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A New Pythagorean Fuzzy Analytic Hierarchy Process Based on Interval-Valued Pythagorean Fuzzy Numbers

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ABSTRACT

The Analytic Hierarchy Process (AHP) is one of the most widely used techniques to determine the priority weights of alternatives from pairwise comparison matrices. Several fuzzy and intuitionistic fuzzy extensions of AHP have been proposed in the literature. However, these extensions are not appropriate to present some real-life situations. For this reason, several researchers extend the AHP to the Pythagorean Fuzzy Analytic Hierarchy Process (PFAHP). In the existing methods, an interval-valued Pythagorean fuzzy pairwise comparison matrix is transformed into a crisp matrix. Then crisp AHP is applied to obtain the normalized priority weights from the transformed crisp matrix. However, it is observed that the transformed crisp matrix, obtained on applying the step of existing methods, violates the reciprocal propriety of pairwise comparison matrices, and the obtained normalized priority weights are the weights of non-pairwise comparison matrices. Therefore, this paper discusses the shortcomings of the existing method, and a modified method is proposed to overcome these shortcomings. Finally, based on a real-life decision-making problem, the superiority of the proposed method over the existing method is shown.

1. Introduction

Saaty [26] developed the concept of the analytic hierarchy process (AHP) which is an effective tool to handle complex decision-making problems [27-29]. This process is based on the three principles, which are decomposition, comparative judgments and synthesis of priorities. The first principle decomposes the complex decision-making problems into a simple hierarchal structure of multi-levels like, objective level, criteria level, sub-criteria level and alternative level. At each level, second principle assists to the decision maker to provide their judgment to compare objects in pairwise comparisons based on the 1-9 fundamental scale [29] and stored in the form a pairwise comparison matrix [27-29]. After the constriction of pairwise comparison matrixes, third

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principle assists the priority weights of alternatives with respect to each criterion and the priority weights of criteria with respect to the objective of the problem are computed. In final stage the global priority weights are synthesized to rank the available alternatives [27-29]. AHP has been wildly applied in scientific engineering, operations research and management science [49], due to its popularity and simplicity of handling complex multi criteria decision making (MCDM) problems [49].

Several extensions of AHP have been proposed successfully in the literature [43-45] under the fuzzy and intuitionistic fuzzy environment. However, due to some limitations of fuzzy set (FS) and intuitionistic fuzzy set (IFS) theory [10, 48], it is unable to deal with the situation. For instance, when a DM gives 0.8 as membership degree and 0.5 degree of non-membership then, obviously their sum is greater than one. Hence, under such circumstance have some types of limitations. In order to address this issue, Yager [64, 47] introduced Pythagorean fuzzy sets (PFSs), an effective tool for describing the uncertainty as sum of squares of membership degree and non-membership degree is less than or equal to one i.e., 〖0.8〗 ^2+〖0.5〗 ^2⩽1 and belongs to the interval [0,1]. After this successful extension of PFSs, Zhang and Xu [50] proposed Pythagorean fuzzy numbers (PFNs), due to the flexibility of PFNs in practical dealing of decision-making problems, several researchers applied in may real life problems [4-30].

On the basis of reviewing AHP with Pythagorean fuzzy sets, firstly, Ilbahar et al. [21], proposed an integrated PFPRA, including Fine Kinney, PFAHP and FIS method for risk assessment process in the field of occupational health and safety. In this integrated method [21], Pythagorean fuzzy sets [47-64] are employed, that provides more flexibility to decision maker than intuitionistic fuzzy sets [10]. Moreover, a general, eleven Step based framework [21, Section 3.5, pp. 128] is proposed, in this framework [21], step 1 and step 2 are used to collect the information from decision maker, in terms of linguistic variables and the linguistic pairwise comparison matrices are constructed. Step 3 of the proposed framework [21] is used to transform the linguistic pairwise comparison matrices into interval valued Pythagorean fuzzy pairwise comparison matrices and then applied the steps of PFAHP proposed by [21, Section 3.2.2, pp. 127] to obtain the weights for two parameters of fine kinney method i.e., probability and severity.

However, after a deep study, it is observed that on applying the steps of the PFAHP proposed by [21, Section 3.2.2, pp. 127], to transform the interval valued Pythagorean fuzzy pairwise comparison matrices into crisp matrices, violates the reciprocal propriety of the pairwise comparison matrices i.e. the obtained crisp matrices are not crisp pairwise comparison matrices. Therefore, it is a well-known fact the crisp AHP method [1-3] can be applied only, to obtain the normalized priority weights, if the transformed crisp matrices are crisp pairwise comparison matrices [1-3]. Recently some authors [2, 3, 4, 11, 12, 13, 24, 22] also, pointed out that to applying the crisp AHP [26-29] on crisp non-pairwise comparison matrix, to obtain the normalized priority weights is a meaningless task and it will mislead to the decision maker.

Hence, Step 4 to Step 11 of the framework proposed by Ilbahar et al. [21] cannot be used. Therefore, the Ilbahar et al.'s integrated method [21] is not valid in its present form and cannot be used to find the solution of any real-life problem. In future, the other researchers/stakeholders may use the same method [21] in numerous real-life problems [15] which lead to problematic decision-making approach and hence may result in a heavy loss in any value-added model. Therefore, keeping the same in mind this paper discusses the shortcomings of the existing method, and a modified method is proposed to overcome these shortcomings.

Focus of the present paper is to make the researchers aware about the flaws of Ilbahara et al.'s integrated method [21] and proposed a modified method to overcome the flaws of existing method [21]. In addition, a decision-making problem is solved and a comparison analysis is given with more valuable outcomes. To accomplish the same, rest of the paper has been organized as follows. Section 2, presents some basic concepts of Pythagorean fuzzy Set, operational laws of Pythagorean fuzzy numbers, and crisp pairwise comparison matrices. Section 3, presents a brief review of the existing method [21] and in Section 4, the flaws of the existing method are discussed. In Section 5, a modified method is proposed to overcome the flaws of the existing method. Section 6 describes the exact transformation of Pythagorean fuzzy pairwise comparison matrix into the corresponding crisp pairwise comparison matrix. Section 7, describes a comparison of the modified method with existing method based on a decision-making problem. Finally, conclusion is given in the last section.

2. Preliminaries

In this section, some basic definitions PFSs as well as the concept of the reciprocal property of pairwise comparison matrices are also discussed.

