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Evaluating the Efficiency of Hospital Emergencies during COVID-19 Pandemic Crisis in the Presence of Undesirable Inputs in DEA

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ABSTRACT

One of the major concerns in healthcare management is the optimal allocation of staffing and resources. The global crisis created by the COVID-19 Pandemic has created extreme strains both on the staffing and the resources available to the healthcare systems. It has geometrically added to the number of patients seeking health services. In addition, the Pandemic has dramatically increased the rate of mortality in the hospitals in an unprecedented way. Therefore, in this research, in order not to sacrifice the quality of the services provided, we have used Data Envelopment Analysis (DEA) model to determine the efficiency of the emergency departments in the hospitals and the possible improvements that could be made to them. As traditional DEA models do not seek to reduce the undesirable outputs and increase the undesirable inputs, in addition to determining the efficiency of decision making units (DMU) despite some undesirable input components, the effect of these units on performance is investigated. To this end, considering the problem assumptions, first, a set of proper production possibilities is defined. Finally, a new method is introduced to determine the system's performance in the presence of some undesirable input components. The impact of undesirable components on determining efficiency is specified, and a real example is provided consisting of emergency rooms in 30 hospitals, in which five desirable inputs and four desirable outputs along with one undesirable input are considered. The example is solved using the presented model, and the efficiency scores are determined.

1. Introduction

Medical care is a complex configuration that includes primary, secondary, and post-care, and hospitals are one of the most influential organizations in this field. The most crowded part of hospitals is the emergency room, which is considered the most vital element in the treatment system [4, 10].

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The hospital's emergency department is a unit that works 24 hours a day all year round to provide treatment for all emergency, semi-emergency, and non-emergency patients in the fastest way possible. This department receives and treats more than 33 million patients in the country every year. As a result, overcrowding in hospitals' emergency rooms is a global issue [10, 22].

This problem has gotten worse since the start of the COVID-19 Pandemic. COVID-19 is an infectious disease that initially appeared in Wuhan, China, in December of 2019. As it is a highly contagious disease, it caused the outbreak of a Worldwide Pandemic afflicting people from all sects of life. During the week 25 to 31 October 2021, a slight upward trend in new weekly cases was observed, with just over 3 million new cases reported. Apart from the European Region, which reported a 6% increase in new weekly cases as compared to the previous week, other regions reported declines or stable trends. New weekly deaths increased by 8% as compared with the previous week, with over 50 000 new fatalities. Cumulatively, over 246 million confirmed cases and nearly 5 million deaths have been reported [23, 35].

In Iran, over 5,944,500 cases of COVID-19 have been reported, of which 126,550 people's lives are lost. This overwhelming number of patients added to the already existing patients visiting the hospitals has placed an extreme strain on the limited and non-existing resources of the healthcare systems in some countries. It is a well-known fact that most hospitalized COVID-19 patients require emergency and Intensive Care Services due to the complex nature of the symptoms of the illness [23, 34]. As we witnessed in India, due to lack of resources, including intensive care unit beds and ventilators, patients were lined up in the halls and the yards of the hospitals; and the staff had to pick and choose who gets the much-needed services. These consequences of the Pandemic can delay providing emergency services to the patients [5].

Studying the performance of these systems in facing the Pandemic is a worthy endeavor to undertake. Charnes et al. [7] have considered that data envelopment analysis (DEA) is an efficient measuring method in many applications, among which is comparing the healthcare systems' performance in different countries. In addition, Zakowskaa and Godycki-Cwirko [33] carried out a systematic review of the application of DEA in the evaluation of primary healthcare. The goal of this study was to standardize this method. As pointed, the metrics and the models used in the study were not conventional; therefore, further research is required [18]. Therefore, identifying hospitals with a low level of preparedness has become more crucial for disaster preparedness planning. Determining their level of disaster preparedness is a critical issue and an essential step in this process. In the line with this need, Ortiz-Barrios et al., [24] carried out a case study of Turkish hospitals and proposed a hybrid fuzzy decision-making model to evaluate the disaster preparedness of hospitals. Alemanno [1] pointed out that the Italian National Health System has suggested the need to optimize the already existing resources and implement them to overcome the crisis caused by the pandemic.

