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Favored Target Setting using a Hybrid Fuzzy Goal Programming and Data Envelopment Analysis

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A B S T R A C T

Data envelopment analysis (DEA) is a method to estimate a relative efficiency of decision making units (DMUs) performing similar tasks in a production system that consumes multiple inputs to produce multiple outputs. The original DEA model does not include a decision maker's (DM's) preference structure while measuring relative efficiency. Regarding to relationship between DEA and multiple objective linear programming (MOLP) this paper propose a method based on fuzzy goal programming to incorporate DM's wishes in evaluation of DMUs then it analyzes the situations that the input-output levels of the estimated benchmark will not or may worsen. A compromised method is suggested that not only considers DM's wishes in target setting but also improve the efficiency of DMUs while none of input-output levels deteriorate.

1. Introduction

Data envelopment analysis (DEA) is a mathematical programming method for evaluating the relative efficiency of decision making units (DMUs) with multiple outputs and multiple inputs that was proposed by Charnes et al. [4]. The usefulness of DEA extends to evaluation and benchmarking against efficient units, target setting and recourse allocation between inputs and outputs. This method provides benchmarking information which can be used to improve the efficiency of the DMU.

The original DEA model does not include a decision maker's (DM's) preference structure or value judgments while measuring relative efficiency, with no or minimal input from the DM [3, 4, 9, 11, 16]. To incorporate DM's preference information in DEA, various techniques have been proposed. Golany [7] suggested a so-called target setting model, which allows DMs to select the preferred set of output levels given the input levels of a DMU. Thanassoulis and Dyson [15] introduced models that can be used to estimate alternative input and output levels. Charnes et al. [3] proposed the cone ratio concept by adjusting the observed input and output level or weight to capture value judgments to belong to a give closed cone. Dyson and

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Thanassoulis [6] introduced weight restrictions and Thompson et al. [16] suggested assurance region. Zhu [21] proposed a model that calculates efficiency score incorporating the DM's preference information.

On the other hand, relationships between DEA and multiple criteria decision analysis (MCDA) have been studied from several viewpoints by many authors [5, 18, 20]. Stewart [14] showed the equivalence between the CCR model and some linear value function model for multiple outputs and multiple inputs.

Joro et al. [8] proved structural correspondences between DEA models and multiple objective linear programming [2, 10, 13] and according to similarity of DEA and MOLP, Yang et al. [20] used interactive methods for assessment and target setting. Yang et al. [19] proposed a hybrid min-max reference point and DEA method to management planning.

In this study, considering the DM's view, first a method based on fuzzy goal programming is presented to find a target for DMUs which has the least deviation from the DM's ideals. But based on the DM's ideals, the determined benchmark may be unfeasible or some inputs or outputs worsen. To solve the problem, a two-stage model is stated that guaranties the obtained target will not worsen. This solution presents the closest benchmark to the DM's ideals and it is close to the efficiency frontier to the extent possible. In short, the current article proposes a framework that considers and incorporates the desires of decision makers in the process of performance measurement.

The paper proceeds as follows. Section 2 provides and reviews the basic models that are needed for the rest of paper. Section 3 proposes hybrid model for favored target setting using DEA and goal programming. Section 4 provides a numerical example to illustrate the proposed approach. Section 5 summarizes the main results obtained and suggests potential extensions.

2. Preliminaries

We summarize below the basic models of goal programming, fuzzy multi objective linear programming, relation between output-oriented CCR DEA model and MOLP.

2.1. Goal Programming

This method seeks to accomplish several purposes. For each objective function an aspiration level is considered and formulized. Then a response will be presented. It minimizes the total deviation of each objective function and its aspiration level.

To do so, suppose that g_k $(k=1,2,...,p)$ was determined as goal or ideal level of P objective functions. Suppose $x_1, x_2, ..., x_n$ are the decision variables and c_{kj} $(k=1,2,...,p,\ j=1,2,...,n)$ are the coefficient of j-th variables in k-th objective function, put $d_k = \sum_{j=1}^{n} c_{kj} x_j - g_k$.