Definition 1. [24] A set $\tilde{P} = \{\langle x, \mu_{\tilde{P}}(x), \ \nu_{\tilde{P}}(x)\rangle \mid x \in X, \ 0 \le \mu_{\tilde{P}}(x) \le 1, 0 \le \nu_{\tilde{P}}(x) \le 1, \ 0 \le \ \mu_{\tilde{P}}(x)^2 + \nu_{\tilde{P}}(x)^2 \le 1\}$, defined on the universal set X, is said to be a Pythagorean fuzzy set (PFS), where, $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ represents the degree of membership and degree of non-membership respectively of the element x in $\tilde{P}$. The pair $\langle \mu_{\tilde{P}}, \nu_{\tilde{P}}\rangle$ is called an PFN with hesitation degree $\pi_{\tilde{P}}(x) = \sqrt{1 - \ \mu_{\tilde{A}}(x)^2 - \nu_{\tilde{A}}(x)^2}$.

Definition 2. [50] Let $\tilde{P}_1 = \langle \mu_1, \nu_1\rangle$ and $\tilde{P}_2 = \langle \mu_2, \nu_2\rangle$ be any two PFNs and $k > 0$ then, the arithmetic operations are defined as follows:

(i) $\tilde{P}_1 \oplus \tilde{P}_2 = \langle \sqrt{{\mu_1}^2 + {\mu_2}^2 - {\mu_1}^2{\mu_2}^2}\ , \ \nu_1\nu_2\rangle$,
(ii) $\tilde{P}_1 \otimes \tilde{P}_2 = \langle \mu_1\mu_2, \ \sqrt{{\nu_1}^2 + {\nu_2}^2 - {\nu_1}^2{\nu_2}^2}\ \rangle$,
(iii) $k \otimes \tilde{P}_1 = \langle \sqrt{1 - (1 - {\mu_1}^2)^k}\ , \ {\nu_1}^k\rangle$,
(iv) ${\tilde{P}_1}^{\,k} = \langle \ {\mu_1}^k, \ \sqrt{1 - (1 - {\nu_1}^2)^k}\rangle$.

Definition 3. [50] Let $\tilde{P}_1 = \langle \mu_1, \nu_1\rangle$ be a PFN, a score function SF of $\tilde{P}_1$is defined as:

$$SF(\tilde{P}_1) = \mu_{\tilde{P}_1}^2 - \nu_{\tilde{P}_1}^2, \ \ SF(\tilde{P}_1) \in \ [-1, 1] \tag{1}$$

and an accuracy function AF is defined as

$$AF(\tilde{P}_1) = \mu_{\tilde{P}_1}^2 + \nu_{\tilde{P}_1}^2, \ \ AF(\tilde{P}_1) \in \ [0, 1]. \tag{2}$$

Definition 4. [18, 19] A set $\tilde{P} = \{\langle x, [\mu_{\tilde{P}}^L(x), \mu_{\tilde{P}}^U(x)], [\nu_{\tilde{P}}^L(x), \nu_{\tilde{P}}^U(x)]\rangle \mid x \in X, \ 0 \le \mu_{\tilde{P}}^L(x) \le \mu_{\tilde{P}}^U(x) \le 1, \ 0 \le \nu_{\tilde{P}}^L(x) \le \nu_{\tilde{P}}^U(x) \le 1, \ \mu_{\tilde{P}}^U(x)^2 + \nu_{\tilde{P}}^U(x)^2 \le 1\}$, defined on the universal set X, is said to be an interval-valued Pythagorean fuzzy set (IVPFS), where, $[\mu_{\tilde{P}}^L(x), \mu_{\tilde{P}}^U(x)]$ and $[\nu_{\tilde{P}}^L(x), \nu_{\tilde{P}}^U(x)]$ represents the intervals of degree of membership and degree of non-membership respectively of the element x in $\tilde{A}$. Moreover, the interval of hesitation is $\pi_{\tilde{P}}(x) = \left[\sqrt{1 - \ \mu_{\tilde{P}}^U(x)^2 - \nu_{\tilde{P}}^U(x)^2}\ , \ \sqrt{1 - \ \mu_{\tilde{P}}^L(x)^2 - \nu_{\tilde{P}}^L(x)^2}\ \right]$ and the pair $\langle [\mu_{\tilde{P}}^L\ , \mu_{\tilde{P}}^U], [\nu_{\tilde{P}}^L, \nu_{\tilde{P}}^U]\rangle$ is called an interval-valued Pythagorean fuzzy number (IVPFN).

Definition 5. [18-19] *Let* $\tilde{P}_1 = \langle [\mu_1^L\ , \mu_1^U], [\nu_1^L, \nu_1^U]\rangle$ *and* $\tilde{P}_2 = \langle [\mu_2^L\ , \mu_2^U], [\nu_2^L, \nu_2^U]\rangle$ *be any two an interval-valued Pythagorean fuzzy numbers (IVPFNs) and* $k > 0$ *then, the arithmetic operations are defined as follows:*

(i) $\tilde{P}_1 \oplus \tilde{P}_2 = \left\langle \begin{bmatrix} \sqrt{(\mu_1^L)^2 + (\mu_2^L)^2 - (\mu_1^L)^2(\mu_2^L)^2}\ , \\ \sqrt{(\mu_1^U)^2 + (\mu_2^U)^2 - (\mu_1^U)^2(\mu_2^U)^2} \end{bmatrix}, [\nu_1^L\nu_2^L, \ \nu_1^U\nu_2^U] \right\rangle$,

(ii) $\tilde{P}_1 \otimes \tilde{P}_2 = \left\langle [\mu_1^L\mu_2^L, \ \mu_1^U\mu_2^U], \ \begin{bmatrix} \sqrt{(\nu_1^L)^2 + (\nu_2^L)^2 - (\nu_1^L)^2(\nu_2^L)^2}\ , \\ \sqrt{(\nu_1^U)^2 + (\nu_2^U)^2 - (\nu_1^U)^2(\nu_2^U)^2} \end{bmatrix} \right\rangle$,

(iii) $k\otimes\tilde{P}_1 = \langle\left[\sqrt{1-\left(1-{\mu_1^L}^2\right)^k}\ ,\sqrt{1-\left(1-{\mu_1^U}^2\right)^k}\right],[(\nu_1^L)^k(\nu_1^U)^k]\rangle$,

(iv) ${\tilde{P}_1}^k = \langle[(\mu_1^L)^k(\mu_1^U)^k],\ \left[\sqrt{1-\left(1-{\nu_1^L}^2\right)^k}\ ,\sqrt{1-\left(1-{\nu_1^U}^2\right)^k}\right]\rangle$.

