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Some Coupled Coincident Point Theorems in Metric Space

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In this present work, we prove coupled coincident point theorem for contractive mappings $F : X \times X \to X$ and $g : X \to X$ such that $F(X \to X) \subseteq g(X)$ in metric spaces that have a nonempty F-invariant and g-invariant complete subset and we prove uniqueness coupled coincidence, then we prove coupled fixed point theorem for contractive mapping F in metric spaces that have a nonempty F-invariant complete subset. Finally, we give an example in this case. Through there are thousands of fixed point theorems in metric space, our theorems is a new type of theorems. because we prove unique coupled fixed point are in F-invariant complete subspace.

1. Introduction

The Banach contraction principle is a basic result in fixed point theory [3]. This principle has been generalized in different directions by many authors, for example Authors in [6] generalized ternary bi-derivations on ternary Banach algebras. Also, Random fixed point theorem in generalized Banach space [14] and Banach fixed point theorems on orthogonal sets [7] are extended by authors.

Bhaskar and Lakshmikantham [4] introduced the notion of a coupled fixed point of a mapping of two variables. Later, Gordji et al. [5] proved some coupled fixed-point theorems for contractions in partially ordered metric spaces. Several extensions of fixed point, coupled fixed and coupled coincidence point by many authors are given in [8, 11, 12]. Furtheremore, some others obtained many results on coupled fixed-point theorems in cone metric spaces (see, e.g., [1, 9, 10]).

Recently in [2], orthogonally ring derivations in Banach algebras with the new type fixed point is studied and in [13] Banach fixed point theorem on orthogonal cone metric spaces is studied and evaluated. In this way, they improved results of Bhaskar and Lakshmikantham [4].

We shall recall their definitions here. Let X be a non-empty set and $F: X \times X \to X$ be a given mapping.

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Definition 1. [4] An element $(x,y) \in X \times X$is called a coupled fixed point of the mapping $F: X \times X \to X$ if $F(x,y) = x, \quad F(y,x) = y$.
Definition 2. [4] An element $(x,y) \in X \times X$is called a coupled coincidence fixed point of the mapping $F: X \times X \to X$and $g: X \to X$ if

$$F(x,y) = x, F(y,x) = g(y).$$

Definition 3. [4] Let X be a non- empty set and $F: X \times X \to X$and $g: X \to X$We say $F$and g are commutative if $g\big(F(x,y)\big) = F\big(g(x), g(y)\big)$for all $x, y \in X$.

Motivated by the results of [4, 5], in this paper, we prove new coupled coincidence point theorem for contractive mapping Fin metric spaces that have a nonempty F -invariant complete subset and obtain several interesting corollaries. An example will be provided to illustrate our results.

2. Main results

Throughout this paper, we will use the following notation:
Let $F: X \times X \to X$be a given mapping. For all $n \in N, n \geq 2$we denote:

$$F^n(x,y) = F\big(F^{n-1}(x,y), F^{n-1}(y,x)\big); \qquad \forall x, y \in X,$$
$$F^n(y,x) = F\big(F^{n-1}(y,x), F^{n-1}(x,y)\big); \qquad \forall x, y \in X.$$

Theorem 1. Let (X,d) be a metric space. Assume there is a function $\varphi: 0, +\infty) \to 0, +\infty)$with $\varphi(t) < t$ and $lim_{r \to t^+} \varphi(r) < t$ for each $t > 0$ and also suppose $F: X \times X \to X$ and $g: X \to X$ are such that $F(X \times X) \subseteq g(X)$and $g$is continuous and commutes with $F$and

$$d\big(F(x,y), F(u,v)\big) \leq \varphi\left(\frac{d\big(g(x), g(u)\big) + d\big(g(y), g(v)\big)}{2}\right)$$

for all $x, y, u, v \in E$and

$$d\big(F(x,y), F(u,v)\big) < \frac{1}{2}\big[d\big(g(x), g(u)\big) + d\big(g(y), g(v)\big)\big]$$

for all $x, y, u, v \in X$ and $g(x) \neq g(u)$ or $g(y) \neq g(v)$. if E be a nonempty F -invariant and $F(E \times E) \subseteq g(E)$and g -invariant complete subset of X; then there exists a unique $(x,y) \in X \times X$such that $F(x,y) = g(x)$ and $F(y,x) = g(y)$. that is, $F$and $g$have a unique coupled coincidence.