In order to reach positive sign for d_k can, we put $d_k = d_k^+ - d_k^-$ and the percent of minimization of the deviation of objective function from goal level is specified. Consequently, the following model is attained:

$$
\begin{aligned}
& Min \sum_{k=1}^{p} |d_k| = \sum_{k=1}^{p} |d_k^+ - d_k^-| = \sum_{k=1}^{p} d_k^+ + d_k^- \\
& s.t. \\
& \sum_{j=1}^{n} c_{kj} x_j - (d_k^+ - d_k^-) = g_k, \qquad k = 1,2....,p, \\
& x_i \geq 0 \qquad\qquad\qquad\qquad , j = 1,2,...,n, \\
& d_k^+, d_k^- \geq 0, \qquad\qquad\qquad k = 1,2,...,n.
\end{aligned}
\tag{1}
$$

2.2. Fuzzy Multi Objective Linear Programming

Let there is a MOLP with p objective functions in generic form:

$$\begin{aligned} & Max \left[z_1(x), z_2(x), \ldots, z_p(x) \right] \\ & s.t \\ & Ax \le b, \\ & x \ge 0. \end{aligned} \tag{2}$$

If $g = \left(g_1, g_2, \ldots, g_p\right)$ is a fuzzy goal level for objective function, So we seek a solution that the objective function will be more than its goal level in fuzzy environment. Considering the stated issues the following model is gained:

$$\begin{aligned} & Find\ x \\ & s.t \\ & z_k(x) \succeq g_k, \\ & Ax \le b, \\ & x \ge 0. \end{aligned} \tag{3}$$

where $\preceq$ is fuzzy form of $\le$ that to be understood essentially less than.

Now, consider the following membership function for the objective function:

$$\mu_k(x) = \begin{cases} 1 & z_k(x) \ge g_k \\ \dfrac{z_k(x) - z_k^-}{g_k - z_k^-} & z_k^- \le z_k(x) < g_k \\ 0 & z_k(x) < z_k^- \end{cases} \tag{4}$$

In which z_k^- is the minimum acceptable value for the k-th objective function.

According to Zadeh's extension [1], assuming $\mu_{\tilde{G}_i}$ is the membership function of i-th fuzzy goal then membership functions of fuzzy decision is as $\mu_{\tilde{D}} = \min_i \left\{ \mu_{\tilde{G}_i} \right\}$.

Considering $0 \le \mu(x) \le 1$, goal 1 is supposed for each of the membership functions ([12, 17]):

$$\frac{z_k(x) - z_k^-}{g_k - z_k^-} + d_k^- - d_k^+ = 1 \tag{5}$$

Based on the previous discussions, ignoring the values more than 1 for the membership functions, we prevent the values less than 1.so the following model is gained:

$$
\begin{aligned}
& Min \sum_{k=1}^{p} d_k^{-} + d_k^{+} \\
& s.t. \\
& \frac{z_k(x) - z_k^{-}}{g_k - z_k^{-}} + d_k^{-} - d_k^{+} = 1, \qquad k = 1,2,\ldots,p, \\
& Ax \le b, \\
& d_k^{-}.d_k^{+} = 0, \\
& d_k^{-}, d_k^{+} \ge 0, \qquad k = 1,2,\ldots,p, \\
& x \ge 0.
\end{aligned} \tag{6}
$$

2.3. Output-oriented CCR Model as an MOLP

Suppose we have n DMU that consume m input x_j to product s output y_j, for $j = 1,2,\ldots,n$ .Let $x_j = (x_{1j}, x_{2j},\ldots, x_{mj})^t$ and $y_j = (y_{1j}, y_{2j},\ldots, y_{sj})^t$, input and output vector are positive.

For assess DMU_{j_0} , $j_0 \in \{1,2,\ldots,n\}$, the under evaluation DMU, the CCR model have been proposed in output-oriented by Charnes et al. [4] as follows:

$$
\begin{aligned}
& Max\ \varphi_{j_0} \\
& s.t \\
& \sum_{j=1}^{n} \lambda_j x_j \le x_{j_0}, \\
& \sum_{j=1}^{n} \lambda_j y_j \ge \varphi_{j_0} y_{j_0}, \\
& \lambda_j \ge 0,\ j = 1,2,\ldots,n.
\end{aligned} \tag{7}
$$

