemma 1. (See [23]) (Zorn's lemma) Suppose a partially ordered set P has the property that every chain in P has an upper bound in P. Then the set P contains at least one maximal element.

Definition 5. Let X be a non-empty subset of R. Then the anti characteristic function of R denoted and defined by

$$\mu_X(a)=\begin{cases}0 & a\in X\\ 1 & a\notin X.\end{cases}$$

Definition 6. (See [1]) An s-norm S is a function $S:[0,1]\times[0,1]\to[0,1]$ having the following four properties for all $x,y,z\in[0,1]$:
(1) $S(x,0)=x$,
(2) $S(x,y)\leq S(x,z) if\ y\leq z$,
(3) $S(x,y)= S(y,x)$,
(4) $S(x,S(y,z))=S(S(x,y),z)$,

We say that S is idempotent if for all $x \in [0,1], S(x,x) = x$.

Example 2. The basic S-norms are

$$S_m(x,y) = max\{x,y\},$$

$$S_b(x,y) = min\{1, x + y\}$$

and

$$S_p(x,y) = x + y - xy$$

for all $x, y \in [0,1]$.

S_m is standard union, S_b is bounded sum, S_p is algebraic sum.

Lemma 2. (See [1]) Let S be a s-norm. Then

$$S(S(x,y), S(w,z)) = S(S(x,w), S(y,z)),$$

for all $x, y, w, z \in [0,1]$.

3. Anti fuzzy bi-ideals under s-norms

Definition 7. A fuzzy subset $\mu : R \to [0,1]$ is called an anti fuzzy bi-ideal of R under s-norm S if
(1) $\mu(a+b) \leq S(\mu(a), \mu(b))$,
(2) $\mu(ab) \leq S(\mu(a), \mu(b))$,
(3) $\mu(abc) \leq S(\mu(a), \mu(c))$,
for all $a, b, c \in R$.
We denote the set of all anti fuzzy bi-ideals of semiring R under s-norm S by $AFBIS(R)$.

Example 3. Let $R = (R, +, .)$ be a semiring of real numbers. Define $\mu : R \to [0,1]$ by

$$\mu(x) = \begin{cases} 0.30 & x \in \mathbb{R}^{\geq 0} \\ 0.50 & x\mathbb{R}^{<0} \end{cases}$$

if $Sp(a,a) = a + b - ab$ for all $a, b \in [0,1]$, then $\mu \in AFBIS(R)$.

Definition 8. Let $\mu, \vartheta \in AFBIS(R)$.
(1) The symbol $\mu \vee \vartheta$ is a fuzzy subset $\mu \vee \vartheta : R \to [0,1]$ and defined dy

$$(\mu \vee \vartheta)(a) = S(\mu(a), \vartheta(a))$$

for all $a \in R$.
(2) The sum of μ and ϑ is a fuzzy subset $\mu + \vartheta : R \to [0,1]$ with

$$(\mu + \vartheta)(a) = \bigwedge_{a=b+c} S(\mu(b), \vartheta(c))$$

for all $a, b, c \in R$.

(3) The product of μ and ϑ is a fuzzy subset $\mu \, o \, \vartheta : R \to [0,1]$ by

$$(\mu\ o\ \vartheta)(a) = \bigwedge_{a=bc} S(\mu(b), \vartheta(c))$$

for all $a, b, c \in R$.

Remark 1. Let $\mu \in AFBIS(R)$ then for all $b, c \in R$ we have that

$$(\mu\ o\ \mu_R)(bc) = \bigwedge_{bc} S\big(\mu(b), \mu_R(c)\big) = \bigwedge_{bc} S(\mu(b), 0) = \mu(b).$$

Proposition 1.
(1) $\mu \in AFBIS(R)$ if and only if $\mu \le \mu + \mu$ and $\mu \le \mu o \mu$ and $\mu \le \mu o \mu_R\ o \mu$.
(2) A non-empty subset B of a semiring R is a bi-ideal of R if and only if the anti characteristic function $\mu_B \in AFBIS(R)$.
(3) Let $\mu, \vartheta \in AFBIS(R)$. Then $\mu \vee \vartheta \in AFBIS(R)$.
(4) Let $\mu, \vartheta \in AFBIS(R)$ such that R be commutative. Then $\mu\ o\ \vartheta \in AFBIS(R)$.
Proof. (1) Let $\mu \in AFBIS(R)$.
If $a = b + c$, then

$$\mu(a) = \mu(b + c) \le S(\mu(b), \mu(c)) \le \bigwedge_{a=b+c} S(\mu(b), \mu(c))$$

for all $a, b, c \in R$ and so $\mu \le \mu + \mu$.
If $a = bc$, then

$$\mu(a) = \mu(b\ c) \le S(\mu(b), \mu(c)) \le \bigwedge_{a=bc} S(\mu(b), \mu(c))$$

for all $a, b, c \in R$ and so $\mu \le \mu\ o\ \mu$.
If $a = bcd$, then

$$\mu(a) = \mu(bcd) \le S(\mu(b), \mu(d)) = S((\mu\ o\ \mu_R)(bc), \mu(d)) \le \bigwedge_{a=bcd} S((\mu\ o\ \mu_R)(bc), \mu(d))$$