Definition 6. [18-19] Let $\tilde{P}_j = \langle[\mu_j^L\ ,\mu_j^U],[\nu_j^L,\nu_j^U]\rangle$ be any collection of IVPFNs and λ_j be the weight vector of $\tilde{P}_j\ (j=1,2,\dots,n)$ such that $\sum_{j=1}^n \lambda_j = 1,\ \lambda_j > 0$. Then, interval-valued Pythagorean fuzzy averaging (IVPFA) operator is defined as:

$$IVPFA(\tilde{P}_1,\tilde{P}_2,\dots,\tilde{P}_n) = \left\langle \begin{matrix}\left[\sqrt{1-\prod_{j=1}^n\left(1-{\mu_j^L}^2\right)^{\lambda_j}}\ ,\sqrt{1-\prod_{j=1}^n\left(1-{\mu_j^U}^2\right)^{\lambda_j}}\right], \\ \left[\prod_{j=1}^n\left({\upsilon_j^L}^2\right)^{\lambda_j},\prod_{j=1}^n\left({\upsilon_j^U}^2\right)^{\lambda_j}\right]\end{matrix}\right\rangle \quad (3)$$

where $\lambda_j = \frac{1}{n}$.

2.1 Pairwise Comparison Matrix

The concept of pairwise comparison matrix is a key in the utilization of the crisp AHP [26-29] method. Pairwise comparison is simply, comparing two objects at a time e.g., if a decision maker D_1 likes an Apple (A) more than a Banana (B) this judgment can be represented by using Saaty's $\left[\frac{1}{9},9\right]$ ratio scale [26] as $\frac{A}{B} = 3$ and obviously the relation between B and A can be represented by the ratio $\frac{B}{A} = \frac{1}{3}$. Therefore, the whole judgment of decision maker D_1 regarding the alternatives A and B can be represented mathematically in the form of a pairwise comparison matrix $T = \begin{matrix}A\\B\end{matrix}\begin{pmatrix}1 & 3\\ \frac{1}{3} & 1\end{pmatrix}$. Hence, the propriety $t_{ij} = \frac{1}{t_{ji}}$ of matrix $T = \left(t_{ij}\right)_{m\times m}$ is known as reciprocal property of the pairwise comparison matrix. Usually, two types of the pairwise comparison matrices are used in the literature [26-29, 24, 25] one is multiplicative pairwise comparison matrix and another is additive pairwise comparison matrix. The following definitions express the situation below.

Definition 7 [26-29] A square matrix $M = \left(m_{ij}\right)_{m\times m}$ of m objects $(o_1,o_2,\dots,o_m)$ is said to be a multiplicative pairwise comparison matrix if it satisfies the conditions $m_{ij} = 1\,;i=j$ and $m_{ij} = \frac{1}{m_{ji}}\,;$ $i\neq j\ \forall\ i,j = 1,2,\dots,n$ (reciprocal propriety) where, the elements m_{ij} represents preference intensity of object o_i over the object o_j i.e., o_i is m_{ij} -times as good as o_j.

Definition 8 [42] A square matrix $A = \left(a_{ij}\right)_{m\times m}$ of m objects $(o_1,o_2,\dots,o_m)$ is said to be a additive pairwise comparison matrix if it satisfies the conditions $a_{ij} = 0.5\,;i=j$ and $a_{ij}+a_{ji} = 1\,;i\neq j\ \forall\ a_{ij}\in[0,1],\ i,j = 1,2,\dots,n$.

Moreover, Fedrizzi and Brunelli [16] showed that an additive pairwise comparison matrix can be transformed into a multiplicative pairwise comparison matrix and vice versa. On applying the following expressions respectively.

$$m_{ij} = 9^{2\times a_{ij}-1};\forall\ a_{ij}\in[0,1];\ i,j = 1,2,\dots,n \quad (4)$$

$$a_{ij} = \frac{1}{2}(1+\log_9\ m_{ij})\,;\forall\ m_{ij}\in\left[\frac{1}{9},9\right]\ ;\ i,j = 1,2,\dots,n\,. \quad (5)$$

2.2. Possibility Degree Measure

In order to transform the interval valued Pythagorean fuzzy pairwise comparison matrix into the crisp pairwise comparison matrix to preserve the reciprocal property of transformed crisp matrix.

In this paper, firstly we transform IVPFNs into the intuitionistic fuzzy numbers (IFNs) based on the equations (1) and (2), score and accuracy function of PFSs as follows:

Definition 9 Let $\tilde{P}_1 = \langle [\mu_{\tilde{P}_1}^L, \mu_{\tilde{P}_1}^U], [\nu_{\tilde{P}_1}^L, \nu_{\tilde{P}_1}^U] \rangle$ be an IVPFN such that $0 \le \mu_{\tilde{P}}^L(x) \le \mu_{\tilde{P}_1}^U(x) \le 1$, $0 \le \nu_{\tilde{P}_1}^L(x) \le \nu_{\tilde{P}_1}^U(x) \le 1$, $\mu_{\tilde{P}_1}^U(x)^2 + \nu_{\tilde{P}_1}^U(x)^2 \le 1$. Then, the corresponding intuitionistic fuzzy number (IFN) is defined as $A = \langle \mu, \nu \rangle$ where, $\mu = \frac{(\mu_{\tilde{P}_1}^U)^2 + (\mu_{\tilde{P}_1}^L)^2}{2}$ and $\nu = \frac{(\nu_{\tilde{P}_1}^U)^2 + (\nu_{\tilde{P}_1}^L)^2}{2}$ are the membership and non-membership degree of A respectively. Note, if $\tilde{P}_1 \ne \tilde{P}_2$ and $\mu = \nu$ then, in this case use $\mu = \frac{(\mu_{\tilde{P}_1}^U)^2 - (\mu_{\tilde{P}_1}^L)^2}{2}$ and $\nu = \frac{(\nu_{\tilde{P}_1}^U)^2 - (\nu_{\tilde{P}_1}^L)^2}{2}$. Moreover, $0 \le \mu + \nu \le 1$ with hesitation degree $\pi = 1 - \mu - \nu$.