Consider the following MOLP in generic:

$$
\begin{aligned}
& Max[f_1(\lambda),\ldots\ldots, f_k(\lambda)] \\
& s.t. \\
& \lambda \in S.
\end{aligned} \tag{8}
$$

Considering an ideal level for *t*-th objective (f_t^{*}) and weighting index w for *t*-th objective function the following model minimize the maximum weighted derivation of each objective function:

$$
\begin{aligned}
& \underset{\lambda}{Min}\ \underset{1\le t\le k}{Max} \{w_t(f_t^{*} - f_t(\lambda))\} \\
& s.t. \\
& \lambda \in S
\end{aligned} \tag{9}
$$

By use of an auxiliary variable model (9) can be rewritten as follow (Yang et al. [20]):

$$
\begin{aligned}
& Min\ \ \theta \\
& s.t. \\
& w_t(f_t^{*} - f_t(\lambda)) \le \theta,\ \ t = 1,2,\ldots,k, \\
& \lambda \in S.
\end{aligned} \tag{10}
$$

Theorem1 [20]: Considering $S = \{\lambda | Y\lambda \ge y_{j_0}, \lambda_j \le 0,\ j = 1,2,\ldots,n\}$ and *s* objective functions (*k*=*s*) in model (8), output-oriented CCR model (7) and model (9) are equivalent.

Therefore the model (7) can be rewritten as the following model:

$$\begin{aligned} & Max\ [\sum_{j=1}^{n}\lambda_j\, y_{1j}, \sum_{j=1}^{n}\lambda_j\, y_{2j}, \ldots\ldots \sum_{j=1}^{n}\lambda_j\, y_{sj}] \\ & s.t. \\ & \sum_{j=1}^{n}\lambda_j\, x_{ij} \le x_{j_0}, \quad i=1,2,\ldots,m, \\ & \lambda_j \ge 0, \qquad\qquad j=1,2,\ldots., n. \end{aligned} \tag{11}$$

3. Favored Target Setting

In the classical performance measurement and target setting processes there is no possibility for considering decision maker's opinions. On the other hand, we face different types of uncertainty when we deal with the human's desires. Thus, in this section, a method based on the discussions of previous section is presented to apply the DM's view in approximation of input-output target levels. The following model uses the procedure of subsection 2.2 for solving model (11):

$$\begin{aligned} & Min \sum_{r=1}^{s} d_r^- + d_r^+ \\ & s.t \\ & \frac{\sum_{j=1}^{n}\lambda_j y_{rj} - z_r^-}{\overline{y}_r - z_r^-} + d_r^- - d_r^+ = 1, \quad r=1,2,\ldots,s, \\ & \sum_{j=1}^{n}\lambda_j x_{ij} \le x_{io}, \qquad\qquad i=1,2,\ldots,m, \\ & d_r^-.d_r^+ = 0, \\ & d_r^-, d_r^+ \ge 0, \qquad\qquad r=1,2,\ldots,s, \\ & \lambda \ge 0. \end{aligned} \tag{12}$$

Now considering $\overline{y}_r$ as an ideal value of manager for r-th output in the evaluation of DMU_{j_0}, model (12) seeks a nearest target to DM's ideal. If it is desired, the objective function model (12) can be modified so that the sum of weighted deviations is minimized that determining the weights is depend on their importance for DM. Manager as a decision maker determines the ideals based on his/her circumstances and is not aware of their feasibility or unfeasibility. It means that the manager doesn't know the possibility of producing ideal output based on accessible inputs. So, the target levels determined by the above model which is based on DM's ideal can be infeasible and DM's ideal levels may be infeasible then determined target may be infeasible too. Moreover in order to improve one of the input or output levels may another input output levels deteriorate. These difficulties obviate with proposing a compromised approach in next issues. Next theorem proposes satiations that considering them, none of inputs and outputs of determined target may worsen.

Theorem 2. If the goal values of model (12) be feasible on (7), none of the inputs and outputs worsen in the benchmark proposed by this model.