Proof. Let x_0, y_0 be two arbitrary points of E . Since $F(E \times E) \subseteq g(E)$, we can choose $x_1, y_1 \in E$ such that$F(x_0, y_0) = g(x_1)$ and $F(y_0, x_0) = g(y_1)$. Again from $F(E \times E) \subseteq g(E)$, we can choose $x_2, y_2 \in E$such that $F(x_1, y_1) = g(x_2)$ and $F(y_1, x_1) = g(y_2)$. Continuing this process, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in Esuch that

$$g(x_{n+1}) = F(x_n, y_n)\,, \quad g(y_{n+1}) = F(y_n, x_n), \quad \forall n \geq 0. \tag{1}$$

We have from (1)

$$d\big(g(x_n), g(x_{n+1})\big) = d\big(F(x_{n-1}, y_{n-1}), F(x_n, y_n)\big) \leq \varphi\left(\frac{d\big(g(x_{n-1}), g(x_n)\big) + d\big(g(y_{n-1}), g(y_n)\big)}{2}\right). \tag{2}$$

Similarly,

$$d\big(g(y_n), g(y_{n+1})\big) = d\big(F(y_{n-1}, x_{n-1}), F(y_n, x_n)\big) \leq \varphi\left(\frac{d\big(g(y_{n-1}), g(y_n)\big) + d\big(g(x_{n-1}), g(x_n)\big)}{2}\right). \tag{3}$$

Now, we denote:

$$\delta_n = d\big(g(x_n), g(x_{n+1})\big) + d\big(g(y_n), g(y_{n+1})\big). \tag{4}$$

From (4) and adding (2) and (3) we obtain

$$\delta_n \leq 2\varphi\left(\frac{d\big(g(y_{n-1}), g(y_n)\big) + d\big(g(x_{n-1}), g(x_n)\big)}{2}\right) = 2\varphi\left(\frac{\delta_{n-1}}{2}\right). \tag{5}$$

Since $\varphi(t) < t$ for $t > 0$, from (5) we have $\delta_n < \delta_{n-1}$.

It follows that a sequence $\{\delta_n\}$ is monotone decreasing.Therefore, there is some $\delta \geq 0$ such that $\lim_{n\to\infty} \delta_n = \delta^+$.

We show that $\delta = 0$. Suppose to the contrary, that$\delta > 0$. Then, taking the limit as $\delta_n \to \delta^+$ of boat sides of (5) and have in mind that $\lim_{r\to t^+}\varphi(r) < t$for all $t > 0$, we have

$$\delta = \lim_{n\to\infty} \delta_n \leq 2 \lim_{n\to\infty} \varphi\left(\frac{\delta_{n-1}}{2}\right) = 2 \lim_{\delta_{n-1}\to\delta^+} \varphi\left(\frac{\delta_{n-1}}{2}\right) < 2\frac{\delta}{2} = \delta$$

This is contradiction. Thus $\delta = 0$. That is,

$$lim_{n\to\infty}\left[d\big(g(x_n), g(x_{n+1})\big) + d\big(g(y_n), g(y_{n+1})\big)\right] = 0. \quad (6)$$

Now, we prove that $\{g(x_n)\}$ and $\{g(y_n)\}$ are Cauchy sequences in E. Suppose, to the contrary, that at least one of $\{g(x_n)\}$ or $\{g(y_n)\}$ is not a Cauchy sequence. Then there exist an $\epsilon > 0$and two subsequences of integer $\{l_k\}$ and $\{m_k\}$ such that $m_k > l_k \geq k$and

$$d\left(g(x_{l_k}),g(x_{m_k})\right) \geq \frac{\epsilon}{2} \quad , \quad d\left(g(y_{l_k}),g(y_{m_k})\right) \geq \frac{\epsilon}{2} \quad \text{for} \quad k \in \mathbb{N}.$$

So,

$$r_k = d\left(g(x_{l_k}), g(x_{m_k})\right) + d\left(g(y_{l_k}), g(y_{m_k})\right) \geq \epsilon \quad for \quad k \in \mathbb{N}. \quad (7)$$

If m_k to be the smallest number exceeding l_k for which (7) holds, then

$$d\left(g(x_{l_k}), g(x_{m_k-1})\right) + d\left(g(y_{l_k}), g(y_{m_k-1})\right) < \dot{o}. \quad (8)$$

From (4), (7), (8) and by the triangle inequality,

$$\epsilon \leq r_k \leq d\left(g(x_{l_k}), g(x_{m_k-1})\right) + d\left(g(x_{m_k-1}), g(x_{m_k})\right) + d\left(g(y_{l_k}), g(y_{m_k-1})\right)$$
$$+d\left(g(y_{m_k-1}), g(y_{m_k})\right) < \epsilon + \delta_{m_k-1}.$$