and *then* $\mu \le \mu\ o\ \mu_R\ o\ \mu$.
Conversely, we prove that $\mu \in AFS(R)$. Since $\mu \le \mu + \mu$ so

$$\mu(a+b) \le (\mu + \mu)(a+b) = \bigwedge_{a+b} S(\mu(a), (\mu(b)) \le S(\mu(a), (\mu(b))$$

and then

$$\mu(a + b) \le S(\mu(a), (\mu(b)) \tag{1}$$

for all $a, b \in R$. As $\mu \le \mu\ o\ \mu$ so

$$\mu(ab) \le (\mu\ o\ \mu)(ab) = \bigwedge_{ab} S(\mu(a), (\mu(b)) \le S(\mu(a), (\mu(b))$$

and then

$\mu(ab) \leq S(\mu(a), (\mu(b))$ (2)
for all $a, b \in R$.
Also since $\mu \leq \mu \, o \, \mu_R \, o \, \mu$ so

$$\mu(abc) \leq (\mu \circ \mu_R \circ \mu)(abc) = \bigwedge_{(ab)c} S\big((\mu \circ \mu_R)(ab), \mu(c)\big) = \bigwedge_{ac} S(\mu(a), \mu(c)) \leq S(\mu(a), \mu(c))$$

and then
$\mu(abc) \leq S(\mu(a), \mu(c))$ (3)

for all $a, b, c \in R$.
Then from (1)-(3) we get that $\mu \in AFBIS(R)$.

(2) Let

$$\mu_B(b) = \begin{cases} 0, & x \in B \\ 1, & x \notin B \end{cases}$$

be the anti characteristic function of B. Let B be a bi-ideal of semiring R such that $b1, b2 \in B$ and $r \in R$. Then $b_1 + b_2 \in B$ and $b_1 b_2 \in B$ and $b_1 r b_2 \in B$ and then

$$\mu_B(b_1 + b_2) = 0 \leq 0 = S(0,0) = S(\mu_B(b_1), \mu_B(b_2))$$

and

$$\mu_B(b_1 b_2) = 0 \leq 0 = S(0,0) = S(\mu_B(b_1), \mu_B(b_2))$$

and

$$\mu_B(b_1 r b_2) = 0 \leq 0 = S(0,0) = S\big(\mu_B(b_1), \mu_B(b_2)\big).$$

Then $\mu_B \in AFBIS(R)$.

Conversely, let $\mu_B \in AFBIS(R)$ and $b_1, b_2 \in B$ and $r \in R$.

As

$$\mu_B(b_1 + b_2) \leq S(\mu_B(b_1), \mu_B(b_2)) = S(0,0) = 0$$

so $\mu_B(b_1 + b_2) = 0$ and then $b_1 + b_2 \in B$. Since

$$\mu_B(b_1 b_2) \leq S(\mu_B(b_1), \mu_B(b_2)) = S(0,0) = 0$$

so $\mu_B(b_1 b_2) = 0$ and then $b_1 b_2 \in B$. Finally

$$\mu_B(b_1 r b_2) \leq S(\mu_B(b_1), \mu_B(b_2)) = S(0,0) = 0$$

so $\mu_B(b_1 r b_2) = 0$ and then $b_1 r b_2 \in B$. Then B is a bi-ideal of a semiring R.

(3) Let $\mu, \vartheta \in AFBIS(R)$ and $a, b, c \in R$. Then

$$(\mu \vee \vartheta)(a + b) = S(\mu(a + b), \vartheta(a + b))$$

$$\leq S(S(\mu(a),\mu(b)),S(\vartheta(a),\vartheta(b)))$$
$$= S(S(\mu(a),\vartheta(a)),S(\mu(b),\vartheta(b)))$$
$$= S((\mu \vee \vartheta)(a),(\mu \vee \vartheta)(b))$$

and so

$$(\mu \vee \vartheta)(a+b) \leq S\big((\mu \vee \vartheta)(a),(\mu \vee \vartheta)(b)\big). \quad (4)$$

Also

$$(\mu \vee \vartheta)(ab) = S(\mu(ab),\vartheta(ab))$$
$$\leq S(S(\mu(a),\mu(b)),S(\vartheta(a),\vartheta(b)))$$
$$= S(S(\mu(a),\vartheta(a)),S(\mu(b),\vartheta(b)))$$
$$= S((\mu \vee \vartheta)(a),(\mu \vee \vartheta)(b))$$

thus

$$(\mu \vee \vartheta)(ab) \leq S\big((\mu \vee \vartheta)(a),(\mu \vee \vartheta)(b)\big). \quad (5)$$

Further

$$(\mu \vee \vartheta)(abc) = S\big(\mu(abc),\vartheta(abc)\big)$$
$$\leq S(S(\mu(a),\mu(c)),S(\vartheta(a),\vartheta(c)))$$
$$= S(S(\mu(a),\vartheta(a)),S(\mu(c),\vartheta(c)))$$
$$= S((\mu \wedge \vartheta)(a),(\mu \wedge \vartheta)(c))$$

then

$$(\mu \vee \vartheta)(abc) \leq S((\mu \wedge \vartheta)(a),(\mu \wedge \vartheta)(c)). \quad (6)$$

Then from (4)-(6) we get that $\mu \vee \vartheta \in AFBIS(R)$.