Proof. Let $(x_0, \overline{y})$ be a feasible solution thus there exists a $\overline{\lambda}$ that $\sum_{j=1}^{n}\overline{\lambda}_j x_{ij} \le x_{io}, \sum_{j=1}^{n}\overline{\lambda}_j y_{rj} \ge \overline{y}_r$ so

$$-\sum_{j=1}^{n}\overline{\lambda}_j y_{rj} \le -\overline{y}_r \Rightarrow z_r^- - \sum_{j=1}^{n}\overline{\lambda}_j y_{rj} \le z_r^- - \overline{y}_r$$

$$\frac{\sum_{j=1}^{n}\bar{\lambda}_j y_{rj} - z_r^-}{\bar{y}_r - z_r^-} \geq 1,\ (z_r^- < \bar{y}_r)$$

Thus, $(\bar{\lambda}, 0, \bar{d}_r^+)$ is a feasible solution of (12). Therefore, if $(\lambda^*, d_r^{-*}, d_r^{+*})$ be optimal solution of (12) then $d_r^{-*} = 0$ (for all r).

Let $(\lambda^*, d_r^{-*}, d_r^{+*})$ is optimal solution of (12) that shows direction of benchmarking so

$$\frac{\sum_{j=1}^{n}\lambda_j^* y_{rj} - z_r^-}{\bar{y}_r - z_r^-} = (1 + d_r^{+*} - d_r^{-*}) \Rightarrow \sum_{j=1}^{n}\lambda_j^* y_{rj} - z_r^- = (\bar{y}_r - z_r^-)(1 + d_r^{+*} - d_r^{-*})$$

$$\Rightarrow \sum_{j=1}^{n}\lambda_j^* y_{rj} = \bar{y}_r + d_r^{+*}(\bar{y}_r - z_r^-) + d_r^{-*}(z_r^- - \bar{y}_r) = \bar{y}_r + d_r^{+*}(\bar{y}_r - z_r^-)$$

$$\Rightarrow \sum_{j=1}^{n}\lambda_j^* y_{rj} \geq \bar{y}_r .$$

This shows that the determined target has a higher output level, namely, the output levels are not worsen. With the same logic and considering the input oriented model, we see that $\sum_{j=1}^{n}\lambda_j^* x_{ij} \leq \bar{x}_i$ that is the input value of targets are not worsen. □

Theorem 2 shows that if goal level of output be feasible then none of the input and output of target may worsen. Next theorem searches for new goal feasible level which is close to DM's goal and efficiency frontier as far as possible.

Theorem 3. Regardless to feasibility and considering $\bar{y}_r$ as a goal value of manager for *r*-th output in the evaluation of DMU_{j_0}, the following model determine the nearest goal feasible to $\bar{y}_r$ on efficiency frontier.

$$\begin{aligned}
& Max\ \delta \\
& s.t. \\
& \sum_{j=1}^{n}\lambda_j x_{ij} \leq x_{ij_0}, \qquad i = 1,2,\ldots,m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \geq y_{rj_0} + \delta(\overline{y_r} - y_{rj_0}),\ r = 1,2,\ldots,s, \\
& \lambda_j \geq 0, \qquad j = 1,2,\ldots,n.
\end{aligned} \tag{13}$$

Proof. Consider the following model for assess the activity $(x_{j_0}, y_{j_0} + \delta^* (\bar{y} - y_{j_0}))$:

$$\begin{aligned}
& Max\ \omega \\
& s.t. \\
& \sum_{j=1}^{n}\lambda_j x_{ij} \leq x_{ij_0}, \qquad i = 1,2,\ldots,m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \geq \omega(y_{rj_0} + \delta^*(\overline{y_r} - y_{rj_0})),\ r = 1,2,\ldots,s, \\
& \lambda_j \geq 0, \qquad j = 1,2,\ldots,n.
\end{aligned} \tag{14}$$

Firstly $(\lambda^*,1)$ is a feasible solution of above model. So, if $(\hat{\lambda},\hat{\omega})$ is an optimal solution of model (14) then $\hat{\omega}\geq 1$. Let negate let $\hat{\omega}>1$ so $\sum_{j=1}^{n}\hat{\lambda}_j y_{rj} \geq \hat{\omega}(y_{rj_0}+\delta^*(\overline{y_r}-y_{rj_0}))$ and $\sum_{j=1}^{n}\hat{\lambda}_j x_j \leq x_{j_0}$. Put $\tilde{\lambda}=\hat{\lambda}$ and $\tilde{\omega}=\hat{\omega}\delta^*$ thus $(\tilde{\lambda},\tilde{\omega})$ is a feasible solution of model (12) in the other hand $\tilde{\omega}>\delta^*$ that is contradiction with optimality of (λ^*,δ^*) hence $\hat{\omega}=1$ so $(x_{j_0}, y_{j_0}+\delta^*(\overline{y}-y_{j_0}))$ is the nearest goal feasible to $\overline{y}_r$ on the efficiency frontier. □