Secondly, the possibility degree measure of IFNs is used [28]. Therefore, the possibility degree measure of any two different IFNs $A_i \ge A_j$; $i, j = 1, 2, \ldots, n$ is denoted by $p(A_i \ge A_j)$ and defined as [20]:

Definition 10 [20] Possibility degree measure $p(A_1 \ge A_2)$ of any two IFNs $A_1 = \langle \mu_{A_1}, \nu_{A_1} \rangle$ and $A_2 = \langle \mu_{A_2}, \nu_{A_2} \rangle$ is defined as:

$$p(A_1 \ge A_2) = min\left\{max\left\{\frac{1 + \mu_{A_1} - 2\mu_{A_2} - \nu_{A_2}}{\pi_{A_1} + \pi_{A_2}},\ 0\right\},\ 1\right\} \tag{6}$$

if either $\pi_{A_1} \ne 0$ or $\pi_{A_2} \ne 0$. Otherwise if $\pi_{A_1} = \pi_{A_2} = 0$, then

$$p(A_1 \ge A_2) = \begin{cases} 1; & \mu_{A_1} > \mu_{A_2} \\ 0; & \mu_{A_1} < \mu_{A_2} \\ 0.5; & \mu_{A_1} = \mu_{A_2} \end{cases} \tag{7}$$

and satisfies the following properties:

(i) $0 \le p(A_1 \ge A_2) \le 1$
(ii) $p(A_1 \ge A_2) = p(A_2 \ge A_1) = 0.5$, if $p(A_1 \ge A_2) = p(A_2 \ge A_1)$
(iii) $p(A_1 \ge A_2) + p(A_2 \ge A_1) = 1$
(iv) $p(A_1 \ge A_2) = 0$ if $\mu_{A_2} - \pi_{A_2} \ge \mu_{A_1}$, $p(A_1 \ge A_2) = 1$, if $\mu_{A_1} - \pi_{A_1} \ge \mu_{A_2}$.

3. Ilbahara et al.'s Proposed Pythagorean Fuzzy AHP

To point out the flaws of the existing method [21], there is a need to discuss the steps of existing method [21]. For the convenience of the readers, instead of explaining the general steps of the existing method [21] the steps of a numerical example are discussed.

The following steps of the existing method [21], are used to obtain the weights of the criteria/alternatives of the decision matrix are as:

Step 1: Construct the interval valued Pythagorean fuzzy pairwise comparison matrix $A = (a_{ij})_{m\times m}$ based on the linguistic scale [21, Table 6, pp. 127], where, $a_{ij} = \langle \mu_{ijL}, \mu_{ijU}, \nu_{ijL}, \nu_{ijU} \rangle$ is a Pythagorean fuzzy number. Also μ_{ijL}, μ_{ijU} and ν_{ijL}, ν_{ijU} are lower, upper membership and non-membership functions respectively. For example, consider the interval valued Pythagorean fuzzy pairwise comparison matrix of criteria C_1 and C_2 are shown in Table 1.

Table 1. Interval valued Pythagorean fuzzy pairwise comparison matrix $\boldsymbol{A}$ of criteria $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$

Criteria	C_1	C_2
C_1	$\langle[0.1965, 0.1965], [0.1965, 0.1965]\rangle$	$\langle[0.65, 0.80], [0.20, 0.35]\rangle$
C_2	$\langle[0.20, 0.35], [0.65, 0.80]\rangle$	$\langle[0.1965, 0.1965], [0.1965, 0.1965]\rangle$

Step 2: Transform the interval valued Pythagorean fuzzy pairwise comparison matrix A into difference matrix $= \left(\langle d_{ijL},\ d_{ijU}\rangle\right)_{m\times m}$, where, $d_{ijL} = \mu_{ijL}^2 - v_{ijU}^2$ and $d_{ijU} = \mu_{ijU}^2 - v_{ijL}^2$. Therefore, the interval valued Pythagorean fuzzy pairwise comparison matrix (shown in Table 1) will be transformed into difference matrix D (shown in Table 2).

Table 2. Difference matrix $\boldsymbol{D}$ of criteria $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$

Criteria	C_1	C_2
C_1	$\langle 0, 0\rangle$	$\langle 0.3000, 0.6000\rangle$
C_2	$\langle -0.6000, -0.3000\rangle$	$\langle 0, 0\rangle$

Step 3: Transform the difference matrix $D = \left(\langle d_{ijL}, d_{ijU}\rangle\right)_{m\times m}$ into the interval multiplicative matrix $S = \left(\left[s_{ijL},\ s_{ijU}\right]\right)_{m\times m}$, where $s_{ijL} = \sqrt{1000^{d_{ijL}}}$ and $s_{ijU} = \sqrt{1000^{d_{ijU}}}$.
Therefore, the difference matrix D (shown in Table 2) will be transformed into the interval multiplicative matrix S (shown in Table 3).

Table 3. Interval multiplicative matrix $\boldsymbol{S}$ of criteria $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$

Criteria	C_1	C_2
C_1	$\langle 1, 1\rangle$	$\langle 2.8184,\ 7.9433\rangle$
C_2	$\langle 0.1259, 0.3548\rangle$	$\langle 1, 1\rangle$

Step 4: Calculate the determinacy value $\tau = \left(\tau_{ij}\right)_{m\times m}$ where, $\tau_{ij} = 1 - \left(\mu_{ijU}^2 - \mu_{ijL}^2\right) - \left(v_{ijU}^2 - v_{ijL}^2\right)$. Therefore, from Table 1, the determinacy matrix $T = \begin{bmatrix} 1.00 & 0.700 \\ 0.700 & 1.00 \end{bmatrix}$.
Step 5: To transform the interval multiplicative matrix $S = \left(\left[s_{ijL},\ s_{ijU}\right]\right)_{m\times m}$ into crisp matrix $= \left(t_{ij}\right)_{m\times m}$, where, $t_{ij} = \left(\frac{s_{ijL}+s_{ijU}}{2}\right)\tau_{ij}$. Therefore, the interval multiplicative matrix S (shown in Table 3) will be transformed into the crisp matrix T (shown in Table 4).

Table 4. Crisp comparison matrix $\boldsymbol{T}$ of criteria $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$

Criteria	C_1	C_2
C_1	1	3.7666
C_2	0.1682	1

Step 6: Finally using the crisp AHP to calculate the normalized priority weights of transformed crisp matrix T, using the relation $W_i = \frac{\sum_{j=1}^{n} t_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} t_{ij}}$. For example, on applying the crisp AHP on the crisp matrix (shown in Table 4), the normalized priority weights of criteria $C_1 = 0.8032$ and $C_2 = 0.1968$.