(4) Let $\mu,\vartheta \in AFS(R)$ and let $a_1,a_2,b_1,b_2,c_1,c_2 \in R$ such that

$$a_1 = b_1c_1, a_2 = b_2c_2, a_3 = b_3c_3, b_1c_2 = b_2c_1 = 0.$$

Then

and so

$$(\mu\, o\, \vartheta)(a_1+a_2) \leq \mathrm{S}((\mu\, \mathrm{o}\, \vartheta)(a_1),(\mu\, \mathrm{o}\, \vartheta)(a_2)) . \quad (7)$$

Also

$$(\mu\, o\, \vartheta)(a_1+a_2) = \bigwedge_{(a_1+a_2)=(b_1+b_2)(c_1+c_2)} S\big(\mu(b_1+b_2),\vartheta(c_1+c_2)\big)$$

$$= \bigwedge_{(a_1+a_2)=b_1c_1+b_2c_2} S\big(\mu(b_1+b_2),\vartheta(c_1+c_2)\big)$$

$$= \bigwedge_{a_1=b_1c_1,a_2=b_2c_2} S\big(\mu(b_1+b_2),\vartheta(c_1+c_2)\big)$$

$$\leq \wedge_{a_1=b_1c_1, a_2=b_2c_2} \mathrm{S}\left(\mathrm{S}\big(\mu(b_1), \mu(b_2)\big), \mathrm{S}\big(\vartheta(c_1), \vartheta(c_2)\big)\right)$$

$$= \bigwedge_{a_1=b_1c_1, a_2=b_2c_2} \mathrm{S}\left(\mathrm{S}\big(\mu(b_1), \vartheta(c_1)\big), \mathrm{S}\big(\mu(b_2), \vartheta(c_2)\big)\right)$$

$$= \mathrm{S}\left(\bigwedge_{a_1=b_1c_1} \mathrm{S}\big(\mu(b_1), \vartheta(c_1)\big), \bigwedge_{a_2=b_2c_2} \mathrm{S}\big(\mu(b_2), \vartheta(c_2)\big)\right)$$

$$= \mathrm{S}\big((\mu \text{ o } \vartheta)(a_1), (\mu \text{ o } \vartheta)(a_2)\big)$$

and

$$(\mu\ o\ \vartheta)(a_1a_2) = \bigwedge_{(a_1a_2)=(b_1b_2)(c_1c_2)} S\big(\mu(b_1b_2), \vartheta(c_1c_2)\big)$$

$$= \bigwedge_{(a_1a_2)=b_1c_1b_2c_2} S\big(\mu(b_1b_2), \vartheta(c_1c_2)\big)$$

$$= \bigwedge_{a_1=b_1c_1, a_2=b_2c_2} S\big(\mu(b_1b_2), \vartheta(c_1c_2)\big)$$

$$\leq \bigwedge_{a_1=b_1c_1, a_2=b_2c_2} \mathrm{S}\left(\mathrm{S}\big(\mu(b_1), \mu(b_2)\big), \mathrm{S}\big(\vartheta(c_1), \vartheta(c_2)\big)\right)$$

$$= \bigwedge_{a_1=b_1c_1, a_2=b_2c_2} \mathrm{S}\left(\mathrm{S}\big(\mu(b_1), \vartheta(c_1)\big), \mathrm{S}\big(\mu(b_2), \vartheta(c_2)\big)\right)$$

$$= \mathrm{S}\left(\bigwedge_{a_1=b_1c_1} \mathrm{S}\big(\mu(b_1), \vartheta(c_1)\big), \bigwedge_{a_2=b_2c_2} \mathrm{S}\big(\mu(b_2), \vartheta(c_2)\big)\right) = \mathrm{S}\big((\mu \text{ o } \vartheta)(a_1), (\mu \text{ o } \vartheta)(a_2)\big)$$

thus

$$(\mu\ o\ \vartheta)(a_1a_2) \ \leq\ \mathrm{S}((\mu \text{ o } \vartheta)(a_1), (\mu \text{ o } \vartheta)(a_2))\,. \tag{8}$$