Now the following two stage model offers a target levels for a unit according to DM's view. On the first stage model (13) determines a feasible ideal levels for DM's ideal and considering the new feasible ideal level the second stage determines the nearest target level to DM's ideal that none of new target input-output level may worsen. The proposed models incorporate the desires of decision makers in the process of efficiency measurement and target setting. However, we face some sort of uncertainty when we deal with the desires that could be different from of decision maker to another. Fuzzy programming helped us in this situation. Therefore, in any target setting process of real world applications that is an important step in the performance improvement. Beside the advantageous of the proposed model, we may face some bias effect due to incorporating decision maker's preferences in the efficiency measurement and target setting. This could be tackled using some multi criteria decision making approach to have more realistic analysis.

4. Numerical Example

In order to illustrate the use of the methodology developed here, five DMUs are considered with one input and two outputs. Units *A*, *B* and *D* are efficient and *C* and *E* are inefficient (Table 1).

Table 1. Inputs and output data

DMU	I	O1	O2
A	1	5	2
B	1	3	5
C	1	3	2
D	1	4	4
E	1	1	3

The CCR dual model (7) offers 1.67 and 5 as the first and second target levels for unit *E* (Figure 1). In order to considering DM's view on target setting, suppose (1.5, 4) and (2, 6) are as ideal output levels which the first one is feasible and second one is infeasible. While solving (12) for incorporating DM's ideal level on target setting, current output level of unit *E* is supposed as minimum acceptable level for target level of unit *E*. This is a coherent minimum acceptable because in DEA literature we are searching for a target with more than or equal output level for DMUs (equality is for a case that DMU is efficient).

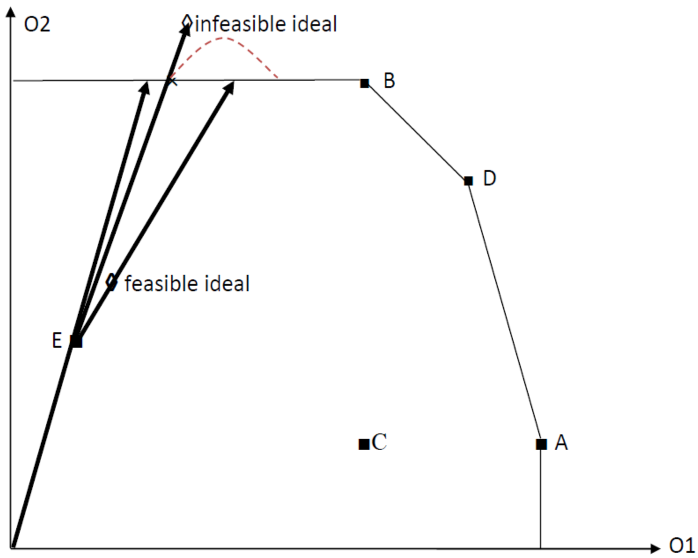

Figure 1. An illustration of target setting

Considering DM's ideal levels, (1.5, 4) and (2, 4.66) are suggested as a target levels that second output of second target is worse.

In order to find a feasible ideal level, model (13) finds (1.67,5) as a new ideal level and regarding to second step of proposed method (1.8, 5) is an efficient target level (segment curve Figure1) which is determined according to DM's ideal level.

5. Conclusion

Efficiency measurement and target setting are very important task in managerial issues which can be done with DEA models but DM's view is neglected in traditional DEA models. This study proposed a two-step DEA and FGP model to incorporate DM's view in process of evaluation and target setting. Considering DM's ideal level this method finds a compromised target levels for inefficient units lying on efficiency frontier and the situations that no level of inputs and outputs of the estimated target may worsen is analyzed. For the future direction research, one may consider solving techniques of multi-objective programming for crisp or uncertain data in the target setting process.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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