4. Flaws in The Ilbahara et al.'s Pythagorean Fuzzy AHP

In order to calculate the priority weights of alternatives/criteria with the help of crisp AHP method [26-29] the following conditions are necessary for its implementation:

(I) Every criteria/alternative matrix should satisfy the reciprocal propriety of pairwise comparison matrix i.e.,

$a_{ij} = 1$, $i = j$ and $a_{ij} = \frac{1}{a_{ji}}$, $i \neq j, \forall\, i, j = 1, 2, \ldots, n$.

(II) The judgment of decision maker should be consistent i.e., the pairwise comparison matrix satisfies the condition $CR = \frac{CI}{RI(n)} < 0.1$ where, $= \frac{\lambda_{max} - n}{n-1}$, λ_{max} is largest eigenvalue, n is order of matrix and $RI(n)$ is random index [26-29].

If the above two conditions will be satisfied for the transformed crisp matrix, then we will apply the crisp AHP method [26-29] to determine the normalized priority weights of criteria/alternative of the crisp pairwise comparison matrix.

However, on applying the steps of Ilbahara et al.'s existing method [21], discussed in Section 3, in Step 5, it can be easily verified that for the elements of the transformed crisp matrix (shown in Table 4) the reciprocal property $a_{ij} = \frac{1}{a_{ji}}$, $i \neq j, \forall\, i, j = 1, 2, \ldots, n$ is not satisfying i.e., $a_{12} = 3.7666 \neq \frac{1}{a_{21}} = 0.1682$. Therefore, the transformed crisp matrix, on applying Step 5, of Ilbahara et al.'s existing method [21], discussed in Section 3, violates the reciprocal propriety of pairwise comparison matrix i.e., $a_{ij} \neq \frac{1}{a_{ji}}$, $i \neq j, \forall\, i, j = 1, 2, \ldots, n$.

Moreover, the transformed crisp matrix, on applying the steps of Ilbahara et al.'s existing method [21], discussed in Section 3, neither satisfying the conditions (I), (II) nor the Definition 7 and Definition 8, so applying the crisp AHP on crisp non-pairwise comparison matrices, to calculate the normalized priority weights is a meaningless task and will mislead to the decision maker, that can result in a heavy loss in any value-added model. Therefore, Step 4 to Step 11 of the existing framework [21, Section 3.5, pp. 128] cannot be used. Hence, the Ilbahara et al. integrated method [21] is not valid in its present form. Keeping the same in mind, in the next section, a modified method is proposed.

5. A New Pythagorean Fuzzy Analytic Hierarchy Process

To overcome the flaws of Ilbahara et al.'s existing method [21], in this section, a modified method is proposed. Using definitions 9 and 10, discussed in Section 2.1, the possibility degree measure [20] is used to preserve the reciprocal property of crisp pairwise comparison matrices. In order to construct the crisp pairwise comparison matrices the steps of the modified method are as follows:

Step 1: Construct the Pythagorean fuzzy pairwise comparison matrix $\tilde{P}_1 = (p_{ij})_{m\times m}$ based on the linguistic scale [21, Table 6, pp. 127], where, $p_{ij} = \langle [\mu_{ijL}, \mu_{ijU}], [v_{ijL}, v_{ijU}] \rangle$ is an IVPFN. Also μ_{ijL}, μ_{ijU} and v_{ijL}, v_{ijU} are lower, upper membership and non-membership functions respectively.

Step 2: Transform the interval valued Pythagorean fuzzy pairwise comparison matrix $\tilde{P}_1$ into the corresponding aggregated column interval valued Pythagorean fuzzy matrix $AP_i = \left(p_{ij} = \langle [\mu_{ijL}, \mu_{ijU}], [v_{ijL}, v_{ijU}] \rangle\right)_{m\times 1}$; $(j = 1, 2, \ldots, n)$, on applying the IVPFA operator:

$$AP_i = IVPFA(\tilde{P}_1, \tilde{P}_2, \ldots, \tilde{P}_n) = \left\langle \begin{array}{c} \left[\sqrt{1 - \prod_{j=1}^{n}\left(1 - \mu_j^{L\,2}\right)^{\lambda_j}}, \sqrt{1 - \prod_{j=1}^{n}\left(1 - \mu_j^{U\,2}\right)^{\lambda_j}}\right], \\ \left[\prod_{j=1}^{n}(v_j^{L\,2})^{\lambda_j}, \prod_{j=1}^{n}(v_j^{U\,2})^{\lambda_j}\right] \end{array} \right\rangle \quad (8)$$

where, $\lambda_j = \frac{1}{n}$.

Step 3: Using the Definition 9, to transform the aggregated column interval valued Pythagorean fuzzy matrix $AP_i = \left(\langle [\mu_{ijL}, \mu_{ijU}], [v_{ijL}, v_{ijU}] \rangle\right)_{m\times 1}$ into the corresponding intuitionistic fuzzy matrix $A_i = \left(\langle \mu_{ij}, v_{ij} \rangle\right)_{m\times 1}$

; $(j = 1, 2, \ldots, n)$, where $\mu_{ij} = \frac{\left(\mu_{ij}^L\right)^2 + \left(\mu_{ij}^U\right)^2}{2}$ and $\nu_{ij} = \frac{\left(\nu_{ij}^L\right)^2 + \left(\nu_{ij}^U\right)^2}{2}$.

Step 4: Construct the possibility degree matrix $P = \left(p_{ij}\right)_{m\times m}$ on utilizing the Definition 10, discussed in Section 2.2, where, $p_{ij} = p\left(A_i \geq A_j\right); (i, j = 1, 2, \ldots, m)$ and $A_i = \langle \mu_{A_i}, \nu_{A_i} \rangle$, $A_j = \langle \mu_{A_j}, \nu_{A_j} \rangle$. If either $\pi_{A_i} \neq 0$ or $\pi_{A_j} \neq 0$ then

$$p\left(A_i \geq A_j\right) = min\left\{max\left\{\frac{1+\mu_{A_i} - 2\mu_{A_j} - \nu_{A_j}}{\pi_{A_i} + \pi_{A_j}}\ ,\ 0\right\},\ 1\right\} \tag{9}$$

Otherwise if $\pi_{A_i} = \pi_{A_j} = 0$, then

$$p\left(A_i \geq A_j\right) = \begin{cases} 1; & \mu_{A_i} > \mu_{A_j} \\ 0; & \mu_{A_i} < \mu_{A_j} \\ 0.5; & \mu_{A_i} = \mu_{A_j} \end{cases}. \tag{10}$$

Moreover, it is easily verified that transformed matrix $P = \left(p_{ij}\right)_{m\times m}$ satisfying the additive reciprocal property of pairwise comparison matrix i.e., $p_{ij} + p_{ji} = 1$ and $p_{ij} \in [0, 1]$.