Furthermore, the type-specific efficiencies are helpful for individual hospitals to make improvements; and the functional productivities are helpful to design case-mix indexes for adjusting measures of different types of hospitals and making them comparable [10]. To this end, Kao et al. [15] identified the most productive types of hospitals by constructing one production frontier for each hospital and another for all types of hospitals. In this study, the relative distance between a type-specific frontier and the overall frontier referred to as functional productivity measures how productive the corresponding type of hospital is.

DEA is a mathematical technique based on linear programming applied to evaluate the efficiency of decision-making units dealing with multiple inputs and outputs. DEA is one of the most popular methods for determining efficiency. The efficiency boundary based on the concept of defective units was first proposed by Pareto, an Italian economist [8]. This concept was used in production by Koopman [31] in 1951 and it was introduced by Farrell to evaluate performance [8]. Jahanshahloo et al. [14] have considered that Charnes et al. [7] identified efficient boundaries using linear planning and utilized them to determine productivity. They used both output-axis and input-axis models in this method. Although these two models are not the only ones used, they are still the most popular DEA models. Many researchers use the DEA method to determine the efficiency boundary and efficiency evaluation. Rezai [27] considered the rank of set efficient units in DEA using two methods namely regular polygon area and TOPSIS. Malekmohammadi [20] suggested an interval DEA to get the overall efficiency for the functions of fuzzy profit of queuing models. Peykani et al. [25] studied empirical evidence from Tehran stock market optimistic and pessimistic with using fuzzy DEA. Santos Ateaga et al. [28] proposed an approach for solving DEA models with intuitionistic fuzzy numbers.

An increase in the number of some inputs of decision-making units' may result in efficiency reduction, leading to increased efficiency. Waste, scrap metal, and glass recycling operations are prime examples of cases where it is necessary to decrease undesirable inputs to improve efficiency. On the other hand, some of the outputs of decision-making units may be such that increasing them would reduce efficiency and decrease them, resulting in increased efficiency. For example, consider the waste of a factory or patients' deaths in hospitals and dismissals of the nurses and the doctors in training centers. These undesirable outputs should be reduced to increase efficiency. Undesirable outputs are generally desirable products; therefore, the outputs can only be improved by reducing them [2, 6].

The methods for importing undesirable outputs into the DEA can be divided into two categories:

1. Direct methods
2. Indirect methods [6].

In indirect methods, undesirable inputs and outputs in each decision-making unit are converted into desirable inputs and outputs by a descending uniform function. Then the unit's efficiency is evaluated using DEA standard models, such as the method introduced by Koopmans [17] in 1951, the method presented by Golany and Roll [12] in 1989 for importing undesirable outputs, and Seiford’s method for importing undesirable inputs and outputs in 1990 [29].

Fare et al. [9] proposed another alternative method for estimating shadow prices using the distance function. A few years later, in 1997, Kao et al. [15] proposed another way to evaluate the productivity and efficiency of undesirable outputs. They proposed using the directional distance function to increase the desirable outputs and simultaneously reduce the undesirable data. Kao et al. [15] have considered that undesirable outputs are generally desirable products; therefore, they can be reduced only by an accompanying reduction in the second product. To understand this concept, the shadow price of an undesirable output must be negative on the one hand and positive for desirable output on the other hand. Based on these conditions, Kao et al. [16] presented a data envelopment analysis model that allows the production units to be evaluated to determine the shadow price for both desirable and undesirable outputs to maximize the measured efficiency score. The proposed model satisfies the assumption of poor usability of outputs. It is also shown that there is a directional function model in a group that has widely been used in undesirable outputs modeling. However, unlike conventional measures of directional distance, the proposed model can provide an efficiency rate within the range of zero to one for easy comparison among the inefficient production units.

Kao et al. [15] developed two previous models to reduce the impact of undesirable outputs. Also, Kao [16] presented another article evaluating efficiency in the presence of undesirable outputs by changing the shadow price. Many other scholars in the field have carried out studies investigating various aspects concerning the DEA model (e.g. Podinovski, [26]; Wu et al., [31]; Malin et al., [21]; Walheer, [30]; Yu et al., [32]; Gómez-Calvet et al., [13]). On the whole, if the performance of the emergency departments in the hospitals were better than it has been, the mortality rate would have been much lower. Therefore, to prove this point, in this case, by using the introduced model, we have evaluated the hospitals' performance under two different sets of criteria: input-oriented and output-oriented.