Thus,

$$\epsilon \leq lim_{k\to\infty} r_k \leq lim_{k\to\infty}(\epsilon + \delta_{m_k-1}).$$

From (6), we have

$$lim_{k\to\infty} r_k = \epsilon^+. \quad (9)$$

By the triangle inequality and (4) and (1)

$$r_k = d\left(g(x_{l_k}), g(x_{m_k})\right) + d\left(g(y_{l_k}), g(y_{m_k})\right) \leq d\left(g(x_{l_k}), g(x_{l_k+1})\right) + d\left(g(x_{l_k+1}), g(x_{m_k+1})\right)$$
$$+d\left(g(x_{m_k+1}), g(x_{m_k})\right) + d\left(g(y_{l_k}), g(y_{l_k+1})\right) + d\left(g(y_{l_k+1}), g(y_{m_k+1})\right) + d\left(g(y_{m_k+1}), g(y_{m_k})\right)$$
$$= \delta_{l_k} + \delta_{m_k} + d\left(g(x_{l_k+1}), g(x_{m_k+1})\right) + d\left(g(y_{l_k+1}), g(y_{m_k+1})\right)$$
$$= \delta_{l_k} + \delta_{m_k} + d\left(F(x_{l_k}, y_{l_k}), F(x_{m_k}, y_{m_k})\right) + d\left(F(y_{l_k}, x_{l_k}), F(y_{m_k}, x_{m_k})\right)$$
$$\leq \delta_{l_k} + \delta_{m_k} + \varphi\left(\frac{d\left(g(x_{l_k}), g(x_{m_k})\right) + d(g(y_{l_k}), g(y_{m_k})}{2}\right) + \varphi\left(\frac{d\left(g(y_{l_k}), g(y_{m_k})\right) + d(g(x_{l_k}), g(x_{m_k})}{2}\right)$$
$$= \delta_{l_k} + \delta_{m_k} + \varphi\left(\frac{r_k}{2}\right) + \varphi\left(\frac{r_k}{2}\right)$$

i.e.

$$r_k \leq \delta_{l_k} + \delta_{m_k} + 2\varphi\left(\frac{r_k}{2}\right)$$

From (6) and (9), we have

$$\epsilon = \lim_{k\to\infty} r_k \leq \lim_{k\to\infty} 2\,\varphi\left(\frac{r_k}{2}\right) = 2 \lim_{r_k\to\epsilon^+} \varphi\left(\frac{r_k}{2}\right) < 2\frac{\epsilon}{2} = \epsilon$$

i.e. $\epsilon < \epsilon$a contradiction. Therefore, $\{g(x_n)\}$and $\{g(y_n)\}$ are Cauchy sequence in E. Since E is complete, there exist $x, y \in E$such that

$$lim_{n\to\infty} g\,(x_n) = x\,, \quad lim_{n\to\infty} g\,(y_n) = y. \tag{10}$$

From (10) and continuity of g,

$$lim_{n\to\infty} g\left(g(x_n)\right) = g\left(lim_{n\to\infty} g\,(x_n)\right) = g(x). \tag{11}$$

$$lim_{n\to\infty} g\left(g(y_n)\right) = g\left(lim_{n\to\infty} g\,(y_n)\right) = g(y). \tag{12}$$

Thus, for all $m \in \mathbb{N}$there exists $n_0 \in \mathbb{N}$such that, for $n \geq n_0$

$$d\left(g\left(g(x_n)\right), g(x)\right) < \frac{1}{4m}\,, \quad d\left(g\left(g(y_n)\right), g(y)\right) < \frac{1}{4m}. \tag{13}$$

Hence, From (1) and (13) and commutativity of $F$and $g$and by the triangle inequality, we have;

$$\begin{aligned} d\left(F(x,y), g(x)\right) &\leq d\left(F(x,y), g\left(g(x_n)\right)\right) + d\left(g\left(g(x_n)\right), g(x)\right) \\ &= d\left(F(x,y), g\left(F(x_{n-1}, y_{n-1})\right)\right) + d\left(g\left(g(x_n)\right), g(x)\right) \\ &= d\left(F(x,y), F\left(g(x_{n-1}), g(y_{n-1})\right)\right) + d\left(g\left(g(x_n)\right), g(x)\right) \\ &\leq \varphi\left(\frac{d\left(g(x), g\left(g(x_{n-1})\right)\right) + d\left(g(y), g\left(g(y_{n-1})\right)\right)}{2}\right) + d\left(g\left(g(x_n)\right), g(x)\right) \\ &< \frac{d\left(g(x), g\left(g(x_{n-1})\right)\right)}{2} + \frac{d\left(g(y), g\left(g(y_{n-1})\right)\right)}{2}) + d\left(g\left(g(x_n)\right), g(x)\right) \\ &< \frac{1}{8m} + \frac{1}{8m} + \frac{1}{4m} = \frac{1}{2m} \to 0 \end{aligned}$$