Step 5: On using the expression (4), to transform the matrix $P = \left(p_{ij}\right)_{m\times m}$ into the multiplicative pairwise comparison matrix $= \left(m_{ij}\right)_{m\times m}$, where $m_{ij} = 9^{2\times p_{ij} - 1}; \forall\, p_{ij} \in [0, 1]$. Therefore, the reciprocal property $m_{ij} = 1; i = j$ and $m_{ij} = \frac{1}{m_{ji}}$; $i \neq j$ will always satisfied for matrix M.

Step 6: Finally using the crisp AHP to calculate the normalized priority weights of transformed crisp pairwise comparison matrix $P = \left(p_{ij}\right)_{m\times m}$, by using the relation:

$$W_i = \frac{\sum_{j=1}^{n} p_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}}\ ;\ (i, j = 1, 2, \ldots, n) \tag{11}$$

and check that $W(A_i) > W(A_j)$ or $W(A_i) < W(A_j)$ or $W(A_i) = W(A_j)$.

Case (i): If $W(A_i) = W(A_j)$ then $A_i = A_j$,

Case (ii): If $W(A)_i > W(A_j)$ then $A_i > A_j$,

Case (iii): If $W(A_i) < W(A_j)$ then $A_i < A_j$.

6. Exact Transformation

In order to obtain the exact weights of criteria/alternatives, we need to transform the interval valued Pythagorean fuzzy pairwise comparison matrix into the crisp pairwise comparison matrix without losing any information, given by the decision maker. Therefore on applying the steps of the modified method proposed in Section 5, for the convenience, it can be easily verified with the help of the same example, discussed in Section 3, that the transformed crisp matrix, on applying Steps of modified method, always preserves both the additive as well multiplicative reciprocal property of crisp pairwise comparison matrix i.e., $a_{ij} + a_{ji} = 1; p_{ij} \in [0, 1]$ and $a_{ij} = 1$, $i = j$; $a_{ij} = \frac{1}{a_{ji}}$, $i \neq j, \forall\, i, j = 1, 2, \ldots, n$ respectively.

Consider the interval valued Pythagorean fuzzy pairwise comparison matrix $\tilde{P}_1$ of criteria C_1 and C_2 as shown in Table 5.

Table 5. Interval valued Pythagorean fuzzy pairwise comparison matrix $\tilde{\boldsymbol{P}}_1$ of criteria $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$

Criteria	C_1	C_2
C_1	$\langle[0.1965, 0.1965], [0.1965, 0.1965]\rangle$	$\langle[0.65, 0.80], [0.20, 0.35]\rangle$
C_2	$\langle[0.20, 0.35], [0.65, 0.80]\rangle$	$\langle[0.1965, 0.1965], [0.1965, 0.1965]\rangle$

Using Step 2 of the modified method proposed in Section 5, to transform the interval valued Pythagorean fuzzy pairwise comparison matrix $\tilde{P}_1 = \left(p_{ij}\right)_{2\times 2}$ of criteria C_1 and C_2 (shown in Table 5) into the corresponding aggregated column interval valued Pythagorean fuzzy matrix $AP_i = \left(a_{ij}\right)_{2\times 1}; (j = 1, 2)$, where a_{ij} is obtain on applying expression (6) as follows:

$$AP_1 = IVPFA(p_{11}, p_{12}) = \langle\left[\sqrt{1-(1-0.1965^2)^{\frac{1}{2}}\times(1-0.65^2)^{\frac{1}{2}}}, \sqrt{1-(1-0.1965^2)^{\frac{1}{2}}\times(1-0.80^2)^{\frac{1}{2}}}\right], \left[(0.1965)^{\frac{1}{2}}\times(0.20)^{\frac{1}{2}}, (0.1965)^{\frac{1}{2}}\times(0.35)^{\frac{1}{2}}\right]\rangle$$

$$= \langle[0.5049,\ 0.6416], [0.1982,\ 0.2622]\rangle.$$

$$AP_2 = IVPFA(p_{21}, p_{22}) = \langle\left[\sqrt{1-(1-0.20^2)^{\frac{1}{2}}\times(1-0.1965^2)^{\frac{1}{2}}}, \sqrt{1-(1-0.35^2)^{\frac{1}{2}}\times(1-0.1965^2)^{\frac{1}{2}}}\right], \left[(0.65)^{\frac{1}{2}}\times(0.1965)^{\frac{1}{2}}, (0.80)^{\frac{1}{2}}\times(0.1965)^{\frac{1}{2}}\right]\rangle$$

$$= \langle[0.1983,\ 0.2855], [0.3574,\ 0.3965]\rangle.$$

Table 6. Aggregated interval valued Pythagorean matrix $\boldsymbol{AP}$ of criteria $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$

Criteria	Aggregated interval valued Pythagorean fuzzy column matrix A_i
C_1	$\langle[0.5049, 0.6416], [0.1982, 0.2622]\rangle$
C_2	$\langle[0.1983, 0.2855], [0.3574, 0.3965]\rangle$

Now using Step 3 of modified method, proposed in Section 5, to transform the aggregated interval valued Pythagorean fuzzy matrix AP (shown in Table 6) into the corresponding intuitionistic fuzzy column matrix $A_i = \left(\langle\mu_{ij},\ \nu_{ij}\rangle\right)_{2\times 1}; (j = 1, 2)$, where μ_{ij} and ν_{ij} are obtain as follows:

$\mu_{11} = \frac{(0.5049)^2+(0.6416)^2}{2} = 0.3333,$ $\nu_{11} = \frac{(0.1982)^2+(0.2622)^2}{2} = 0.0540,$ $\pi_{11} = 0.6127$ and $\mu_{21} = \frac{(0.19839)^2+(0.2855)^2}{2} = 0.0604,$ $\nu_{21} = \frac{(0.3574)^2+(0.3965)^2}{2} = 0.1425,$ $\pi_{21} = 0.7971$ therefore $A = \begin{pmatrix}\langle 0.3333 & 0.0540\rangle \\ \langle 0.0604 & 0.1425\rangle\end{pmatrix}$.