The paper proceeds as follows. Section 2 gives the concept of Production Possibility Set and introduces the main definitions required to build the intuitionistic new DEA model. Section 3 proposed the new DEA model with undesirable outputs. Section 4 illustrates the proposed model using a case study in performance evaluation of hospitals. Section 5 summarizes the main results obtained and suggests potential extensions.

2. Production Possibility Set

Suppose we have n observations on n DMUs with input and output vectors (x_j, y_j), $j = 1,...,n$. Let $x_j = (x_{1j},...,x_{mj})^T$ and $y_j = (y_{1j},...,y_{sj})$. All $x_j \in R^m$, $y_j \in R^s$ and $x_j > 0$, $y_j > 0$, $j = 1,...,n$. The input matrix X and output matrix Y can be represented as

$$X = [x_1, ..., x_j, ...x_n], Y = [y_1, ..., y_j, ...y_n]$$

Where X is an $(m \times n)$ matrix and Y is an $(s \times n)$ matrix.

The production possibility set T is generally defined as:

$$T = \{(x, y) | x \text{ can produce } y\} \quad (1)$$

In DEA, the production possibility set under a Variable Return to Scale (VRS) technology is constructed using the observed data $(x_j, y_j), j = 1, ..., n$ as follows:

$$T = \left\{ (x, y) \middle| x \geq \sum_{j=1}^{n} \lambda_j x_j \,,\ y \leq \sum_{j=1}^{n} \lambda_j y_j \,, \lambda_j \geq 0 \,,\ \sum_{j=1}^{n} \lambda_j = 1, \ j = 1, ..., n \right\} \quad (2)$$

In the absence of undesirable factors, when $DMU_o, o \in \{1, 2, ..., n\}$ is the unit under evaluation, we can use the following BCC model [6]:

$$\begin{aligned} \theta_o^* = Min \quad & \theta_o \\ st. \quad & \\ & \sum_{j=1}^{n} \lambda_j X_{ij} \leq \theta_o X_{io} \quad && i = 1, 2, ..., m \\ & \sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{ro} \quad && r = 1, 2, ..., s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0 \quad && j = 1, ..., n \end{aligned} \quad (3)$$

Corresponding to each output, y, $L(y)$ is defined as follows:

$$L(y_j) = \{x | (x_j, y_j) \in T\} \quad (4)$$

In fact, $L(y_j)$ is a function in which y_j portrays to a subset of inputs so that inputs can produce y_j.

Now, suppose that some inputs are undesirable. Therefore, input matrix X can be represented as $X = (X^g, X^b)^T$ where $X^g = (x_{1j}^g, ..., x_{m_1 j}^g), j = 1, ..., n$ and $X^b = (x_{1j}^b, ..., x_{m_1 j}^b)$, $j = 1, ..., n$ are $(m_1 \times n)$ and $(m_2 \times n)$ matrixes that represent desirable (good) and undesirable (bad) inputs, respectively. Similarly, suppose that some outputs are undesirable. In such a case, Matrix Y can be represented as $Y = (Y^g, Y^b)^T$ where $Y^g = (y_{1j}^g, ..., y_{s_1 j}^g), j = 1, ..., n$ and $Y^b = (y_{1j}^b, ..., y_{s_2 j}^b)$, $j = 1, ..., n$ are $(s_1 \times n)$ and $(s_2 \times n)$ matrixes representing desirable (good) and undesirable (bad) inputs, respectively [29].