as $m \to \infty$.So, $F(x,y) = g(x)$. Similarly we can show $F(y,x) = g(y)$. To show uniqueness in E, assume there exist $u, v \in E$such that $F(u,v) = g(u)$and$F(v,u) = g(v)$. Therefore

$$\begin{aligned} d\left(g(x), g(u)\right) &= (d\left(F(x,y), F(u,v)\right) \leq \varphi\left(\frac{d\left(g(x), g(u)\right) + d\left(g(y), g(v)\right)}{2}\right) \\ &< \frac{1}{2}\left[d\left(g(x), g(u)\right) + d\left(g(y), g(v)\right)\right]. \end{aligned} \tag{14}$$

Similarly,

$$d\left(g(y), g(v)\right) = (d\left(F(y,x), F(v,u)\right) \leq \varphi\left(\frac{d\left(g(y), g(v)\right) + d\left(g(x), g(u)\right)}{2}\right) < \frac{1}{2}\left[d\left(g(y), g(v)\right) + d\left(g(x), g(u)\right)\right]. \tag{15}$$

Adding (14) and (15) , we obtain

$$d\left(g(x), g(u)\right) + d\left(g(y), g(v)\right) < d\left(g(x), g(u)\right) + d\left(g(y), g(v)\right).$$

This is a contradiction . Thus there exists a unique $(x,y) \in E \times E$ such that $F(x,y) = g(x)$and $F(y,x) = g(y)$. (i) Let there exist $u, v \in X - E$such that $g(x) \neq g(u)$or $g(y) \neq g(v)$and $F(u,v) = g(u)$and$F(v,u) = g(v)$. Then, we have

$$d\left(g(x), g(u)\right) = (d\left(F(x,y), F(u,v)\right) < \frac{1}{2}\left[d\left(g(x), g(u)\right) + d\left(g(y), g(v)\right)\right]. \tag{16}$$

Similarly,

$$d\left(g(y), g(v)\right) = (d\left(F(y,x), F(v,u)\right) < \frac{1}{2}\left[d\left(g(y), g(v)\right) + d\left(g(x), g(u)\right)\right]. \tag{17}$$

Adding (16) and (17) , we obtain

$$d\big(g(x),g(u)\big)+d\big(g(y),g(v)\big)<d\big(g(x),g(u)\big)+d\big(g(y),g(v)\big).$$

This is a contradiction. Also, if one of the following conditions hold:

(ii) There exist $u \in X-E$ and $v \in E$ such that $g(x) \neq g(u)$ or $g(y) \neq g(v)$ and $F(u,v) = g(u)$ and $F(v,u) = g(v)$,

(iii) There exist $u \in E$ and $v \in X-E$ such that $g(x) \neq g(u)$ or $g(y) \neq g(v)$ and $F(u,v) = g(u)$ and $F(v,u) = g(v)$,

Similarly, we have a contradiction. Thus, there exists a unique $(x,y) \in X \times X$ such that $F(x,y) = g(x)$ and $F(y,x) = g(y)$.

Corollary 2. Let (X,d) be a metric space. Assume $F: X \times X \to X$ and $g: X \to X$ are such that $F(X \times X) \subseteq g(X)$ and g *is* continuous and commutes with F. Let there exists *a* $k \in [0,1)$ with

$$\left(F(x,y()\,F(u,v())) \leq \frac{k}{2[d(g(x()\,g(u())) + d(g(y()\,g(v())))]}\right)$$

for all $x,y,u,v \in E$ and

$$d\big(F(x,y),F(u,v)\big) < \frac{1}{2}\big[d\big(g(x),g(u)\big)+d\big(g(y),g(v)\big)\big]$$

for all $x,y,u,v \in X$ such that $g(x) \neq g(u)$ or $g(y) \neq g(v)$.