Using the expressions (7) and (8) of the Step 4, of modified method, proposed in Section 5, the obtained possibility degree matrix $P = \begin{pmatrix}0.5000 & 0.7590 \\ 0.2410 & 0.5000\end{pmatrix}$ and using Step 5 of modified method discussed in Section 5, to transform the matrix $P = \left(p_{ij}\right)_{m\times m}$ into the multiplicative pairwise comparison matrix $M = \begin{pmatrix}1 & \frac{1551}{497} \\ \frac{497}{1551} & 1\end{pmatrix}$.

Therefore, it can be easily verified that the matrix M as well as the matrix P both are satisfying the additive as well as the multiplicative reciprocal property of pairwise comparison matrix i.e., $p_{12} + p_{21} = 1$; $p_{ij} \in [0,1]$ as well as $m_{12} = \frac{1551}{497}$ and $m_{21} = \frac{497}{1551}$ preserves the reciprocal property $p_{ij} + p_{ji} = 1$; $p_{ij} \in [0,1]$ and $a_{ij} = 1, i = j$; $a_{ij} = \frac{1}{a_{ji}}, i \neq j, \forall\, i, j = 1, 2, \dots, n$ of crisp pairwise comparison matrices respectively.

Finally, using Step 6 of modified method discussed in Section 5, to obtain normalized priority weights of criteria C_1 and C_2 are 0.6295 and 0.3705 respectively. And it can be easily verified that $C_1 + C_2 = 1$ i.e., the obtained weights are normalized weights.

7. A practical Multi Criteria Decision Making Problem

An information technology institute (Minhaj Technologies) in a rural village Awaneera Zainapora, of Jammu and Kashmir located in north-India. The institute Facilitate the young generation with the knowledge of computer education and want to provide Placements for rural candidates especially the Girl candidates. Annually, the institute trained more the 750 candidates. Due to the large role of candidates the institute wants to purchase more desktop computers with a maximum usability with high performance and minimum cost. To select the best desktop computer from a set of four different alternatives: (i) Alternative first is Dell desktop computer, very expensive with faster processer, (ii) Alternative second is HP desktop computer, moderate expensive with faster processer, (iii) Alternative third is Asus desktop computer, with faster processer and moderate expensive, (iv) Alternative fourth is Asser desktop computer and (v) Alternative fifth is Toshiba desktop computer, slow and very cheap.

To select the best alternative from a set of available alternatives $A = \{A_1, A_2, A_3, A_4, A_5\}$, based on the criteria $C = \{C_1 = \text{Cost},\ C_2 = \text{maximum usability with high performance}\}$.To apply the proposed method for the selection of a best desktop computer, the following computational process is required.

Step 1: The information provided by the decision maker regarding the criterion with respect to the goal of the problem is represented in the form of an interval valued Pythagorean fuzzy pairwise comparison matrix as shown in Table 7. Similarly, for the alternatives with respect to the criterion C_1 and C_2 as shown in Table 8 and Table 9 respectively.

Table 7. Interval valued Pythagorean fuzzy pairwise comparison matrix $\widetilde{\boldsymbol{P}}_{\mathbf{C}}$ of criteria $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$

Criteria	C_1	C_2
C_1	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩	⟨[0.55 , 0.65], [0.35 , 0.45]⟩
C_2	⟨[0.35 , 0.45], [0.55 , 0.65]⟩	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩

Table 8. Interval valued Pythagorean fuzzy pairwise comparison matrix $\widetilde{\boldsymbol{P}}_{\mathbf{A}}$of alternatives with respect to criteria $\boldsymbol{C_1}$

Alternatives	A_1	A_2	A_3	A_4	A_5
A_1	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩	⟨[0.65 , 0.80], [0.20 , 0.35]⟩	⟨[0.80 , 0.90], [0.10, 0.20]⟩	⟨[0.65 , 0.80], [0.20 , 0.35]⟩	⟨[0.55 , 0.65], [0.35 , 0.45]⟩
A_2	⟨[0.20 , 0.35], [0.65 , 0.80]⟩	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩	⟨[0.55 , 0.65], [0.35 , 0.45]⟩	⟨[0.35 , 0.45], [0.55 , 0.65]⟩	⟨[0.20 , 0.35], [0.65 , 0.80]⟩
A_3	⟨[0.0 , 0.0], [0.9 , 1]⟩	⟨[0.35 , 0.45], [0.55 , 0.65]⟩	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩	⟨[0.20 , 0.35], [0.65 , 0.80]⟩	⟨[0.0 , 0.0], [0.9 , 1]⟩
A_4	⟨[0.20 , 0.35], [0.65 , 0.80]⟩	⟨[0.55 , 0.65], [0.35 , 0.45]⟩	⟨[0.65 , 0.80], [0.20 , 0.35]⟩	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩	⟨[0.35 , 0.45], [0.55 , 0.65]⟩
A_5	⟨[0.35 , 0.45], [0.55 , 0.65]⟩	⟨[0.65 , 0.80], [0.20 , 0.35]⟩	⟨[0.80 , 0.90], [0.10, 0.20]⟩	⟨[0.55 , 0.65], [0.35 , 0.45]⟩	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩

Table 9. Interval valued Pythagorean fuzzy pairwise comparison matrix $\widetilde{\boldsymbol{P}}_{\mathbf{A}}$ alternatives with respect to criteria $\boldsymbol{C_2}$.

Alternatives	A_1	A_2	A_3	A_4	A_5
A_1	⟨[0.1965 , 0.1965 [0.1965 , 0.196	⟨[0.1 , 0.2], [0.8, 0.9]⟩	⟨[0.20 , 0.35], [0.65 , 0.80]⟩	⟨[0.35 , 0.45], [0.55 , 0.65]⟩	⟨[0.55 , 0.65], [0.35 , 0.45]⟩
A_2	⟨[0.80 , 0.90], [0.10, 0.20]⟩	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩	⟨[0.55 , 0.65], [0.35 , 0.45]⟩	⟨[0.65 , 0.80], [0.20 , 0.35]⟩	⟨[0.9 , 1], [0 , 0]⟩
A_3	⟨[0.0 , 0.0], [0.9 , 1]⟩	⟨[0.35 , 0.45], [0.55 , 0.65]⟩	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩	⟨[0.55 , 0.65], [0.35 , 0.45]⟩	⟨[0.80 , 0.90], [0.10, 0.20]⟩
A_4	⟨[0.55 , 0.65], [0.35 , 0.45]⟩	⟨[0.20 , 0.35], [0.65 , 0.80]⟩	⟨[0.35 , 0.45], [0.55 , 0.65]⟩	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩	⟨[0.65 , 0.80], [0.20 , 0.35]⟩
A_5	⟨[0.35 , 0.45], [0.55 , 0.65]⟩	⟨[0 , 0], [0.9 , 1]⟩	⟨[0.1 , 0.2], [0.8, 0.9]⟩	⟨[0.20 , 0.35], [0.65 , 0.80]⟩	⟨[0.1965 , 0.1965], [0.1965 , 0.1965]⟩