Further

$$(\mu\ o\ \vartheta)(a_1a_2a_3) = \bigwedge_{(a_1a_2a_3)=(b_1b_2b_3)(c_1c_2c_3)} S\big(\mu(b_1b_2b_3), \vartheta(c_1c_2c_3)\big)$$

$$= \bigwedge_{(a_1a_2a_3)=b_1c_1b_2c_2b_3c_3} S\big(\mu(b_1b_2b_3), \vartheta(c_1c_2c_3)\big)$$

$$= \bigwedge_{a_1=b_1c_1, a_2=b_2c_2, a_3=b_3c_3} S\big(\mu(b_1b_2b_3), \vartheta(c_1c_2c_3)\big)$$

$$\leq \wedge_{a_1=b_1c_1, a_2=b_2c_2, a_3=b_3c_3} \mathrm{S}\left(\mathrm{S}\big(\mu(b_1), \mu(b_3)\big), \mathrm{S}\big(\vartheta(c_1), \vartheta(c_3)\big)\right)$$

$$= \bigwedge_{a_1=b_1c_1, a_3=b_3c_3} \mathrm{S}\left(\mathrm{S}\big(\mu(b_1), \vartheta(c_1)\big), \mathrm{S}\big(\mu(b_3), \vartheta(c_3)\big)\right)$$

$$= \mathrm{S}\left(\bigwedge_{a_1=b_1c_1} \mathrm{S}(\mu(b_1),\vartheta(c_1)), \bigwedge_{a_3=b_3c_3} \mathrm{S}(\mu(b_2),\vartheta(c_2))\right)$$

$$= \mathrm{S}((\mu \text{ o } \vartheta)(a_1),(\mu \text{ o } \vartheta)(a_3))$$

and then

$$(\mu\ o\ \vartheta)(a_1a_2a_3) \leq \mathrm{S}((\mu \text{ o } \vartheta)(a_1),(\mu \text{ o } \vartheta)(a_3)). \quad (9)$$

Thus from (8)-(9) we obtain that $\mu\ o\ \vartheta\ \in AFBIS(R)$.

Proposition 2. *Let $\mu,\vartheta\ \in AFBIS(R)$ and X, Y be two non-empty subsets of semiring R.*
(1) If $\mu \geq\ \vartheta$, then $\mu\ o\ \beta\ \geq\ \vartheta\ o\ \beta$ and $\beta\ o\ \mu\ \geq\ \beta\ o\ \vartheta$.
(2) $\mu_X\ o\ \ \mu_Y = \mu_{XY}$.
(3) $\mu_X\ \vee\ \mu_Y\ =\ \mu_{X\cap Y}$.
(4) $\mu_X +\ \mu_X =\ \mu_{X+Y}$.

Proof. Let $x, y, z\ \in R$.

(1) As $\mu \geq\ \vartheta$ *so* $\mu(y) \geq\ \vartheta(y)$. *Then*

$$(\mu \text{ o } \beta)(\mathrm{x}) = \bigwedge_{x=yz} S(\mu(y),\beta(z)) \geq \bigwedge_{x=yz} S(\vartheta(y),\beta(z)) = (\vartheta \text{o } \beta)(x)$$

and then μ o $\beta\ \geq \vartheta$o β. Also as $\mu\ \geq\ \vartheta$, then $\mu(z) \geq \vartheta(z)$ and

$$(\beta o \mu\ )(x) = \bigwedge_{x=yz} S(\beta(y),\mu(z)) \geq \bigwedge_{x=yz} S(\beta(y),\vartheta(z)) = (\beta o \vartheta)(x)$$

and thus $\beta\ o\ \mu \geq\ \beta\ o\ \vartheta$.

(3) We know that $\mu_X(x) = \begin{cases} 0, & x \in X \\ 1, & x \notin X \end{cases}$ and $\mu_Y(y) = \begin{cases} 0, & y \in Y \\ 1, & y \notin Y \end{cases}$ and

$$\mu_{XY}(z) = \begin{cases} 0, & z \in XY \\ 1, & z \notin XY \end{cases} = \begin{cases} 0, & z = xy \in XY \\ 1, & z = xy \notin XY \end{cases} = \begin{cases} 0, & x \in X, y \in Y \\ 1, & x \notin X, y \notin Y. \end{cases}$$

If $x\ \in X$ and $y\ \in Y$, then $xy\ \in XY$ therefore

$$(\mu_X o\ \mu_X\ )(z)\ = \bigwedge_{z=xy} S(\mu_X(x),\mu_Y(y)) = \bigwedge_{z=xy} S(0,0) = 0 = \mu_{XY}(\mathrm{z} = \mathrm{xy}).$$

If $x\ \in X$ and $y\ \notin Y$, then $xy \notin XY$ and so

$$(\mu_X o\ \mu_X\ )(z)\ = \bigwedge_{z=xy} S(\mu_X(x),\mu_Y(y)) = \bigwedge_{z=xy} S(0,1) = 1 = \mu_{XY}(\mathrm{z} = \mathrm{xy}).$$

If $x\ \notin X$ and $y \in Y$, then $xy \notin XY$ and then

$$(\mu_X o\ \mu_X)(z) = \bigwedge_{z=xy} S(\mu_X(x), \mu_Y(y)) = \bigwedge_{z=xy} S(1,0) = 1 = \mu_{XY}(\mathrm{z = xy}).$$

If $x \notin X$ and $y \notin Y$, then $xy \notin XY$ then

$$(\mu_X o\ \mu_X)(z) = \bigwedge_{z=xy} S(\mu_X(x), \mu_Y(y)) = \bigwedge_{z=xy} S(1,1) = 1 = \mu_{XY}(\mathrm{z = xy}).$$

Thus $\mu_X\ o\ \mu_X = \mu_{XY}$.