Definition 1. Let DMU $(x_1^g, x_1^b, y_1^g, y_1^b)$ be dominant to DMU $(x_2^g, x_2^b, y_2^g, y_2^b)$ if $(x_1^g \leq x_2^g, x_1^b \geq x_2^b, y_1^g \geq y_2^g)$ and $y_1^b \leq y_2^b$; and the unequal be strict at least in a component. So that,

$$\begin{pmatrix} -x_1^g \\ x_1^b \\ y_1^g \\ -y_1^b \end{pmatrix} \geq \begin{pmatrix} -x_2^g \\ x_2^b \\ y_2^g \\ -y_2^b \end{pmatrix}$$

Definition 2. DMU_o is efficient if in T there is no DMU to be dominant over it.

We consider the properties of the Production Possibility Set as follows:

(1) T is convex.

(2) T is closed.

(3) The monotony property of desirable inputs and outputs, so that [22]:

$$\forall u \in R_+^{m_1}, v \in R_+^{s_1}, (x^g, x^b, y^g, y^b) \in T \Rightarrow (x^g + u, x^b, y^g - v, y^b) \in T$$

This is not necessarily established for undesirable factors, as, in this case, T has no efficient DMU. We can define the Production Possibility Set T satisfying (1) through (3) by

$$T=\left\{(x^g,x^b,y^b,y^g)\middle| x^g\geq\sum_{j=1}^{n}\lambda_j x_j^g, x^b=\sum_{j=1}^{n}\lambda_j x_j^b, y^b=\sum_{j=}^{n}\lambda_j y_j^b, y^g\leq\sum_{j=1}^{n}\lambda_j y_j^g, \sum_{j=1}^{n}\lambda_j=1, \lambda_j\geq 0, j=1,\dots,n\right\} \quad (5)$$

3. New DEA Model with Undesirable Inputs

To determine the efficiency of the unit under evaluation in input-oriented data, we seek to improve the unit undergoing evaluation by reducing the desirable input and increasing the undesirable input. Conversely, we increase desirable output and decrease the undesirable output in output-oriented data. Fare [9] introduced a model to increase and decrease desirable and undesirable outputs, respectively. However, the nonlinearity of his model was a problem.

The method introduced by Ali and Seiford [3] increases desirable output and decreases undesirable outputs simultaneously, but the problem with this method is that the efficiency size depends on value. The increased value will increase efficiency scores for inefficient decision-making units.

There are some other methods, such as [WD] and [MLT] that were introduced by Golany and Roll [12], respectively. In some methods such as [WD] and [MLT] decreased undesirable outputs are only accompanied by decreased desirable outputs. However, we believe that efficiency improvements are achieved if the desirable output increases or the undesirable output decreases [19]. These new models introduced by us, which will be evaluated using the data Covid-19 related data gathered in the emergency department of the hospitals and will determine the functionality of the model, will utilize input approaches in the presence of undesirable input. In the following sections, they are explained in detail [23].

Suppose $DMU_o=(x_o^g,x_o^b,y_o^g,y_o^b)$ to be unit under evaluation corresponding to the output $y_o=(y_o^g,y_o^b)$. In this case, using (2), $L(y_o^g,y_o^b)$ is defined as follows [22]:

$$L(y_o^g,y_o^b)= (x^g,x^b)\big|(x^g,x^b,y_o^g,y_o^b)\in T \quad (6)$$

Also, we consider the subset of $L(y_o^g,y_o^b)$ as:

$$\partial^p L(y_o^g,y_o^b)= (x^g,x^b)\big|\forall(u,v)\geq 0,(u,v)\neq 0 \quad \Rightarrow \quad (x^g-u,x^b+v)\notin L(y_o^g,y_o^b) \quad (7)$$

where $\partial^s L(y_o^g,y_o^b)$ includes all inputs of the efficient DMUs capable of producing (y_o^g,y_o^b).

In the new model to evaluate the efficiency of DMU_o, the most decrease of x_o^g and the most increase of x_o^b is defined as follows [22]:

$$d_o{}^g=x_o{}^g,\ d_o{}^b=x_o{}^b-x_{\max}^b,\ (x_{\max}^b)_i=Max_j\ \ x_{ij}^b$$