If E be a nonempty F - invariant and g - invariant complete subset of X and $F(E \times E) \subseteq g(E)$; then there exists a unique $(x,y) \in X \times X$ such that $F(x,y) = g(x)$ and $F(y,x) = g(y)$, that is, F and g have a unique coupled coincidence.

Proof. Taking $\varphi(t) = kt$ with $k \in 0,1)$. Then $\varphi(t) < t$ and $lim_{r \to t^+}\varphi(r) = kt < t$ for each $t > 0$. Thus conditions of Theorem 1 is satisfying. Therefore there exists a a unique $(x,y) \in X \times X$ such that

$$F(x,y) = g(x), \quad F(y,x) = g(y).$$

Corollary 3. Let (X,d) be a metric space. Assume there is a function $\varphi: 0,+\infty) \to 0,+\infty)$ with $\varphi(t) < t$ and $lim_{r \to t^+}\varphi(r) < t$ for each $t > 0$ and also suppose $F: X \times X \to X$ be a mapping and

$$d\big(F(x,y),F(u,v)\big) \leq \varphi\left(\frac{d(x,u)+d(y,v)}{2}\right)$$

for all $x,y,u,v \in E$ and

$$d\big(F(x,y),F(u,v)\big) < \frac{1}{2}[d(x,u)+d(y,v)]$$

for all $x,y,u,v \in X$ where $x \neq u$ or $y \neq v$. If E be a nonempty F - invariant complete subset of X, then there exists a unique $(x,y) \in X \times X$ such that $F(x,y) = x$ and $F(y,x) = y$.

Proof. Give $g(x) = x$ for all $x \in X$ in theorem1, we obtain the corollary 2.

Corollary 4. *Let* (X,d) *be a metric space and* $F: X \times X \to X$ *be a mapping. Assume that there exists a* $X - k \in 0,1)$ *with*

$$\left(d(\left(F(x,y() F(u,v())) \leq \frac{k}{2}[d(x,u)+d(y,v)]\right)$$

for all $x,y,u,v \in E$ and

$$d\big(F(x,y),F(u,v)\big) < \frac{1}{2}[d(x,u)+d(y,v)]$$

for all $x,y,u,v \in X$with $x \neq u$or $y \neq v$.

If Ebe a nonempty F - invariant complete subset of X, then there exists a unique $(x,y) \in X \times X$such that $F(x,y) = x$and $F(y,x) = y$.

Proof. Taking $\varphi(t) = kt$ with $k \in 0,1)$ and $g(x) = x$ for all $x \in X$. Then the conditions of Theorem 1 is satisfying. Thus, there exists a a unique $(x,y) \in X \times X$such that

$$F(x,y) = x, \quad F(y,x) = y.$$

Example 1. Put $X = \{0\}U\left\{1+\frac{1}{n}; n \in N\right\}$ and define a metric$d$on X by $d\big(x,y()|x|||y|||\big)$for all $x,y \in X$ with $x \neq y$. Define a mapping $F: X \times X \to X$ by

$$F(x,y) = \begin{cases} 0 & if\ \ x=0\ \ or\ \ y=0 \\ 1+\dfrac{1}{N+1} & if\ \ x = 1+\dfrac{1}{n}\ \ and\ \ y = 1+\dfrac{1}{m} \end{cases}$$

whenever $N = max\{n,m\}$. Then F satisfies the assumption in Theorem 1; however $\{F^n(2,2)\}$ does not converge.

It is obvious that (X,d) is a complete metric space and $(0,0)$ is a unique coupled fixed point of F. put $E = 0$and $k = 0$; then F satisfies the assumption in theorem 1. Because if $x,y,u,v \in E$ then

$$d(F(x,y),F(u,v)) = 0) \leq \frac{k}{2[d(x,u() d(y,v())) = 0]}$$

Thus, F satisfies the assumption (1) of Theorem 1.

Now, let $x,y,u,v \in X$and $x \neq u$and $y \neq v$. Then one the following states hold:

- $(i) x,y \in E$and $u,v \in X - E$.
- $(ii) x,v \in E$and $u,y \in X - E$.
- $(iii) u,y \in E$and $x,v \in X - E$.
- $(iv) u,v \in E$and $x,y \in X - E$.
- $(v) v \in E$and $x,y,u \in X - E$.
- $(vi) u \in E$and $x,y,v \in X - E$.
- $(vii) y \in E$and $x,u,v \in X - E$.
- $(ix) x \in E$and $y,u,v \in X - E$.
- $(x) x,y,u,v \in E - y$