(4) We have that

$$\mu_{X\cap Y}(z) = \begin{cases} 0, & z \in X \cap Y \\ 1, & otherwise \end{cases} = \begin{cases} 0, & z \in X, z \in Y \\ 1, & otherwise. \end{cases}$$

If $z \in X$ and $z \in Y$, then $z \in X \cap Y$ then

$$(\mu_X \vee \mu_Y)(z) = S(\mu_X(z), \mu_Y(z)) = S(0,0) = 0 = \mu_{X\cap Y}(z).$$

If $z \in X$ and $z \notin Y$, then $z \notin X \cap Y$ so

$$(\mu_X \vee \mu_Y)(z) = S(\mu_X(z), \mu_Y(z)) = S(0,1) = 1 = \mu_{X\cap Y}(z).$$

If $z \notin X$ and $z \in Y$, then $z \notin X \cap Y$ then

$$(\mu_X \vee \mu_Y)(z) = S(\mu_X(z), \mu_Y(z)) = S(1,0) = 1 = \mu_{X\cap Y}(z).$$

If $z \notin X$ and $z \notin Y$, then $z \notin X \cap Y$ thus

$$(\mu_X \vee \mu_Y)(z) = S(\mu_X(z), \mu_Y(z)) = S(1,1) = 1 = \mu_{X\cap Y}(z).$$

Then $\mu_X \vee \mu_Y = \mu_{X\cap Y}$.

(1) We get that

$$\mu_{X+Y}(z) = \begin{cases} 0, & z \in X + Y \\ 1, & otherwise \end{cases} = \begin{cases} 0, & z = x + y \in X + Y \\ 1, & otherwise \end{cases} = \begin{cases} 0, & x \in X, y \in Y \\ 1, & otherwise. \end{cases}$$

If $x \in X$ and $y \in Y$, then $x + y \in X + Y$ thus

$$(\mu_X + \mu_Y)(z) = \bigwedge_{z=x+y} S(\mu_X(x), \mu_Y(y)) = \bigwedge_{z=x+y} S(0,0) = 0 = \mu_{X+Y}(z = x + y).$$

If $x \in X$ and $y \notin Y$, then $x + y \notin X + Y$ so

$$(\mu_X + \mu_Y)(z) = \bigwedge_{z=x+y} S(\mu_X(x), \mu_Y(y)) = \bigwedge_{z=x+y} S(0,1) = 1 = \mu_{X+Y}(z = x + y).$$

If $x \notin X$ and $y \in Y$, then $x + y \notin X + Y$ then

$$(\mu_X + \mu_Y)(z) = \bigwedge_{z=x+y} S(\mu_X(x), \mu_Y(y)) = \bigwedge_{z=x+y} S(1,0) = 1 = \mu_{X+Y}(z = x + y).$$

If $x \notin X$ and $y \notin Y$, then $x + y \notin X + Y$ then

$$(\mu_X + \mu_Y)(z) = \bigwedge_{z=x+y} S(\mu_X(x), \mu_Y(y)) = \bigwedge_{z=x+y} S(1,1) = 1 = \mu_{X+Y}(z = x + y).$$

4. Prime, strongly prime, semiprime, irreducible and strongly irreducible of $AFBIS(R)$

Definition 9. Let $\mu \in AFBIS(R)$.
(1) μ is called a prime if for any $\alpha, \beta \in AFBIS(R)$, if $\alpha\ o\ \beta \geq \mu$, then $\alpha \geq \mu$ or $\beta \geq \mu$.
(2) μ is called a strongly prime if for any $\alpha, \beta \in AFBIS(R)$, if $(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \geq \mu$, then $\alpha \geq \mu$ or $\beta \geq \mu$.
(3) μ is called idempotent if $\mu = \mu\ o\ \mu = \mu^2$.
(4) μ is said to be a semiprime if $\alpha\ o\ \alpha = \alpha^2 \geq \mu$ implies $\alpha \geq \mu$ for every $\alpha \in AFBIS(R)$.
(5) μ is said to be an irreducible if for any $\alpha, \beta \in AFBIS(R)$, if $\alpha \vee \beta = \mu$, then $\alpha = \mu$ or $\beta = \mu$.

(6) μ is said to be a strongly irreducible if for any $\alpha, \beta \in AFBIS(R)$, if $\alpha \vee \beta \geq \mu$, then $\alpha \geq \mu$ or $\beta \geq \mu$.