Step 2: Using Step 2 of the modified method proposed in Section 5, to transform the interval valued Pythagorean fuzzy pairwise comparison matrix $\tilde{P}_C = (p_{ij})_{2\times 2}$ of criteria C_1 and C_2 (shown in Table 7) into the corresponding aggregated column interval valued Pythagorean fuzzy matrix $AP_i = (a_{ij})_{2\times 1}; (j = 1, 2)$, where a_{ij} is obtain on applying expression (8) as follows:

$$AP_1 = IVPFA(p_{11}, p_{12}) = \langle \left[\sqrt{1-(1-0.1965^2)^{\frac{1}{2}}\times(1-0.55^2)^{\frac{1}{2}}}, \sqrt{1-(1-0.1965^2)^{\frac{1}{2}}\times(1-0.65^2)^{\frac{1}{2}}}\right], \left[(0.1965)^{\frac{1}{2}}\times(0.35)^{\frac{1}{2}}, (0.1965)^{\frac{1}{2}}\times(0.45)^{\frac{1}{2}}\right]\rangle$$

$$= \langle [0.4256, 0.5049], [0.2622, 0.2974]\rangle.$$

$$AP_2 = IVPFA(p_{21}, p_{22}) = \langle \left[\sqrt{1-(1-0.35^2)^{\frac{1}{2}}\times(1-0.1965^2)^{\frac{1}{2}}}, \sqrt{1-(1-0.45^2)^{\frac{1}{2}}\times(1-0.1965^2)^{\frac{1}{2}}}\right], \left[(0.55)^{\frac{1}{2}}\times(0.1965)^{\frac{1}{2}}, (0.65)^{\frac{1}{2}}\times(0.1965)^{\frac{1}{2}}\right]\rangle$$

$$= \langle [0.2855, 0.3527], [0.3287, 0.3574]\rangle.$$

Table 10. Aggregated interval valued Pythagorean matrix $\boldsymbol{AP}$ of criteria $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$

Criteria	Aggregated interval valued Pythagorean fuzzy column matrix A_i
C_1	$\langle [0.4256, 0.5049], [0.2622, 0.2974]\rangle$
C_2	$\langle [0.2855, 0.3527], [0.3287, 0.3574]\rangle$

Step 3: Using Step 3 of modified method, proposed in Section 5, to transform the aggregated interval valued Pythagorean fuzzy matrix AP (shown in Table 10) into the corresponding intuitionistic fuzzy column matrix $A_i = (\langle \mu_{ij}, \nu_{ij}\rangle)_{2\times 1}; (j = 1, 2)$, where μ_{ij} and ν_{ij} are obtain as follows:

$\mu_{11} = \frac{(0.4256)^2+(0.5049)^2}{2} = 0.2180$, $\nu_{11} = \frac{(0.2622)^2+(0.2974)^2}{2} = 0.0786$, $\pi_{11} = 0.7034$ and $\mu_{21} = \frac{(0.19839)^2+(0.2855)^2}{2}$ =0.1029, $\nu_{21} = \frac{(0.3574)^2+(0.3965)^2}{2} = 0.1179$ $\pi_{21} = 0.7792$ therefore $A = \begin{pmatrix}\langle 0.2180 & 0.0786\rangle \\ \langle 0.1029 & 0.1179\rangle\end{pmatrix}$.

Step 4: Using the expressions (9) and (10) of the Step 4, of modified method, proposed in Section 5, the obtained possibility degree matrix $P = \begin{pmatrix} 0.5000 & 0.6032 \\ 0.3968 & 0.5000 \end{pmatrix}$ and using Step 5 of modified method discussed in Section 5, to transform the matrix $P = (p_{ij})_{m\times m}$ into the multiplicative pairwise comparison matrix $M = \begin{pmatrix} 1 & \frac{2291}{1456} \\ \frac{1456}{2291} & 1 \end{pmatrix}$.

Step 5: Finally, on using the expression (11) of Step 6 of the modified method, discussed in Section 5, to obtain normalized priority weights of criteria C_1 and C_2 are 0.5516 and 0.4484 respectively.

Similarly, the normalized priority weights of the alternatives corresponding to the criteria C_1 and C_2 are presented in Table 11.

Table 11. Normalized priority weights of the alternatives corresponding to the criteria $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$

Alternatives	Priority weights corresponding C_1	Priority weights corresponding C_2
A_1	0.2977	0.1313
A_2	0.1420	0.3600
A_3	0.1136	0.2298
A_4	0.1848	0.1678
A_5	0.3056	0.1112

Step 6: Finally, the ranking of alternatives based on the global priority weights i.e., product of criteria and alternatives shown in Table 12. Moreover, the ranking order of the alternatives, obtained by considering the Ilbahar et al.'s existing method [21] and the proposed modified method are shown in Table 12.

Table 12. Overall ranking order of the alternatives

Alternatives	Ilbahar et al.'s existing Method [21]		Proposed modified method	
	C_i	Rank	C_i	Rank
A_1	0.1756	3	0.2231	2
A_2	0.3870	1	0.2398	1
A_3	0.1802	2	0.1657	5
A_4	0.1226	4	0.1772	4
A_5	0.1348	5	0.2184	3

8. Conclusion

This paper develops a modified Pythagorean fuzzy analytic hierarchy process based on IVPFNs which overcomes the flaws of the Ilbahar et al.'s existing method [21]. Moreover, an important property of pairwise comparison matrix have been investigated in detail and found that the existing method fails to preserve the reciprocal property of pairwise comparison matrix. Therefore, the impact of this property is clearly shown in the final raking of the decision-making problem. Finally, based on the proposed method, a real life multicriteria decision-making problem is solved and a comparison is given with the existing method. In future the proposed approach will be integrated to other decision-making approaches and solve some complex real lifer problems.

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