We prove that (i) ,(iii) , $(v)$and (x) satisfy is similarly. For(i), if $u = 1+\frac{1}{n}$and $v = 1+\frac{1}{m}$with $n \neq m$and $N = max\{m,n\}$, we have ;

$$d\big(F(x,y),F(u,v)\big) = 1+\frac{1}{N+1} < 1+\frac{1}{N} = \frac{1}{2}\left(1+\frac{1}{N}\right)+\frac{1}{2}\left(1+\frac{1}{N}\right) < \frac{1}{2}\left(1+\frac{1}{n}\right)+\frac{1}{2}\left(1+\frac{1}{m}\right) = \frac{1}{2}[d(x,u)+d(y,v)].$$

For (iii) if $x = 1+\frac{1}{n}$and $v = 1+\frac{1}{m}$; we have:

$$d\big(F(x,y),F(u,v)\big) = 0 < \frac{1}{2}\left[1+\frac{1}{n}+1+\frac{1}{m}\right] = \frac{1}{2}[d(x,u)+d(y,v)].$$

For (v), let $x = 1 + \frac{1}{n}$and $v = 1 + \frac{1}{m}$and $u = 1 + \frac{1}{l}$such that $n \neq l$and $N = max\{n, m\{\}\}$, then we have:

$$d\big(F(x,y), F(u,v)\big) = 1 + \frac{1}{N+1} < 1 + \frac{1}{N} < 1 + \frac{1}{N} < \frac{1}{2}\left(1 + \frac{1}{n}\right) + \frac{1}{2}\left(1 + \frac{1}{m}\right)$$

$$< \frac{1}{2}\left[1 + \frac{1}{n} + 1 + \frac{1}{m} + 1 + \frac{1}{l}\right] = \frac{1}{2}[d(x,u) + d(y,v)].$$

For(x), let $x = 1 + \frac{1}{n}$and $y = 1 + \frac{1}{m}$and $u = 1 + \frac{1}{l}$and $v = 1 + \frac{1}{p}$such that $n \neq l$and $m \neq p$and $N_1 = max\{n, m\}$ and $N_2 = max\{l, p\}$, then we have:

$$d\big(F(x,y), F(u,v)\big) = 1 + \frac{1}{N+1} + 1 + \frac{1}{N+2} < 1 + \frac{1}{N_1} < 1 + \frac{1}{N_2}$$

$$= \frac{1}{2}\left(1 + \frac{1}{N_1}\right) + \frac{1}{2}\left(1 + \frac{1}{N_1}\right) + \frac{1}{2}\left(1 + \frac{1}{N_2}\right) + \frac{1}{2}\left(1 + \frac{1}{N_2}\right)$$

$$= \frac{1}{2}\left[1 + \frac{1}{n} + 1 + \frac{1}{m} + 1 + \frac{1}{l} + 1 + \frac{1}{p}\right] = \frac{1}{2}[d(x,u) + d(y,v)].$$

Also, we have:

$$F^2(2,2) = F\big(F(2,2), F(2,2)\big) = F\left(1 + \frac{1}{2}, 1 + \frac{1}{2}\right) = 1 + \frac{1}{3},$$

$$F^3(2,2) = F\big(F^2(2,2), F^2(2,2)\big) = F\left(1 + \frac{1}{3}, 1 + \frac{1}{3}\right) = 1 + \frac{1}{4},$$

$$\vdots$$

$$F^n(2,2) = F\big(F^{n-1}(2,2), F^{n-1}(2,2)\big) = F\left(1 + \frac{1}{n}, 1 + \frac{1}{n}\right) = 1 + \frac{1}{n+1} \to 1,$$

as $n \to \infty$. but $1 \notin X$. Thus $\{F^n(2,2)\}$ dose not converge in X.

This example should that F has a unique coupled fixed point but $\{F^n(x,y)\}$ dose not necessarily converge to the coupled fixed point.

3. Conclusion

In this work, we prove coupled coincident point theorem for contractive mappings $F: X \times X \to X$ and $g: X \to X$ in metric spaces that have a nonempty F - invariant and g - invariant complete subset and we prove uniqueness coupled coincidence, then we prove coupled fixed point theorem for contractive mapping F in metric spaces that have a nonempty F - invariant complete subset. The resulys of our work are very interesting and result in an example that shows the mapping F has a unique coupled fixed point but $\{F^n(x,y)\}$ dose not necessarily converge to the coupled fixed point.

In future we will generalize these results and prove some coupled coincidence and fixed point theorems for contractive mapping in fuzzy metric spaces, then we present its applications.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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