Proposition 3. *We have the following assertions.*
(1) Let $\mu \in AFBIS(R)$ be strongly prime then it will be prime.
(2) If $\mu \in AFBIS(R)$ be prime, then it will be semiprime.
(3) Let $\mu_1, \mu_2 \in AFBIS(R)$ be prime. Then $\mu_1 \vee \mu_2$ will *be semiprime.*
(4) Let $\mu \in AFBIS(R)$ be strongly irreducible semiprime then it willbe strongly prime.
Proof. Let $\mu, \alpha, \beta \in AFBIS(R)$.
(1) Let μ be strongly prime and $\alpha\ o\ \beta \geq \mu$ then $(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \geq \mu$, and so $\alpha \geq \mu$ or $\beta \geq \mu$. This implies that μ will be prime.
(2) Let μ be prime and $\alpha\, o\, \alpha \geq \mu$ then $\alpha \geq \mu$ and then μ is semiprime.
(3) Let μ_1 and μ_2 be primes and α o $\beta \geq \mu_1 \vee \mu_2$. Then $\alpha\ o\ \beta \geq \mu_1$ or $\alpha\ o\ \beta \geq \mu_2$. This implies that $\alpha \geq \mu_1$ or $\beta \geq \mu_1$and $\alpha \geq \mu_2$ or $\beta \geq \mu_2$. Thus $\alpha \geq \mu_1 \vee \mu_2$ and $\beta \geq \mu_1 \vee \mu_2$. Therefore $\mu_1 \vee \mu_2$will be prime.
(4) Let $\mu \in AFBIS(R)$ be strongly irreducible semiprime such that $(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \geq \mu$. As $\alpha \vee \beta \geq \alpha$ and $\alpha \vee \beta \geq \beta$ so

$$(\alpha \vee \beta) o (\alpha \vee \beta) = (\alpha \vee \beta)^2 \geq \alpha o \beta.$$

Also $\alpha \vee \beta \geq \beta$ and $\alpha \vee \beta \geq \alpha$ so

$$(\alpha \vee \beta)\ o\ (\alpha \vee \beta) = (\alpha \vee \beta)^2 \geq \beta\ o\ \alpha.$$

Thus

$$(\alpha \vee \beta)^2 \geq (\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \geq \mu.$$

Since μ is a semiprime so $\alpha \vee \beta \geq \mu$. Now since μ is a strongly irreducible then $\alpha \geq \mu$ or $\beta \geq \mu$. Hence μ is a strongly prime.

Proposition 4. Let $\mu \in AFBIS(R)$ with $\mu(a) = \varepsilon > 0$ for all $a \in R$ and $\varepsilon \in (0,1]$. Then there exists an irreducible $\beta \in AFBIS(R)$ such that $\mu \leq \beta$ and $\beta(a) = \varepsilon$ for all $a \in R$ and $\varepsilon \in (0,1]$.

Proof. Let $P = \{\alpha : \alpha \in AFBIS(R) : \mu \leq \alpha : \alpha(a) = \varepsilon > 0\}$. As $\mu \in P$ so $P \neq \emptyset$. Let

$$H = \{ h_i : h_i \in AFBIS(R) : h_i(a) = \varepsilon : \mu \le h_i : \forall i \in I \}$$

be any totally ordered subset of P. Now we prove that $\bigvee_{i\in I} h_i \in AFBIS(R)$ such that $\mu \le \bigvee_{i\in I} h_i$. Assume that $a, b, c \in R$ and as $h_i \in AFBIS(R)$ then

(1)
$$(\bigvee_{i\in I}) h_i(a+b) = \bigvee_{i\in I}\big(h_i(a + b)\big) \le \bigvee_{i\in I} S\big(h_i(a), h_i(b)\big) = S(\bigvee_{i\in I} h_i(a), \bigvee_{i\in I} h_i(b)).$$

(2)
$$(\bigvee_{i\in I}) h_i(ab) = \bigvee_{i\in I}(h_i(a\, b)) \le \bigvee_{i\in I} S(h_i(a), h_i(b)) = S(\bigvee_{i\in I} h_i(a), \bigvee_{i\in I} h_i(b)).$$

(3)
$$(\bigvee_{i\in I}) h_i(abc) = \bigvee_{i\in I}(h_i(a\, bc)) \le \bigvee_{i\in I} S(h_i(a), h_i(c)) = S(\bigvee_{i\in I} h_i(a), \bigvee_{i\in I} h_i(c)).$$

Thus $(\bigvee_{i\in I}) h_i \in AFBIS(R)$.
Since $\mu \le h_i$ for all $i \in I$ then $\mu \le (\bigvee_{i\in I}) h_i$. Also $(\bigvee_{i\in I}) h_i(a) = \bigvee_{i\in I}(h_i)(a) = \varepsilon$ with $\varepsilon \in (0,1]$. Therefore $\bigvee_{i\in I}(h_i) \in P$ and $\bigvee_{i\in I}(h_i)$ is an upper bound of H. Now by Zorn's lemma, there exists a $\beta \in AFBIS(R)$ which is maximal with respect to the property $\mu \le \beta$ and $\beta(a) = \varepsilon$. Now we show that β is an irreducible. Let $\beta_1, \beta_2 \in AFBIS(R)$ such that $\beta_1 \vee \beta 2 = \beta$ then $\beta \ge \beta_1$ and $\beta \ge \beta_2$. We claim that $\beta = \beta_1$ or $\beta = \beta_2$. By the contrary, assume that $\beta \ne \beta_1$ and $\beta \ne \beta 2$. This implies $\beta > \beta_1$ and $\beta > \beta_2$. So $\beta(a) \ne \beta_1(a)$ and $\beta(a) \ne \beta_2(a)$. Hence $(\beta_1 \vee \beta 2)(a) = \beta(a) = \varepsilon$. Which is a contradiction to the fact that $(\beta_1 \vee \beta 2)(a) = \beta(a) \ne \varepsilon$. Hence either $\beta = \beta_1$ or $\beta = \beta_2$.