Therefore, according to the definition of inefficiency, we have:

$$
\begin{aligned}
&\theta_o^* = Max \quad \theta_o \\
&st. \\
&\quad \sum_{j=1}^{n} \lambda_j x_j^g + s^- = x_o^g - \theta_o d_o{}^g \\
&\quad \sum_{j=1}^{n} \lambda_J x_j^b = x_o^b - \theta d_o{}^b \\
&\quad \sum_{j=1}^{n} \lambda_j y_j^g - s^+ = y_o^g \\
&\quad \sum_{j=1}^{n} \lambda_j y_j^b = y_o^b \\
&\quad \sum_{j=1}^{n} \lambda_j = 1 \\
&\quad \lambda_j \geq 0 \qquad for \quad all \quad j = 1, \ldots, n
\end{aligned}
\tag{8}
$$

Theorem 1. The DMU_o in model (8) is efficient if and only if

1) $\theta_O^* = 1$

2) All slacks are zero for all optimal solutions.

Theorem 2. If all optimal solutions of model (8) are to be θ^*, s^{-*}, then

$$(x^g - \theta^* d^g - s^{-*}, x^b - \theta^* d^b) \in \partial^p L(y_o^b, y_o^g)$$

where s^{-*} is one of the optimal solutions.

4. Case Study

Because a large percentage of patients go to public hospitals, they usually waste much time as a result of the formation of long queues and lack of resources. Consequently, sometimes, some critically ill patients die, leading to the dissatisfaction of the patient's relatives. In the present study, the emergency center of public and private hospitals in Tehran province has been selected as the research sample. The data was collected in one month, and the queue time has been collected using the queuing device. Due to the prevalence of the Covid-19 virus in the world and the country, the number of patients visiting the hospitals as well as the number of deaths has gone up. Up to this date, the Covid-19 Pandemic has entered its fifth peak. Over 3,623,840 people have tested positive, and over 88,063 souls have been lost to this illness. Lack of import of equipment and raw materials for in-house production has resulted in a shortage of equipment in hospitals for treating or diagnosing the disease. The hospital officials have been forced to sterilize some equipment and reuse them numerous times before discarding them. Therefore, this action does not disrupt the production possibility set and the feasibility status in the studied system. In this practical study, data nature was considered quantitatively and evaluated in the following example using the basic and the designed model: an input-oriented.

We considered 30 decision-making units with five desirable inputs, including nurses, general practitioners, specialist doctors or emergency medicine practitioners, number of hospital beds, and waiting time. In addition, sterilization of medical equipment to be used on patients was considered as an undesirable input.

They were used to produce four desirable outputs, including improvement of the patients' condition, the number of the discharged patients from the hospital, outpatient treatment in less than 4 hours, stay time between 4 and 12 hours, and the number of patients having to be hospitalized for more than 12 hours. The data associated with these decision-making units, along with undesirable inputs, is displayed in Table 1. Furthermore, the implementation of the model for emergency departments of hospitals is demonstrated in Table 2.

Table 1. Data associated with hospitals' emergency departments with an undesirable input

DMU	Inputs					undesirable input	Outputs			
DMU_S	Nurse X_1	Doctor X_2	Specialist doctor X_3	BedX_4	waiting timeX_5	Sterilization equipment X_6	Cleared persons Y_1	Stay less than 4 hours Y_2	Stay between 4 to 12 hours Y_3	Stay more than 12 hours y_4
DMU_1	18	1	1	38	27	19	1155	295	265	32
DMU_2	19	2	1	41	15	4	1254	338	305	30
DMU_3	21	2	2	42	17	9	1259	325	261	28
DMU_4	19	2	1	39	21	14	1244	320	263	29
DMU_5	20	2	1	40	25	17	1254	323	271	29
DMU_6	22	2	2	42	34	7	917	125	169	22
DMU_7	21	2	1	41	26	15	1245	332	237	28
DMU_8	21	2	1	41	18	11	1254	323	270	28
DMU_9	20	2	1	40	19	8	1204	340	265	27
DMU_{10}	20	2	1	39	17	6	1254	315	270	29
DMU_{11}	20	2	1	39	18	10	1260	324	272	29
DMU_{12}	19	1	2	39	29	17	944	192	246	30
DMU_{13}	18	1	2	38	29	9	985	194	240	28
DMU_{14}	19	1	2	40	30	13	1085	295	226	32
DMU_{15}	19	1	2	39	30	12	764	162	116	20
DMU_{16}	20	1	2	41	31	5	691	150	244	19
DMU_{17}	20	1	2	42	31	8	994	192	246	28
DMU_{18}	20	1	2	41	25	9	931	201	256	29
DMU_{19}	21	1	2	42	26	18	941	188	274	28
DMU_{20}	20	1	2	41	25	16	1145	284	275	27
DMU_{21}	21	1	2	41	25	19	948	193	212	32
DMU_{22}	18	1	1	38	27	6	994	305	266	28
DMU_{23}	20	1	2	39	26	5	941	245	246	27
DMU_{24}	19	1	2	39	27	15	984	189	274	29
DMU_{25}	20	1	2	41	27	7	948	193	247	28
DMU_{26}	22	2	2	43	14	11	1259	335	271	30
DMU_{27}	23	2	2	44	32	5	1015	224	261	24
DMU_{28}	21	2	1	42	13	13	1370	365	322	35
DMU_{29}	22	2	1	42	15	18	1244	320	270	29
DMU_{30}	23	2	2	44	14	8	1154	314	272	31