Proposition 5. Let S be idempotent s-norm. Then for a semiring R the following assertions are hold:
(1) If R is both regular and intra-regular, then $\mu\, o\, \mu = \mu$ for every $\mu \in AFBIS(R$.
(2) If $\mu\, o\, \mu = \mu$ for every $\mu \in AFBIS(R)$, then $\alpha \vee \beta = (\alpha o \beta) \vee (\beta o \alpha)$ for all $\alpha, \beta \in AFBIS(R)$.
Proof. (1) Let R be both regular and intra-regular and $a \in R$. Then there exist elements $x, y_i, z_i \in R$ such that $a = axa$ and $a = \sum_{i=1}^{n} y_i a^2 z_i$ and then

$$a = axa = axaxa = ax\left(\sum_{i=1}^{n} y_i a^2 z_i\right) xa = \sum_{i=1}^{n} (axy_i a)(az_i xa).$$

Now

$$\begin{aligned}(\mu o\, \mu)(a) &= (\mu\, o\, \mu)\left(\textstyle\sum_{i=1}^{n} (axy_i a)(az_i xa)\right) \\ &= \textstyle\sum_{i=1}^{n} (\mu o\, \mu)(axy_i a)(az_i xa)) \\ &= \textstyle\sum_{i=1}^{n} \wedge S\big(\mu(axy_i a), \mu(az_i xa)\big) \\ &\le \sum_{i=1}^{n} S\Big(S\big(\mu(a), \mu(a)\big), S\big(\mu(a), \mu(a)\big)\Big) \\ &= \sum_{i=1}^{n} S\big(\mu(a), \mu(a)\big) \\ &= \sum_{i=1}^{n} \mu(a)\end{aligned}$$

$$= \mu(a).$$

Thus $\mu\ o\ \mu \leq \mu$.
Also as $\mu \in AFBIS(R)$ so by Proposition 1 (part 1) we get that $\mu\ o\ \mu \geq \mu$.
Thus $\mu\ o\ \mu = \mu$.
(2) Let $\alpha, \beta \in AFBIS(R)$ then by Proposition 1 (part 3)we have that $\alpha \vee \beta \in AFBIS(R)$. Thus by hypothesis, we have $\alpha \vee \beta = (\alpha \vee \beta)\ o\ (\alpha \vee \beta)$. As $\alpha \vee \beta \geq \alpha$ and $\alpha \vee \beta \geq \beta$ so $\alpha \vee \beta = (\alpha \vee \beta)\ o\ (\alpha \vee \beta) \geq \alpha\ o\ \beta$. Similarly, $\alpha \vee \beta \geq \beta\ o\ \alpha$. Then

$$\alpha \vee \beta \geq (\alpha\ o\ \beta) \vee (\beta\ o\ \alpha). \tag{10}$$

Now from Proposition 1 (part 4) we have that $\alpha\ o\ \beta \in AFBIS(R)$ and $\beta\ o\ \alpha \in AFBIS(R)$ and by Proposition 1 (part 3) we get that $(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \in AFBIS(R)$. Thus by hypothesis, we have

$$(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) = [(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha)]\ o\ [(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha)]$$

$$\geq (\alpha\ o\ \beta) o\ (\beta\ o\ \alpha) = \alpha\ o\ \underbrace{\beta\ o\ \beta}_{\beta} o\alpha = \alpha\ o\ \underbrace{\beta}_{\geq \mu_R}\ o\ \alpha$$

$$\geq \alpha\ o\ \mu_R\ o\alpha \geq \alpha = \sum_{i=1}^{n} \mu(a) = \mu(a).$$

Similarly, $(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \geq \beta$ and then

$$(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \geq \alpha \vee \beta. \tag{11}$$

Then from (10) and (11) we get that $(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \alpha \vee \beta$.

Corollary 1. Let R be a regular and intra-regular semiring and $\mu \in AFBIS(R)$. Then μ is a strongly prime if and only if μ be a strongly irreducible.
Proof. Let R be a regular and intra-regular semiring and $\mu, \alpha, \beta \in AFBIS(R)$. Then from Proposition 5 we get that $\alpha \vee \beta = (\alpha\ o\ \beta) \vee (\beta\ o\ \alpha)$. Now if μ be a strongly prime, then $\alpha \vee \beta = (\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \geq \mu$ and then $\alpha \geq \mu$ or $\beta \geq \mu$ and so μ will be a strongly irreducible. Conversely, if μ is a strongly irreducible, then $(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) = \alpha \vee \beta \geq \mu$ wich means that $\alpha \geq \mu$ or $\beta \geq \mu$ and then μ is a strongly prime.

Corollary 2. Let R be a regular and intra-regular semiring. If the set $A = \{\mu : \mu \in AFBIS(R)\}$ is totally ordered by inclusion $(\leq)$, then each $\mu \in A$ is a strongly prime.

Proof. Let $\alpha, \beta, \mu \in A$ and we prove that μ will be a strongly prime. Since the set of A is totally ordered by inclusion so $\alpha \leq \beta$ or $\beta \leq \alpha$ and this gets that $\alpha \vee \beta = \alpha$ or $\alpha \vee \beta = \beta$. As R be a regular and intra-regular semiring so from Proposition 5 we get that $\alpha \vee \beta = (\alpha\ o\ \beta) \vee (\beta\ o\ \alpha)$. Now let $(\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \geq \mu$ then

$$\alpha = \alpha \vee \beta = (\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \geq \mu$$

or

$$\beta = \alpha \vee \beta = (\alpha\ o\ \beta) \vee (\beta\ o\ \alpha) \geq \mu$$

and this implies that μ will be a strongly prime.

Corollary 3. Let R be a regular and intra-regular semiring. If the set $A = \{\mu : \mu \in AFBIS(R)\}$ is totally ordered by inclusion $(\leq)$, then each $\mu \in A$ is a prime.

Proof. Let $\alpha, \beta, \mu \in A$ and we prove that μ will be a prime. Since the set of A is totally ordered by inclusion so from Proposition 5 we get that $\alpha o \alpha = \alpha$ and $\beta o \beta = \beta$. Now let $\alpha\ o\ \beta \geq \mu$ and as the set of A is totally ordered by inclusion so $\alpha \geq \beta$ or $\beta \geq \alpha$. If $\alpha \geq \beta$, then $\alpha = \alpha\ o\ \alpha \geq \alpha\ o\ \beta \geq \mu$ and if $\beta \geq \alpha$, then $\beta = \beta\ o\ \beta \geq \alpha\ o\ \beta \geq \mu$. Thus $\alpha \geq \mu$ or $\beta \geq \mu$ and then μ will be a prime.

Proposition 6. Let R be a semiring. Then the following assertions are equivalent:
(1) Set $A = \{\mu : \mu \in AFBIS(R)\}$ is totally ordered by inclusion ($\leq$).
(2) Each $\mu \in A$ *is strongly irreducible.*
(3) Each $\mu \in A$ is an irreducible.

Proof. (1) ⇒ (2) Let $\alpha, \beta, \mu \in A$ and we prove that μ will be a strongly irreducible. Since the set of A is totally ordered by inclusion so $\alpha \geq \beta$ or $\beta \geq \alpha$ and then $\alpha \vee \beta = \alpha$ or $\alpha \vee \beta = \beta$. Now let $\alpha \vee \beta \geq \mu$ then $\alpha = \alpha \vee \beta \geq \mu$ or $\beta = \alpha \vee \beta \geq \mu$. Thus $\alpha \geq \beta$ and $\beta \geq \mu$ and then μ will be a strongly irreducible.

(2) ⇒ (3) Let $\alpha, \beta, \mu \in A$ and let μ be a strongly irreducible. Let $\alpha \vee \beta = \mu$. Since $\alpha \vee \beta \geq \alpha$ and $\alpha \vee \beta \geq \beta$ so $\mu \geq \alpha$ and $\mu \geq \beta$. Also since μ is a strongly irreducible so $\alpha \vee \beta = \mu \geq \mu$ and then $\alpha \geq \mu$ and $\beta \geq \mu$. Therefore we obtain that $\alpha = \mu$ or $\beta = \mu$ and this implies that μ is irreducible.

(3) ⇒ (1) Let $\alpha, \beta \in A$ and we must prove that $\alpha \geq \beta$ or $\beta \geq \alpha$. By Proposition 1 (part 3) we have that $\alpha \vee \beta \in AFBIS(R)$ and then will be an irreducible. As $\alpha \vee \beta = \beta \vee \alpha$ then $\alpha = \alpha \vee \beta$ or $\beta = \alpha \vee \beta$ and so $\alpha \geq \beta$ or $\beta \geq \alpha$. This means that the set of A is totally ordered by inclusion ($\leq$).

5. Conclusion

In this study, we define the notion of anti fuzzy bi-ideals in semirings with respect to *s*-norms and we and investigate some properties of them. Next we introduce anti prime fuzzy bi-ideals, anti strongly prime fuzzy bi-ideals, anti semiprime fuzzy bi-ideals, anti irreducible fuzzy bi-ideals, anti strongly irreducible fuzzy bi-ideals with respect to *s*-norms and obtained some results about them and we investigate them under regular and intra-regular semirings. We characterize them under totally ordered by inclusion. Now one can define anti fuzzy bi-rings with respect to *s*-norms and investigate them as we did for bi-ideals and this can be an open problem.

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