Table 2. Results of the proposed model (undesirable input) for emergency departments of the hospitals

DMUs	θ_o^*	**BCC**	**DMUs**	θ_o^*	**BCC**	**DMUs**	θ_o^*	**BCC**
DMU_1	1	1	DMU_{11}	0.9905	1	DMU_{21}	1	1
DMU_2	0.8952	1	DMU_{12}	1	0.9375	DMU_{22}	0.8859	1
DMU_3	0.9122	0.919	DMU_{13}	0.9163	0.9048	DMU_{23}	0.9663	1
DMU_4	0.9889	1	DMU_{14}	0.9963	1	DMU_{24}	0.9959	1
DMU_5	1	0.9711	DMU_{15}	0.7068	0.6615	DMU_{25}	0.8759	0.9169
DMU_6	0.7732	0.6693	DMU_{16}	0.7692	0.8873	DMU_{26}	0.9463	0.919
DMU_7	1	0.944	DMU_{17}	0.9039	0.9146	DMU_{27}	0.7901	0.8106
DMU_8	0.9477	0.9405	DMU_{18}	0.9898	1	DMU_{28}	0.9893	1
DMU_9	0.9763	1	DMU_{19}	1	0.9986	DMU_{29}	1	0.908
DMU_{10}	0.9689	1	DMU_{20}	1	1	DMU_{30}	0.7964	0.8857

According to the results displayed in Table 2, the units that have the most undesirable inputs, such as DMU1, DMU20 and DMU21 units ... either their efficiency is maintained or like DMU5 and DMU7 units, etc. has slightly decreased. Some units that use fewer adverse inputs than others have changed from efficient to inefficient, or the number of efficiencies has decreased.

Therefore, we conclude that any undesirable input in the units undergoing evaluation allow the production facility (taking into account that the equipment cannot be sterilized more than two or three times in the hospitals to avoid compromising the feasibility of the collection, i.e., Excessive use the undesirable input to get out of PPS) increases efficiency, and vice versa. In addition, it could be concluded that these factors have an impact on determining efficiency. Therefore, according to the real example of hospitals, the proposed model is acceptable, and the same is true about the model.

5. Conclusion

Nowadays, with the prevalence of Covid -19 virus pandemic, infliction of many people across the world with this deadly virus, and their endangered health, reducing the waiting time of the patients in the hospitals' emergency room, identifying queuing points and essential sources, combining resources, and reducing Patient mortality were the main objectives of this study. Our proposed models determine the efficiency of the decision-making units while assuming that some of their input components may be undesirable. Real numerical examples show that this model ensures the effectiveness of undesirable input factors in determining the efficiency of the decision-making units undergone evaluation. Furthermore, they are compared with a unit corresponding to the efficient frontier set. Reducing undesirable output and increasing desirable output can improve the efficiency of the decision-making units. It can also lead these units to the efficient frontier and improve and enhance the efficiency frontier. Therefore, it can be concluded that the presence of undesirable factors in the efficient frontier is effective. For future research, it is proposed to upgrade models for undesirable output factors and fuzzy.

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